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Single spin logic: Computing with single electron spins

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Preamble

- Traditional computing has always been “charge based”. Electric charge encodes computable information
- Charge is a scalar. It has a “magnitude” but no “direction”
- Therefore, if you want to encode binary logic bits 0 and 1 in charge, then they must be encoded in two different amounts (or “magnitude”) of charge
- Anytime the bit is switched, one must change the magnitude of the charge encoding logic levels. Changing the magnitude invariably involves current flow.
- Current flow (or physical motion of charge) always leads to excessive power dissipation (I^2R loss)
- An example is the transistor: When the transistor contains a lot of charge, it is conducting (or “on) and this state could represent the bit 1. When the transistor is depleted of charge, it is “off” and could represent the bit 0. Switching the transistor dissipates an energy of $(1/2)CV^2$

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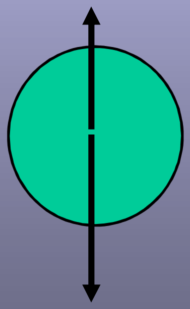
Spin

- Spin is a pseudo vector with a fixed magnitude and a variable direction (or polarization)
- You can make the polarization bistable by placing the electron in a static magnetic field
- These two polarizations (orthogonal states) can represent binary bits 0 and 1
- Switching involves flipping the spin, without physically moving charge in space.



Much lower dissipation

Power dissipation

- Today's transistors dissipate about 40,000 – 50,000 kT (at room temperature) when they switch between logic levels
- Energy dissipated is 1 keV/transistor, or 0.16 fJ/transistor
- Pentium IV has a transistor density of 10^8/cm^2
- Energy dissipated on the Pentium IV chip is 16 nJ/cm^2
- Assume a clock frequency of 2 GHz
- Power dissipated is 32 Watts/cm^2
- Pentium IV actually dissipates about 80 Watts

Why do we worry about power dissipation?

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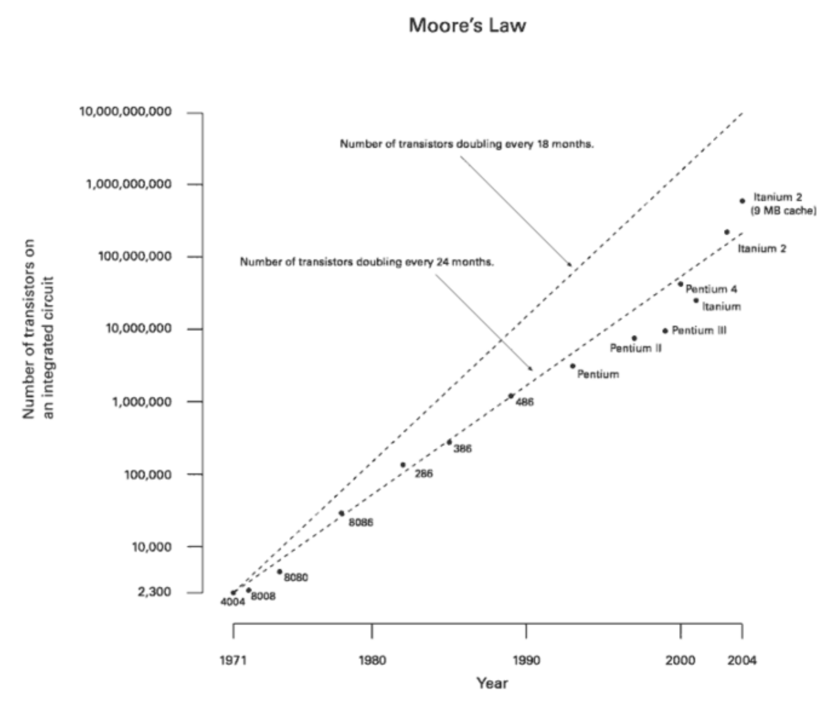
Threatening Moore

Formulated by Gordon Moore, one of the founders of Intel

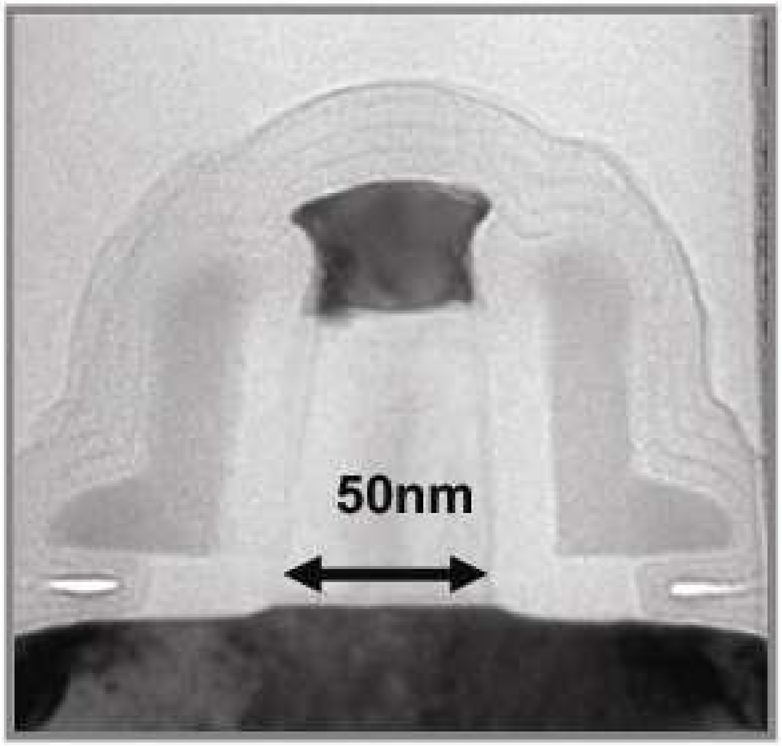
Device density on a chip doubles every 18 months



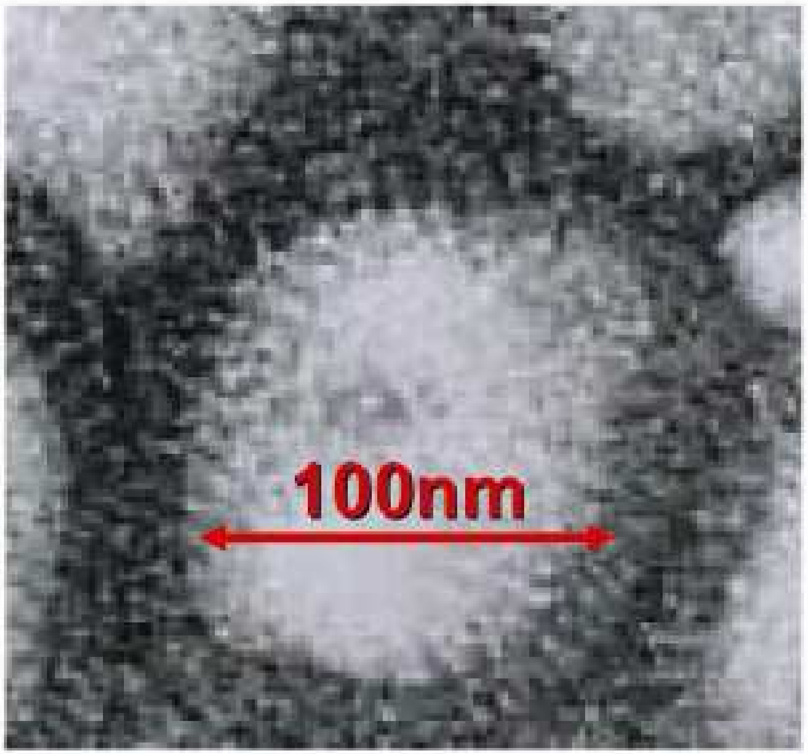
What does the future hold?

- If device density doubles every 18 months and this trend continues, then in 15 years after Pentium IV, i.e. by 2015, the transistor density on a chip should be 10^{11}/cm^2. Such small transistors exist!
- If transistors still dissipate 1 keV, then energy dissipated will be 16 μJ/cm^2
- With a 2 GHz clock, the power dissipated will be 32 kW/cm^2
- No known heat sinking technology can cope with that
- Chip meltdown!

Production Transistors Smaller Than Virus



Si transistor in the 90nm
logic technology node:
currently in production

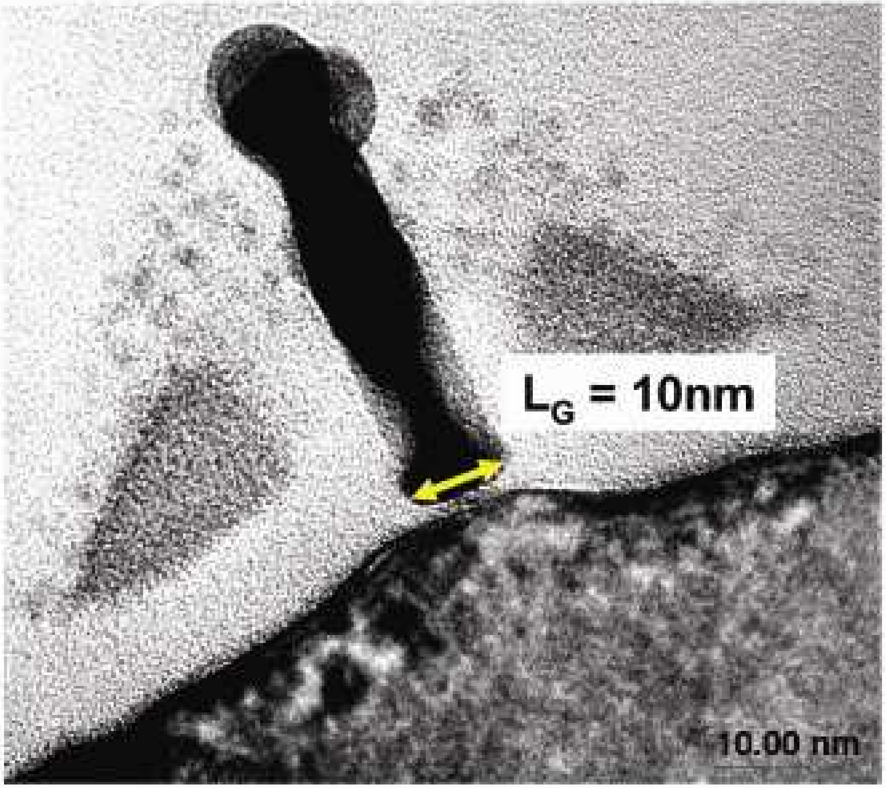


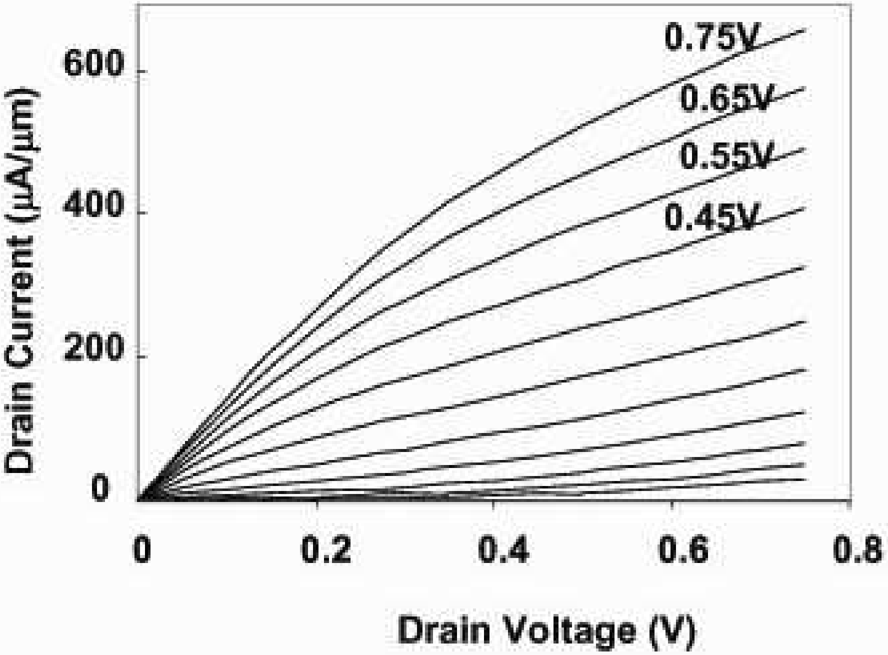
Influenza virus

Source: CDC

Courtesy: Suman Datta, Intel Corp.

Experimental 10nm Si MOS Transistor





- 10nm transistor still behaves like a transistor !

Courtesy: Suman Datta, Intel Corp.

Silicon Transistor Scaling and Moore's Law will Continue through 2015 ……

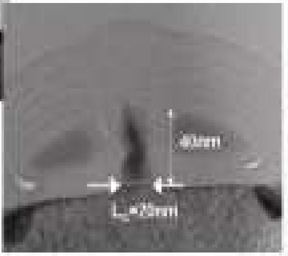
90nm Node
2003

50nm Length

65nm Node
P1264
2005

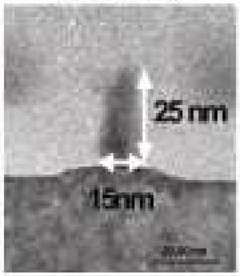
30nm
Prototype

45nm Node
P1266
2007



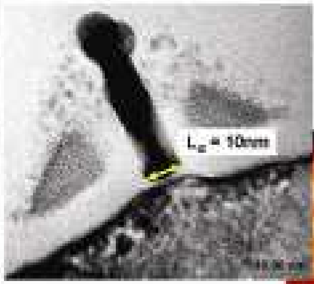
20nm Prototype

32nm Node
2009



15nm Prototype

22nm Node
2011



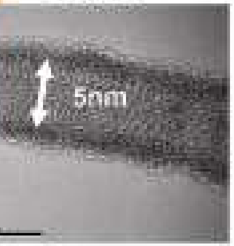
10nm Prototype

New materials integrated onto Si

16 nm node
2013

7nm
C-nanotubee
Prototype

11nm node
2015



5nm
Si-Nanowire
Prototype

New electronic materials, nano-technologies and nano-architecture introduced along the way

R.Chau, S. Datta, Intel, Nov20 2003

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Why do nano-transistors not permeate the marketplace?

- Because of the heat dissipation problem
- Cannot cool the chips efficiently

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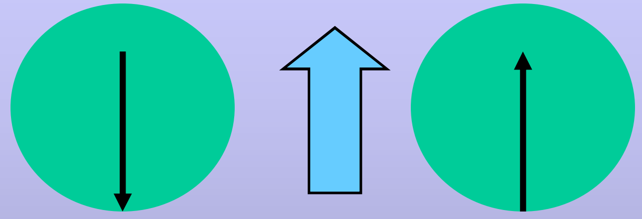
Spins to the rescue

- 1994: Proposal for Single Spin Logic (SSL).
- Single electrons confined in quantum dots and placed in a magnetic field
- Magnetic field makes the spin polarization in each dot “bistable”

$$H = H_0 - (g/2)\mu_B \vec{B} \bullet \vec{\sigma}$$

- These two polarizations encode (classical) logic bits 0 and 1
- Switch by flipping spin, without moving charge in space
- By arranging the quantum dots in suitable patterns, one can engineer the exchange interaction between neighboring spins in such a way that
  - If you designate some dots as input ports and align the spins in those dots externally to conform to input bits, then the system goes to an excited state
  - Let the system relax to ground state by emitting phonons, magnons, etc.
  - When ground state is reached, the spin orientations in certain chosen dots always represent the correct outputs conforming to the truth table of a specific logic gate
  - **S. Bandyopadhyay, B. Das and A. E. Miller, Nanotechnology, 5, 113 (1994).**

Example: Inverter



Place two electrons in two quantum dots such that their wavefunctions overlap

$J > g\mu_B B$

One spin is the input bit and the other is the output bit.

The ground state is the “singlet” state (Heitler-London)

The output is “naturally” the logic complement of the input

Two electrons in two coupled quantum dots form a natural inverter (the most primitive logic gate)

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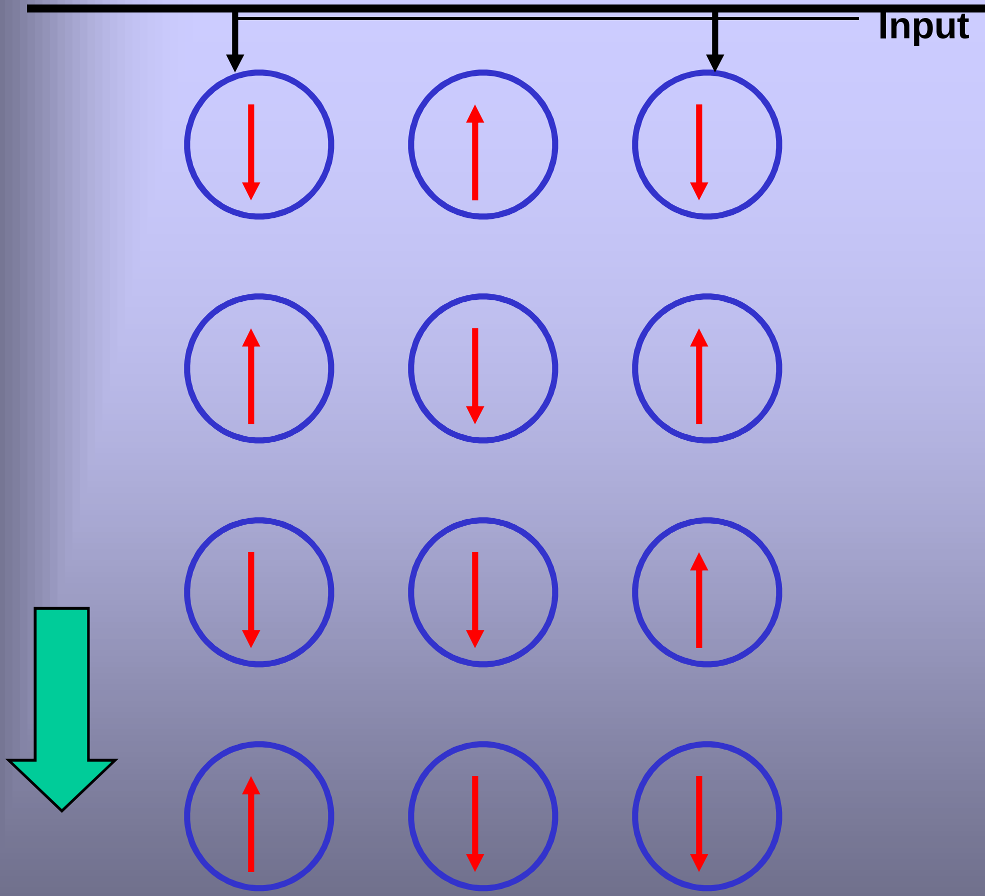
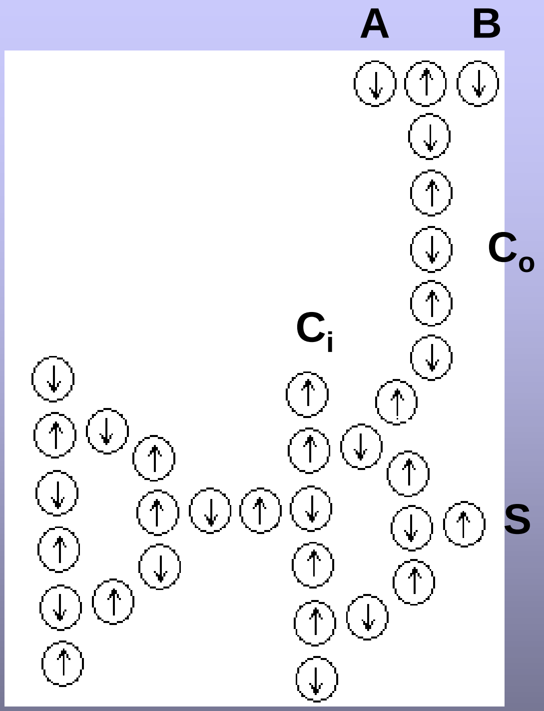
Universal spin gate - NAND



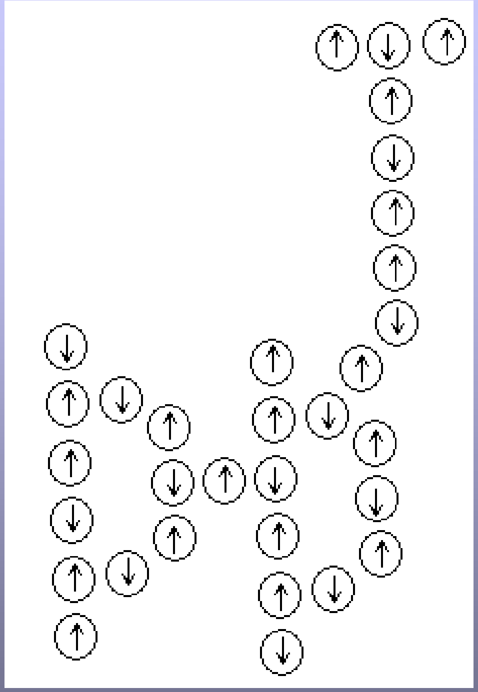
Downspin (parallel to the field) is logic 1, and upspin is 0

Input 1	Input 2	Output
1	1	0
0	0	1
1	0	1
0	1	1

Complex combinational circuits



Full adder



Full subtractor

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The calculation for the NAND gate

- Start with the Hubbard Hamiltonian and simplify to Heisenberg

$$H_{Heisenberg} = \sum_{<ij>} J_{ij}^{\parallel} \sigma_{zi}\sigma_{zj} + \sum_{<ij>} J_{ij}^{\perp} \left(\sigma_{xi}\sigma_{xj} + \sigma_{yi}\sigma_{yj}\right) + \sum_{\text{input dots}} \sigma_{zi} h_{zi}^{inputs} + \sum_{i} \sigma_{zi} h_{zi}^{global}$$

- The spins are polarized in the +z or –z-direction (assuming the global magnetic field is in the z-direction).
- The 3-spin basis states are $|\downarrow\downarrow\downarrow\rangle, |\downarrow\downarrow\uparrow\rangle, |\downarrow\uparrow\downarrow\rangle, |\downarrow\uparrow\uparrow\rangle, |\uparrow\downarrow\downarrow\rangle, |\uparrow\downarrow\uparrow\rangle, |\uparrow\uparrow\downarrow\rangle, |\uparrow\uparrow\uparrow\rangle$
- The Hamiltonian in the bases of these states is

$$\begin{pmatrix}
2J - h_A - h_C - 3Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -h_A + h_C - Z & 2J & 0 & 0 & 0 & 0 & 0 \\
0 & 2J & -2J - h_A - h_C - Z & 0 & 2J & 0 & 0 & 0 \\
0 & 0 & 0 & -h_A + h_C + Z & 0 & 2J & 0 & 0 \\
0 & 0 & 2J & 0 & h_A - h_C - Z & 0 & 0 & 0 \\
0 & 0 & 0 & 2J & 0 & -2J + h_A + h_C + Z & 2J & 0 \\
0 & 0 & 0 & 0 & 0 & 2J & h_A - h_C + Z & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2J + h_A + h_C + 3Z
\end{pmatrix}$$

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When inputs are (1,1), i.e. $h_A = h_c = h$

$$\Delta_1 = \sqrt{(h+J)^2 + 8J^2}$$

$$\Delta_2 = \sqrt{(h-J)^2 + 8J^2}$$

$$\alpha_1 = -J - h - \Delta_1$$

$$\alpha_3 = J - h + \Delta_2$$

$$\alpha_6 = -J - h + \Delta_1$$

$$\alpha_7 = J - h - \Delta_2$$

$$\beta_n = \sqrt{(\alpha_n / J)^2 + 8}$$

Eigenenergies (E_n)	**Eigenstates**
$- J - h - Z - \Delta_1$	$[0, 2/\beta_{1,} \alpha_1/(J\beta_1), 0, 2/\beta_1, 0, 0, 0]$
$2J - 2h - 3Z$	$[1, 0, 0, 0, 0, 0, 0, 0]$
$- J + h + Z - \Delta_2$	$[0, 0, 0, 2/\beta_3, 0, - \alpha_3/(J\beta_3), 2/\beta_3, 0]$
$-Z$	$1/\sqrt{}2[0, 1/, 0, 0, -1/, 0, 0, 0]$
Z	$1/\sqrt{}2[0, 0, 0, -1/, 0, 0, 1/, 0]$
$- J - h - Z + \Delta_1$	$[0, 2/\beta_{6,} \alpha_6/(J\beta_6), 0, 2/\beta_6, 0, 0, 0]$
$- J + h + Z + \Delta_2$	$[0, 0, 0, 2/\beta_7, 0, - \alpha_7/(J\beta_7), 2/\beta_7, 0]$
$2J + 2h + 3Z$	$[0, 0, 0, 0, 0, 0, 0, 1]$

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Ground state wavefunction when inputs are (1, 1)

$$\psi^{11}_{ground} = \frac{2}{\beta_1}\left|\downarrow\downarrow\uparrow\right\rangle + \frac{\alpha_1}{J\beta_1}\left|\downarrow\uparrow\downarrow\right\rangle + \frac{2}{\beta_1}\left|\uparrow\downarrow\downarrow\right\rangle$$

Desired state

$$\psi^{11}_{desired} = \left|\downarrow\uparrow\downarrow\right\rangle$$

$$\psi^{11}_{ground} \approx \psi^{11}_{desired}$$

$$\left|\frac{\alpha_1}{2J}\right| = \frac{h+J+\sqrt{(h+J)^2+8J^2}}{2J} \gg 1$$ or if $h >> J$

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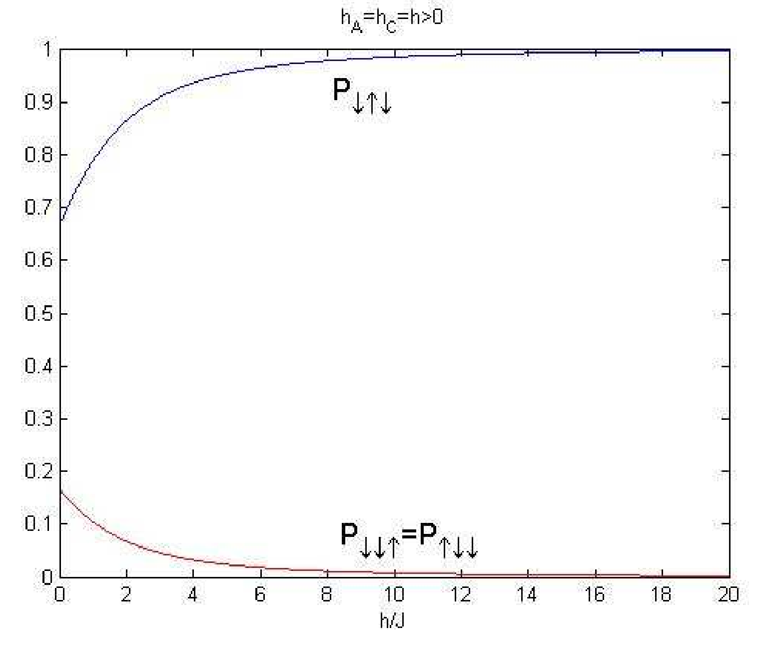
In all cases

- Correct output is produced in ground state if

  $h >> J$, i.e. if $g\mu_B B_i >> J$



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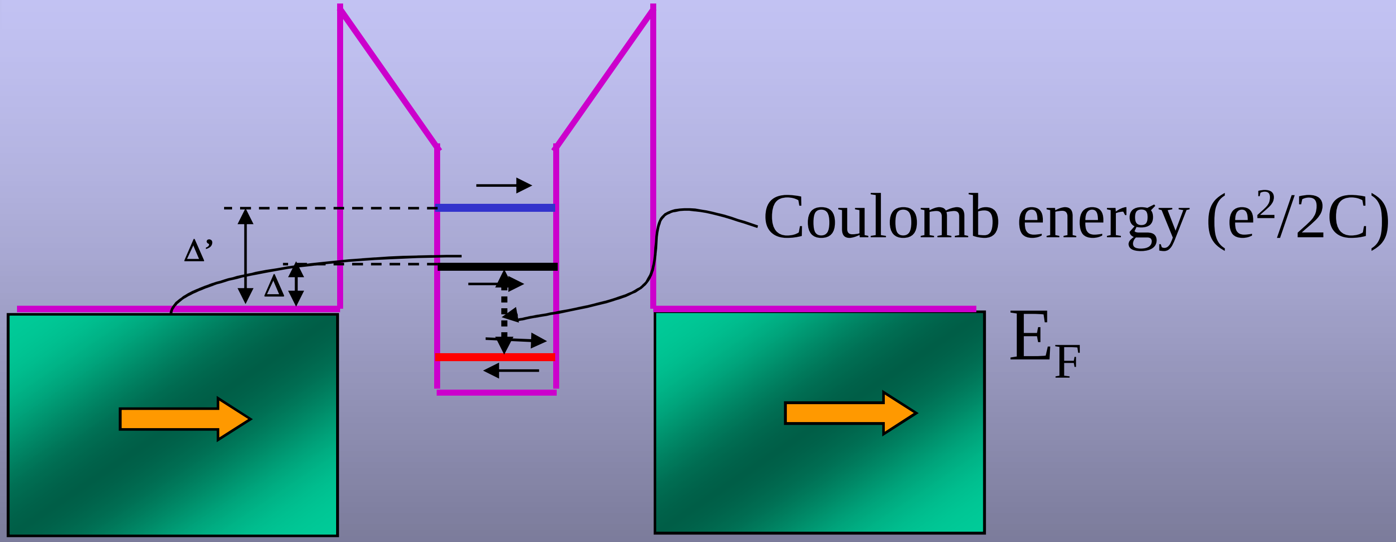
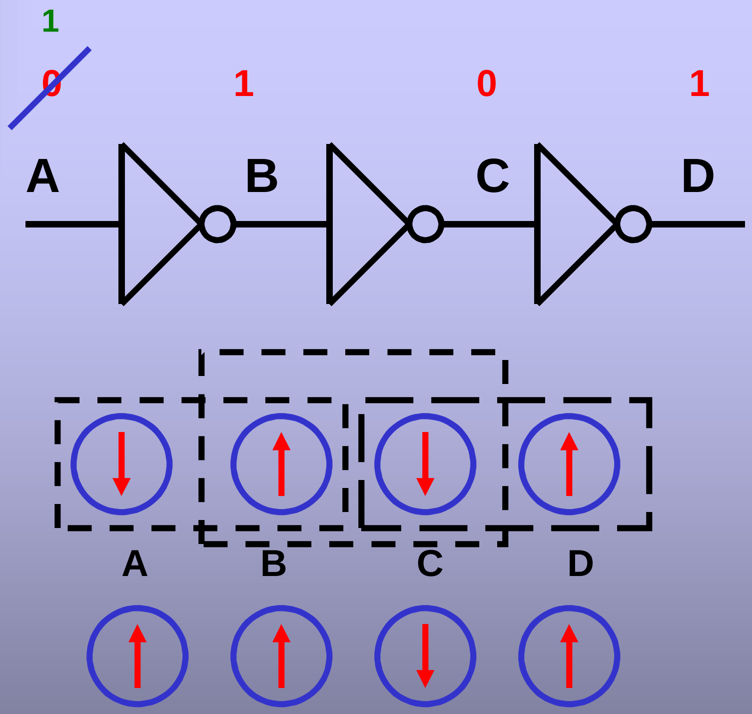
Some estimates

- J in lithographically defined quantum dots is ~ 1 meV (Melnikov and Leburton, PRB, 73, 155301 (2006).
- Therefore, h = 10 meV
- Use InSbN type materials with g-factor of ~ 900 (Zhang, et al., APL, 90, 193111 (2007))
- $g\mu_B B_i = 10$ meV
- Need $B_i = 0.2$ Tesla for writing
- Can generate with local magnetic fields (as in MRAM chips) - *writing*

Reading a single spin



Problem!



- Logic needs unidirectional signal flow
- Isolation between input and output of logic device

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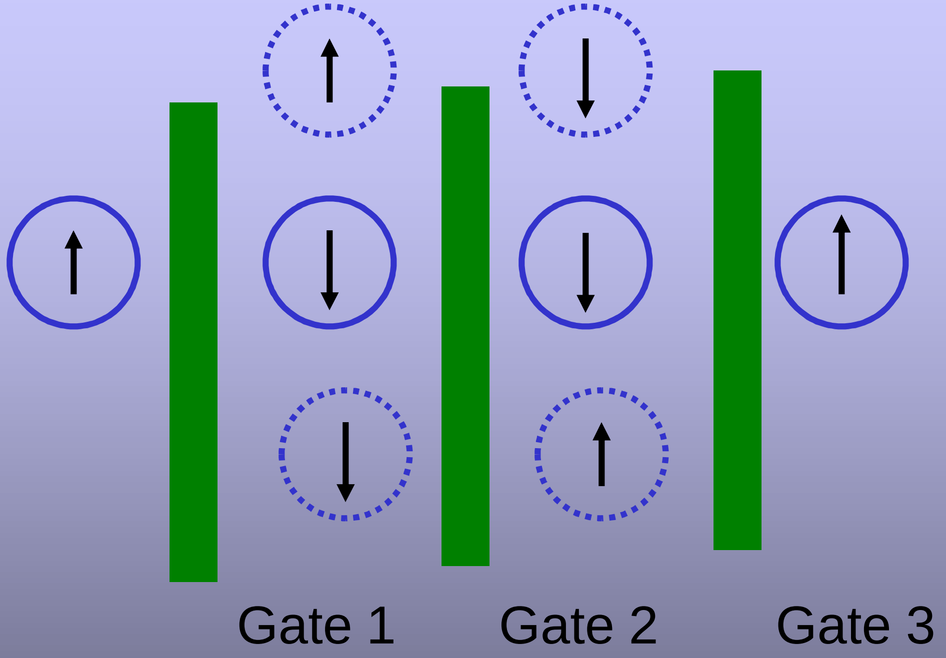
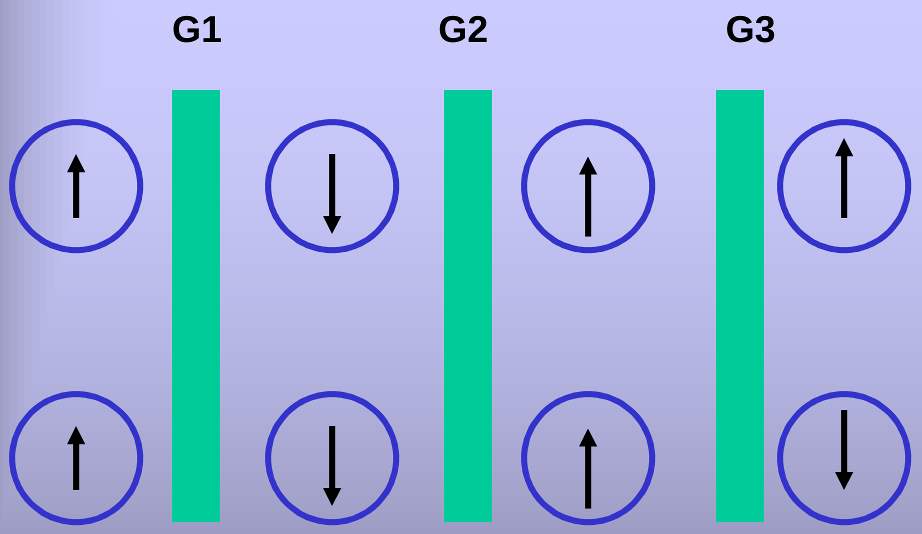
Clocking….

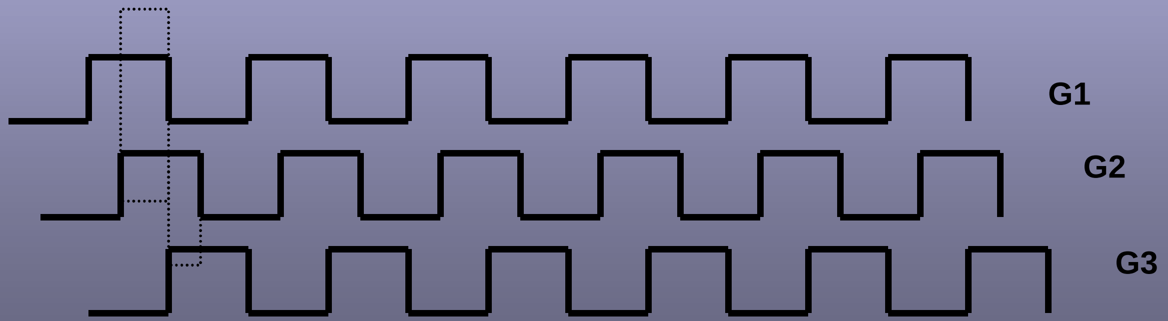


Clocking.. Contd..



First two gates raised pairwise

Second two gates raised pairwise



3-phase clock

Low power

Intrinsic gate error

- Natural tendency of any physical system is to be in ground state.
- However, the probability of being in ground state is not unity!
- The probability is given by Fermi Dirac if the system is in equilibrium with the surrounding phonon bath. This probability is less than 1
- Errors happen when a spin strays from the ground state to the excited state
- Probability of straying into the first excited state from the ground state is $p_i = exp[-(E_{excited} – E_{ground})/kT] = exp[-2Z/kT] = exp[-g\mu_B B/kT]$
- Energy dissipated in flipping (or switching) is $g\mu_B B = kTln(1/p)$
- Landauer-Shannon limit
- With $p_i = 10^{-9}$, the energy dissipated is $kTln(10^9) = 21kT$
- Compare with *40,000 – 50,000 kT* for today's transistors
- May perpetuate Moore's law into the next few decades

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Extrinsic gate error

- If bit flips spontaneously during a clock period, then extrinsic gate error.
- That probability $p_e = 1 - \exp[-T/T_1]$, where T is clock period
- Keep this at 10^{-9}: then we need $T_1 \sim 1$ second if clock is 1 GHz!

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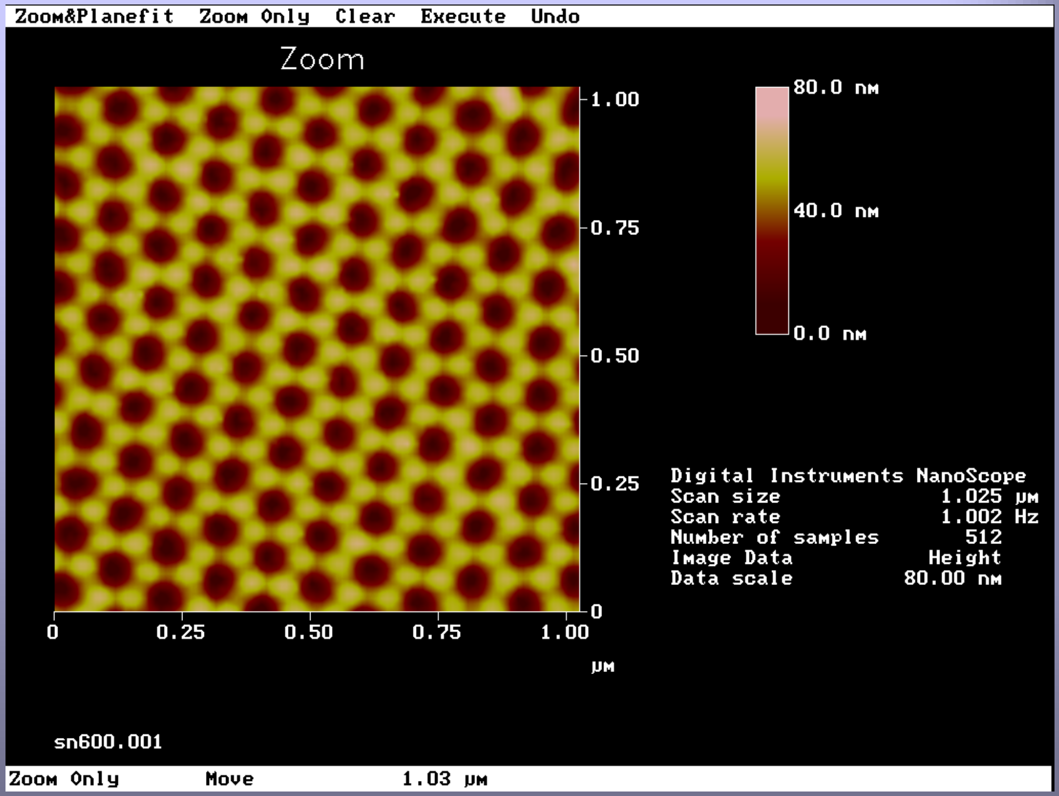
Find a system with 1 second spin relaxation time

- Does such a system exist?
- Yes! GaAs quantum dots have T_1 = 1 second, but at a few millikelvins (Kastner group at MIT)
- What about “reasonable” temperatures?
- Yes, but use organic nanostructures
- Organics have very weak spin orbit interaction

Nanoscale self assembly

- **Anodize Al foil in oxalic acid**
- **Pores 50 nm diameter**
- **Fill up the pores with material**
- **We used Alq_3**



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Digital Instruments NanoScope
Scan size              1.025 µm
Scan rate              1.002 Hz
Number of samples        512
Image Data             Height
Data scale           80.00 nm
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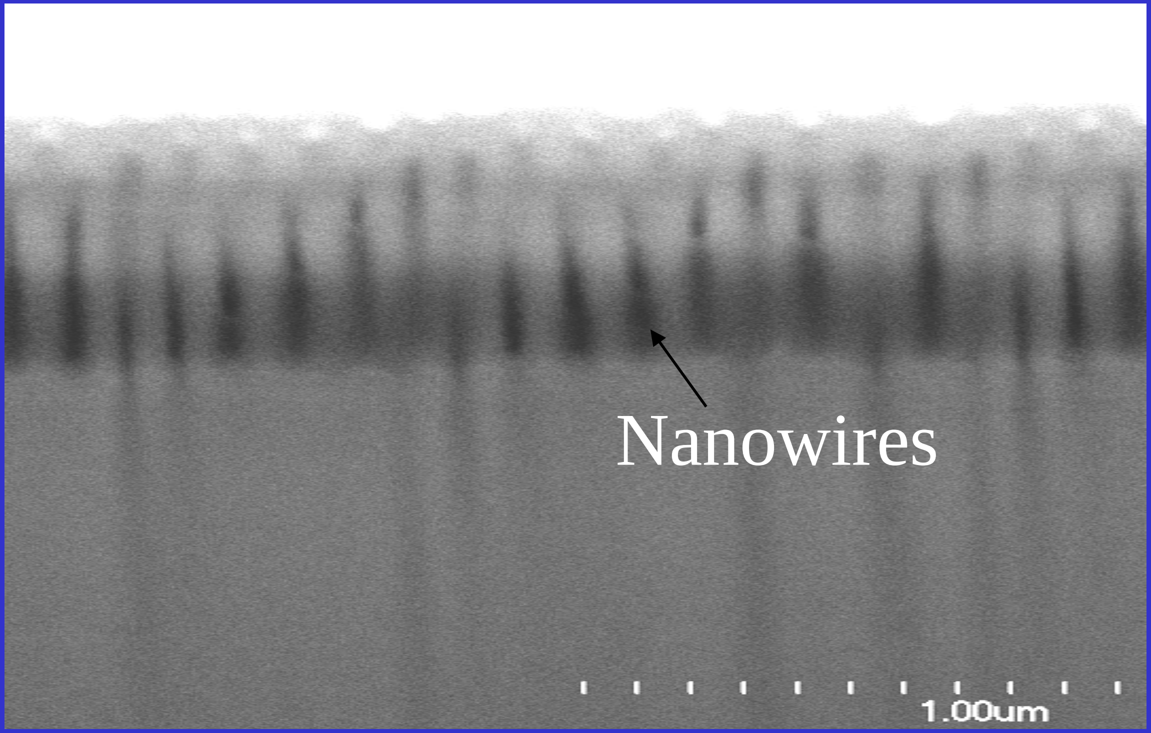
Cross-section FE-SEM



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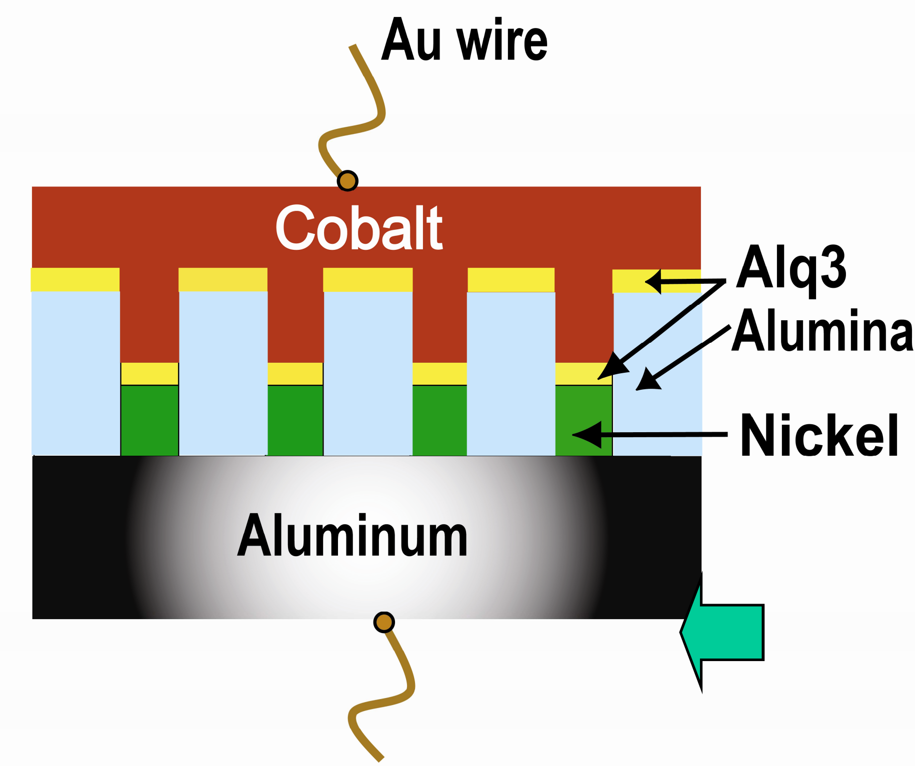
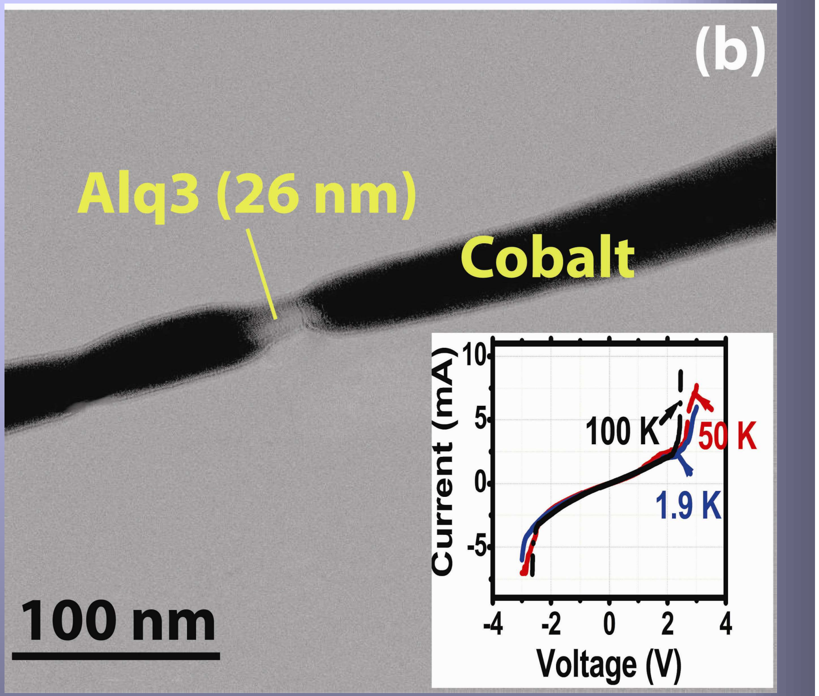
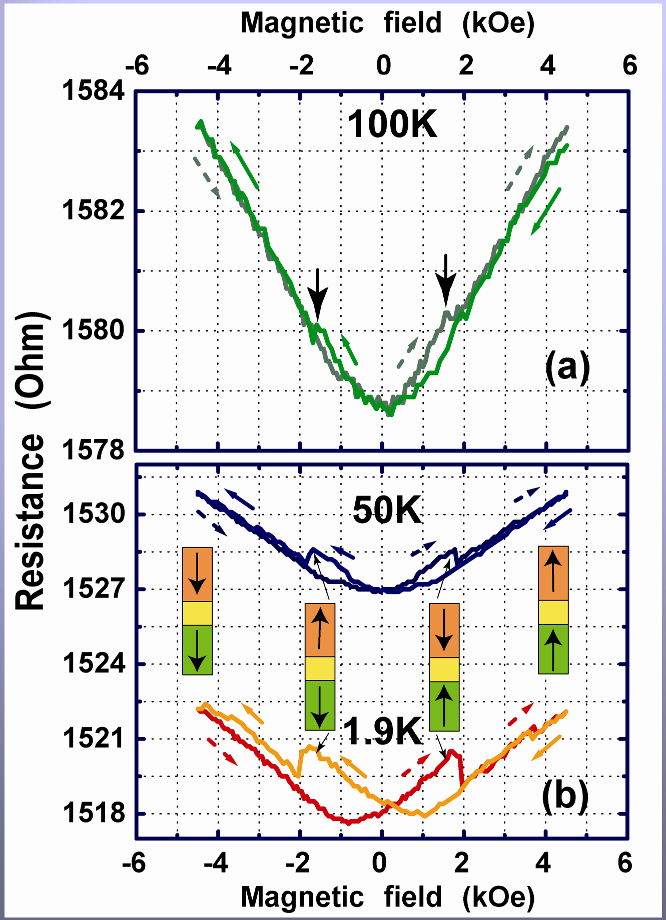
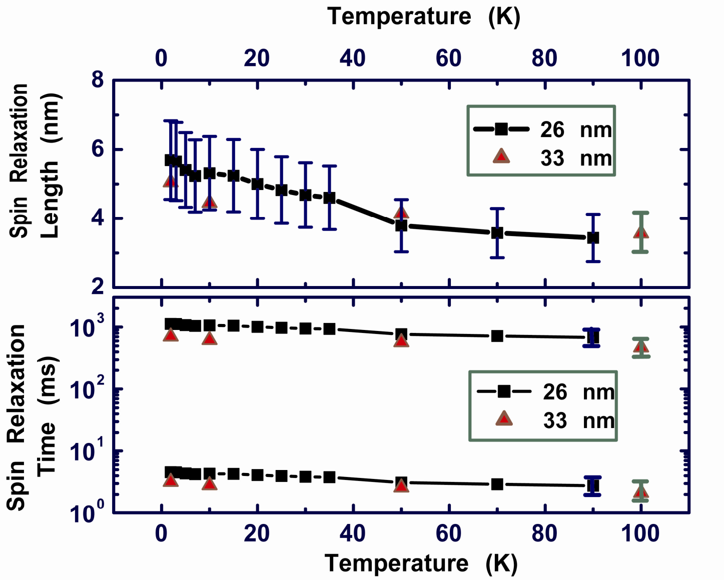
Self assembling a nanoscale “spin-valve”





Experimental measurement of T_1 time





Nature Nanotech., 2, 216 (2007); T_1 = 1 sec at 100 K!

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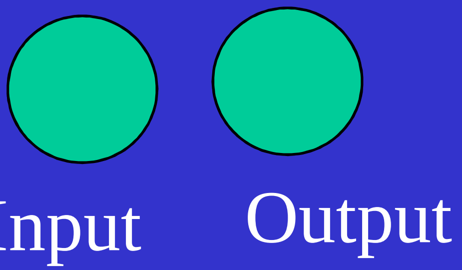
Quantum Computing

- Quantum Inverter
- 1996: Proposed the notion of using the spin of a single electron in a quantum dot for a “qubit” (Proc. Conf. Superlat. Microstruct., Liege, Belgium, July 1996. Later published in Superlat. Microstruct., 22, 411 (1997))



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Quantum Inverter

- Analyzed with Hubbard Hamiltonian

$$H = \sum_{i\sigma} \left(\varepsilon_0 n_{i\sigma} + g\mu_B B_i sign(\sigma)\right) + \sum_{<ij>} t_{ij}\left(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c\right) + \sum_i U_i n_{i\uparrow} n_{i\downarrow} + \sum_{<ij>\alpha\beta} J_{ij} c_{i\alpha}^{\dagger} c_{i\beta} c_{j\beta}^{\dagger} c_{j\alpha} + H_z \sum_{i\sigma} g\mu_B n_{i\sigma} sign(\sigma)$$

- Key results:
  - Fix the input spin and let the output evolve according to $e^{-iHt/\hbar}$
  - Correct output is produced periodically (Poincare recurrence) provided $h_A = 2J$
  - The period is $t_d = h/(8\sqrt{2}J)$
  - The system must be "halted" at the precise moment to harvest the correct output, otherwise the system continues to evolve and will stray into the wrong result.
  - With J = 1 meV, the switching time is 0.3 ps… very fast!

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Need long phase coherence time

- For quantum dynamics, the important time constant is the transverse spin relaxation time T_2 as opposed to T_1.
- Used the same material Alq_3 since it has weak S-O interaction
- Put the molecules in pores of anodic alumina and wash several times with dichloro-ethane solution
- Removes the molecule from pores, but leaves behind some in cracks in the alumina
- The cracks are 1-2 nm in size, whereas the molecule is 0.8 nm
- Therefore only *one or two* molecules in cracks
- Can probe almost non-aggregated single molecules

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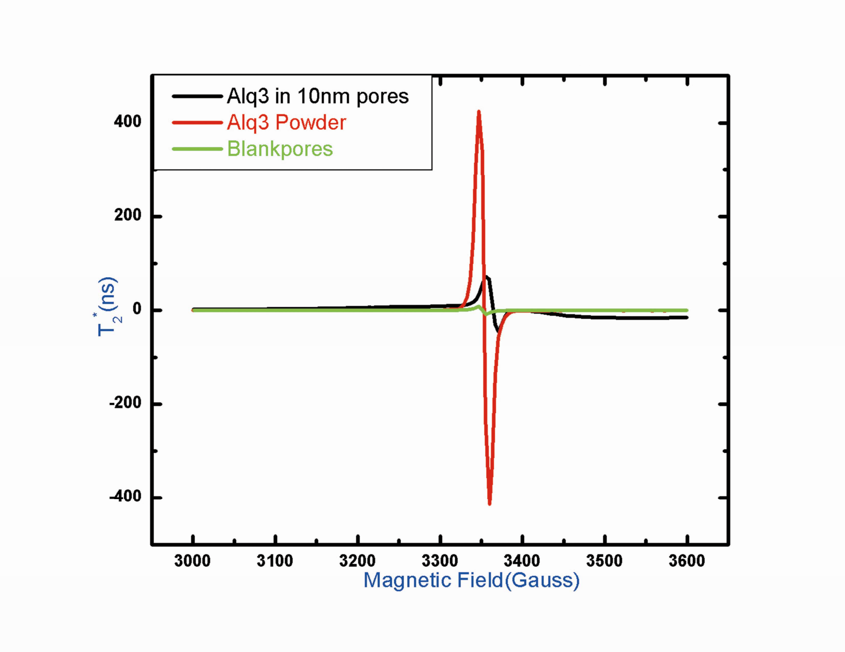
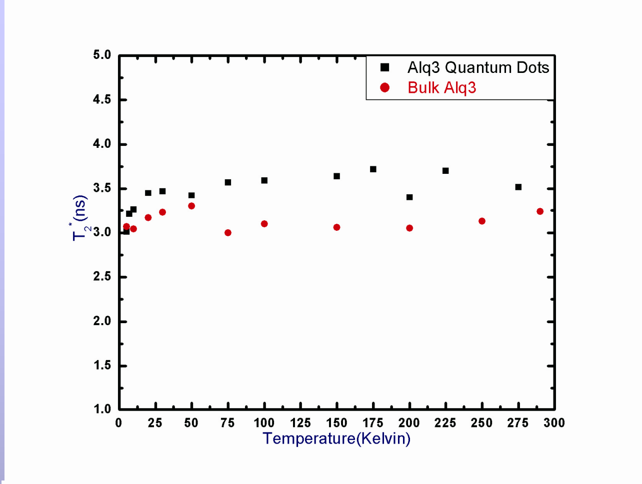
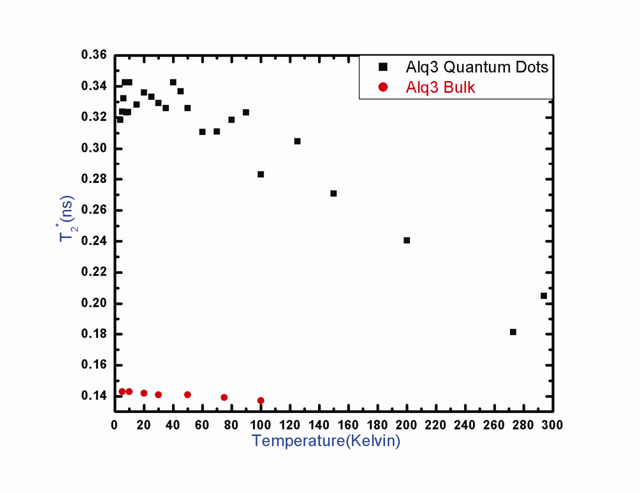
ESR spectroscopy







First observation of phonon bottleneck effect in molecules?

Conclusion

- Single spin logic is a very low power paradigm that can extend Moore’s law into the next few decades
- Very long longitudinal spin relaxation time of 1 second in organic molecules at the relatively balmy temperature of 100 K
- Long transverse relaxation time of few nsec at room temperature
- Possible first observation of the phonon bottleneck effect in organic molecules.