Magnetism in ultracold gases

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*Theoretical* condensed matter and atomic physics

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Outline

- Magnetism in condensed matter
- Ultracold atomic physics
- Magnetism in Bose gases
- Spontaneous symmetry breaking
Magnetism in condensed matter

- Magnetism is one of the pillars of condensed matter
  - One of the most studied solid states of matter
    (along with metals, superconductors, insulators, etc.)
  - Huge technological importance *e.g.* magnetic storage
  - A *quantum* phenomenon - depends on electron spin

Lodestone (circa 600 BC)

Thales of Miletus
Magnetism comes in many flavors

- Plain vanilla magnet is *Ferromagnet* (like Fe!)
  - Electron spins line up (exchange interaction from Pauli)

- Can also have *Antiferromagnetism* (e.g. MnO)
Basic research in magnetism

- Asks the question: is magnetic order inevitable? (at low temperatures)
- Not all materials freeze at $T=0$
  - Helium remains liquid due to quantum fluctuations
- Can same happen to magnetic moments of spins?
  - *Spin liquid* - hypothetical state of matter
Ultracold atomic gases

- Fe becomes ferromagnetic at $T=1043 \text{ K}$
- Ultracold atomic physics takes place at $<10^{-6} \text{ K}$

Quantum effects determine collective (i.e. material) properties when

\[
\text{thermal wavelength} \approx \text{interparticle separation}
\]

Atomic gases are *heavier* and *less dense* than gas of electrons in Fe

*Vast differences in scale no obstacle to theorists!*
So what’s new?

- In the solid state we (mostly) care about the quantum mechanics of electrons. These are *fermions*.
- By contrast, atoms (considered as particles) may be *bosons* or *fermions*.
- Possibility of *Bose-Einstein condensation* - bosons accumulate in lowest energy state.

Nobel prize 2001
Exotic magnetism

- $^{87}\text{Rb}$ has nuclear spin $I=3/2$, electron spin $S=1/2$
  - Possible total spin $F=1$ or $2$

- What are magnetic properties of $F=1$ or $2$ Bose gas?
Magnetism in Bose gases

- BEC: (nearly) all atoms sit in same quantum state
- This state $\Psi$ is called the \textit{condensate wavefunction}
- But what if lowest energy state is degenerate?

Condensate wavefunction is a spin vector (\textit{spinor}) and must pick a direction in spin space

Bose condensates with spin are \textit{always} magnets
Why higher spin is fun

- Spin 1/2 (e.g. of electron) points in some direction

\[ \chi_{1/2} = \begin{pmatrix} e^{-i\phi/2} \cos \theta/2 \\ e^{i\phi/2} \sin \theta/2 \end{pmatrix} \]

- To make electron magnetism more interesting need non-trivial arrangements on lattice (e.g. antiferromagnetism)

- Spin 1 doesn’t necessarily “point” anywhere

\[ \chi_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \]

\[ \chi_1^\dagger (\hat{n} \cdot \mathbf{F}) \chi_1 = 0 \]

- $F_i$ spin-1 matrices

- Yet evidently there is still an axis involved!
Which spin state wins?

- Must consider interatomic interactions
- Atoms can collide with total spin 0 or 2
  - Total spin 1? Antisymmetric and blocked by Bose statistics
  - Spin 2
  - Spin 0
Spin dependent interactions

\[ H_{\text{int}} = \sum_{i<j} \delta(\mathbf{r}_i - \mathbf{r}_j) (g_0 \mathcal{P}_0 + g_2 \mathcal{P}_2) \]
\[ = \sum_{i<j} \delta(\mathbf{r}_i - \mathbf{r}_j) (c_n + c_F \mathbf{F}_i \cdot \mathbf{F}_j) \]

- Energy of state \( \varphi_{f_1}(\mathbf{r}_1) \cdots \varphi_{f_N}(\mathbf{r}_N) \) includes a piece

\[ \langle H_{\text{spin}} \rangle = \frac{N n c_F}{2} (\varphi^\dagger \mathbf{F} \varphi) \cdot (\varphi^\dagger \mathbf{F} \varphi) \]

- For \( c_F < 0 \) (e.g. \(^{87}\text{Rb}\)): maximize \( \langle \mathbf{F} \rangle \) \hspace{1cm} \text{Ferromagnet}
- For \( c_F > 0 \) (e.g. \(^{23}\text{Na}\)): minimize \( \langle \mathbf{F} \rangle \) \hspace{1cm} \text{Polar state}
The Bose ferromagnet: $^{87}\text{Rb}$

– Stamper-Kurn group, Berkeley
Polar condensates and a paradox

\[ \langle H_{\text{spin}} \rangle = \frac{N n c_F}{2} (\varphi^\dagger F \varphi) \cdot (\varphi^\dagger F \varphi) \]

- For \( c_F > 0 \) minimize \( \langle F \rangle \). Pick a quantization (z) axis

\[ \varphi = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \]

\[ \varphi^\dagger (\hat{n} \cdot F) \varphi = 0 \]

- Problem: for a more general state \( \chi_{f_1 f_2 \ldots f_N} \varphi(r_1) \cdots \varphi(r_N) \)

seek ground state of \( H_{\text{spin}} = \frac{c_F n}{2N} \mathbf{F}_{\text{tot}} \cdot \mathbf{F}_{\text{tot}} \)

Must be a singlet \( \mathbf{F}_{\text{tot}} = 0 \)
The parable of the chair

- Chair has (rotational) Hamiltonian
  \[ H = \frac{L_z^2}{2I_{\text{chair}}} \]

- States \( \psi_n(\theta) \propto e^{in\theta} \) very unlike the chair we see
- Tiny energy differences swamped by perturbations
Symmetry breaking in atomic gases

- The same goes for

\[ H_{\text{spin}} = \frac{c_F n}{2N} \mathbf{F}_{\text{tot}} \cdot \mathbf{F}_{\text{tot}} \]

- From excitations on top of singlet ground state, we can build a state

\[ \varphi_{f_1}(\mathbf{r}_1) \cdots \varphi_{f_N}(\mathbf{r}_N) \]

\[ \varphi = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \]

at little cost, with a definite axis (but still \( \langle \mathbf{F} \rangle = 0 \))

Spontaneous symmetry breaking