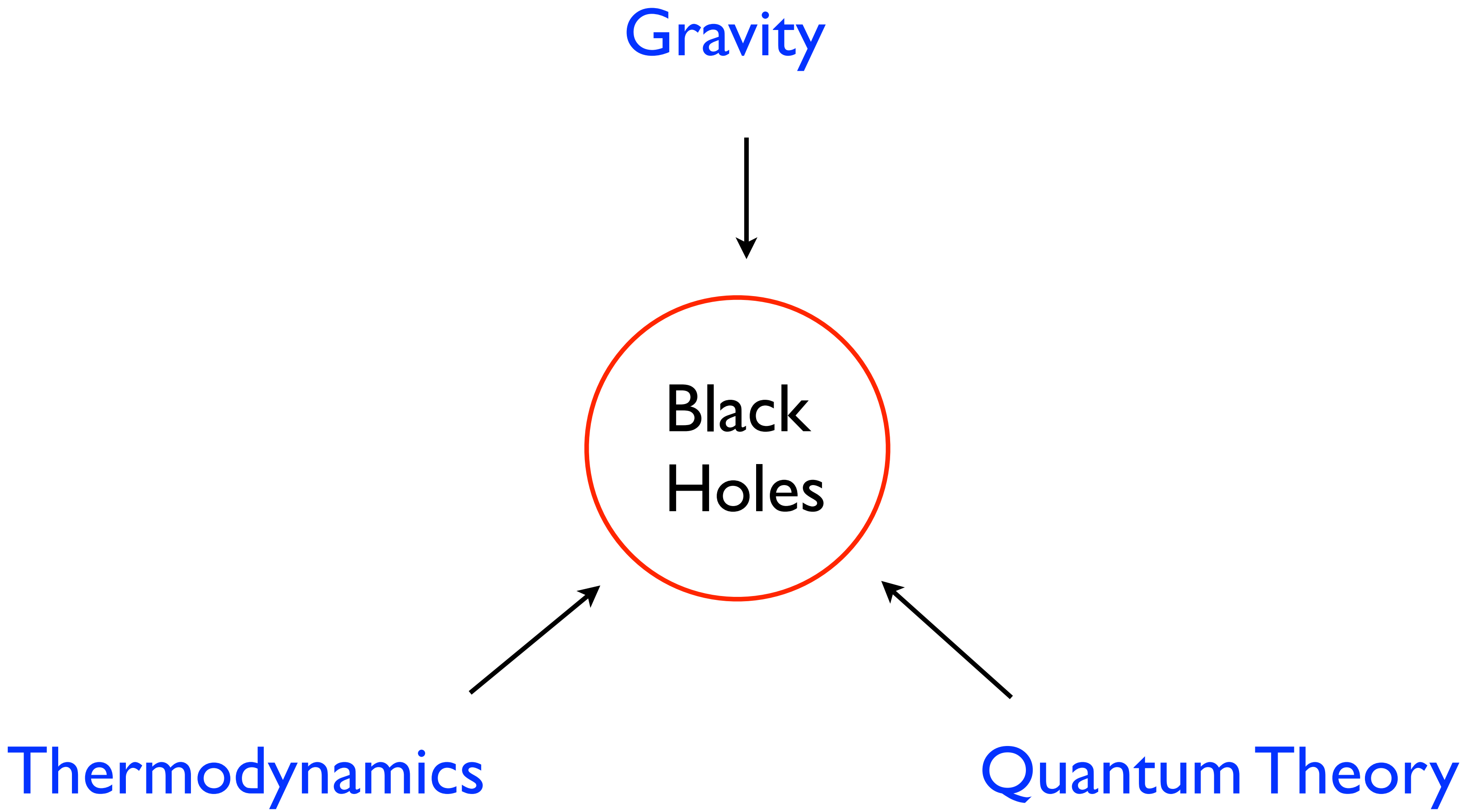
The black hole information paradox

Samir D. Mathur

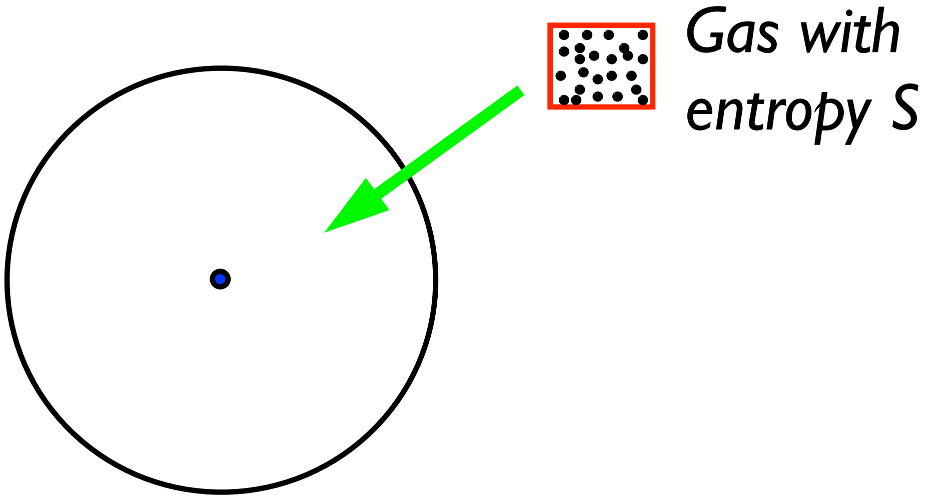

Gravity
Black
Holes
Thermodynamics
Quantum Theory

Black hole entropy

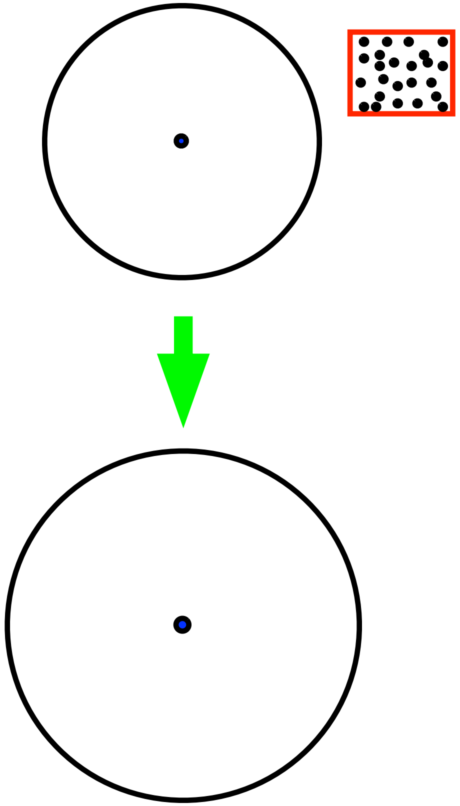
Suppose we throw a box of gas into a black hole

The gas disappears, and we seem to have reduced the entropy of the Universe

Have we violated the second law of thermodynamics ?



(Bekenstein '72,
Hawking '74)



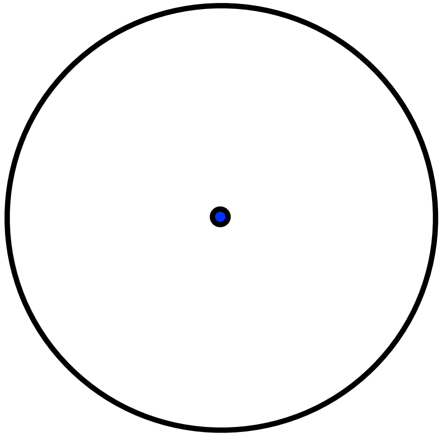
Suppose we assume that the hole has an entropy

$$\boxed{S = \frac{c^3}{\hbar}\frac{A}{4G} \to \frac{A}{4G} \qquad (c = \hbar = 1)}$$

Then
$$\frac{dS_{matter}}{dt} + \frac{dS_{hole}}{dt} \geq 0$$

So we save second law of thermodynamics ...

The entropy puzzle



The black hole has entropy $S = \frac{c^3}{\hbar}\frac{A}{4G}$

Statistical Mechanics says that

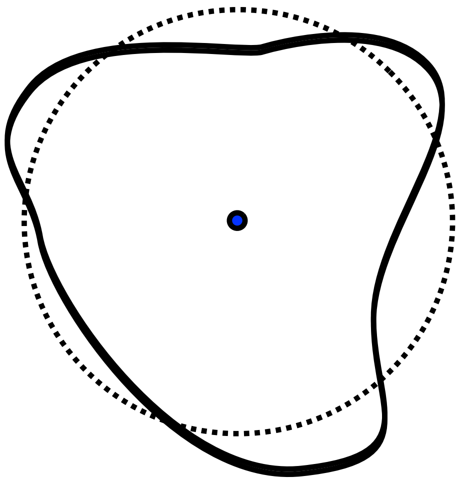
$$\boxed{S \; = \; \ln(\#\ states)}$$

Thus the black hole should have e^S states

Solar mass : $10^{10^{77}}$ states

Where are these states ?

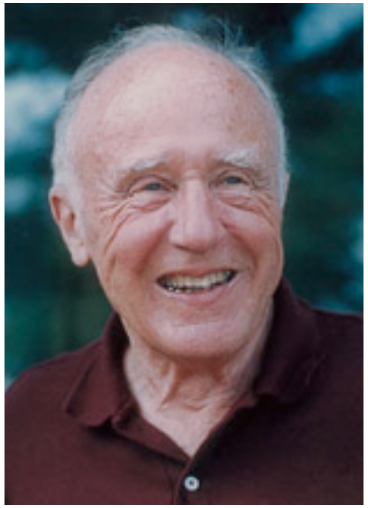
Fix M, look for small distortions of the hole ...



Find NO allowed distortions:

Black hole geometry completely fixed by its conserved quantum numbers:
mass, charge, angular momentum

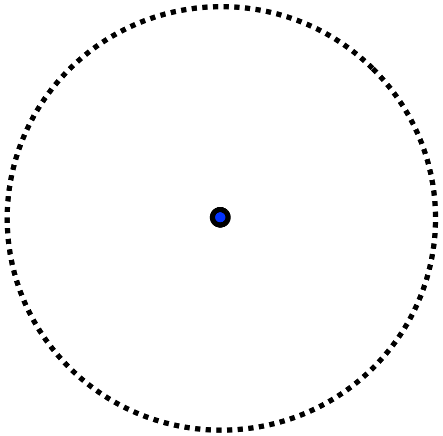
This would mean $S = \ln 1 = 0$



'Black holes have no hair' (John Wheeler)

Where are the states of the black hole ?

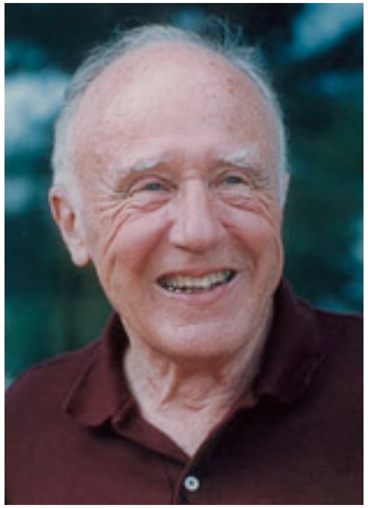
Fix M, look for small distortions of the hole ...



Find NO allowed distortions:

Black hole geometry completely fixed by its conserved quantum numbers:
mass, charge, angular momentum

This would mean $S = \ln 1 = 0$

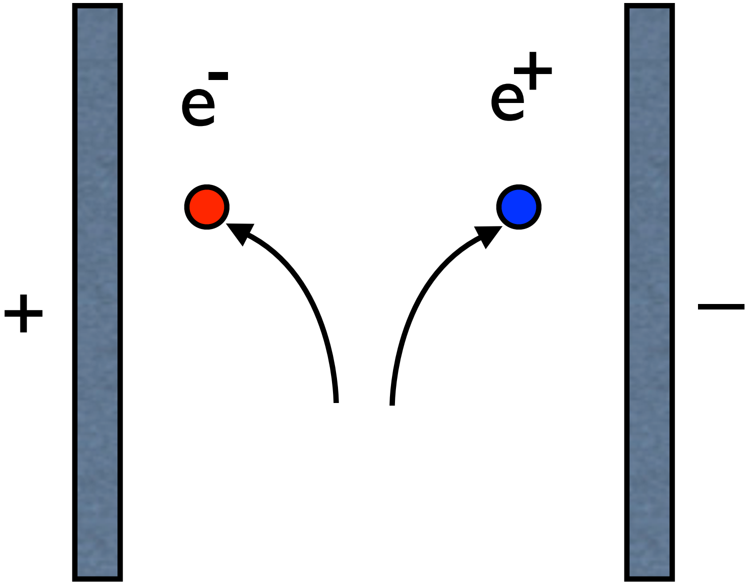


‘Black holes have no hair’ (John Wheeler)

Where are the states of the black hole ?

A more serious (but related) problem:

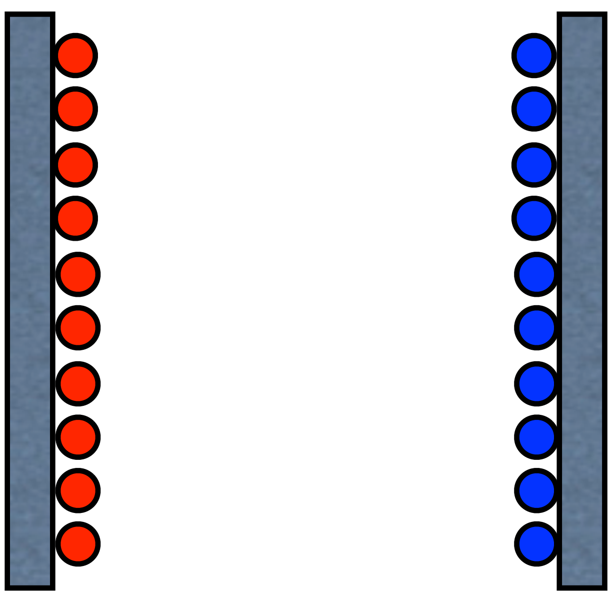
The information paradox



Schwinger pair production process
(interesting, but not a problem !)

State of created quanta is entangled

$$\uparrow\downarrow - \downarrow\uparrow$$



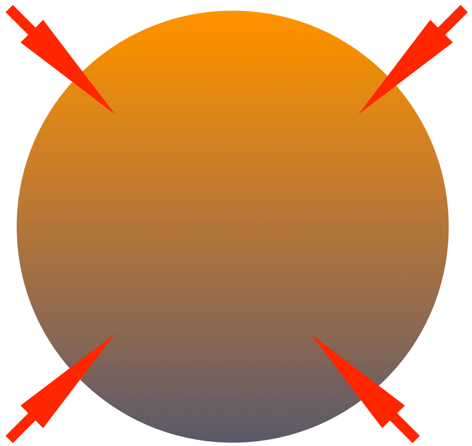
Entanglement entropy

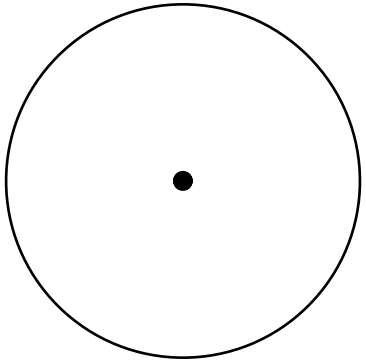
$$S_{ent} = \ln 2$$

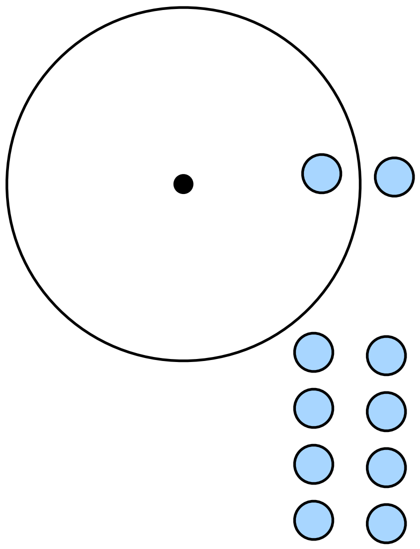
After N steps

$$S_{ent} = N \ln 2$$

The information problem







$$\Psi_M$$

$$\otimes \quad |0\rangle_1 |0\rangle_{1'} + |1\rangle_1 |1\rangle_{1'}$$

$$\otimes \quad |0\rangle_2 |0\rangle_{2'} + |1\rangle_2 |1\rangle_{2'}$$

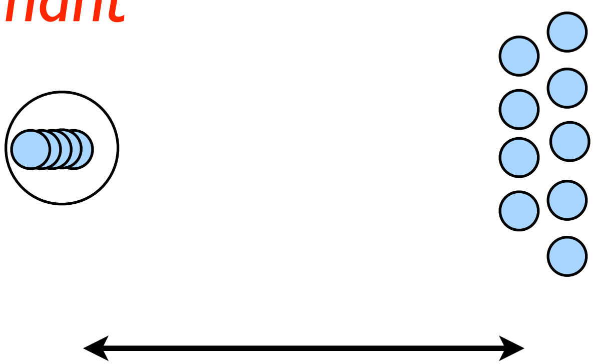
$$\ldots$$

$$\otimes \quad |0\rangle_n |0\rangle_{n'} + |1\rangle_n |1\rangle_{n'}$$

Schwinger process in the gravitational field

Possibilities

Planck mass remnant

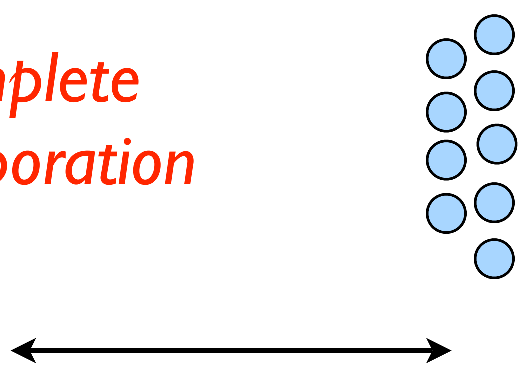


$$S_{ent} = N \ln 2$$

To have this entanglement, the remnant should have at least 2^N internal states

But how can we have an unbounded degeneracy for objects with a given mass ?

Complete evaporation



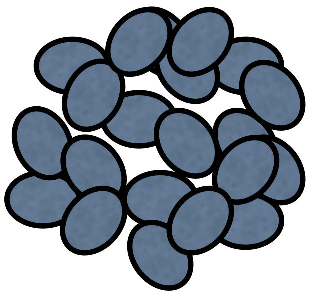
$$S_{ent} = N \ln 2$$

The radiated quanta are in an entangled state, but there is nothing that they are entangled with !

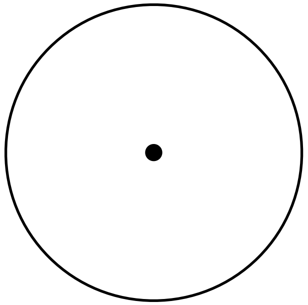
They cannot be described by any wavefunction, but only by a density matrix

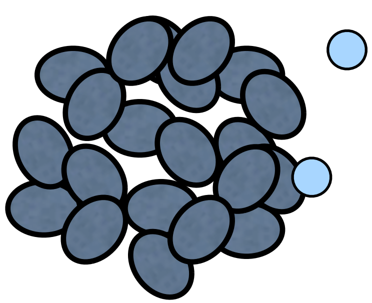
$\longrightarrow$ *failure of quantum mechanics*

The two problems are related ...

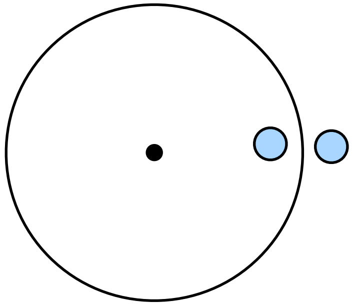


A normal body has visible microstates, so the entropy can be found by counting them





A normal body emits radiation from its surface, so that the radiation depends on the microstate. In the black hole the radiation is pulled from the vacuum



What do string theorists say ?

The entropy problem :

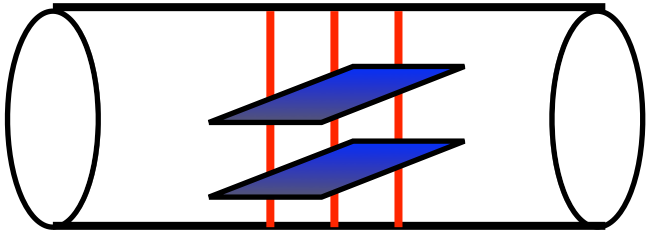
Does $S_{bek} = \frac{A}{4G}$ represent a count of states ?

(A) There are 10 - 4 = 6 compact directions in string theory

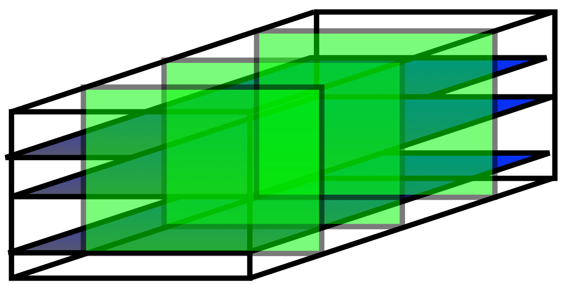
(B) The coupling constant g is a free modulus that we can vary

(C) If we look at states with mass = charge, then their number does not change with g (supersymmetric protection of BPS states)

$$g \to 0$$



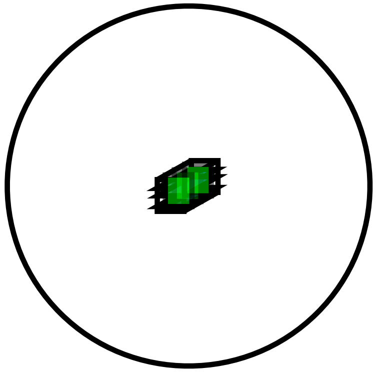
All charges are obtained by wrapping strings, branes etc along compact directions



Entropy is given through number of intersection points

$$S_{micro} = 2\pi\sqrt{n_1 n_2 n_3}$$

$$g \quad large$$



Expect that gravitational field will extend around branes to make a black hole

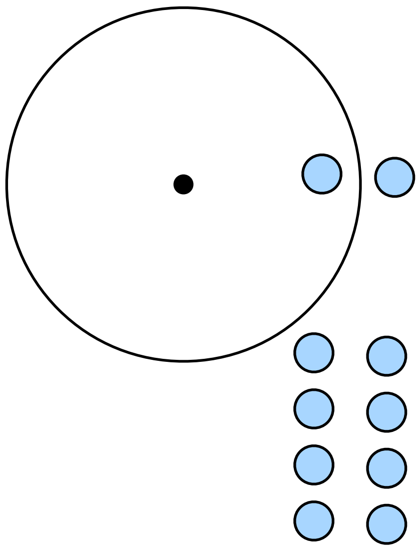
Find

$$S_{bek} \equiv \frac{A}{4G} = 2\pi\sqrt{n_1 n_2 n_3}$$

(Susskind 93, Sen 95, Strominger-Vafa 96 ...)

Thus S_{bek} is a count of states

The information problem : What happens to the entanglement ?



$$\Psi_M$$

$$\otimes \quad |0\rangle_1|0\rangle_{1'} + |1\rangle_1|1\rangle_{1'}$$

$$\otimes \quad |0\rangle_2|0\rangle_{2'} + |1\rangle_2|1\rangle_{2'}$$

$$\dots$$

$$\otimes \quad |0\rangle_n|0\rangle_{n'} + |1\rangle_n|1\rangle_{n'}$$

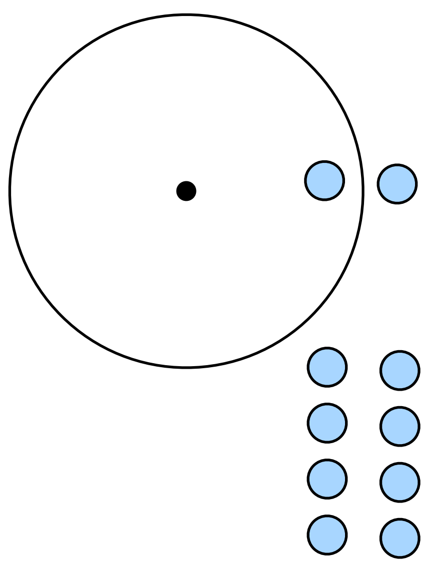
String theorists: String theory is unitary, so there should be no problem, really

GR folks: But what is the resolution to the problem? Hawking has an explicit computation, and you have not shown us what is wrong with it !!

There will of course be small corrections to the leading order Hawking computation

String theorists

Small correction to a large number of pairs will (hopefully) disentangle the inner and outer quanta



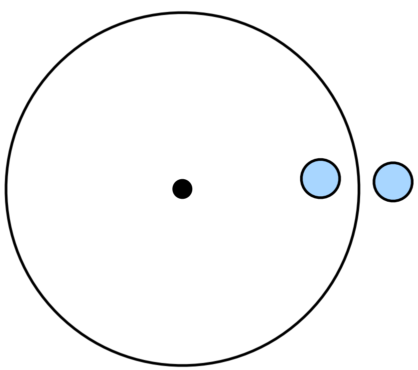
$$
\begin{aligned}
&\Psi_M \\
&\otimes \quad |0\rangle_1|0\rangle_{1'} + |1\rangle_1|1\rangle_{1'} + \delta\psi_1 \\
&\otimes \quad |0\rangle_2|0\rangle_{2'} + |1\rangle_2|1\rangle_{2'} + \delta\psi_2 \\
&\qquad \qquad \cdots \\
&\otimes \quad |0\rangle_n|0\rangle_{n'} + |1\rangle_n|1\rangle_{n'} + \delta\psi_n
\end{aligned}
$$

GR folks

You must think we are such fools.

We know there will always be small corrections from quantum gravity.

If that could remove the entanglement, why would we be worrying about the information paradox for 35 years ?



$$\Psi_M$$

$$\otimes \quad |0\rangle_1|0\rangle_{1'} + |1\rangle_1|1\rangle_{1'} + \delta\psi_1$$

$$\otimes \quad |0\rangle_2|0\rangle_{2'} + |1\rangle_2|1\rangle_{2'} + \delta\psi_2$$

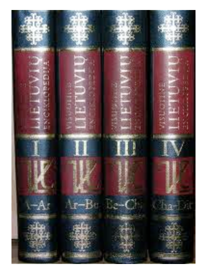
$$\dots$$

$$\otimes \quad |0\rangle_n|0\rangle_{n'} + |1\rangle_n|1\rangle_{n'} + \delta\psi_n$$

In 2005, Stephen Hawking surrendered his bet to John Preskill, based on such an argument of 'small corrections' ...

(Subleading saddle points in a Euclidean path integral give exponentially small corrections to the leading order evaporation process)

Stephen Hawking



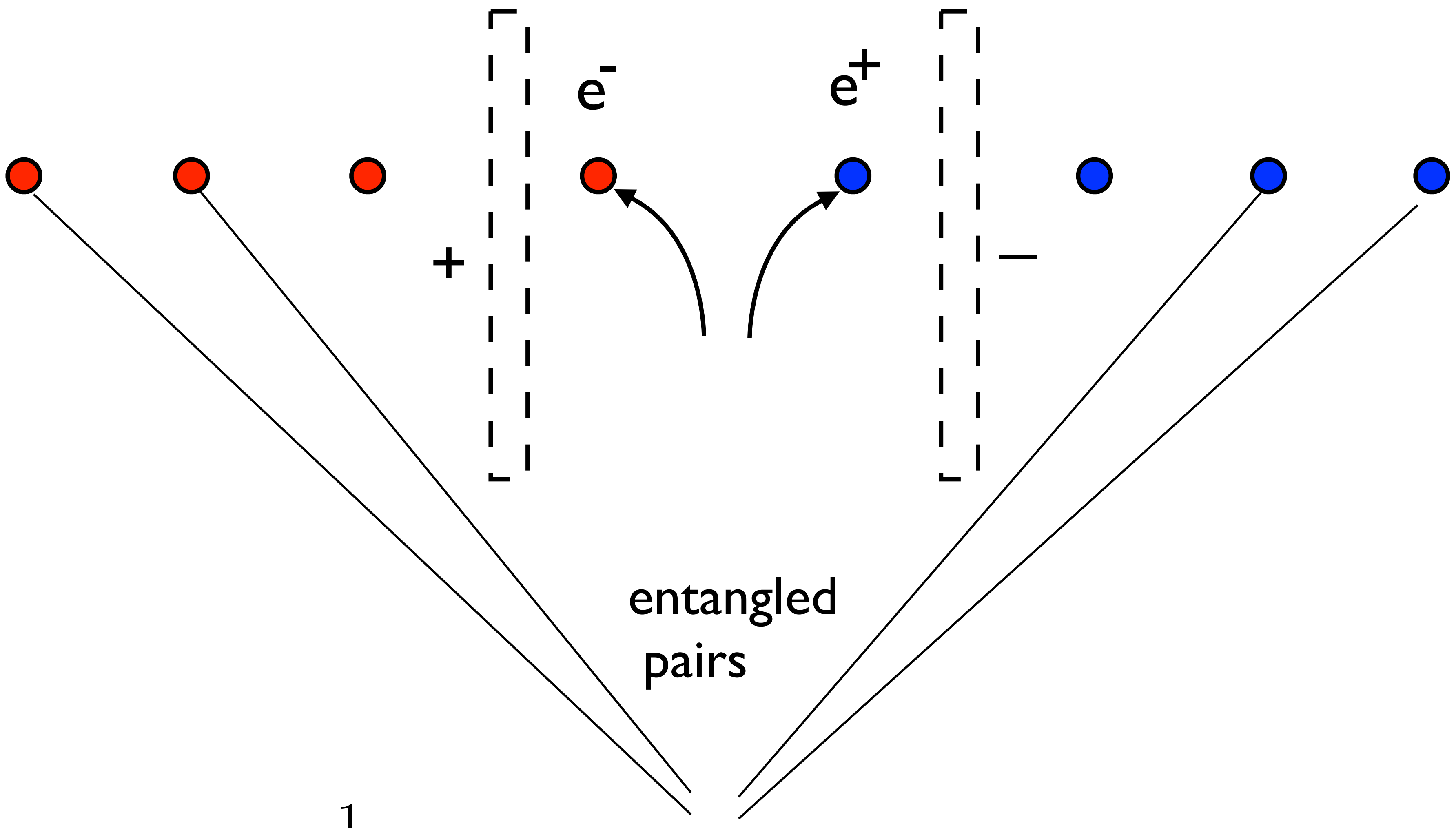
John Preskill

Kip Thorne

But Kip Thorne did not agree to surrender the bet ...

So who is right ?

Schwinger process

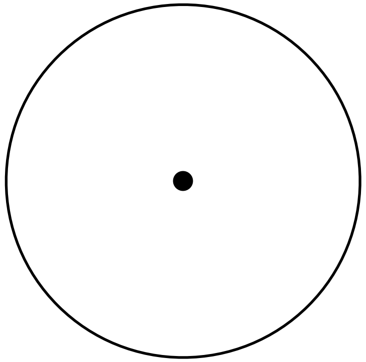


$$\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\ldots$$

$$S_{ent} \approx N \ln 2$$

+ corrections

The black hole



The black hole is described by the Schwarzschild metric

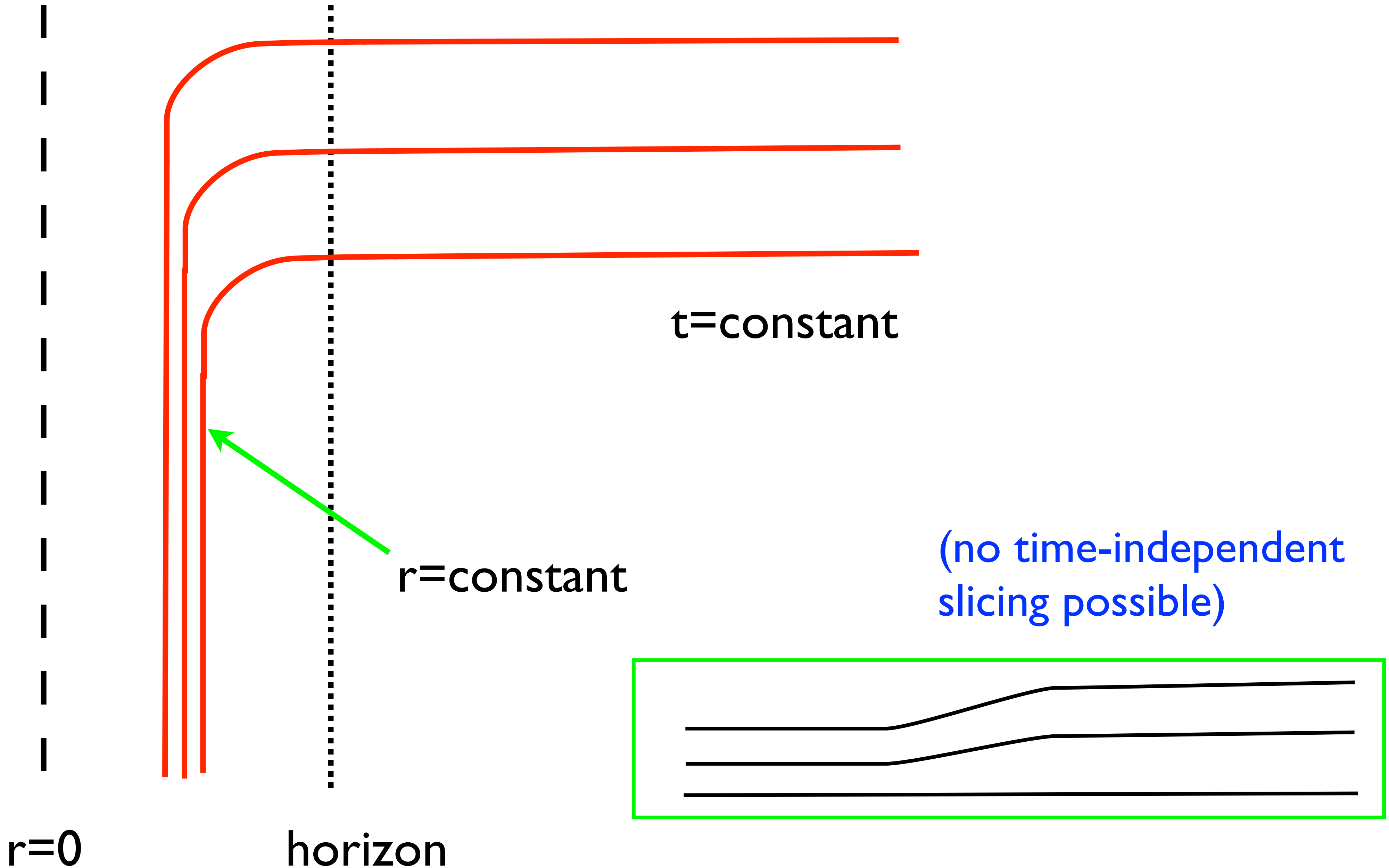
$$ds^2 \;=\; -(1-\frac{2M}{r})dt^2 \;+\; \frac{dr^2}{(1-\frac{2M}{r})} \;+\; r^2(d\theta^2+\sin^2\theta d\phi^2)$$

Crucial point about the black hole:

For $r>2M$ the surface $t=constant$ is spacelike

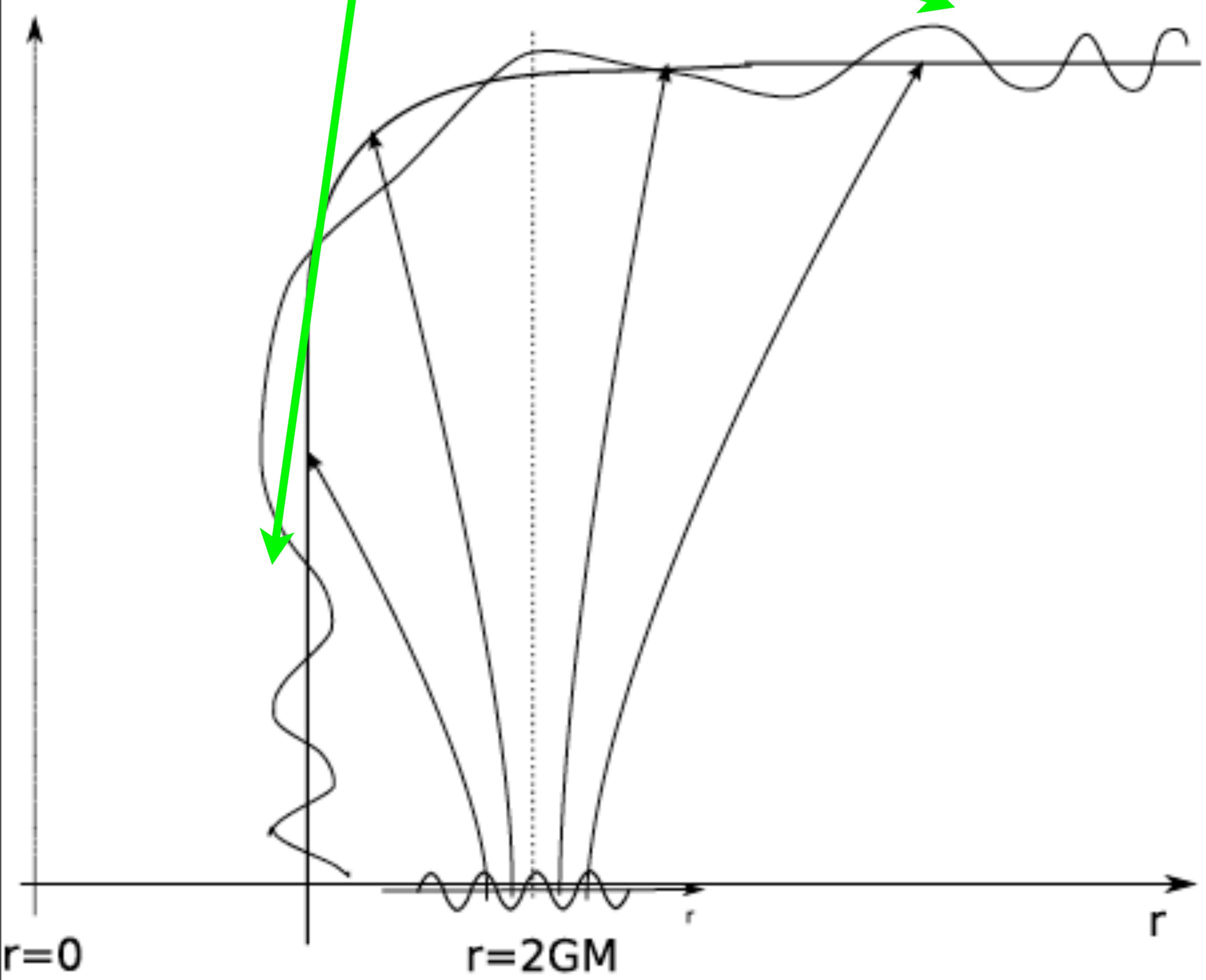
For $r<2M$ the surface $r=constant$ is spacelike

We have to draw spacelike slices to foliate spacetime

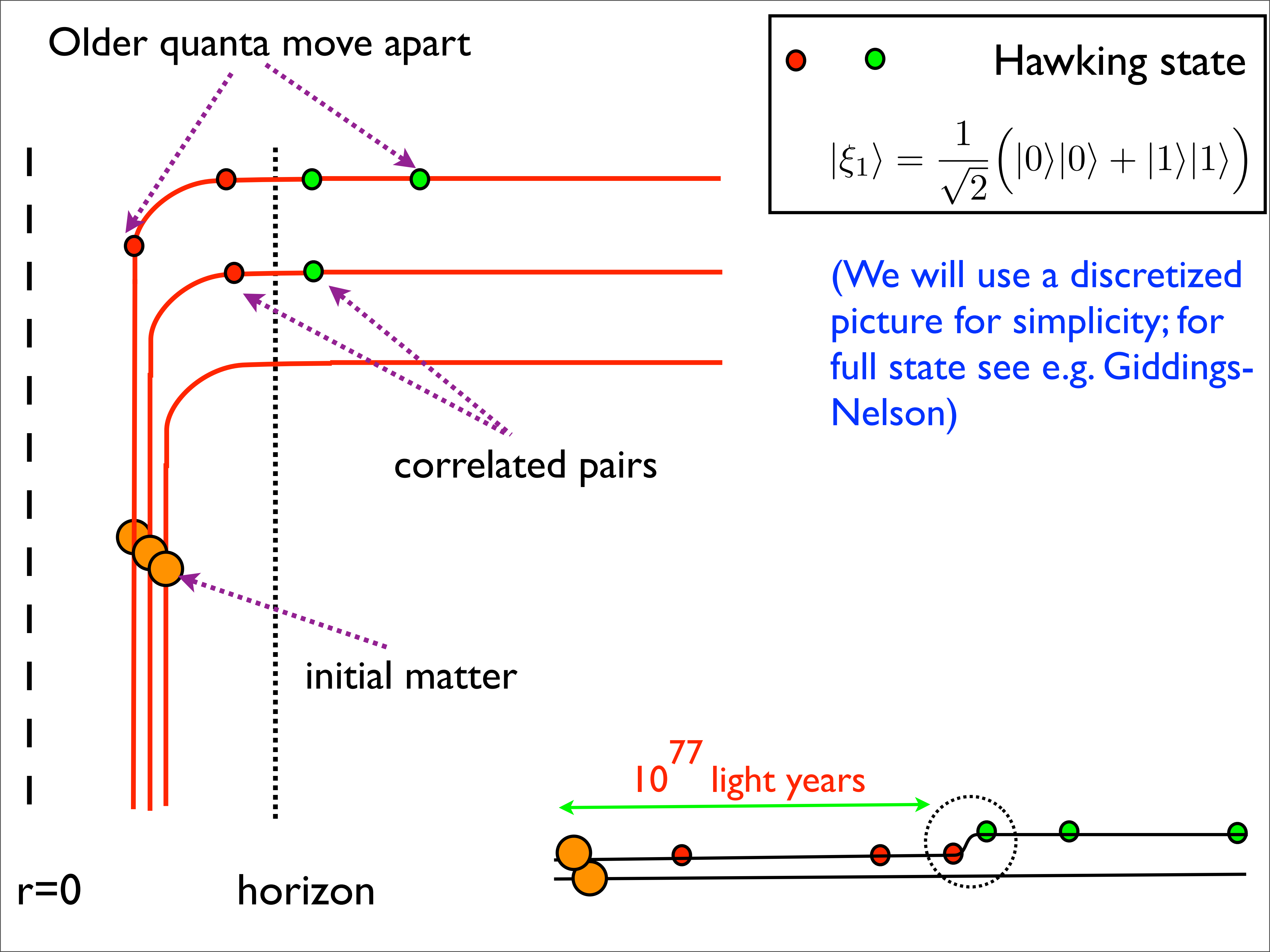


The Hawking process

Entangled pairs

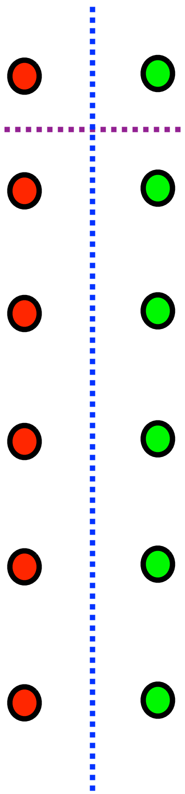


Stretching of spacetime causes field modes to get excited



Older quanta move apart
Hawking state
$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle|0\rangle + |1\rangle|1\rangle\Big)$
(We will use a discretized picture for simplicity; for full state see e.g. Giddings-Nelson)
correlated pairs
initial matter
10^{77} light years
r=0
horizon

Hawking's argument



$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle|0\rangle + |1\rangle|1\rangle\Big)$$

$$S_{N+1} = S_N + \ln 2$$

S_N: Entanglement entropy after N pairs have been created

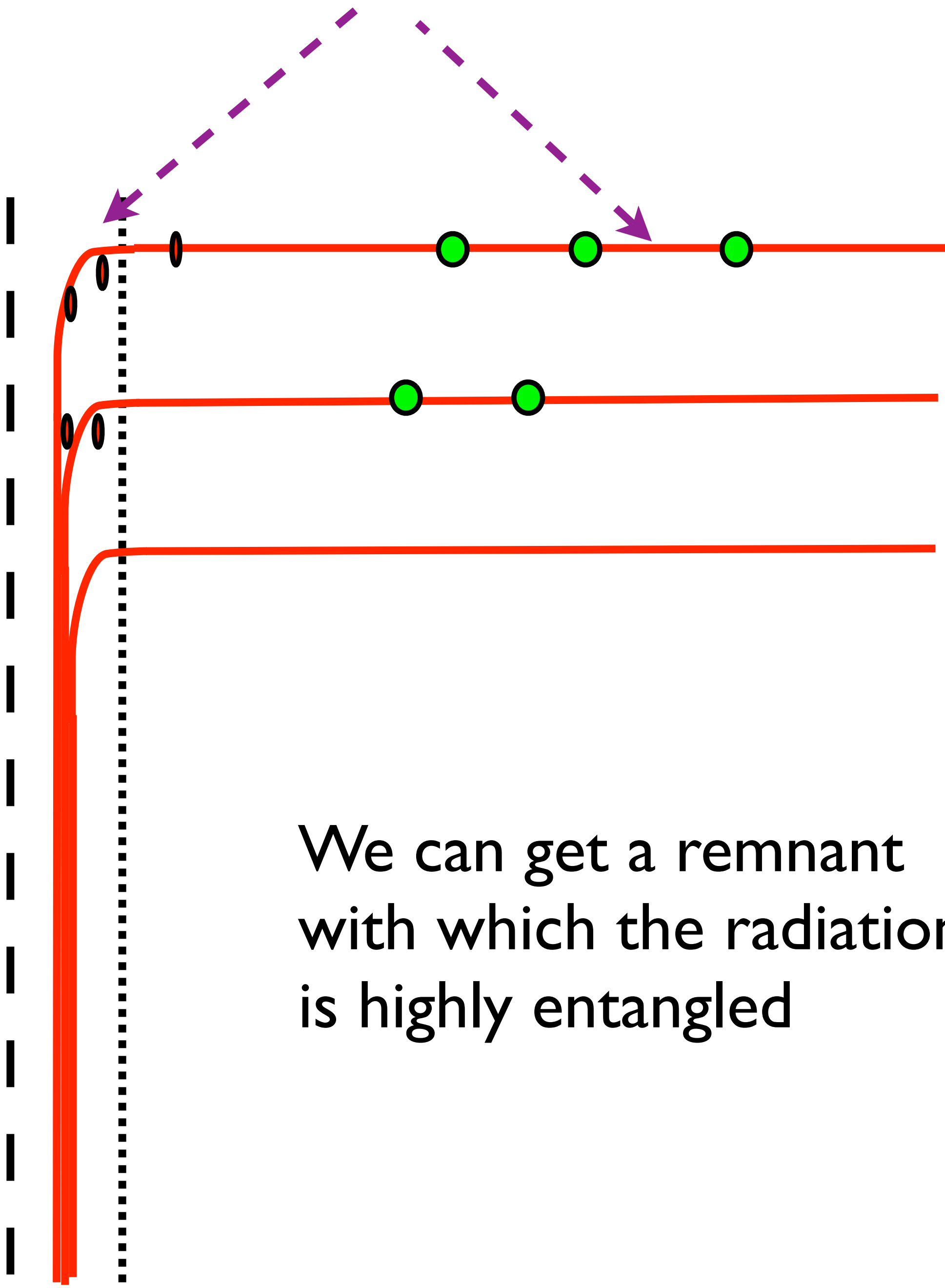
$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle|0\rangle + |1\rangle|1\rangle\Big)$$

$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle|0\rangle + |1\rangle|1\rangle\Big)$$

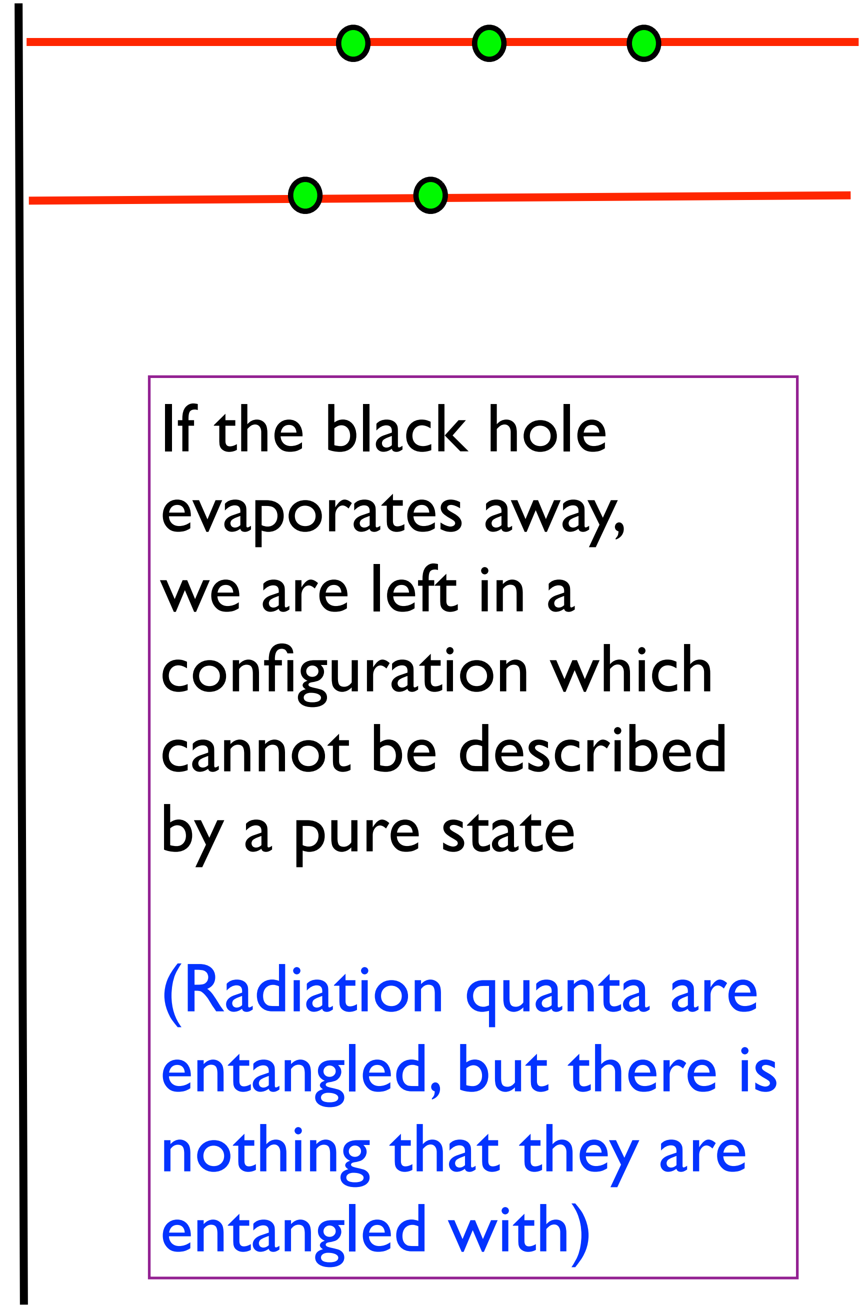
The radiation state (green quanta) are highly entangled with the infalling members of the Hawking pairs (red quanta)

$$S = N_{total} \ln 2$$

Entangled state



We can get a remnant with which the radiation is highly entangled



If the black hole evaporates away, we are left in a configuration which cannot be described by a pure state

(Radiation quanta are entangled, but there is nothing that they are entangled with)

So what can small corrections do ?

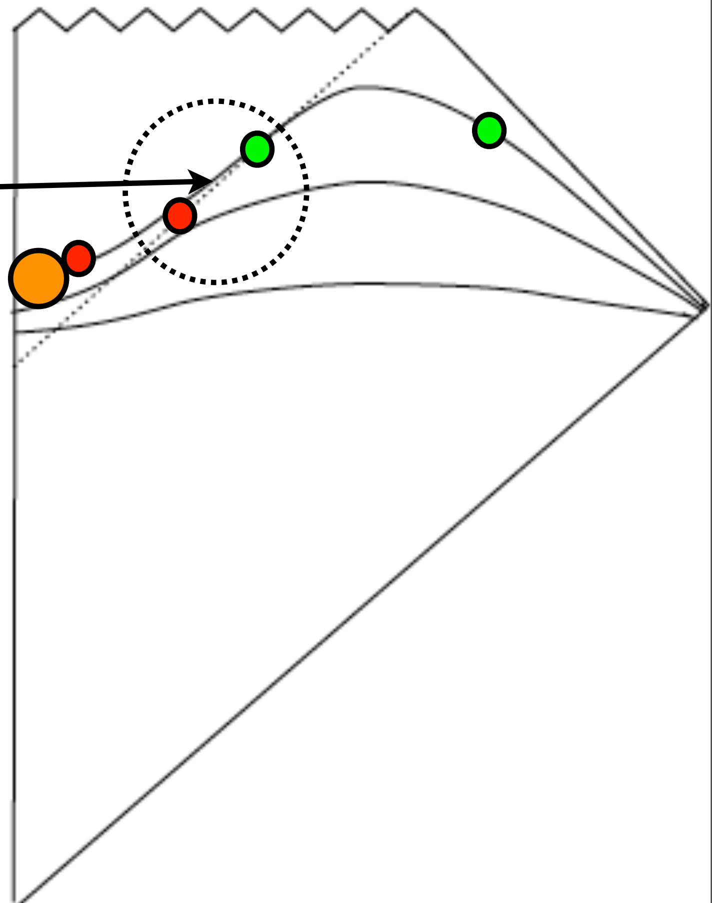
Rules of the game:

(a) The region where pairs are being produced has 'normal evolution'; i.e. vacuum modes evolve as expected upto corrections of order ϵ

(b) The stuff inside the hole can be reshuffled in any way we want, but the quanta that have left cannot be altered

$$\frac{1}{\sqrt{2}}\Big(|0\rangle_1|0\rangle_{1'} + |1\rangle_1|1\rangle_{1'}\Big)$$

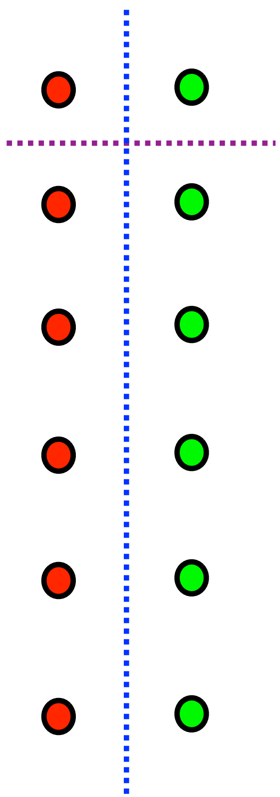
$$+ \epsilon\,\frac{1}{\sqrt{2}}\Big(|0\rangle_1|0\rangle_{1'} - |1\rangle_1|1\rangle_{1'}\Big)$$



$$\epsilon \ll 1$$

Making rigorous the statement that 'Nothing happens at the horizon'

Theorem: Small corrections to Hawking's leading order computation do NOT remove the entanglement

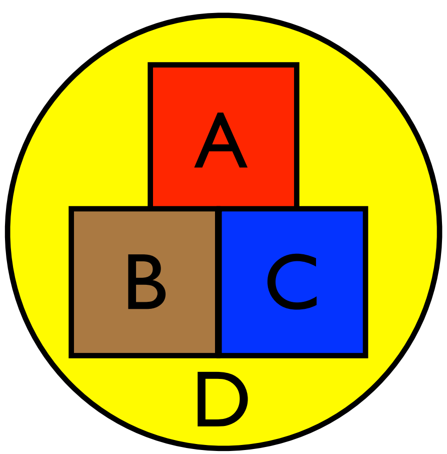


$$\frac{\delta S_{ent}}{S_{ent}} < 2\epsilon$$

(SDM 09)

Bound does not depend on the number of pairs N

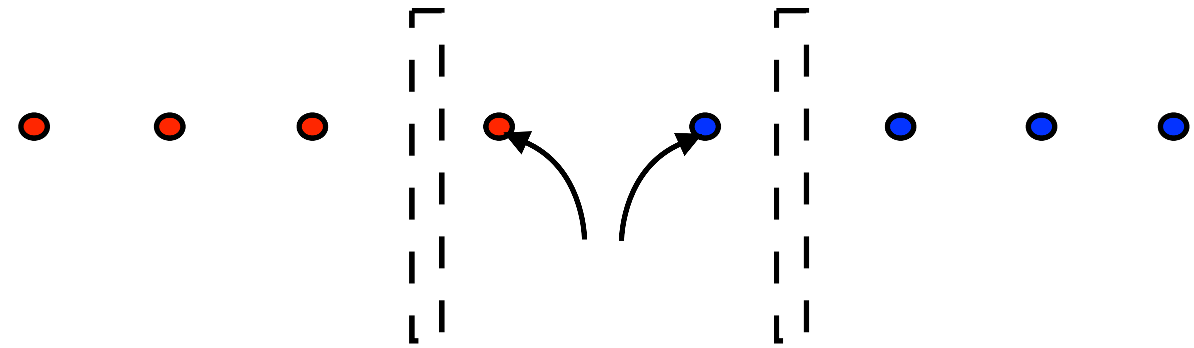
Basic tool : Strong Subadditivity (Lieb + Ruskai '73)



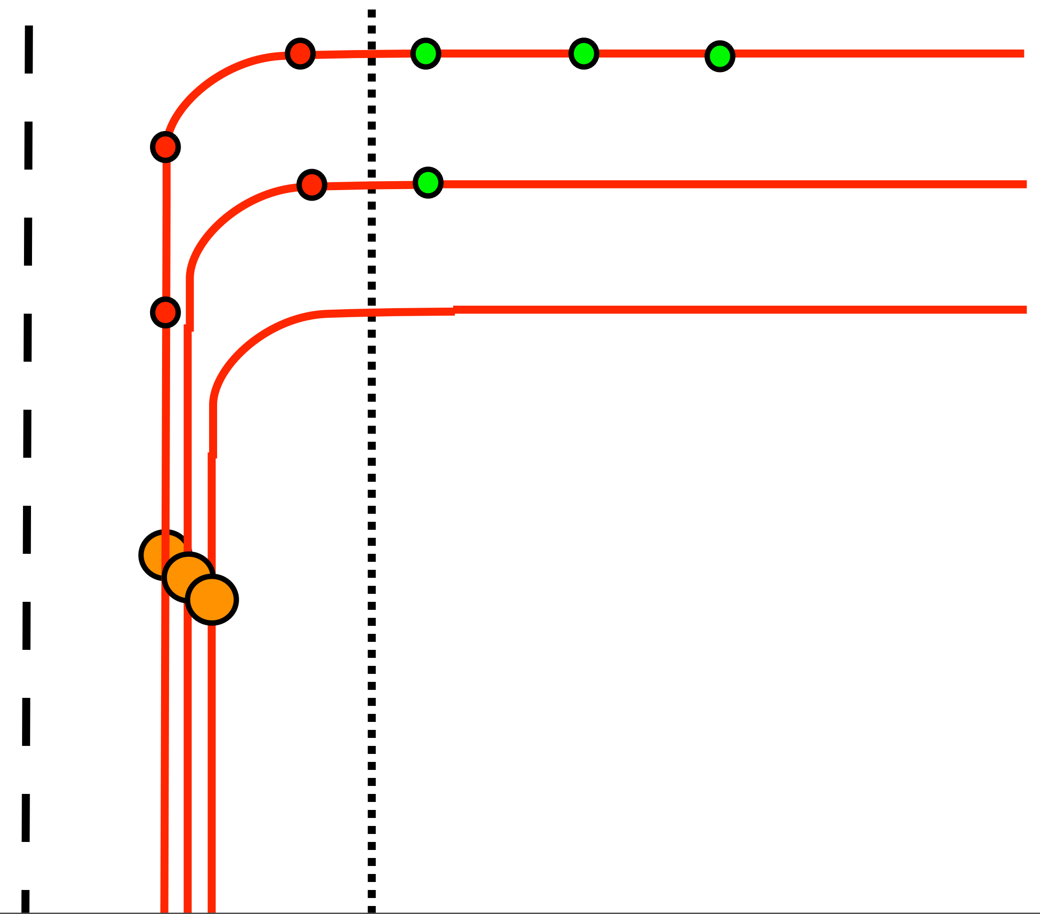
$$S(A) = -Tr[\rho_A \ln \rho_A]$$ etc.

$$S(A+B) + S(B+C) \geq S(A) + S(C)$$

Conclusion:

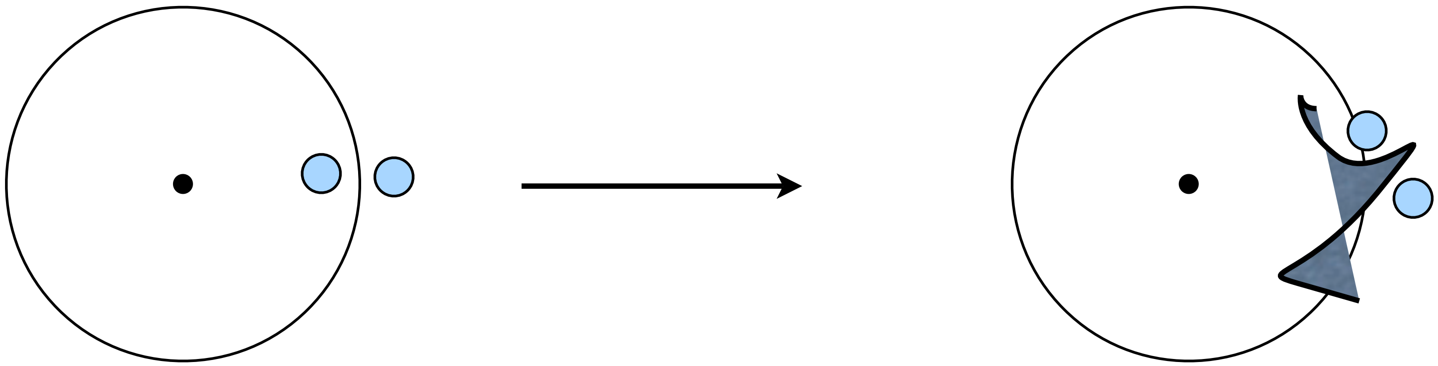


In Schwinger process or in black hole:
Entanglement rises with each emission



Cannot resolve the problem as long as corrections to low energy dynamics are small at the horizon

An order unity correction at the horizon means that we need 'hair' ..



Thus we have a conflict between two 'theorems':

(a) The Hawking argument, supplements by the inequality that shows its robustness to small corrections

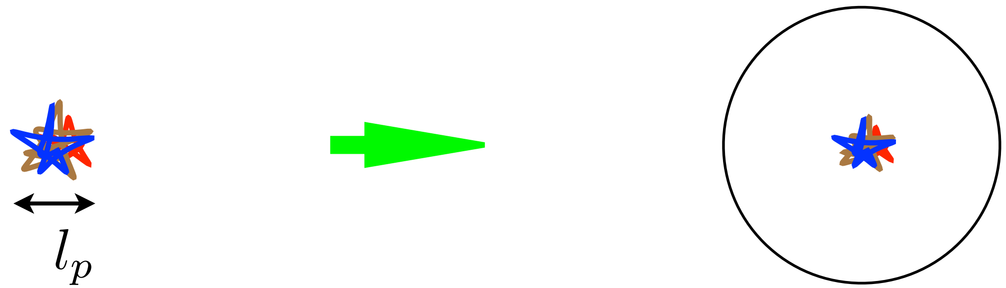
(b) The 'no hair theorem', which encodes our failure to find any alternative to the black hole geometry

So, what is the resolution?

(Avery, Balasubramanian, Bena, Chowdhury, de Boer, Gimon, Giusto, Keski-Vakkuri, Levi, Maldacena, Maoz, Park, Peet, Potvin, Ross, Ruef, Saxena, Skenderis, Srivastava, Taylor, Turton, Warner ...)

The traditional expectation ...

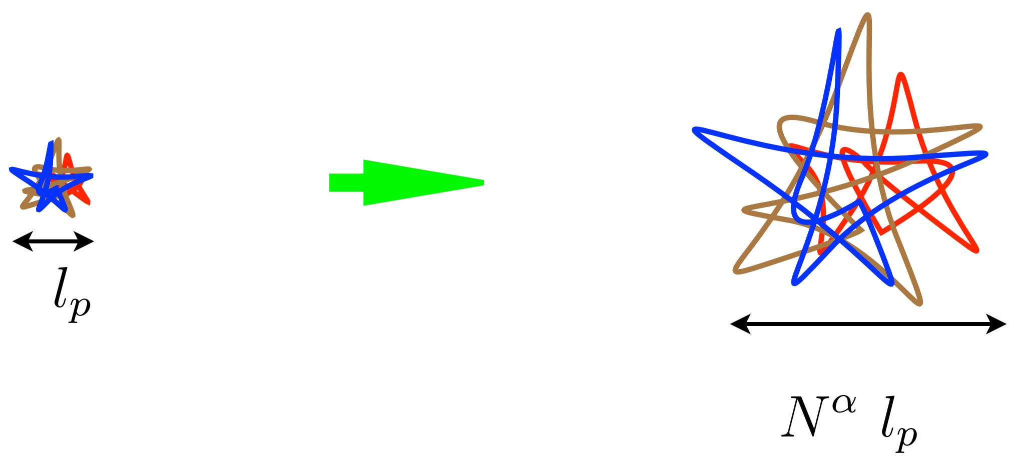
weak coupling



strong coupling

But it seems in string theory the opposite happens ...

weak coupling

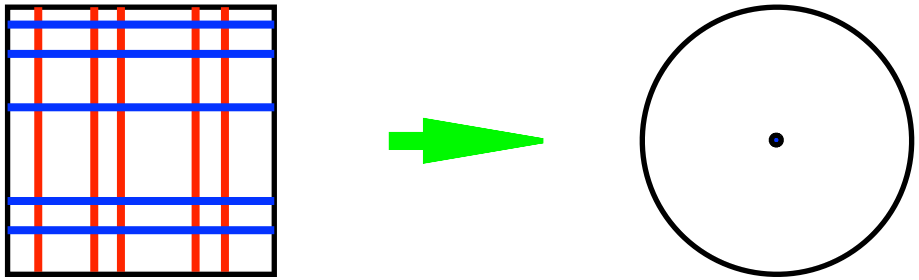


strong coupling

Size of bound states grows with coupling, number of branes ...

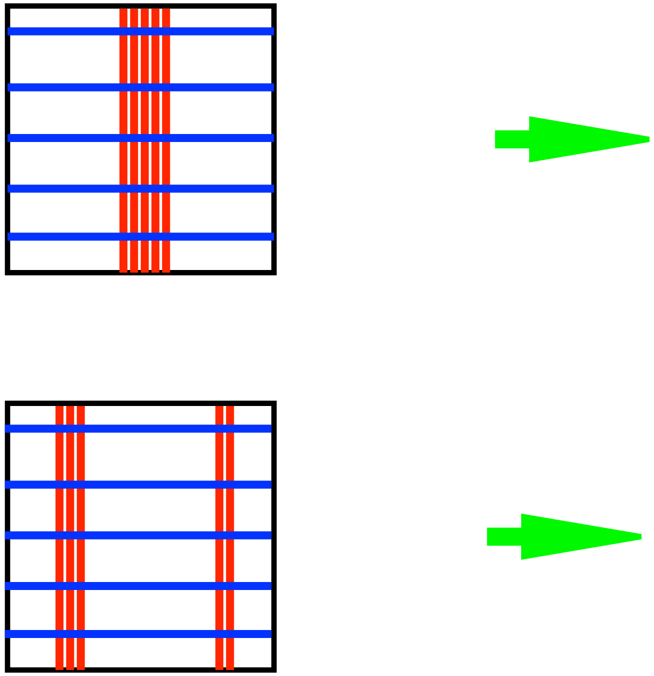
(SDM 97)

Recall that weak coupling states are described by intersecting branes wrapped on compact directions



Start with the simplest microstates:

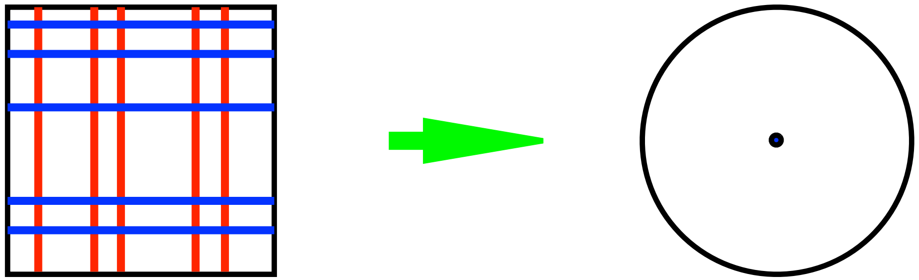
weak coupling



strong coupling

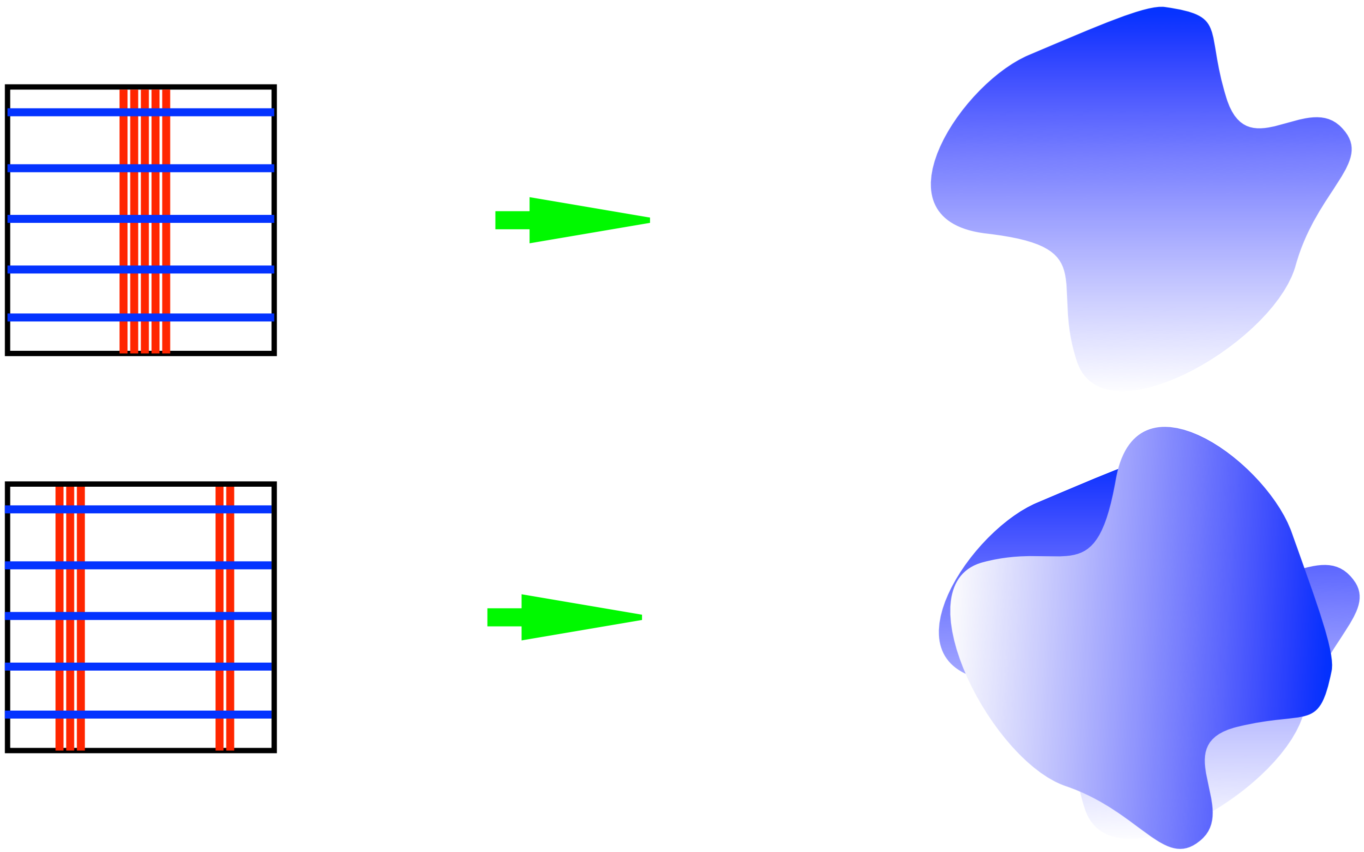
No horizon, no singularity ... so we do not get the traditional hole ...

Recall that weak coupling states are described by intersecting branes wrapped on compact directions



Start with the simplest microstates:

weak coupling

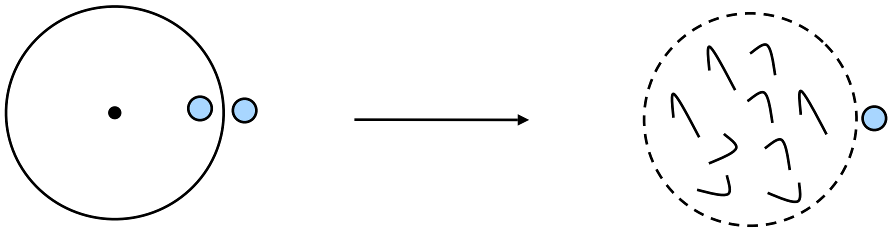


strong coupling

No horizon, no singularity ... so we do not get the traditional hole ...

Many such constructions have been done ...

General lesson: Eigenstates in string theory do not have a traditional horizon



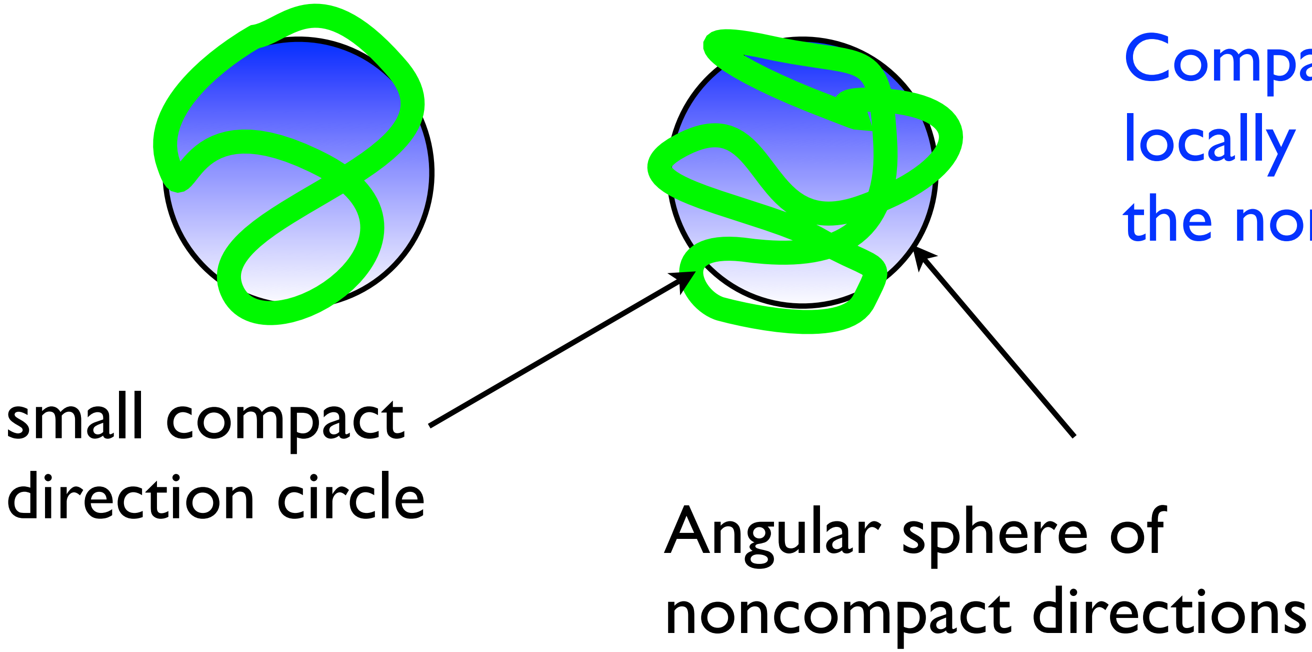
That is, there is no smooth region straddling the horizon where low energy pair modes evolve as expected on gently curved spacetime

Geometry can be different from the traditional hole, or more generally, there is no geometry, just a quantum 'fuzz'

$\longrightarrow$ fuzzballs

Thus we have finally found the ‘hair’ for black holes ...

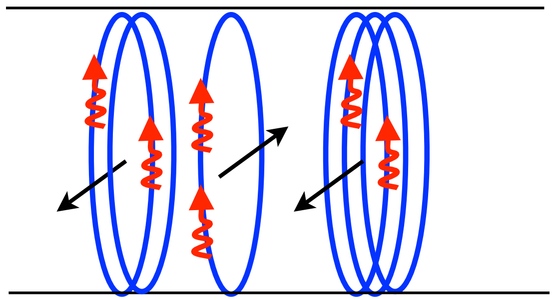
Nature of the hair:



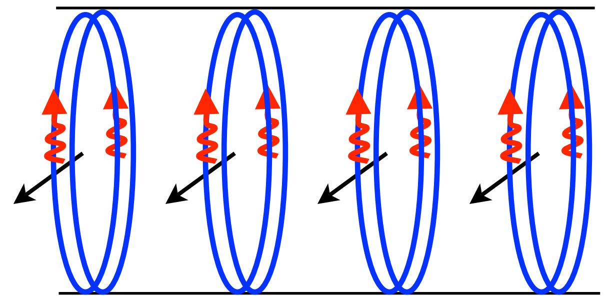
Compact directions make locally nontrivial fibrations over the noncompact directions

$e^{S_{bek}}$ states upon quantization

Thus the hair are fundamentally a nonperturbative construct involving the extra dimensions ...

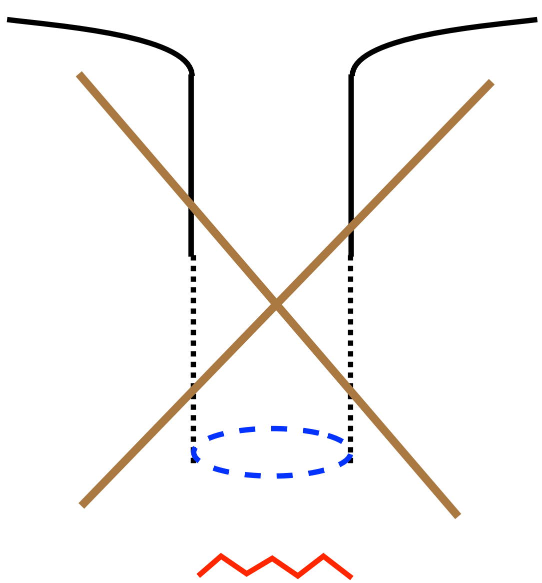


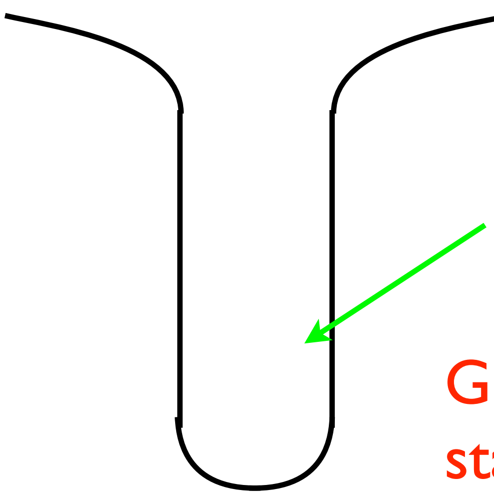
Generic D1D5P CFT state



Simple states: all components the same, excitations fermionic, spin aligned

$$|k\rangle^{total} = (J^{-,total}_{-(2k-2)})^{n_1 n_5} (J^{-,total}_{-(2k-4)})^{n_1 n_5} \dots (J^{-,total}_{-2})^{n_1 n_5}\ |1\rangle^{total}$$





$AdS_3 \times S^3 \times T^4$

Geometry for simple state (winding =1)

$$
\begin{aligned}
ds^2 &= -\frac{1}{h}(dt^2 - dy^2) + \frac{Q_p}{hf}(dt-dy)^2 + hf\left(\frac{dr_N^2}{r_N^2 + a^2\eta} + d\theta^2\right) \\
&+ h\left(r_N^2 - na^2\eta + \frac{(2n+1)a^2\eta Q_1 Q_5 \cos^2\theta}{h^2 f^2}\right)\cos^2\theta d\psi^2 \\
&+ h\left(r_N^2 + (n+1)a^2\eta - \frac{(2n+1)a^2\eta Q_1 Q_5 \sin^2\theta}{h^2 f^2}\right)\sin^2\theta d\phi^2 \\
&+ \frac{a^2\eta^2 Q_p}{hf}\left(\cos^2\theta d\psi + \sin^2\theta d\phi\right)^2 \\
&+ \frac{2a\sqrt{Q_1 Q_5}}{hf}\left[n\cos^2\theta d\psi - (n+1)\sin^2\theta d\phi\right](dt - dy) \\
&- \frac{2a\eta\sqrt{Q_1 Q_5}}{hf}\left[\cos^2\theta d\psi + \sin^2\theta d\phi\right] dy + \sqrt{\frac{H_1}{H_5}}\sum_{i=1}^{4} dz_i^2
\end{aligned}
$$

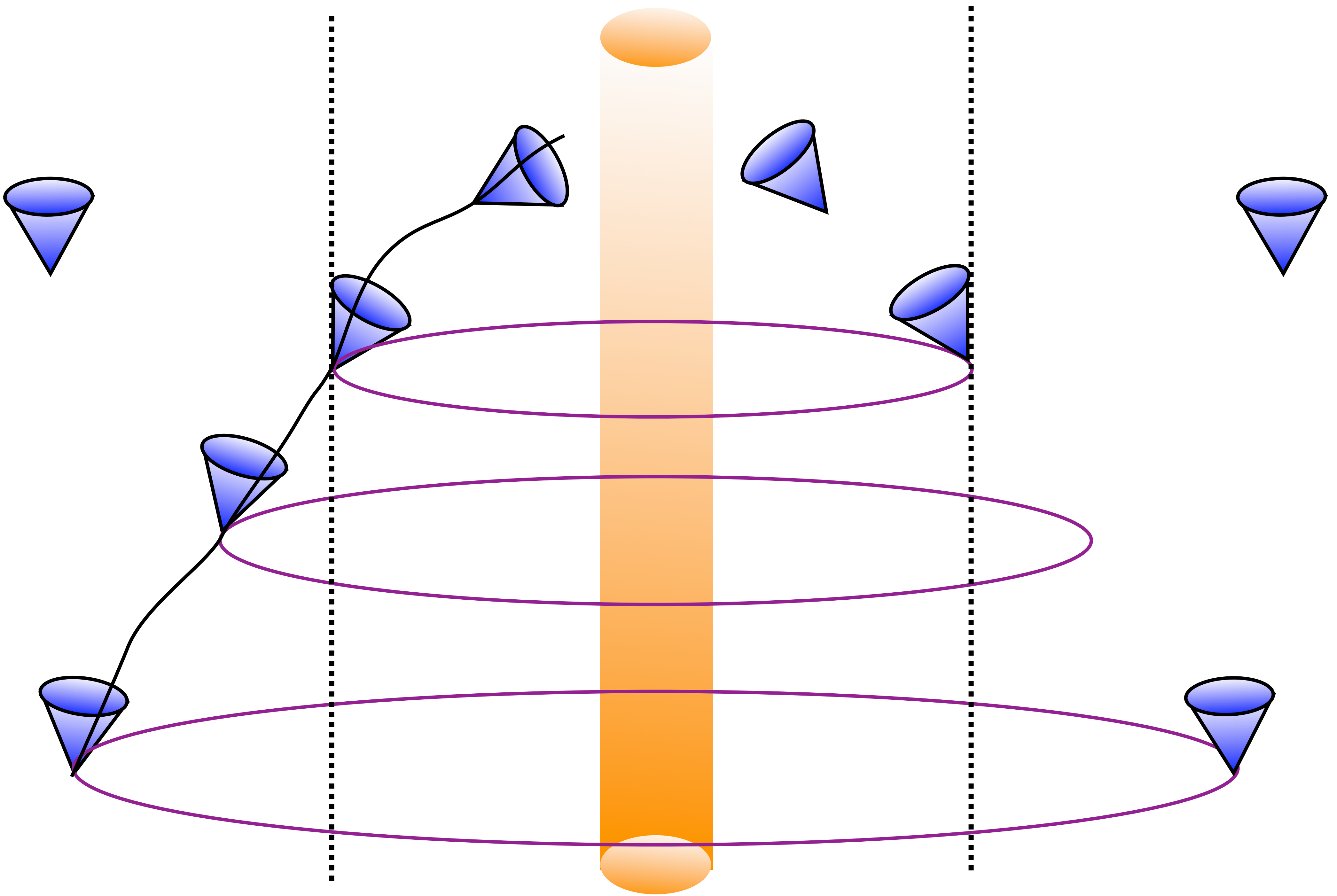
$$
\begin{aligned}
f &= r_N^2 - a^2\eta\, n\sin^2\theta + a^2\eta\,(n+1)\cos^2\theta \\
h &= \sqrt{H_1 H_5},\ H_1 = 1 + \frac{Q_1}{f},\ H_5 = 1 + \frac{Q_5}{f}
\end{aligned}
$$

$$
\eta \equiv \frac{Q_1 Q_5}{Q_1 Q_5 + Q_1 Q_p + Q_5 Q_p}
$$

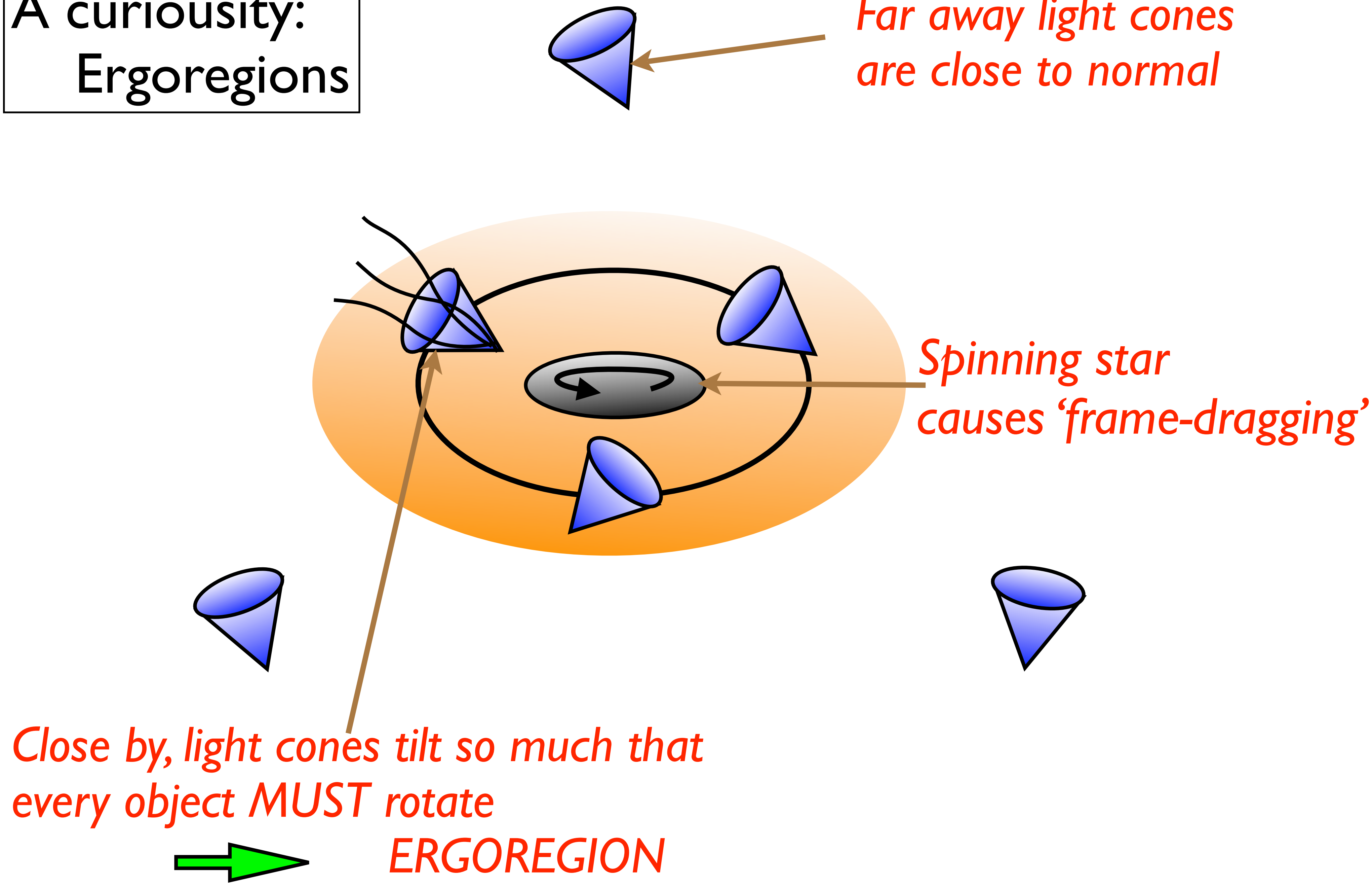
(Giusto SDM Saxena 04)

How does Hawking radiation arise ?

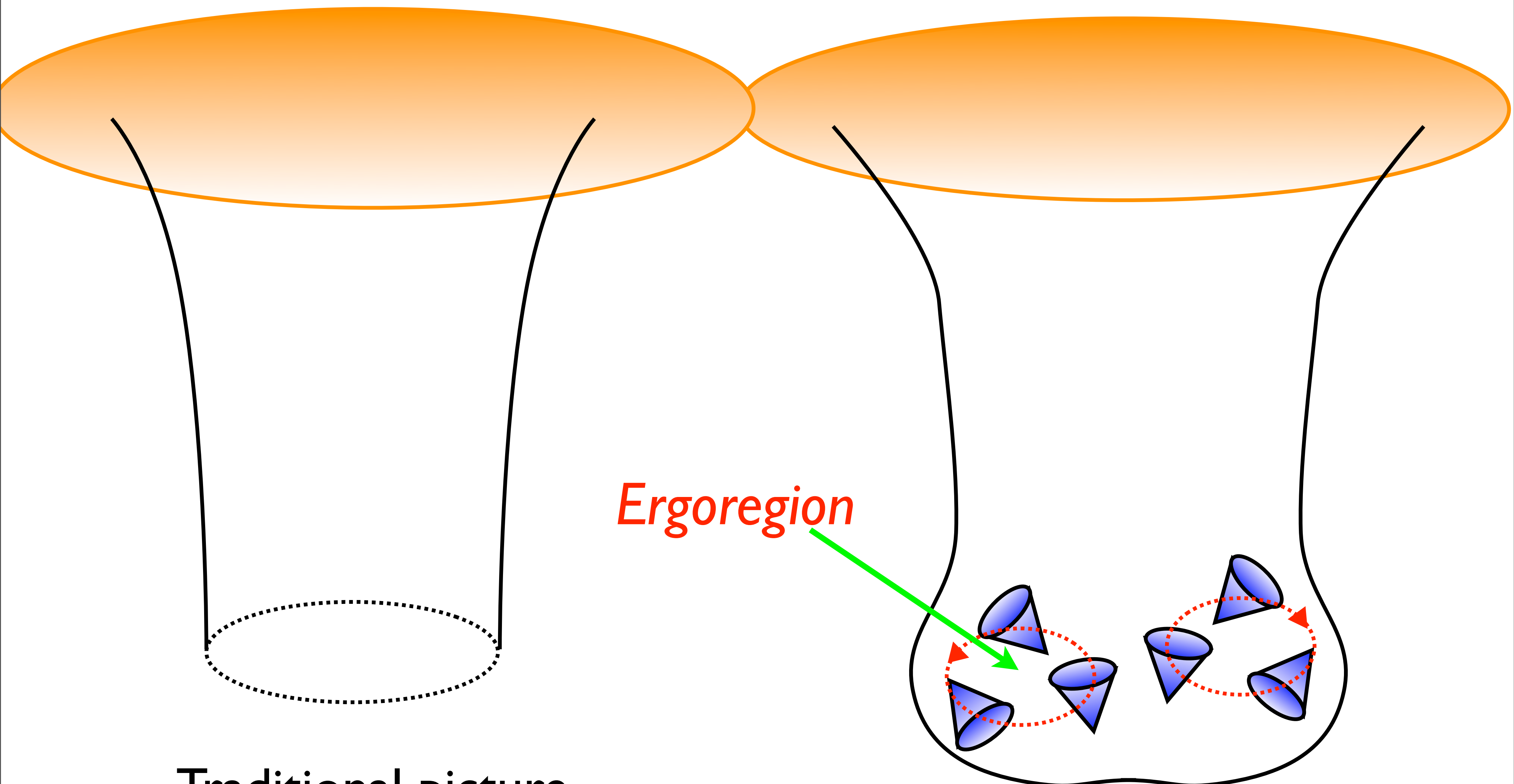
The Black Hole



A curiousity:
Ergoregions



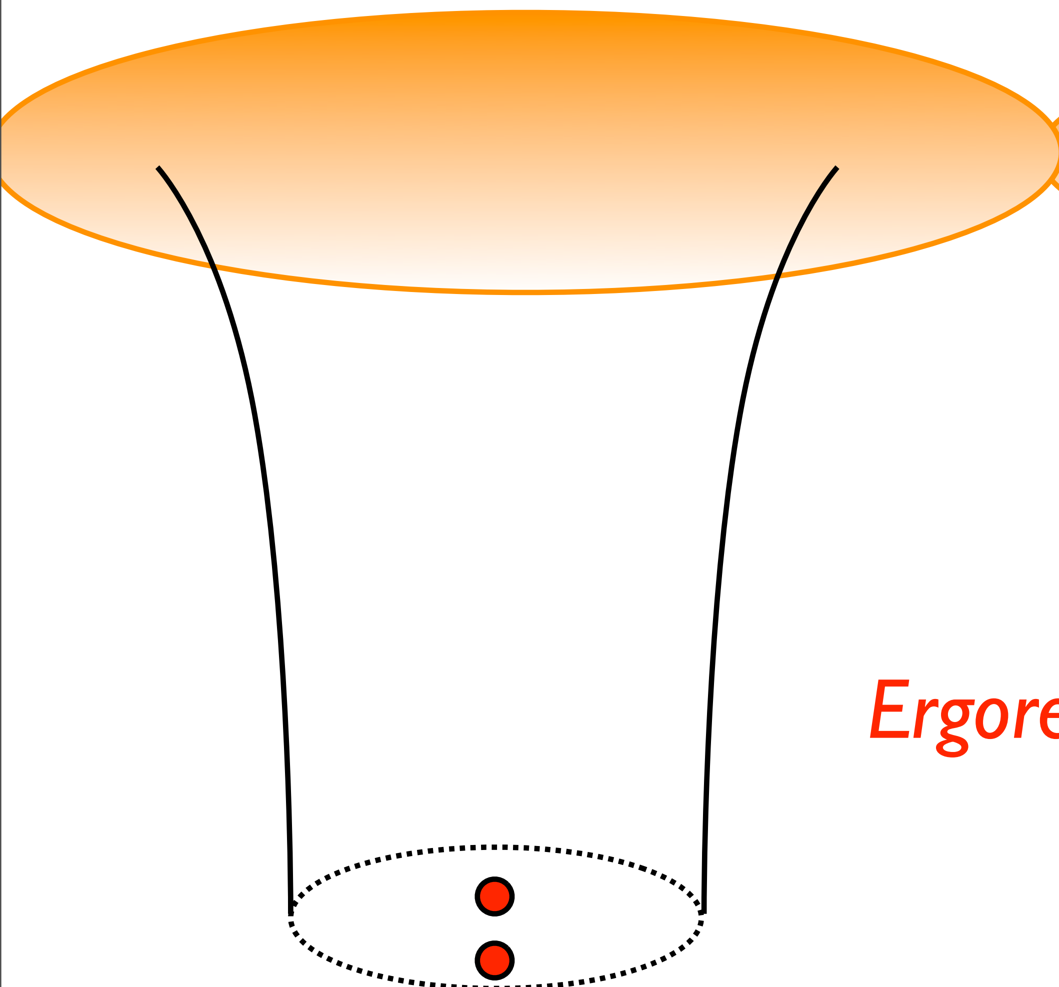
Hawking radiation



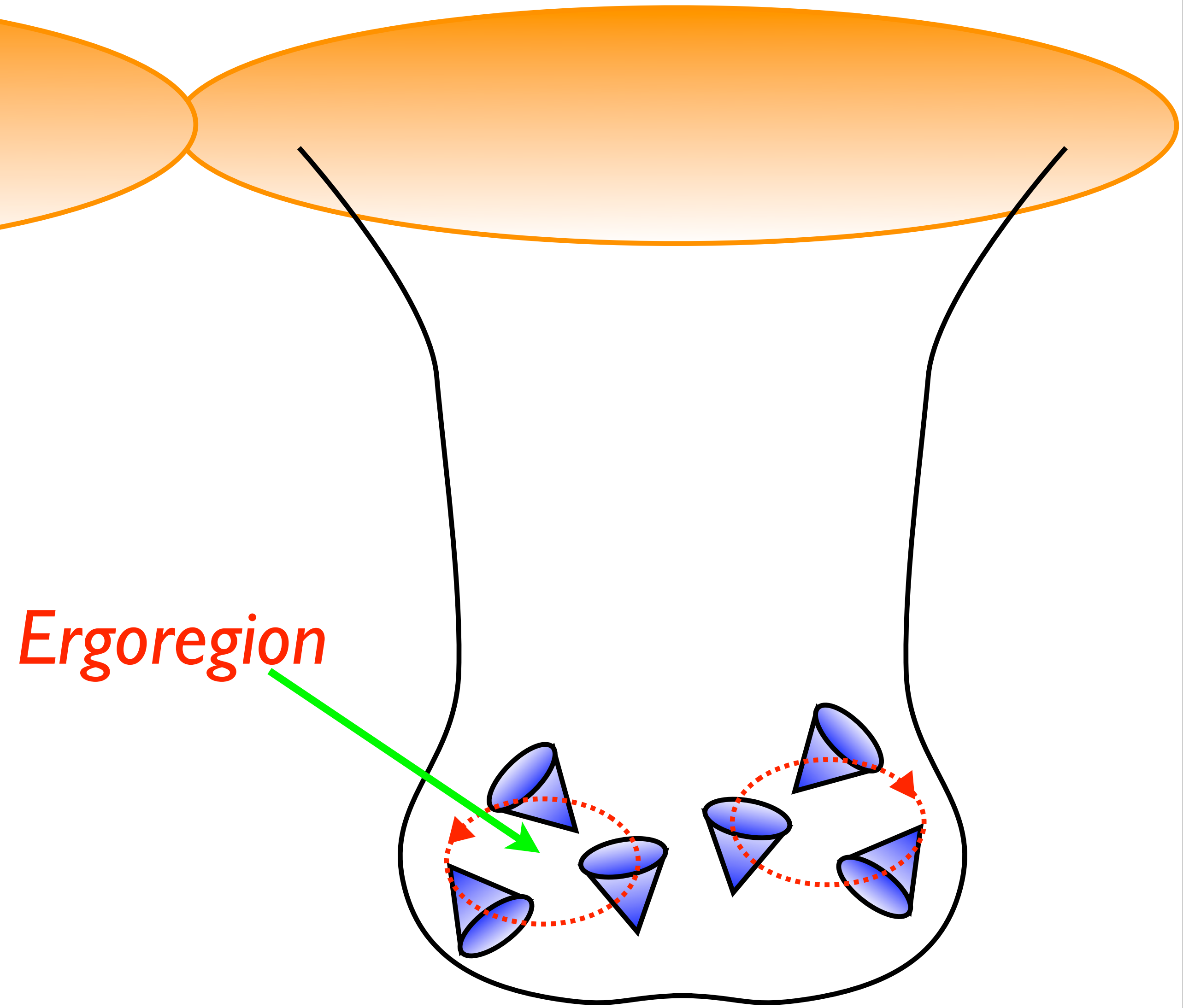
Traditional picture

Actually radiation comes out just like from any other object, not from the vacuum

Hawking radiation

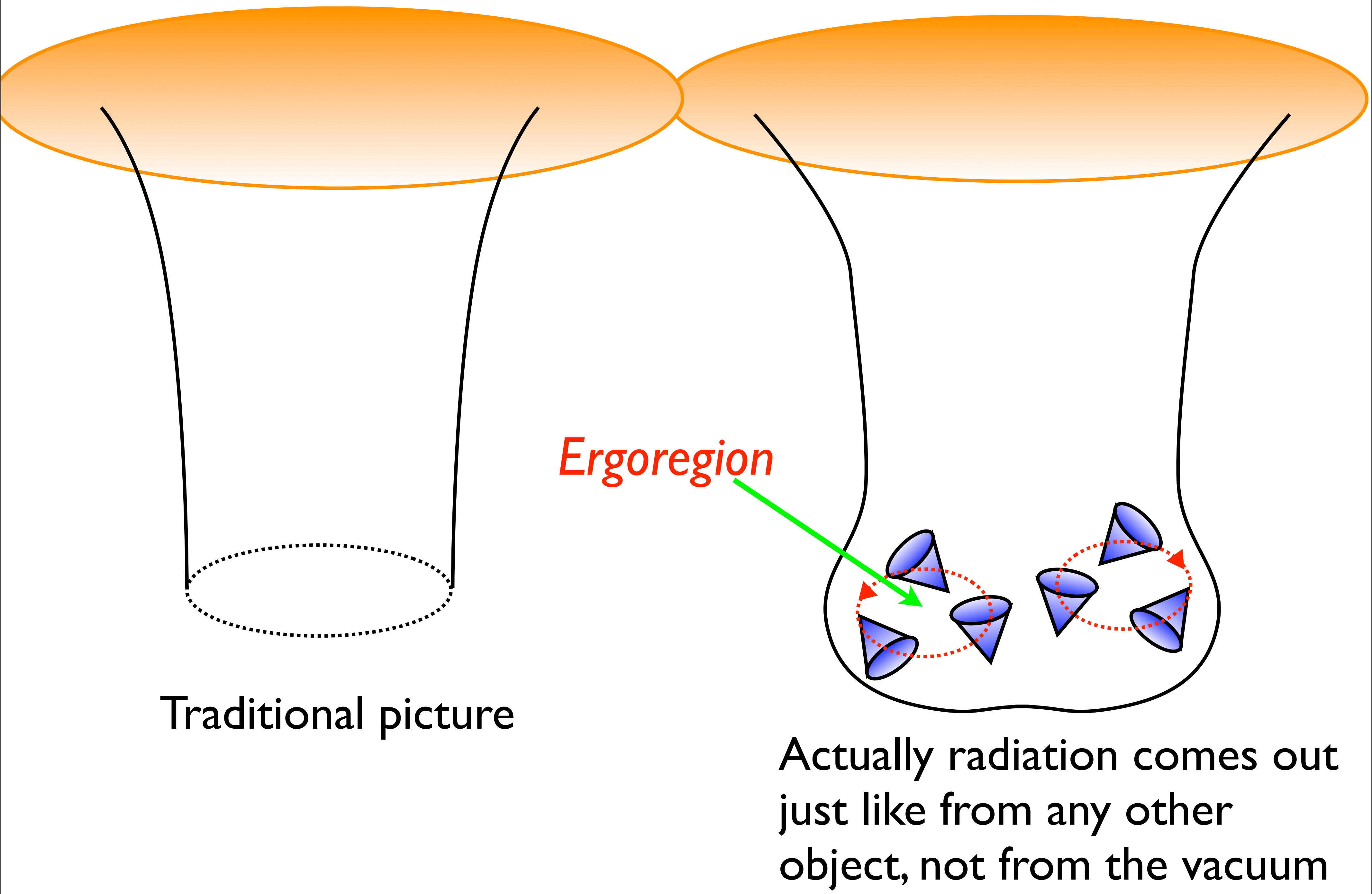


Traditional picture

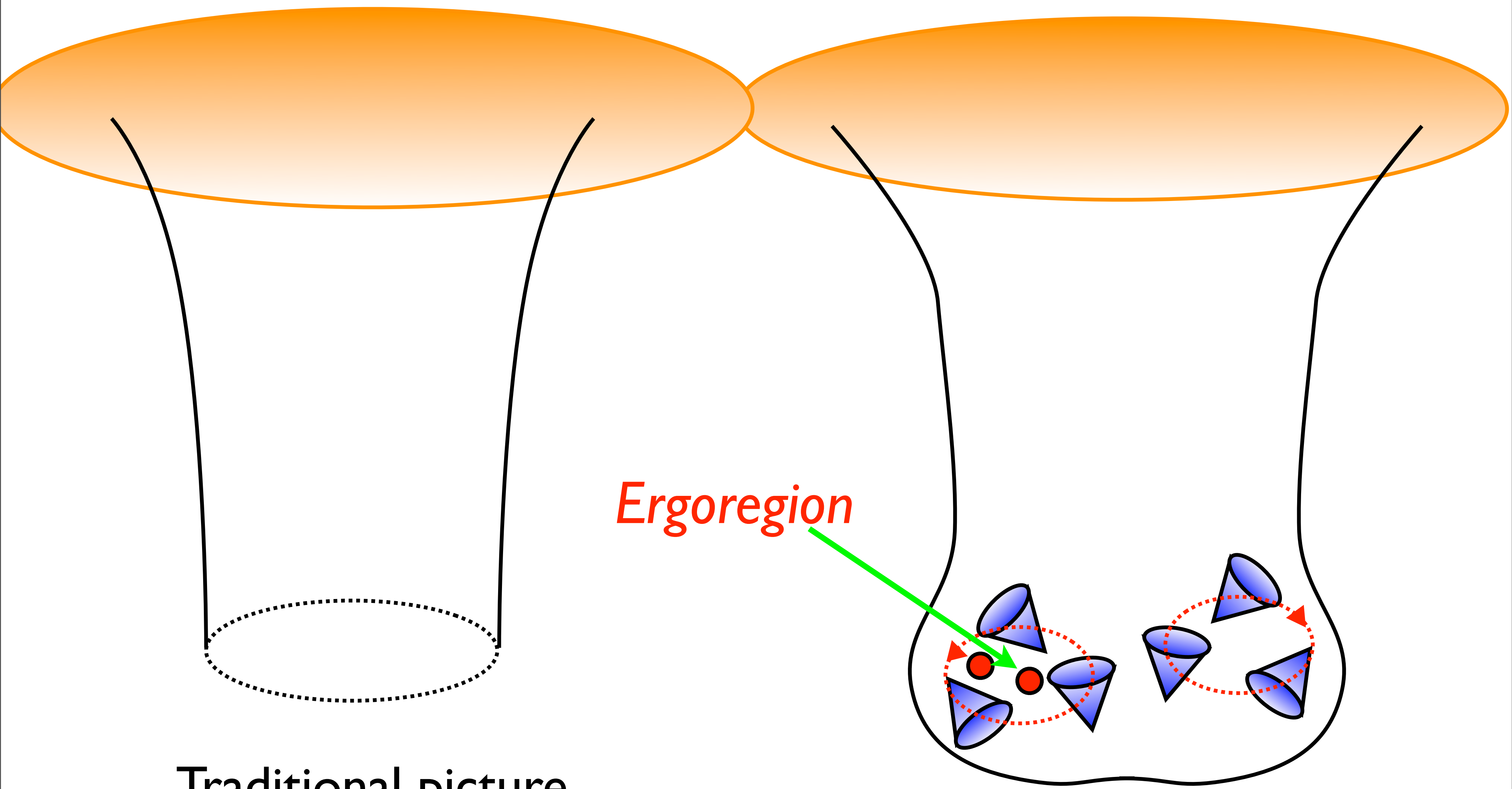


Actually radiation comes out just like from any other object, not from the vacuum

Hawking radiation



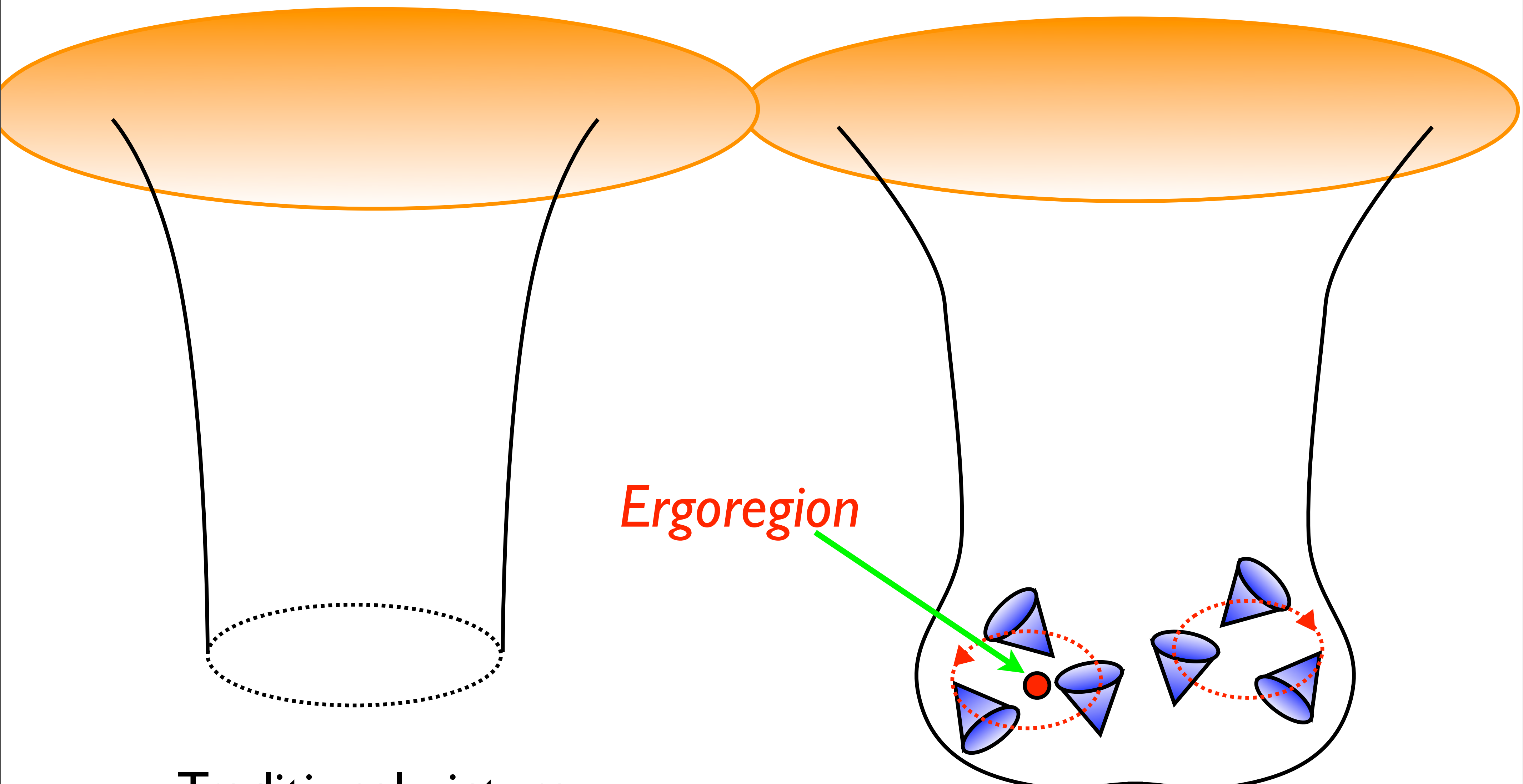
Hawking radiation



Traditional picture

Actually radiation comes out just like from any other object, not from the vacuum

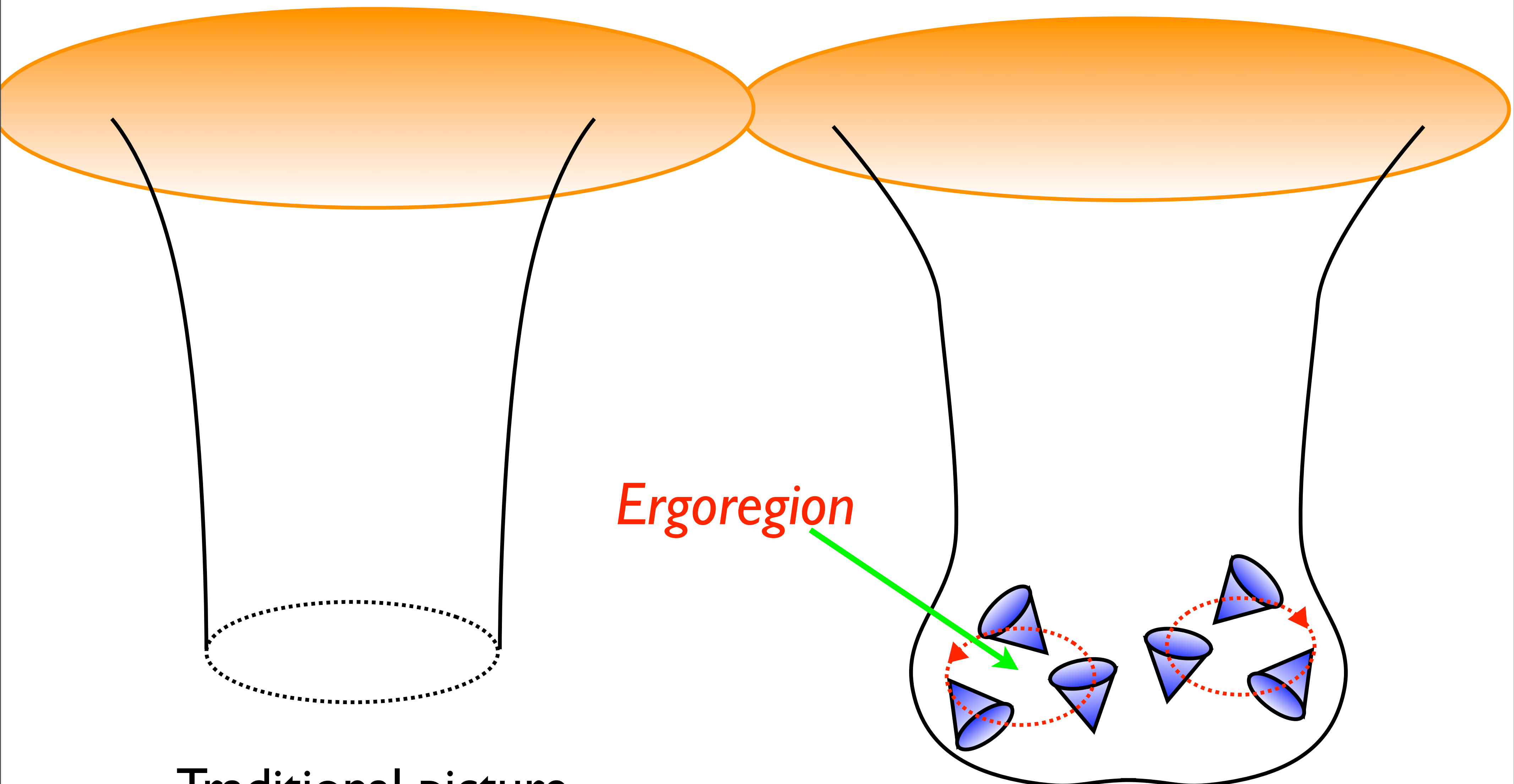
Hawking radiation



Traditional picture

Actually radiation comes out just like from any other object, not from the vacuum

Hawking radiation

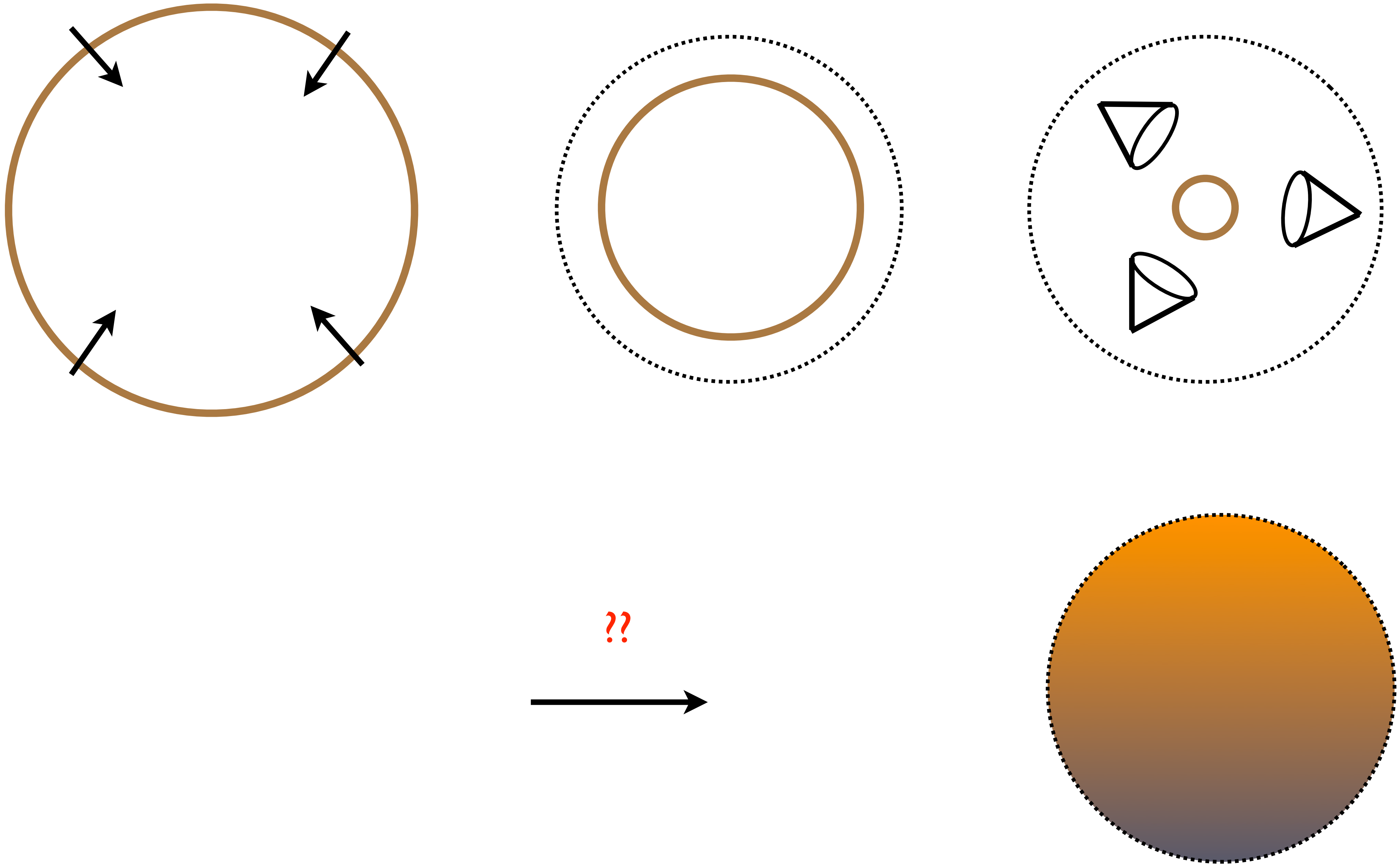


Traditional picture

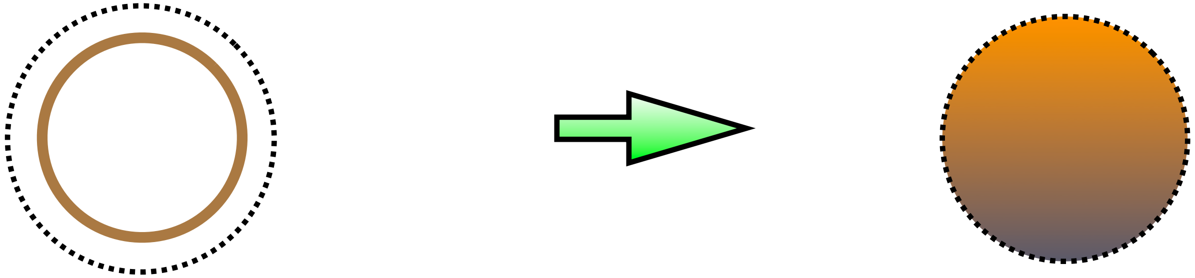
Actually radiation comes out just like from any other object, not from the vacuum

Why does semiclassical intuition fail ?

Shell collapses to make a black hole ...



Consider the amplitude for the shell to tunnel to a fuzzball state

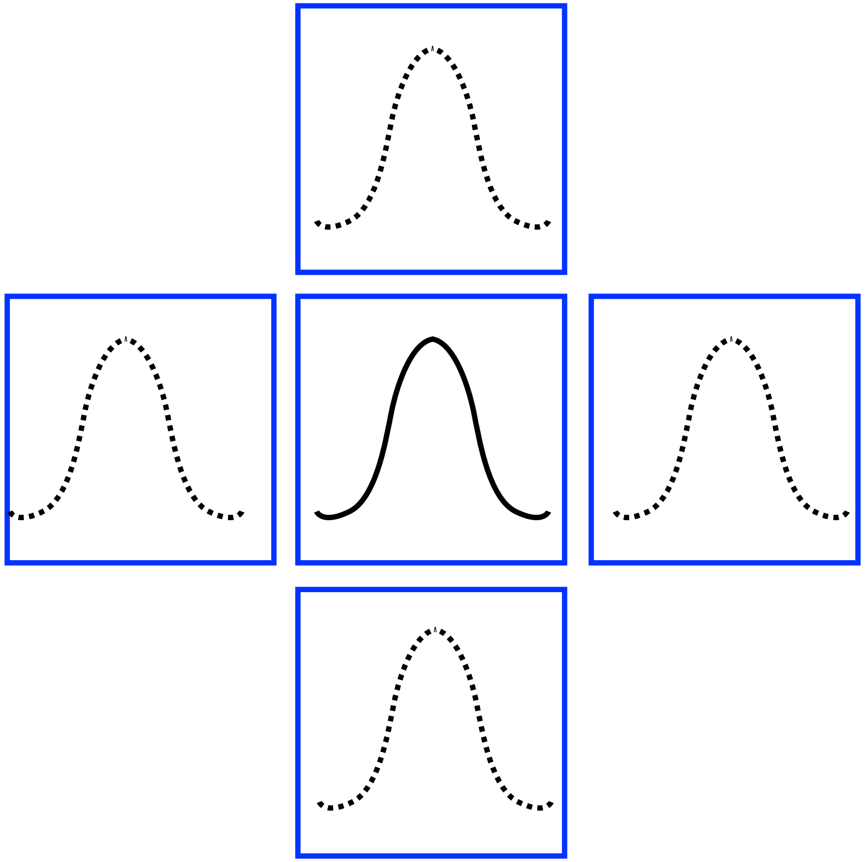


$$S_{tunnel} \sim \frac{1}{G}\int R d^4x \sim \frac{1}{G}\frac{1}{(GM)^2}(GM)^4 \sim GM^2$$

$\mathcal{A} \sim e^{-S_{tunnel}}$ Amplitude to tunnel is very small

$\mathcal{N} \sim e^{S_{bek}} \sim e^{GM^2}$ But the number of states that one can tunnel to is very large !

Toy model: Small amplitude to tunnel to a neighboring well, but there are a correspondingly large number of adjacent wells



In a time of order unity, the wavefunction in the central well becomes a linear combination of states in all wells (SDM 07)

Path integral

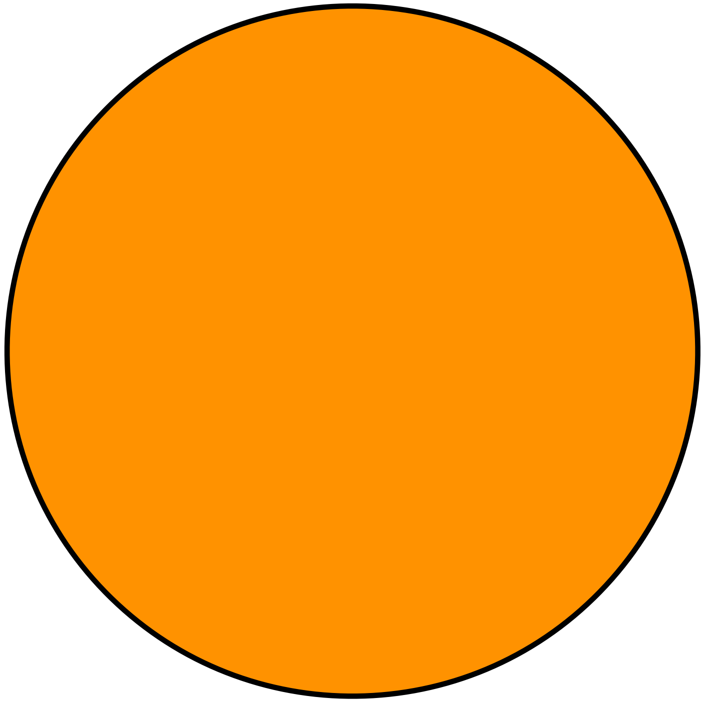
$$\boxed{Z = \int D[g] e^{-\frac{1}{\hbar} S[g]}}$$

Measure has degeneracy of states

Action determines classical trajectory

For traditional macroscopic objects the measure is order $\hbar$ while the action is order unity

But for black holes the entropy is so large that the two are comparable ...



We have a failure of the semiclassical approximation ...

Path integral

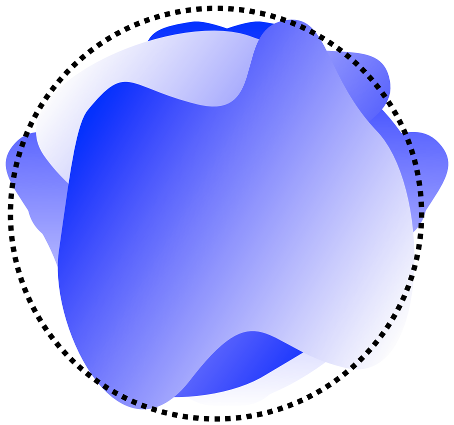
$$\boxed{Z = \int D[g] e^{-\frac{1}{\hbar} S[g]}}$$

Measure has degeneracy of states

Action determines classical trajectory

For traditional macroscopic objects the measure is order $\hbar$ while the action is order unity

But for black holes the entropy is so large that the two are comparable ...



We have a failure of the semiclassical approximation ...

What happens if someone falls into a black hole ?

(a) Low energy dynamics $(E \sim kT)$

No horizon, radiation from ergoregions, so radiation like that from any warm body

no information loss since radiation depends on choice of microstate ψ_k

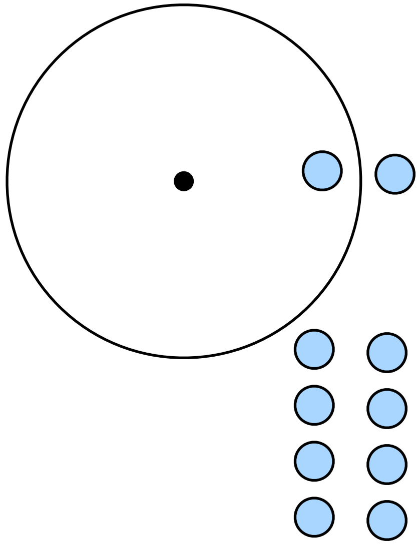
(b) Correlators in high energy infalling frame $(E \gg kT)$

Collective oscillations of fuzzball gives Green's functions that equal Green's functions in empty space ....

(SDM+Pumberg 2011)

Summary

(A) The black hole information paradox is a serious problem: *It does not allow GR to be consistent with quantum unitarity*



$$\Psi_M$$

$$\otimes \quad |0\rangle_1 |0\rangle_{1'} + |1\rangle_1 |1\rangle_{1'}$$

$$\otimes \quad |0\rangle_2 |0\rangle_{2'} + |1\rangle_2 |1\rangle_{2'}$$

$$\ldots$$

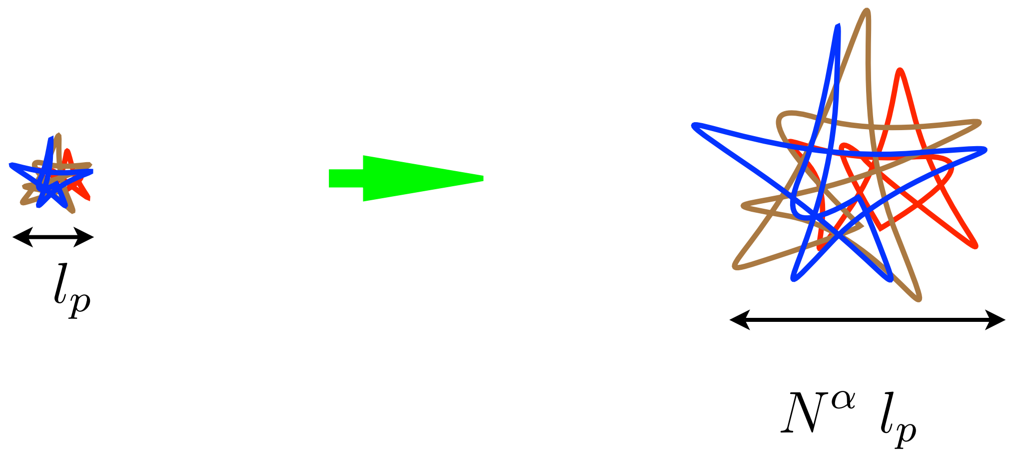
$$\otimes \quad |0\rangle_n |0\rangle_{n'} + |1\rangle_n |1\rangle_{n'}$$

Inequality shows that there is no way around this problem unless we have an order unity change to low energy evolution at the horizon

(B) The problem then is the 'no hair theorem' which prevented us from finding any significant change to the evolution at the horizon

(C) In String theory we can now make explicit constructions of the microstates of the black hole. It turns out that they do have 'hair'; in fact a horizon never forms

weak coupling

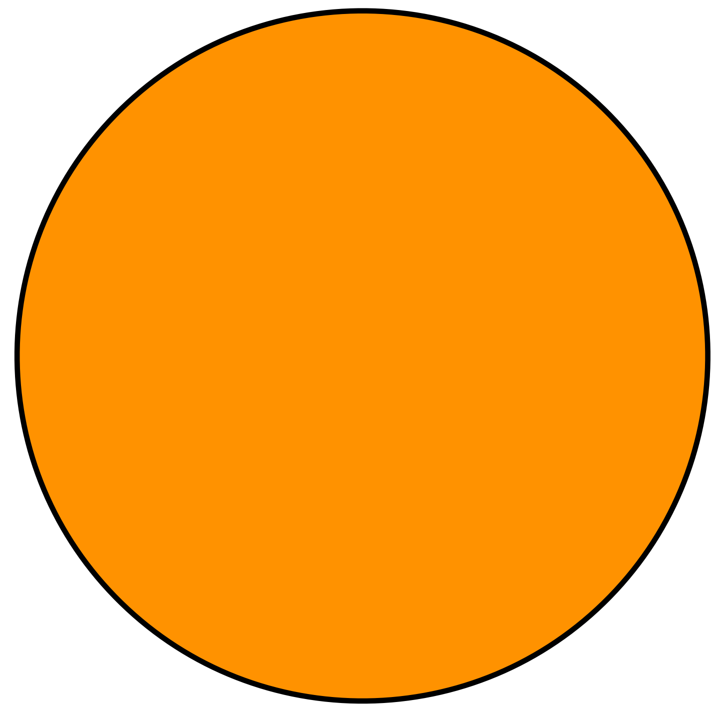


strong coupling

(D) One can then ask how semiclassical intuition failed.

The reason can be traced to the large entropy of gravitational states of the black hole, which made the measure in the path integral compete with the classical action

$$\boxed{Z = \int D[g] e^{-\frac{1}{\hbar} S[g]}}$$

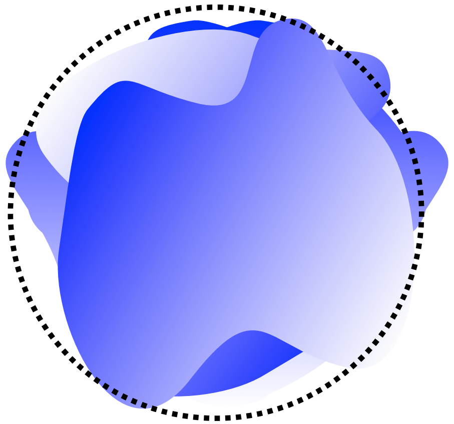


This should be a basic lesson for the behavior of quantum gravity in general, in all situations where we have a sufficiently large density of quanta ...

(D) One can then ask how semiclassical intuition failed.

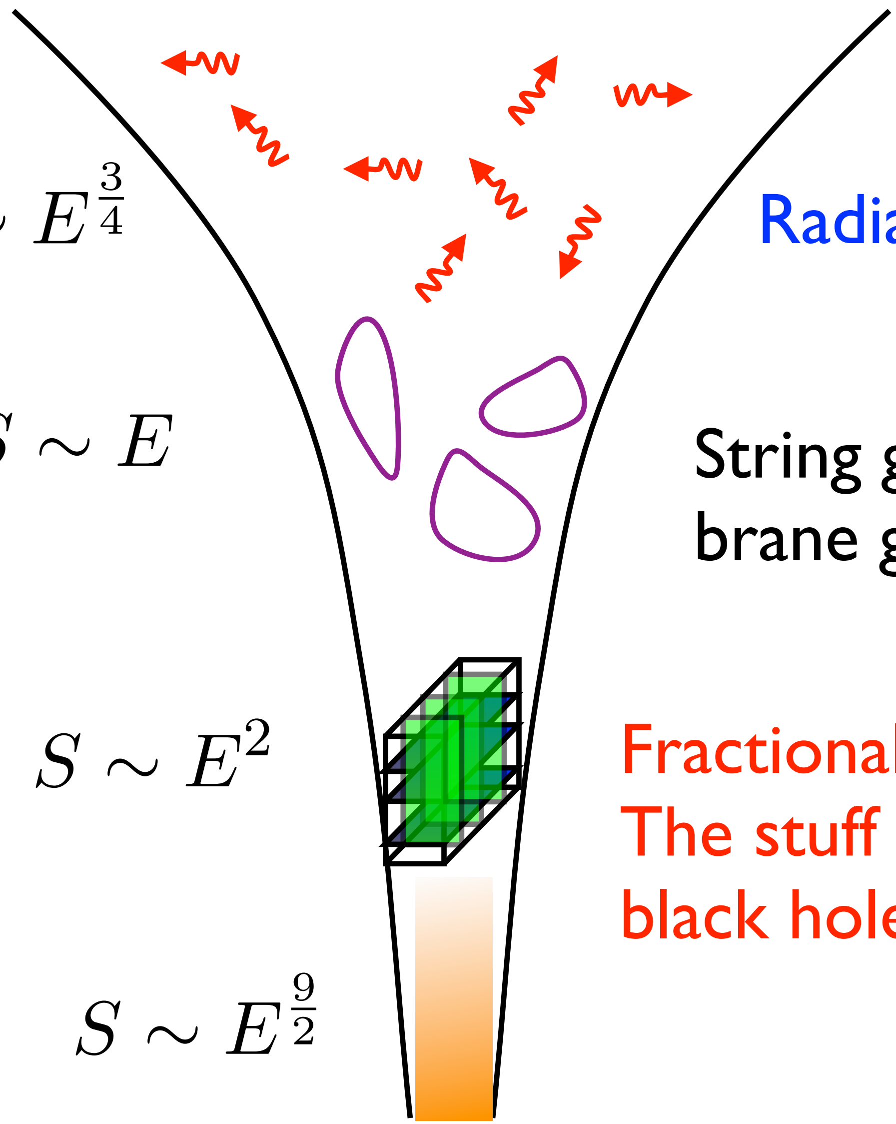
The reason can be traced to the large entropy of gravitational states of the black hole, which made the measure in the path integral compete with the classical action

$$\boxed{Z = \int D[g] e^{-\frac{1}{\hbar} S[g]}}$$



This should be a basic lesson for the behavior of quantum gravity in general, in all situations where we have a sufficiently large density of quanta ...

(E) Cosmology



$S \sim E^{\frac{3}{4}}$ Radiation

$S \sim E$ String gas/ brane gas ?

$S \sim E^2$ Fractional brane gas: The stuff inside black holes ⁇

$S \sim E^{\frac{9}{2}}$

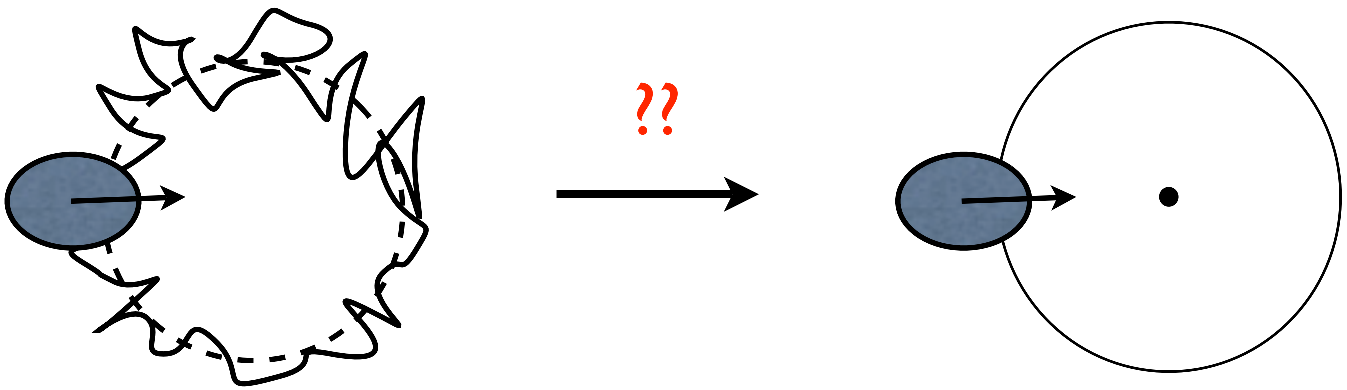
Matter is also crushed to high densities in the early Universe ...

So these lessons on phase space may radically change our picture of the dynamics there ...

The infall problem

What happens to an object (E >> kT) that falls into the black hole?

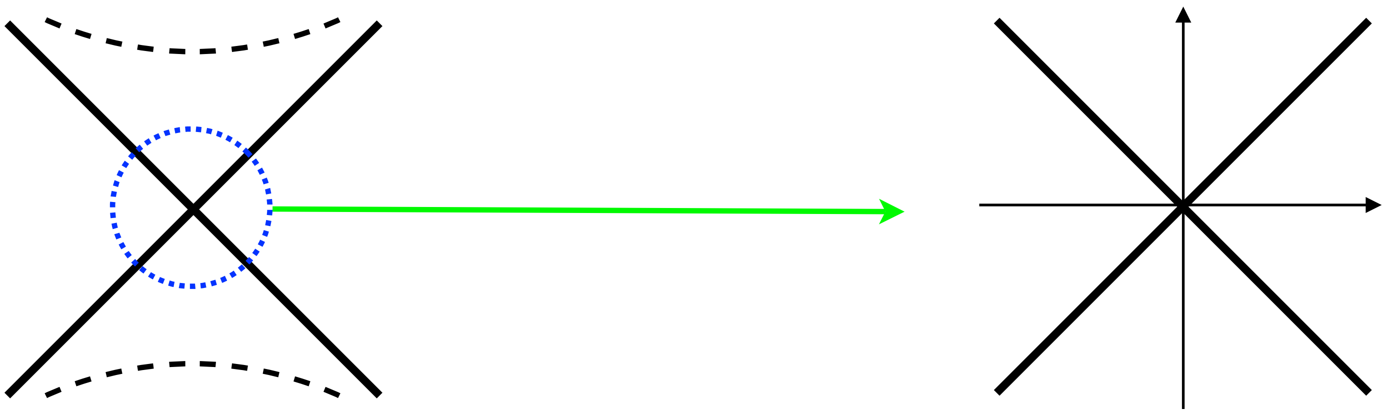
What does an infalling observer ‘feel’ ?



Low energy radiation modes are corrected by order unity, no information loss in process of creation

Is it possible that the dynamics of high energy infalling objects can approximated by the traditional black hole geometry in some way?

Now we know that black hole microstates are fuzzballs. let us see if we can do any better ...

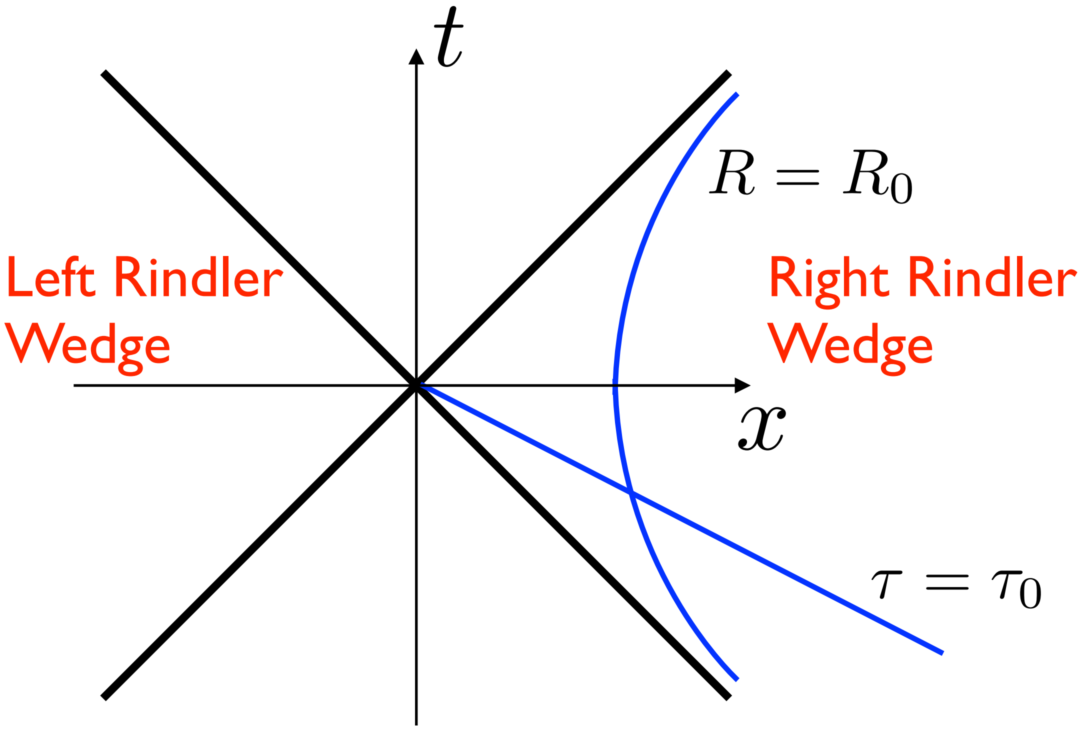


Central part of eternal black hole diagram looks like a piece of Minkowski spacetime, Horizons look like Rindler horizons

So complementarity looks as strange as asking that we get destroyed at a Rindler horizon, and in a dual description we continue past the horizon

Rindler space: Accelerated observers see a thermal bath

Minkowski spacetime



$$t = R \sinh \tau$$

$$x = R \cosh \tau$$

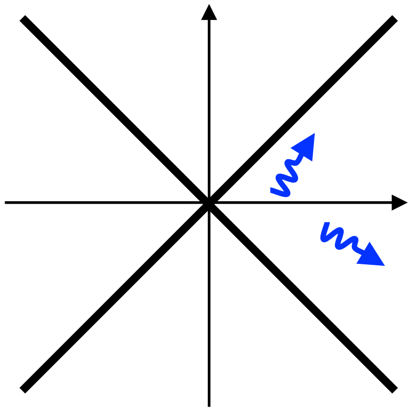
An observer moving along $R = R_0$ sees a temperature

$$T = \frac{1}{2\pi R_0}$$

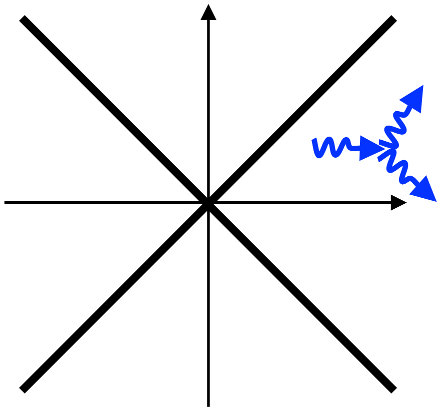
The Minkowski vacuum can be written an an entangled sum of Rindler states

$$|0\rangle_M = \sum_k e^{-\frac{E_k}{2\pi}} \, |E_k\rangle_L \; {}_R\langle E_k|$$

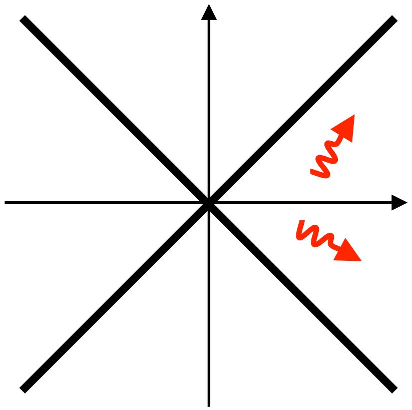
An observation



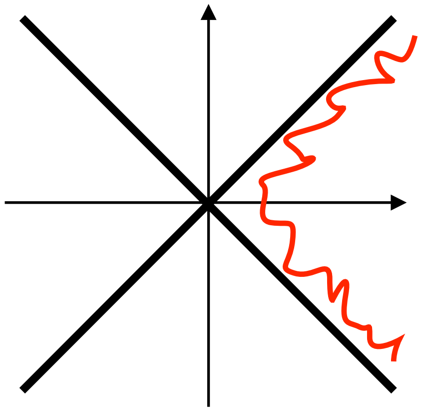
If there is a scalar field ϕ, then the Rindler states will have a bath of scalar quanta



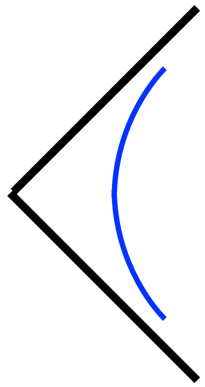
If ϕ has a ϕ^3 interaction, then this bath of scalar quanta will be interacting

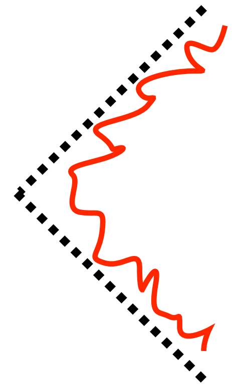


The graviton is a field that is always present, so we will have a bath of (interacting) gravitons



Thus expect fully nonlinear quantum gravity near Rindler horizon



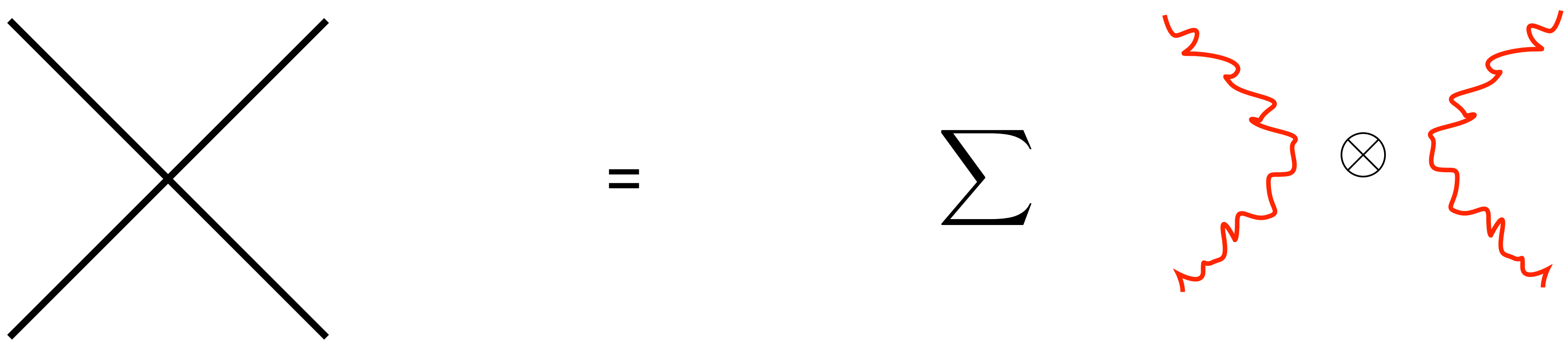


Rindler coordinates 'end' at the boundary of the wedge

Thus it is logical to expect that the gravity solution for Rindler states should also 'end'

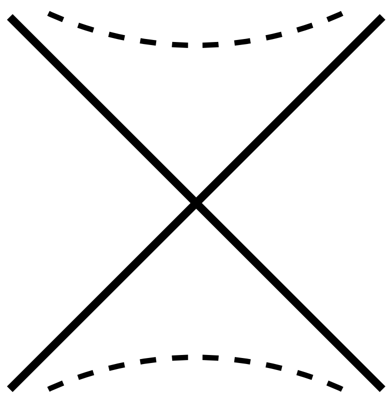
But this is exactly what fuzzball microstates do !

Thus we expect :

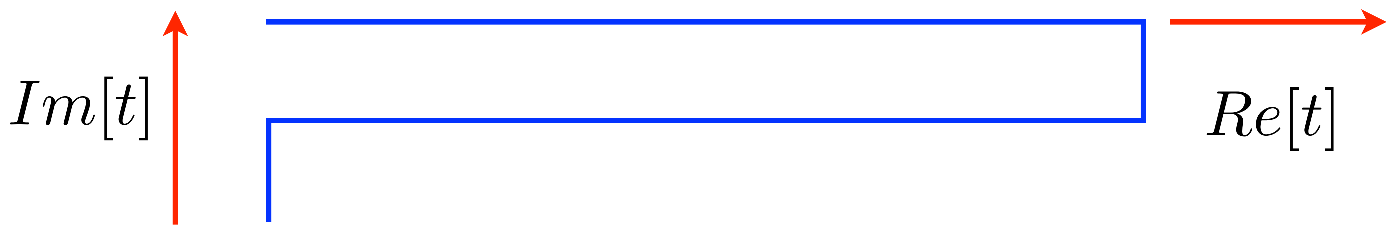


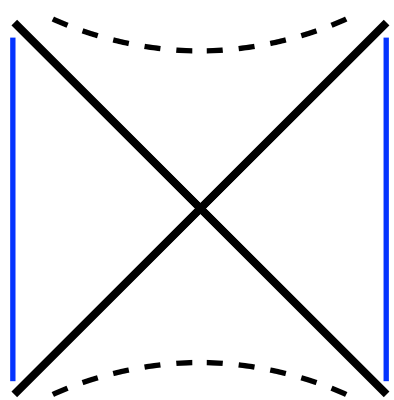
$$|0\rangle_M = \sum_k e^{-\frac{E_k}{2\pi}} \, |E_k\rangle_L \; {}_R\langle E_k|$$

Black Holes :

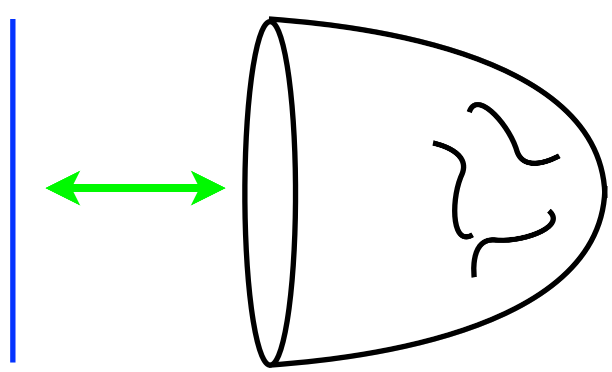


Israel (1976): The two sides of the eternal black hole are the two entangled copies of a thermal system in thermo-field-dynamics



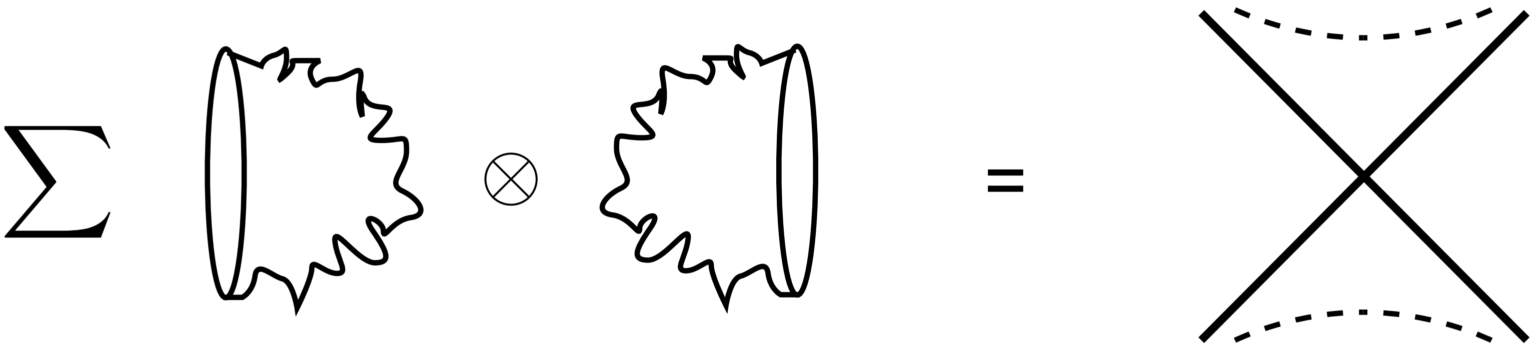


Maldacena (2001): This implies that the dual to the eternal black hole is two entangled copies of a CFT



Van Raamsdonk (2009): CFT states are dual to gravity solutions ... so we should be able to write an entangled sum of CFT states as an entangled sum of gravity states ...

Thus we can expect that summing over fuzzball microstates will generate the eternal black hole spacetime



(SDM + Plumberg 2011)

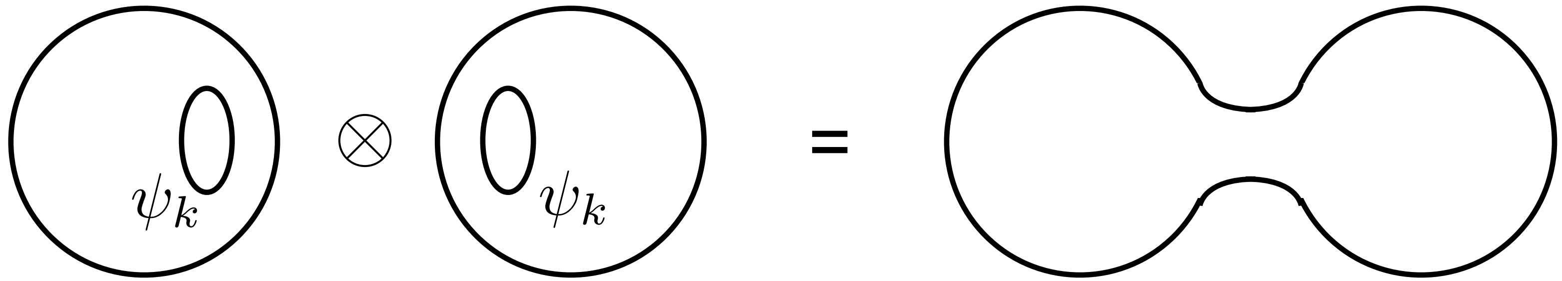
The fuzzball microstates do not have horizons, but the eternal black hole spacetime does ...

Is it reasonable to expect that sums over (disconnected) gravitational solutions can be a different (connected) gravitational solution ?

Something like this happens in 2-d Euclidean CFT ...

‘Sewing’ process in CFT

$$\sum_k e^{-\tau h_k - \bar{\tau}\bar{h}_k}$$



(a) Low energy dynamics $(E \sim kT)$

No horizon, radiation from ergoregions, so radiation like that from any warm body

no information loss since radiation depends on choice of microstate ψ_k

(b) Correlators in high energy infalling frame $(E \gg kT)$

$$\langle\psi_k|\hat{O}_1\hat{O}_2|\psi_k\rangle \approx \sum_m e^{-\beta E_m}\langle\psi_m|\hat{O}_1\hat{O}_2|\psi_m\rangle$$

for generic states ψ_k