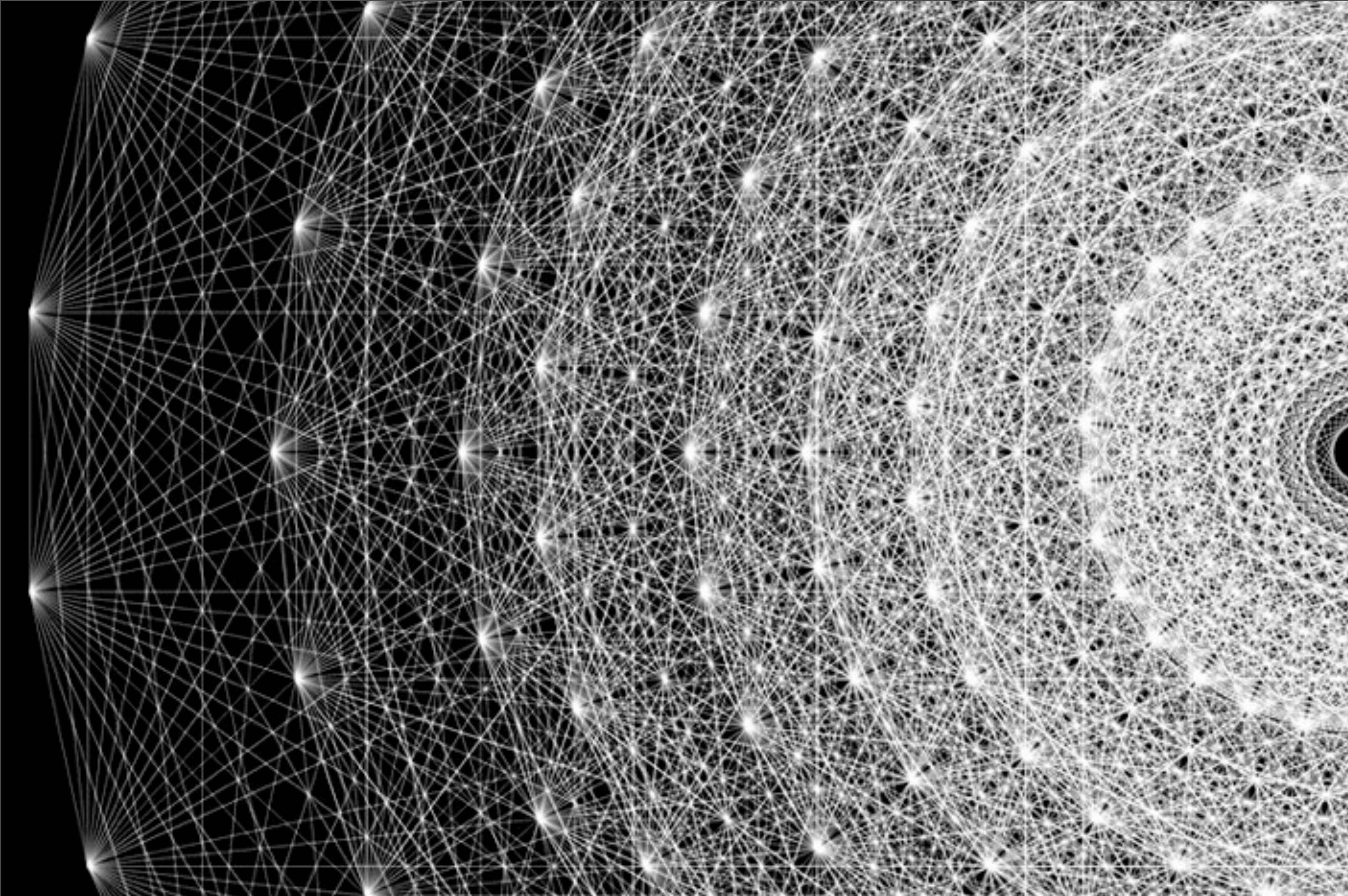
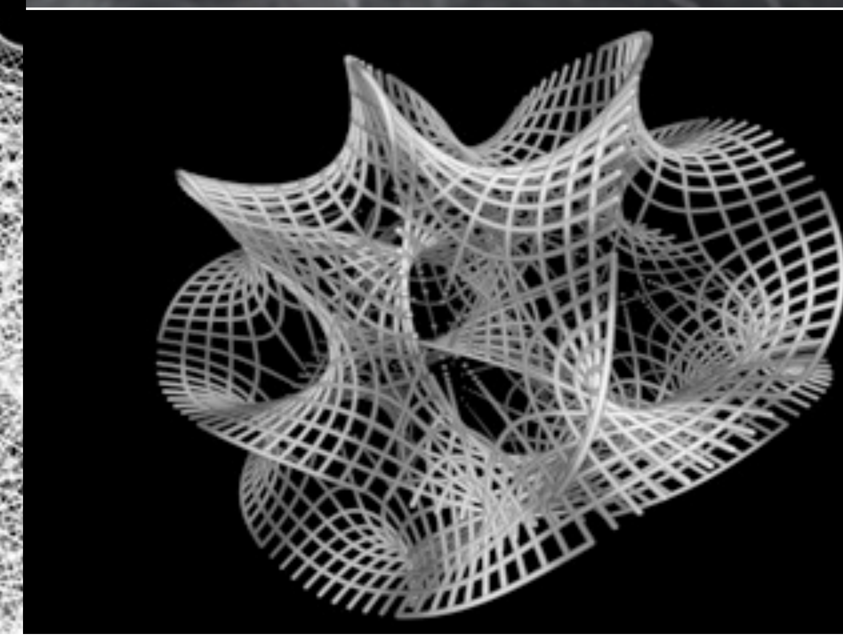




An algorithmic approach to string compactification

Lara B. Anderson

Virginia Tech

March 25th, 2015

Work in this done in collaboration with:

Model Building:

- J. Gray, A. Lukas and E. Palti: arXiv:1106.4804, 1202.1757, 1307.487, 15??.?????

- A. Constantin, S.J. Lee and A. Lukas: arXiv:1411.0034

Moduli Stabilization:

- J. Gray, A. Lukas and B. Ovrut: arXiv:1010.0255, 1102.0111, 1107.5076,1304.2704

What this talk is about...

- String theory is a powerful extension of quantum field theory, but extracting low-energy physics from string geometry is mathematically challenging...

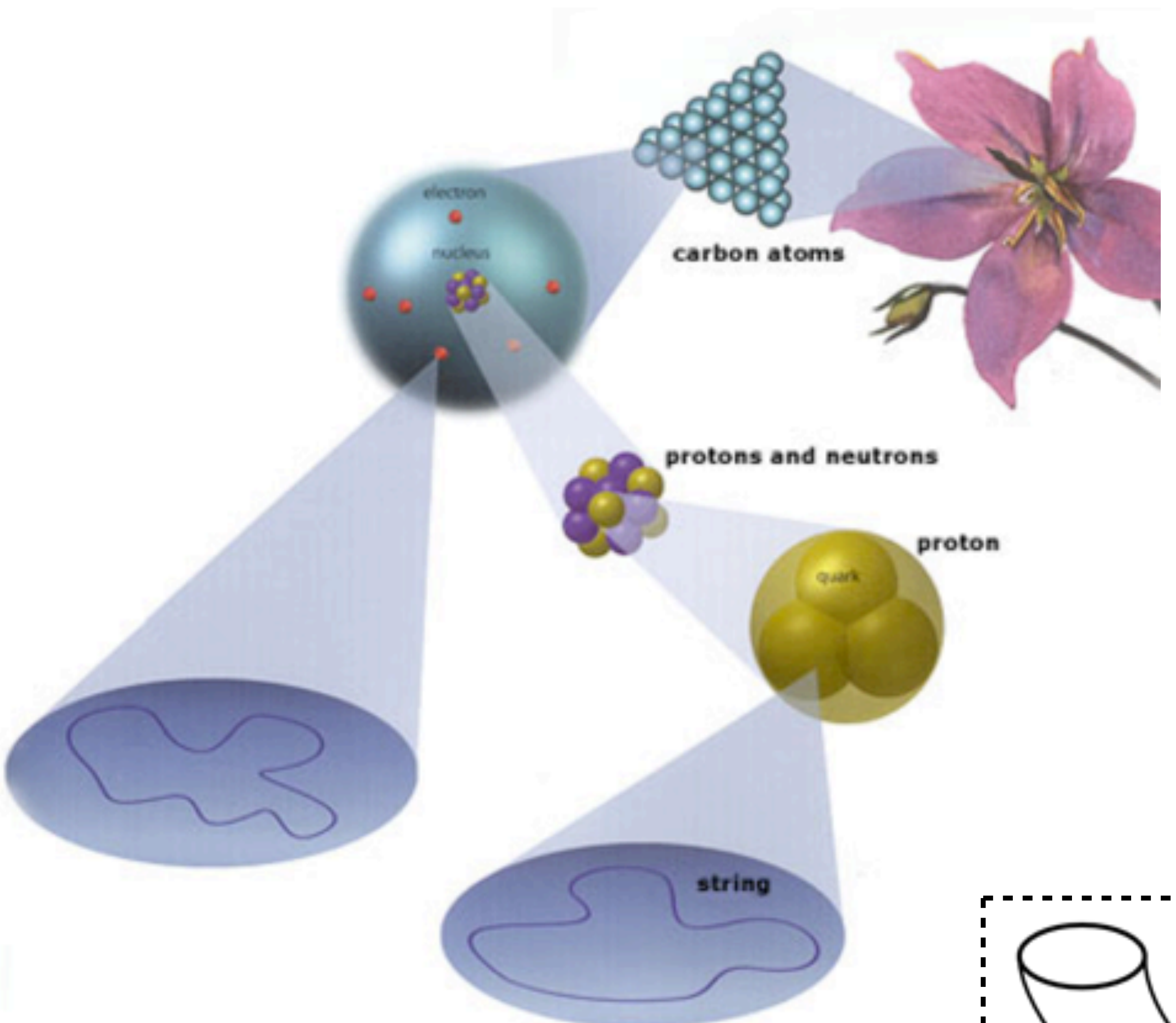
- Need a good toolkit in any corner of string theory to extract the full low energy physics: (missing structure in the low-energy lagrangian, couplings, unphysical massless scalars (moduli), etc)

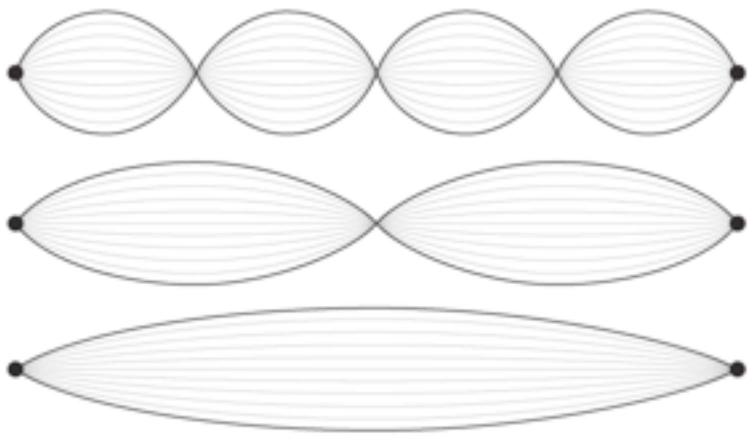
- Rules for “top down” model building? Patterns/Constraints/Predictions?

- An algorithmic approach: Rather than attempting to engineer/tune a single model, can we develop general techniques? Produce a large number (i.e. billions) of consistent global models and then scan for desired properties? Identify patterns?

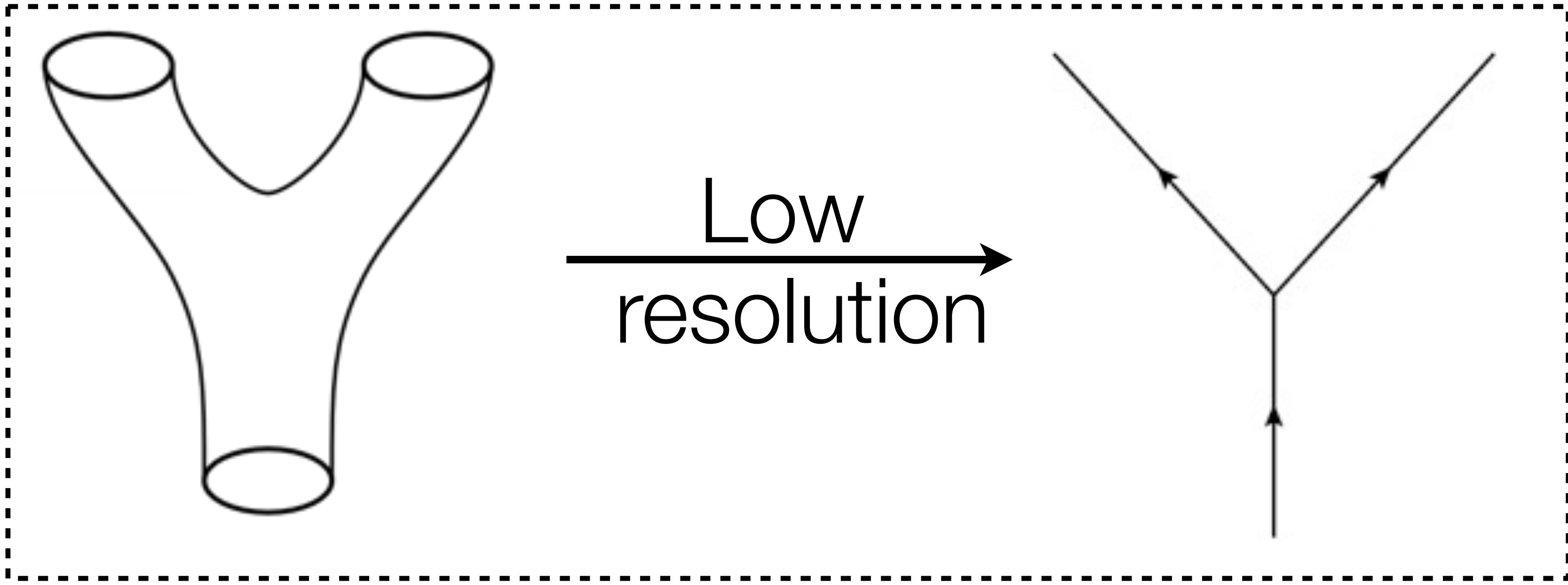
- I’ll describe the state of the art: what works and what doesn’t yet

String Theory... The basic idea



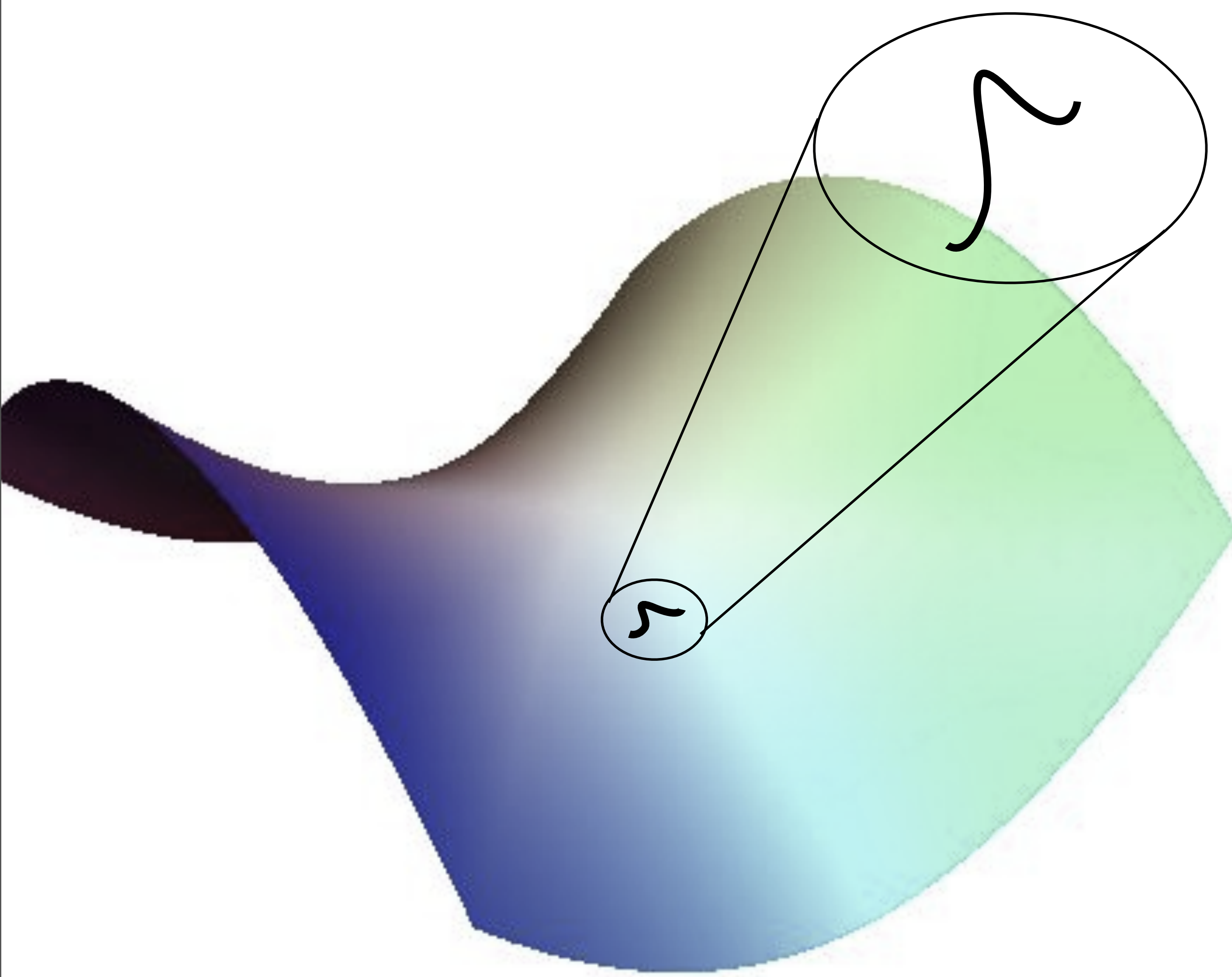


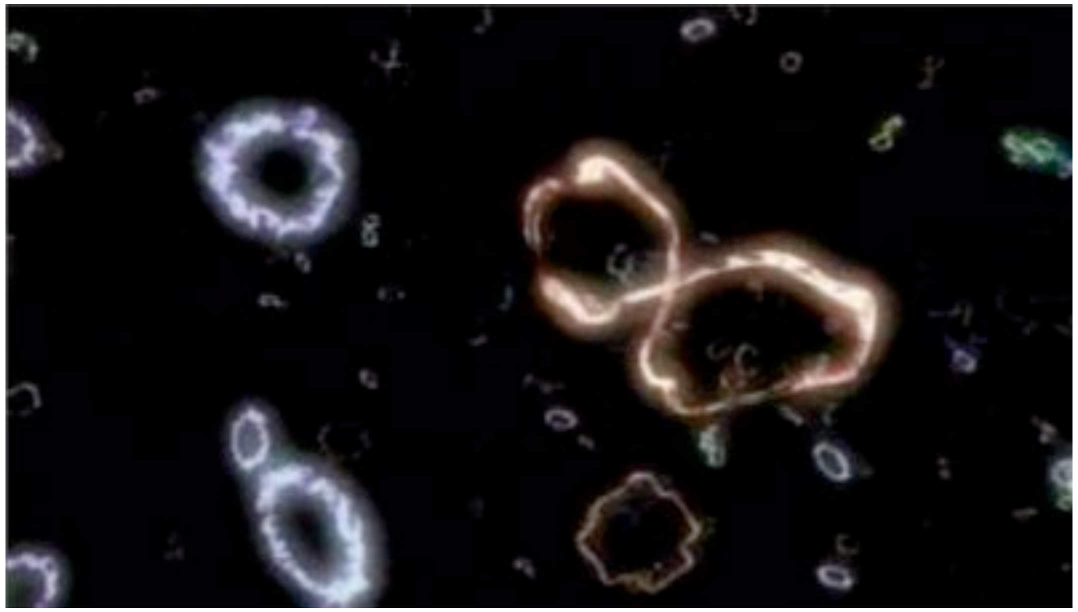
Strings -> QFT



What's the big idea? String Theory and Gravity

- A quantum particle can move in any kind of space and obey many kinds of ``rules'' for gravity





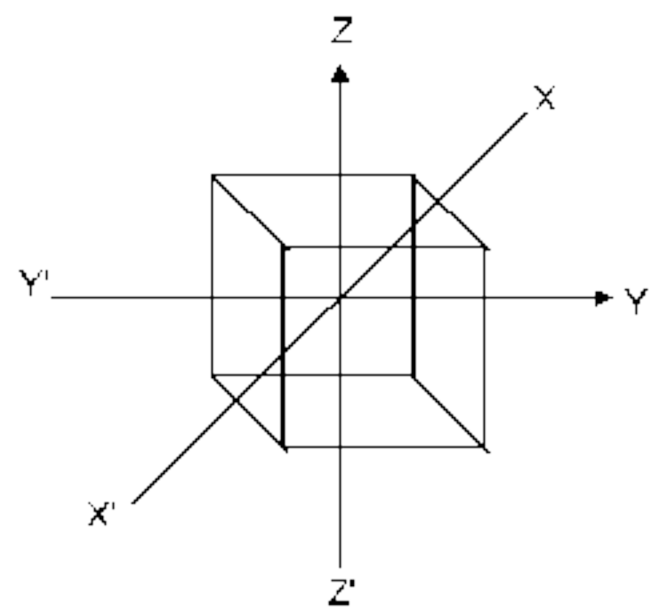
Cool Fact:

A quantum "string" can only move in a space that obeys Einstein's Theory of Gravity

Quantum gravity is consistent and compulsory in string theory.

However, it comes at a price...

Two Big "ifs"...

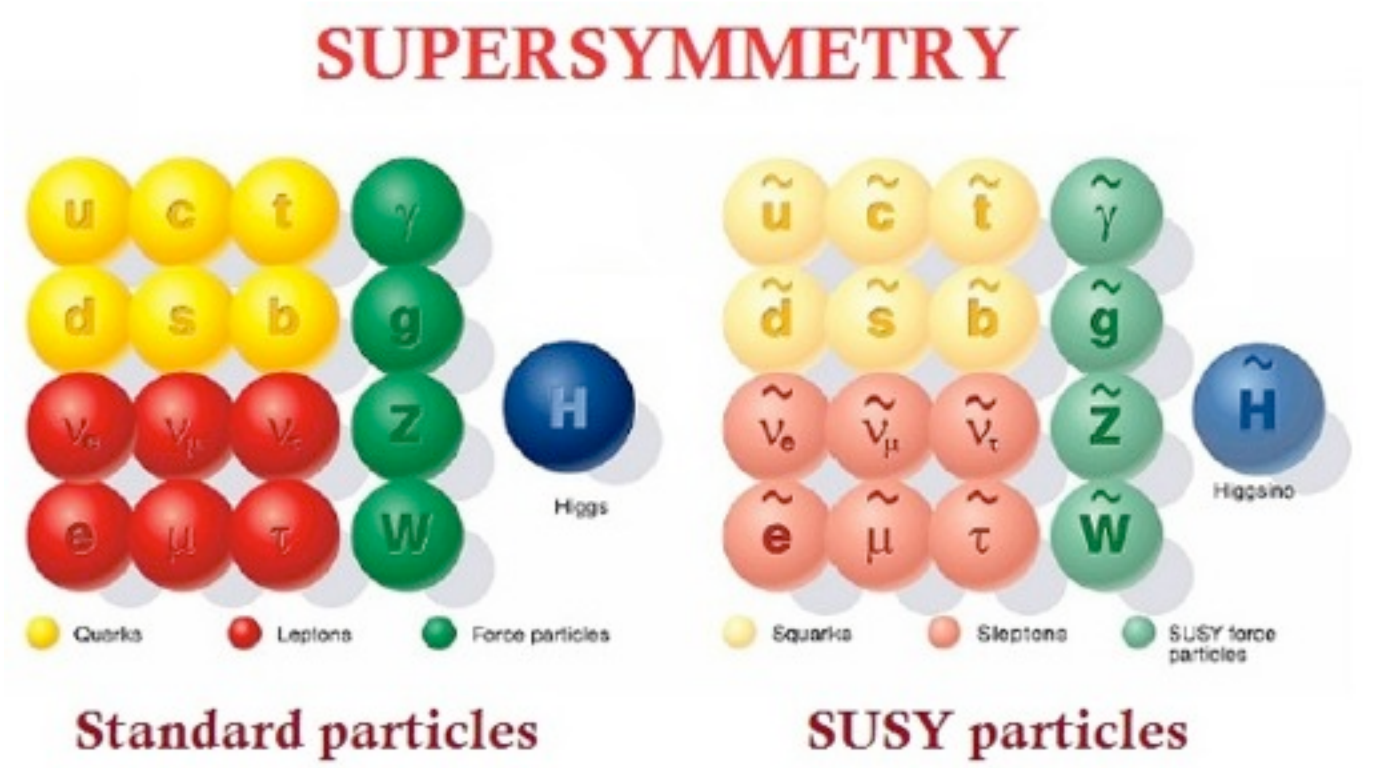


+???

1) It only works if the universe has more than the 3 dimensions of space and 1 of time that we see around us....

In fact, a lot more... String theory works when there are 10 (or 11, 12) dimensions

2) Theory is only free of tachyons if it has more than the Standard Model particles...Only consistent if



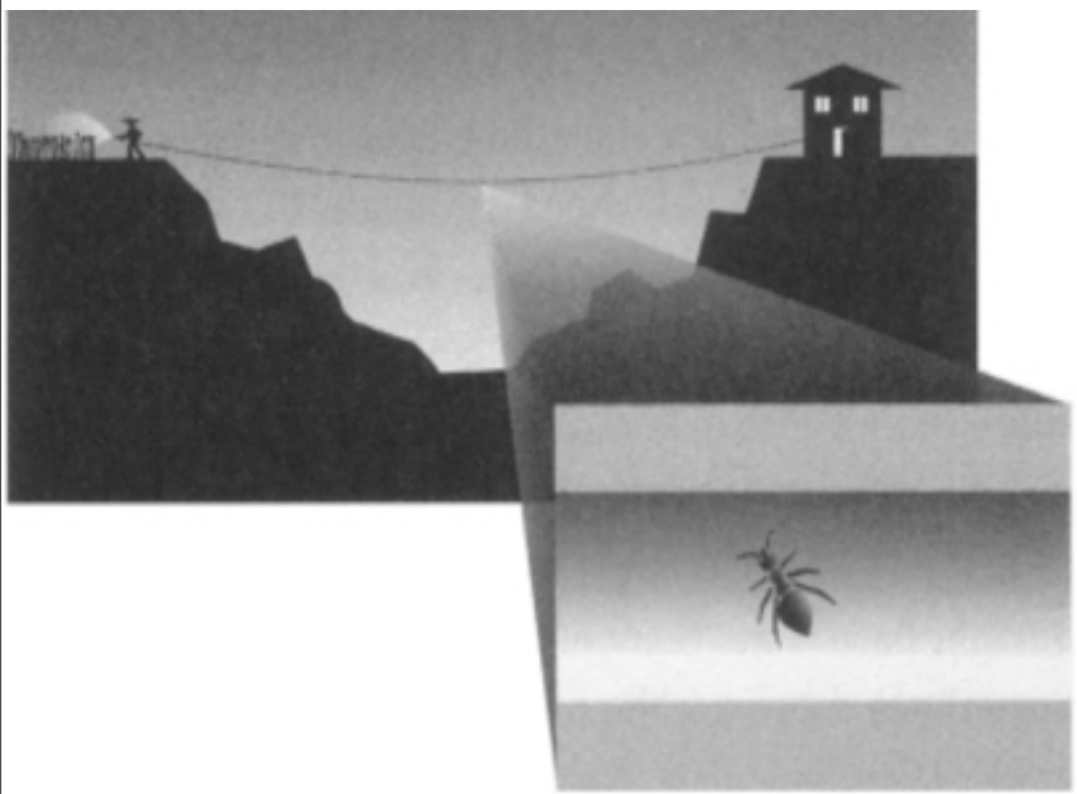
Supersymmetry

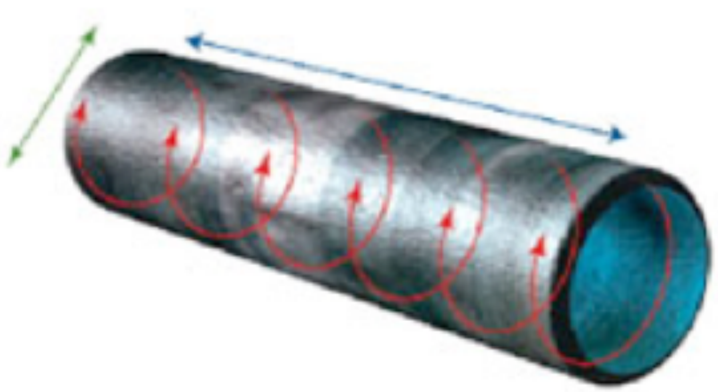
exists (at some scale)

First, what to do with higher dimensions?

Compactification

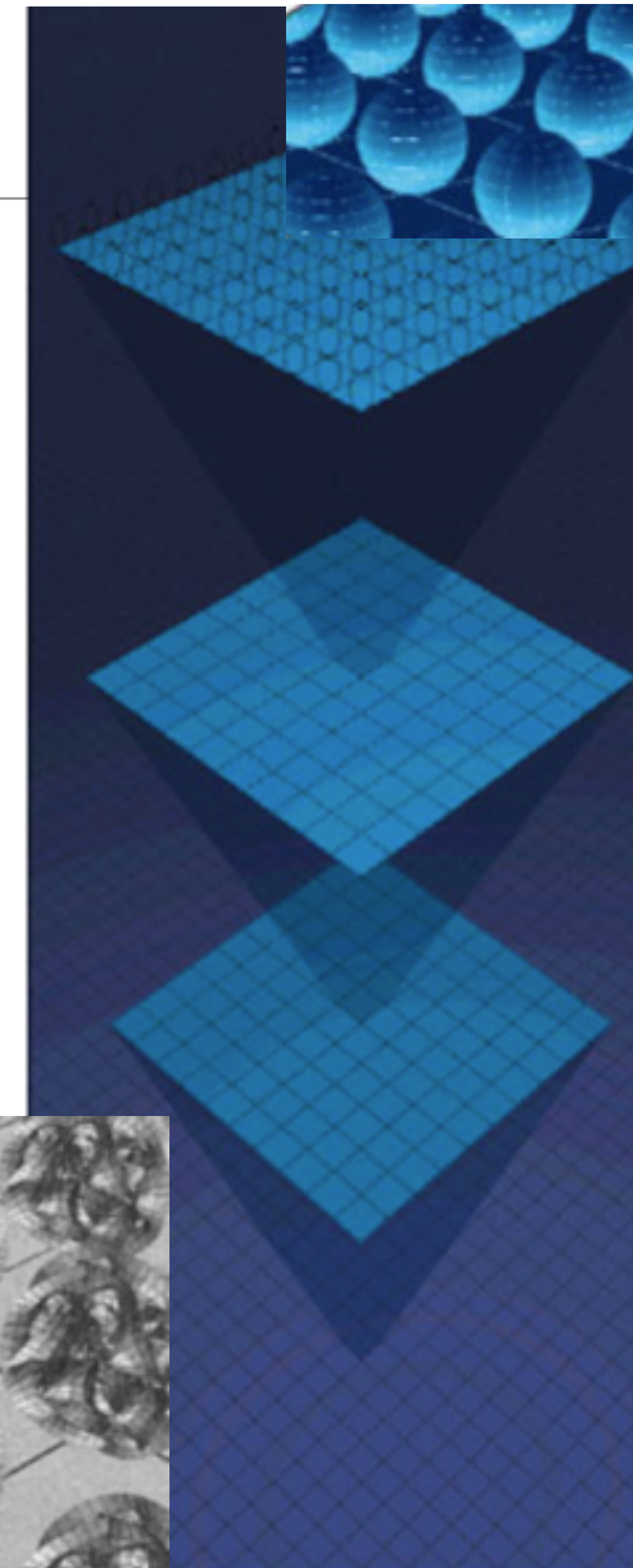
The idea...



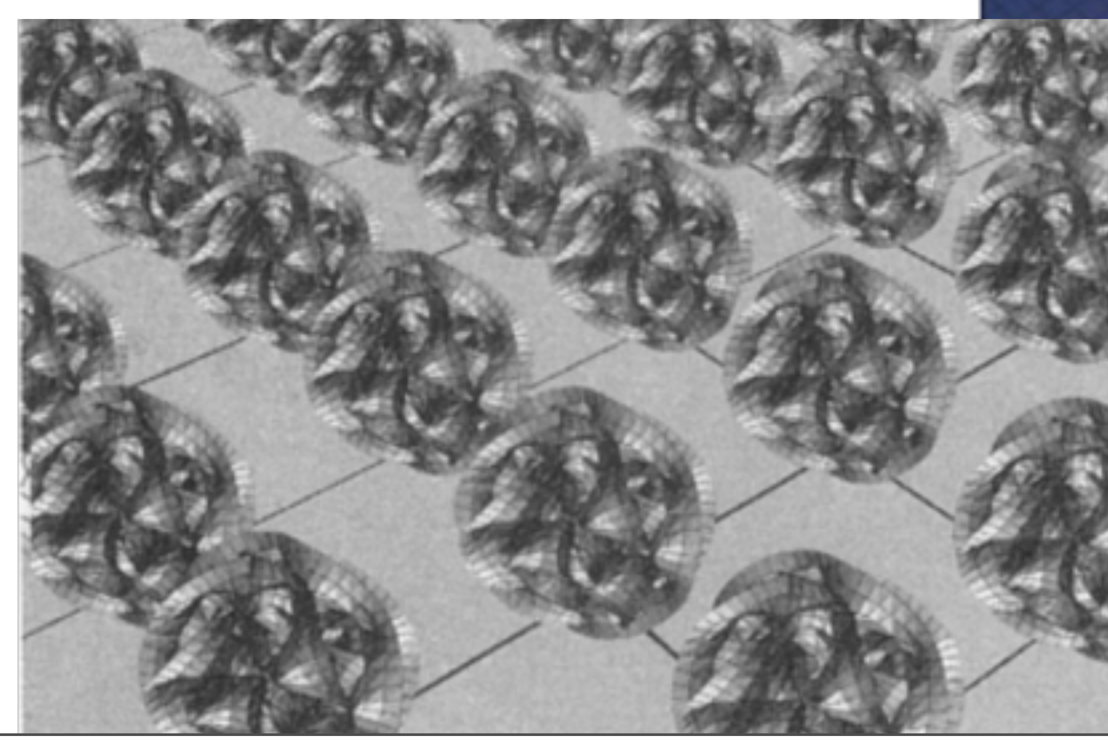


The “Shape” of extra dimensions could be

Simple...



Or complicated...



What this talk is about:
Laws of physics change with extra dimensions...

String theory theory predicts extra dimensions, we need to figure out:

- What kinds/shapes of extra dimensions are allowed in string theory?
- How do these different geometries change the laws of physics and particles we would see in the 4D world around us?
- Which geometries agree with the physics we already know? And what other particles/physics do they predict to exist?

This area of investigation is called “String Phenomenology”

- But before we look at this in detail, we need to encode a bit more information in this “geometry”...

Symmetries:
U(1) Gauge Theory

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial B}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

can be written more compactly... $\longrightarrow$

(F "Harmonic")

$$dF = 0$$

$$d * F = 0$$

$$F = dA$$

$$A = A_\mu dx^\mu, \quad A_\mu = (-\phi, \mathbf{A})$$

Can redefine the potentials

$$\mathbf{A}' = \mathbf{A} + \nabla\lambda$$

$$\phi' = \phi - \frac{\partial \lambda}{\partial t}$$

$\lambda(\mathbf{x}, t)$ gauge freedom drops out of all the equations

Gauge symmetry of the equations can be used to characterize the equations themselves!

Introduction to Vector Bundles

- Symmetries of physical laws can be encoded in “geometry” in the form of a Vector Bundle

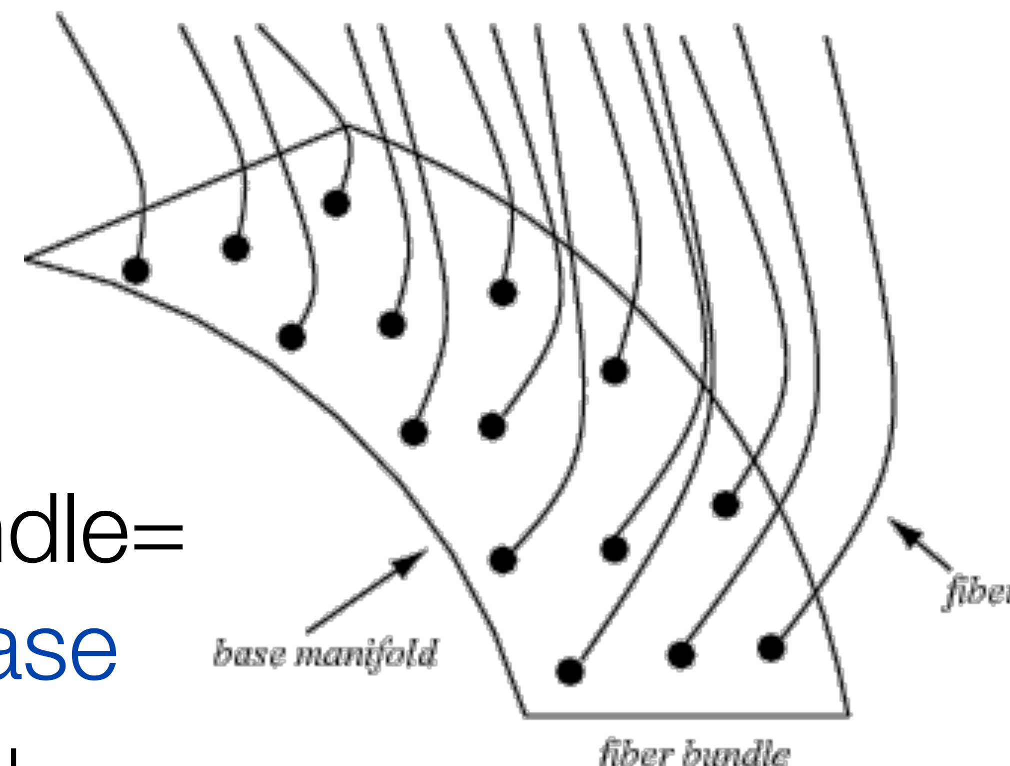
- Vector bundles “keep track” at each point in a space of whatever information you need

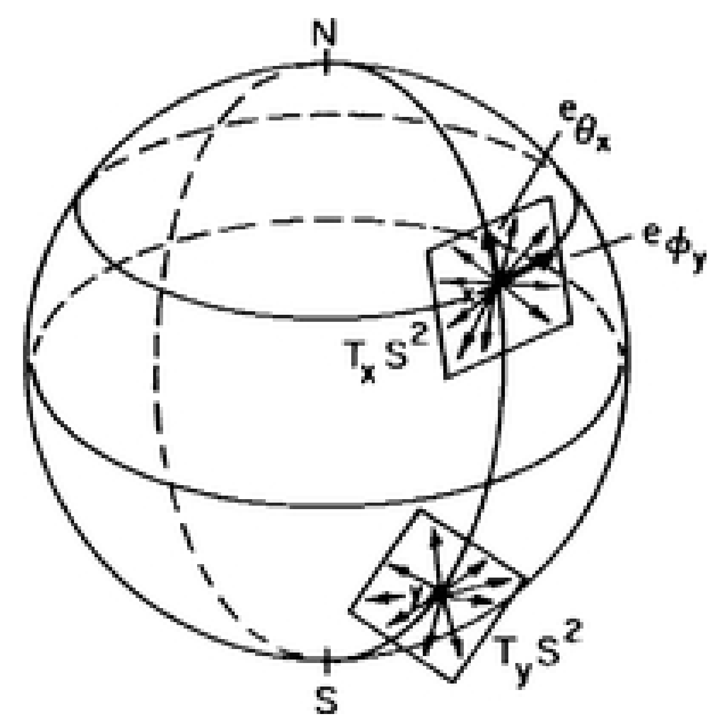
- Examples: The Temperature or Windspeed at every point in a room...

- Example: The tangent bundle keeps track of the tangent vector space at each point

- Example: U(1) gauge freedom

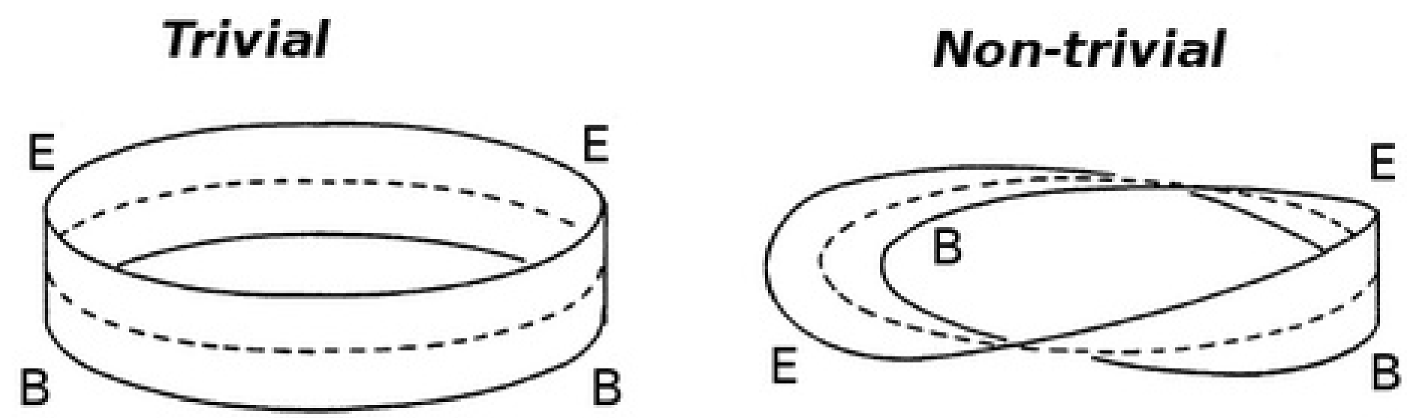
Bundle=
Base
+
Fiber



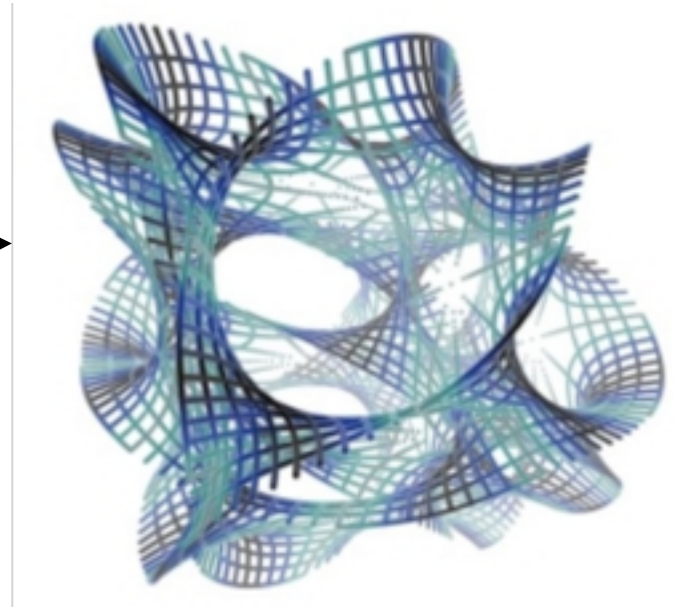


Base space and fiber geometry constrain one another...

Neat fact:
The "shape" of the underlying space and the shape of the fibers constrain one another



SM Symmetry:
SU(3) x SU(2) x U(1)?



Can use this to figure out...

a) What gauge theories can arise when we have compact directions?

b) How can the vector bundles constrain the size/ shape of those new directions?

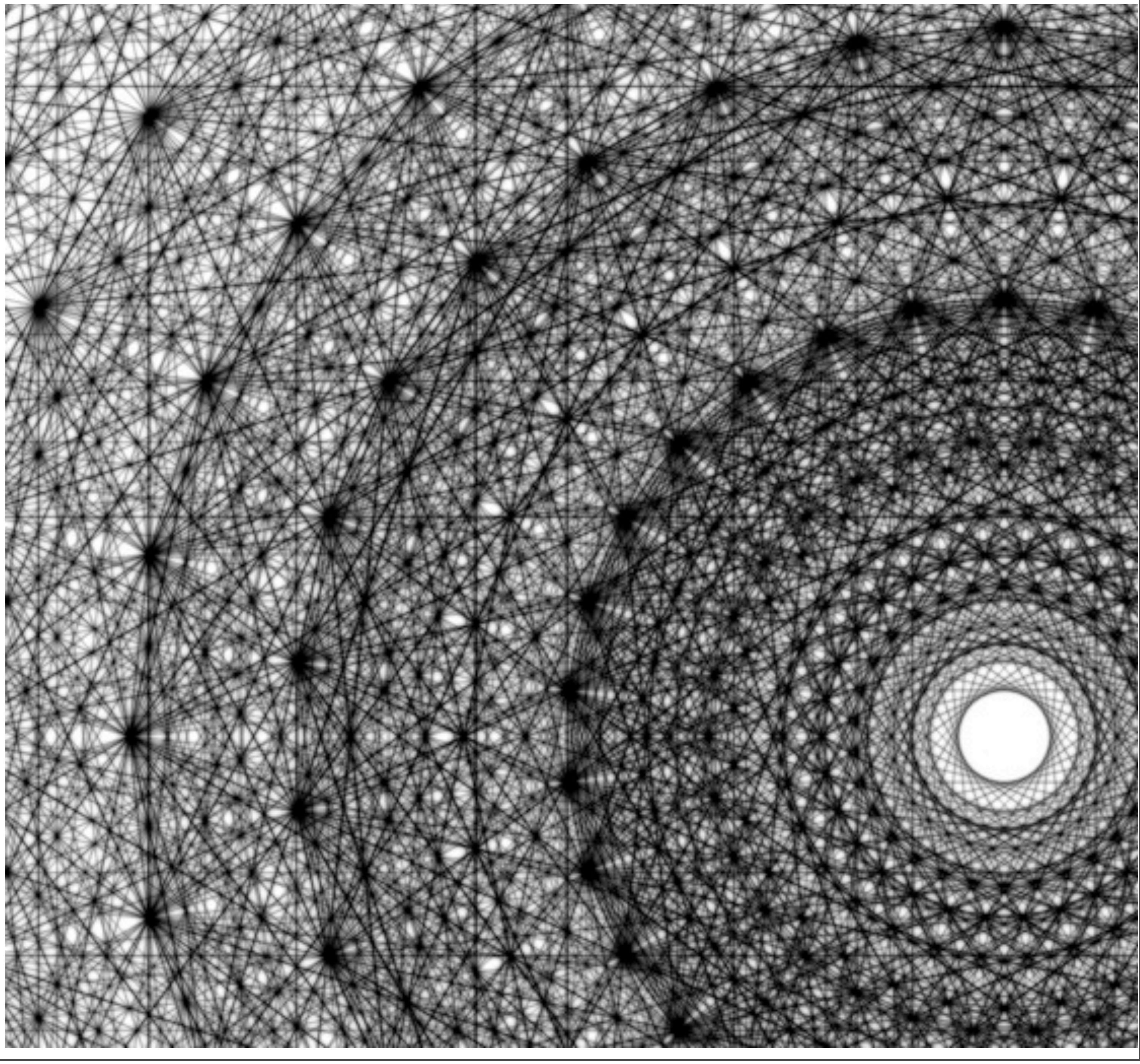
what bundles?

What symmetries/bundles arise in string theory? $E_8 \times E_8$ Heterotic String Theory...

- Comes equipped with vector bundles in 10 dimensions. The Symmetry that describes the higher dimensional physics is called "E_8". It has 248 dimensions and a rich and complicated structure!

- This bundle/symmetry can be broken into parts that live over the the compact space, X, and parts that are ``left over" in our 4-dimensions

- The Hard Part: Choose the bundle on the compact directions to "leave behind" a piece of E_8 that gives the symmetry and particles of the Standard Model in our world



Dimensional Reduction

- Start in 10D:

$$S_{het} = \frac{1}{2\kappa^2}\int d^{10}x(-G)^{1/2}[R - \frac{\kappa^2}{4g^2\phi}Tr(\mid F\mid^2) + \ldots]$$

- Look solutions of the form: $\mathbb{R}^{1,3} \times X_6$

- "Integrate out" The dependence on X --> 4D Effective Field theory!

- E.O.M. (for the simplest class of solutions):

X_6 is a complex manifold w/
SU(3) Holonomy

$\updownarrow$

Tr(R)=0 (Ricci Flat, Kahler)

$\updownarrow$

A Calabi-Yau Manifold

Introducing complex coordinates (a,b=1,2,3) on X...

Hermitian Yang-Mills Eqs:

$$g^{a\bar{b}}F_{a\bar{b}} = 0$$

$$F_{ab} = F_{\bar{a}\bar{b}} = 0$$

The origin of matter...

All charged matter in the 4D theory must come from the 10D gauge field (Adjoint-valued=**248**). Let A,B=0,..9

4D Gauge fields $$A^B_{10D} = (\boxed{A^\mu_{4D}}, A^a_{6D})$$

4D Scalar fields $$A_{\bar{a}} = A^0_{\bar{a}} + \boxed{\delta C_x} T^{xi}\omega_{i\bar{a}}$$

where T^{xi} are structure constants of $G \times H \subset E_8$

For 4D (massless modes): $$\bar{D}\omega = \bar{D} * \omega = 0$$

(harmonic H-valued 1-forms on X)

(We're solving a V-twisted Dirac Eq on X):

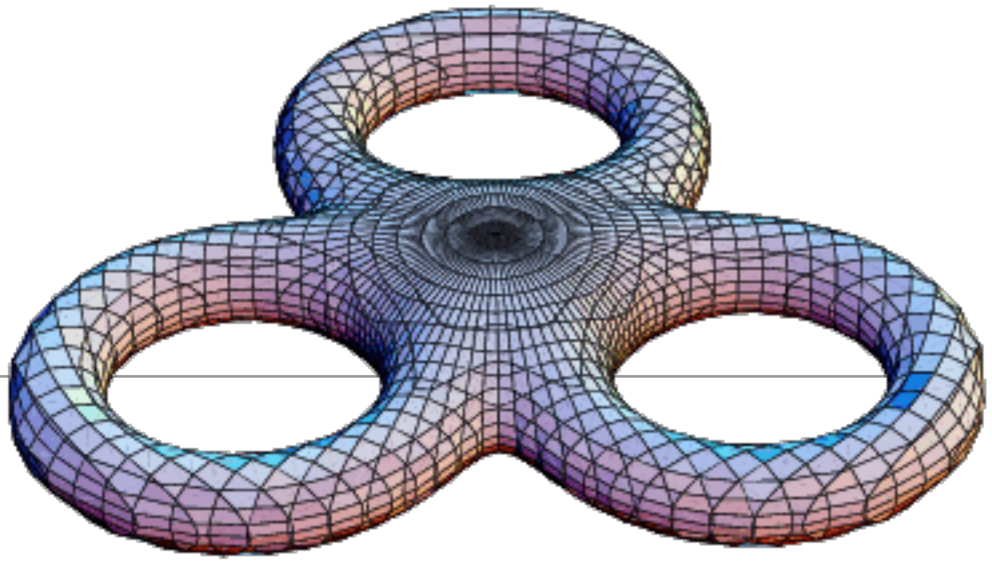
$$\not{\nabla}_X \Psi = 0$$

$$F = F^0 + 2\bar{D}\delta A$$

(So a harmonic perturbation doesn't change the E.O.M --> Flat directions in the potential)

Gauge field vevs valued in H, break the 4D symmetry to G

Matter + Moduli



E.g. $E_8 \to SU(5) \times SU(5)$

$$\mathbf{248}_{E_8} \to [(\mathbf{1},\mathbf{24}) \oplus (\mathbf{5},\overline{\mathbf{10}}) \oplus (\bar{\mathbf{5}},\mathbf{10}) \oplus (\mathbf{10},\mathbf{5}) \oplus (\overline{\mathbf{10}},\bar{\mathbf{5}}) \oplus (\mathbf{24},\mathbf{1})]_{\mathbf{SU(5)\times SU(5)}}$$

This tells the type of matter that could exist in 4D, but not how much of it there is?...(i.e. what multiplicity? $\longleftrightarrow$ How many harmonic forms in the basis?

$$(\delta C_\rho \omega^\rho)$$

The space of all such "closed but not exact" 1-forms is called a Cohomology Group, $H^1(X,V)$

All these cohomology groups are finite, and we need to calculate different groups for different representations of the the 4D symmetry group, G:

SU(5)-Charged Matter	$n_{10} = h^1(V), n_{\overline{10}} = h^1(V^*), n_5 = h^1(\wedge^2 V^*), n_{\bar{5}} = h^1(\wedge^2 V)$
Moduli (n_1)	$X \Rightarrow h^{1,1}(X)$ Kähler (size), $h^{2,1}(X)$ Complex Struct.(shape) $V \Rightarrow h^1(X, V \otimes V^*)$ (Bundle moduli)

The 4D potential...

We can see schematically the form of the 4D scalar potential

$$S_{partial} \sim \int_{M_{10}} \sqrt{-g}\{(F_{a\bar{b}}g^{a\bar{b}})^2 + (F_{ab}F_{\overline{a}\overline{b}}g^{a\overline{a}}g^{a\overline{a}})\} + \ldots$$

Very hard to analyze directly since we don't know (g,F) analytically! But we'll come back to this...

What do we have so far?...

A compact CY manifold X and a vector bundle V on it can reduce the 10D E_8 x E_8 heterotic string theory to $G \subset E_8$

1) A Yang-Mills theory coupled to gravity in 4D with symmetry group H (commutant of $G \subset E_8$

2) All the matter in the 4D theory comes from (g, A) in 10D. Number of fields is fixed by the number of harmonic forms (i.e. cohomology groups) on X

3) Can we get the Standard Model? $SU(3) \times SU(2) \times U(1)$ +3 families of quarks/leptons?

SUSY GUTs

We are lead naturally to SUSY GUTS (An idea that pre-dates string theory)

GUTs are 4D theories that unify the strong, weak and electromagnetic forces into a single more fundamental interaction at high energies. E.g.'s: $E_6, SO(10), SU(5) \supset SU(3) \times SU(2) \times U(1)$

Some Successes	Some Problems
Explanation of the relative strengths of the fundamental forces	Problematic relationships enforced between Yukawa couplings
Explanation of the quantum numbers of the particles of the standard model	"Doublet-triplet splitting" leads to issues with proton decay

SU(5) for example...

$$\begin{pmatrix} & & \\ & SU(5) & \\ & & \end{pmatrix} \supset \begin{pmatrix} \begin{pmatrix} SU(3) \end{pmatrix} & \\ & \begin{pmatrix} SU(2) \end{pmatrix} \end{pmatrix}$$

$$+ \quad \mathrm{diag}(e^{2i\psi}, e^{2i\psi}, e^{2i\psi}, e^{-3i\psi}, e^{-3i\psi})$$

$$\underbrace{}_{(e,\nu)_L} \qquad \underbrace{}_{\overline{d}_L}$$

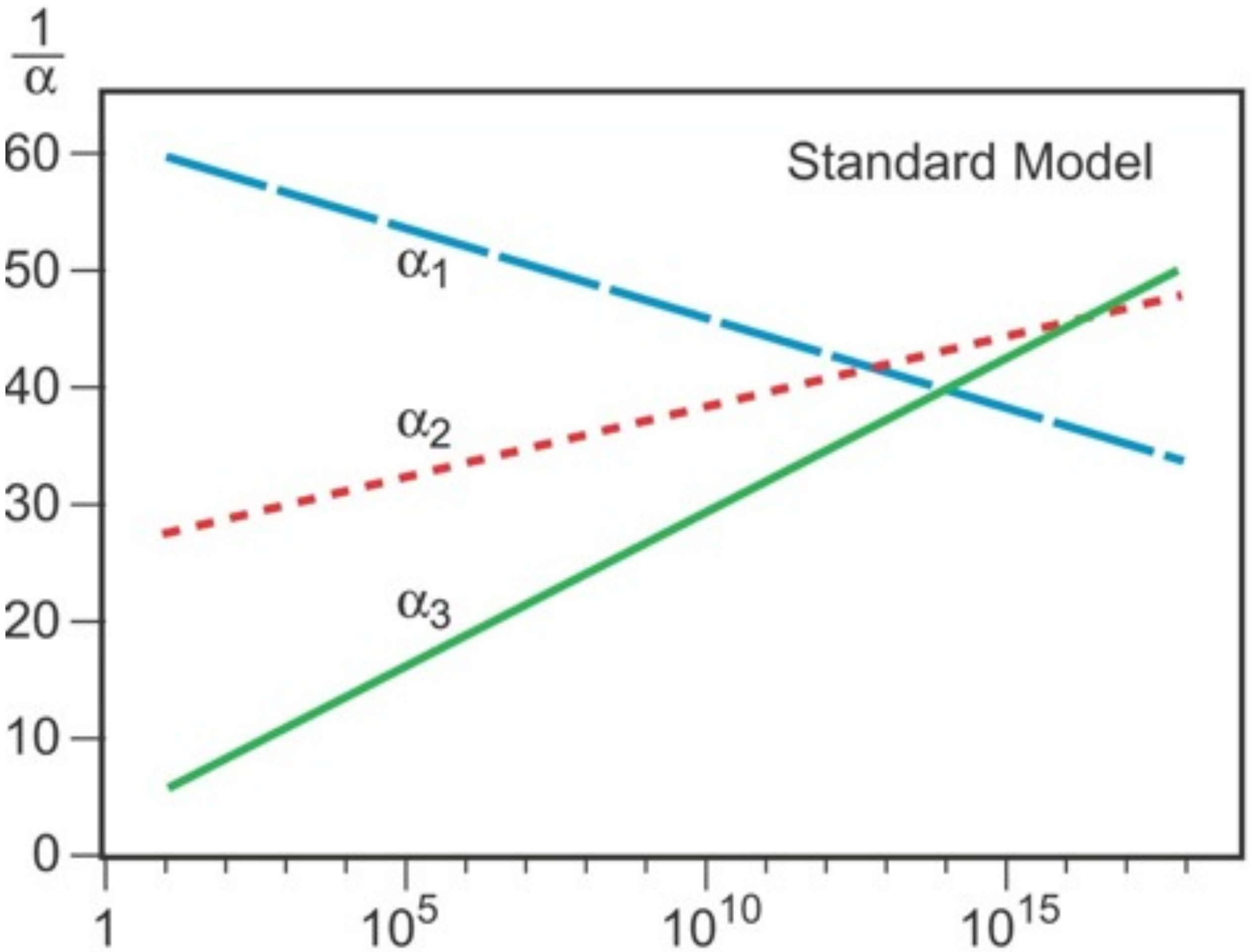
$$\overline{\mathbf{5}} = (\mathbf{2}, \mathbf{1})_{-1} + (\mathbf{1}, \overline{\mathbf{3}})_{2/3}$$

$$\overline{u}_L \qquad e_L^+ \qquad (u,d)_L$$

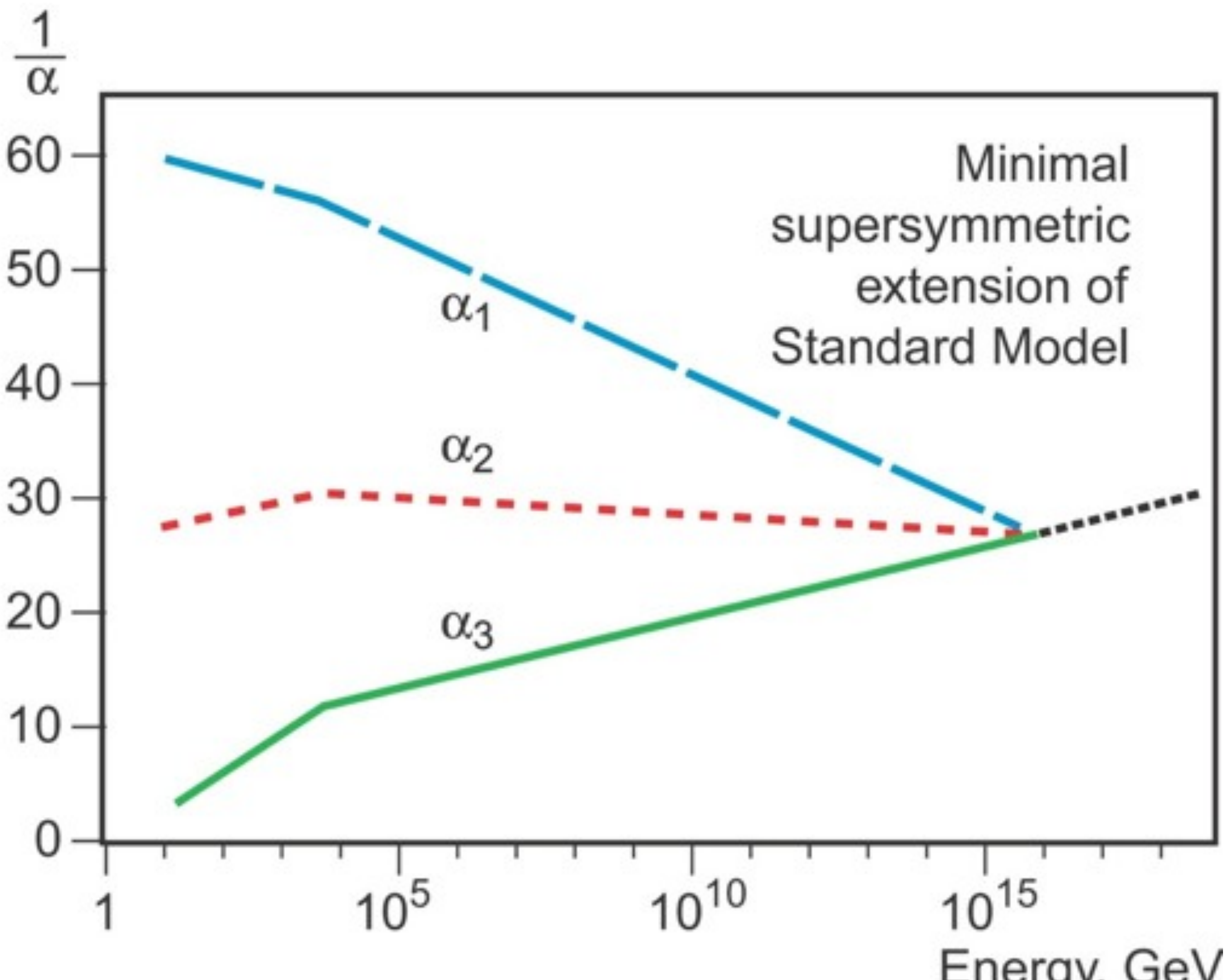
$$\mathbf{10} = (\mathbf{1}, \overline{\mathbf{3}})_{-4/3} + (\mathbf{1}, \mathbf{1})_2 + (\mathbf{2}, \mathbf{3})_{1/3}$$

Standard Model families fit into small representations of SU(5)

SUSY GUTs



Gauge coupling unification

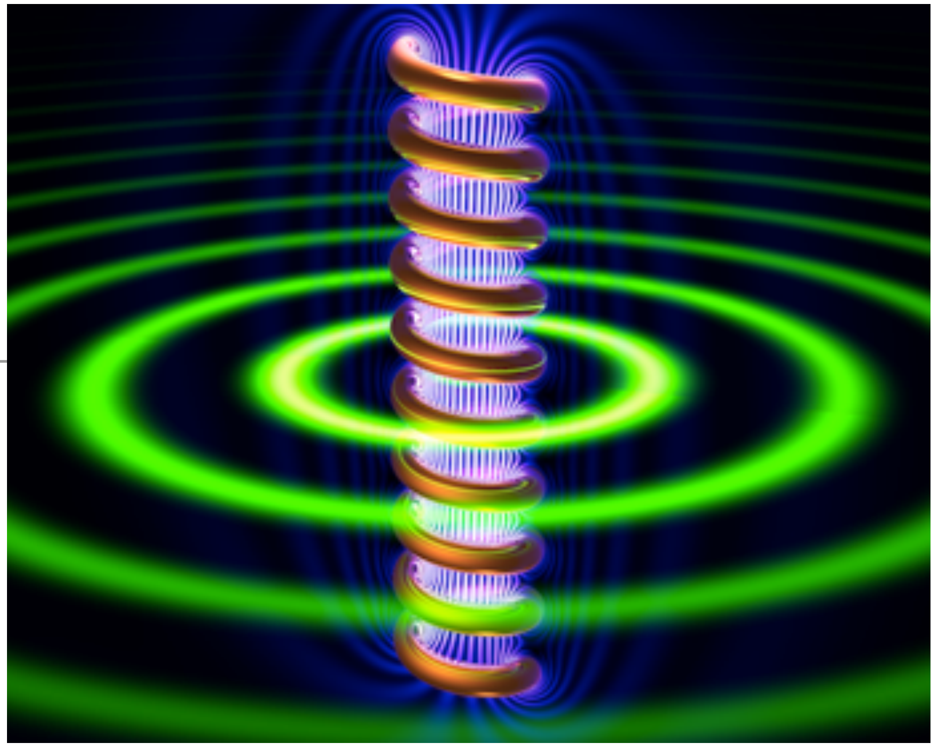


Color Triplets

$$\begin{array}{ccc} & \overline{H}_2 & \overline{H}_3 \\ \overline{\mathbf{5}} = & (\mathbf{2},\mathbf{1})_{-1} + & (\mathbf{1},\overline{\mathbf{3}})_{2/3} \end{array}$$

$$\begin{array}{ccc} & H_2 & H_3 \\ \mathbf{5} = & (\mathbf{2},\mathbf{1})_{1} + & (\mathbf{1},\mathbf{3})_{-2/3} \end{array}$$

GUT-breaking with Wilson Lines



- In field-theoretic GUTS, separating out the mass scales of the Higgs doublet/triplets involved lots of fine-tuning (breaking through <**24**>)
- In addition, problematic relationships between the couplings because of “parent” SU(5) structure
- In theories of extra dimensions this is different
- The GUT group is broken through ``Wilson Lines” (the Hosotani Mechanism) in the extra dimensions
- Symmetry is broken by an Aharanov-Bohm type of gauge field configuration in the extra dimensions!

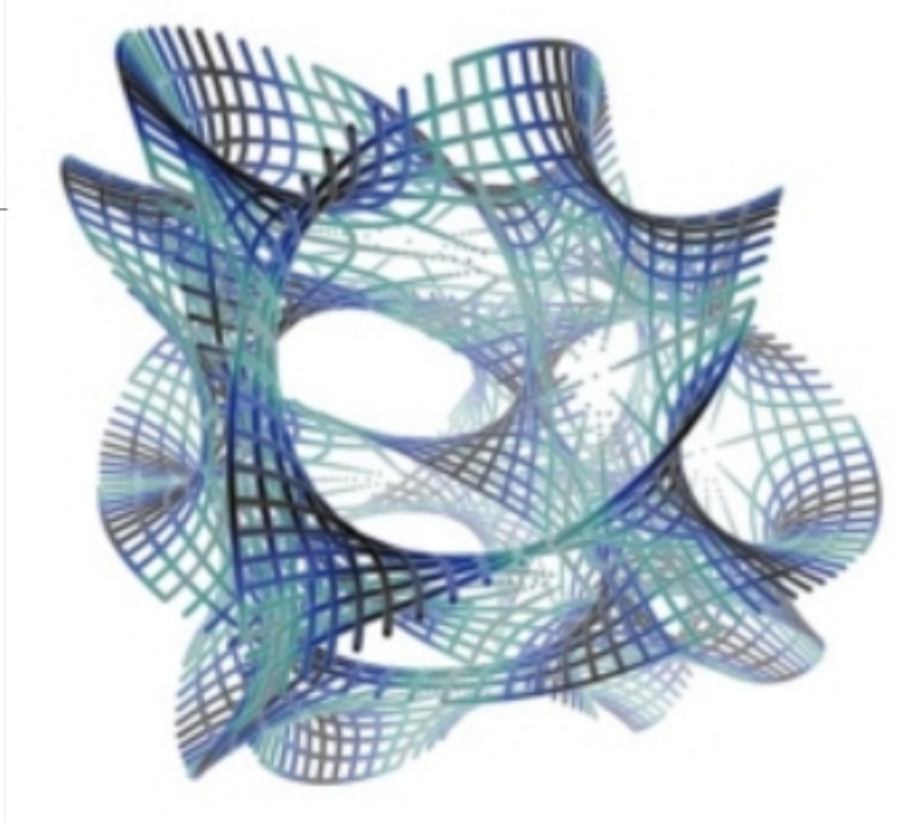
Gauge fields with

$$\langle A \rangle \propto \mathrm{diag}(2,2,2,-3,-3)$$

$$\langle F \rangle = 0$$

Symmetry breaking...

- The effects of the Wilson line gauge field are determined by topology. Locally it can always be gauged away
- Topology -----> Integers -----> No fine tuning!
- GUT Relations between couplings also broken by Wilson lines
- We've seen where symmetries arise (GUTs) and are broken (Wilson lines) and where the matter comes from (i.e. 248 of E8 in 10D)
- So what's next?...



Wilson line breaking only possible when

$$\pi_1(X) \neq 0$$

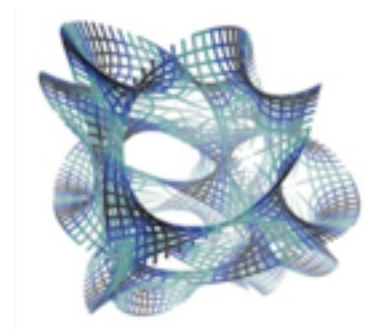
$$\rho : \pi_1(X) \to G_{GUT}$$

$$e^{2\pi i \int_\gamma A \cdot dx}$$

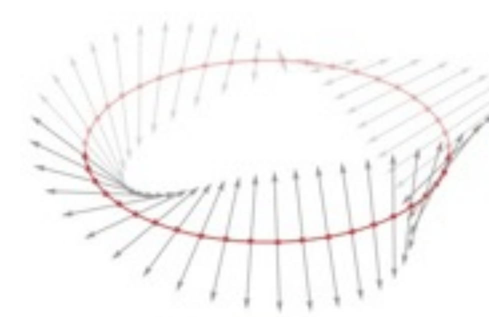
Goals of Modern String Phenomenology

- We'll begin in 10d with the following geometric input: 6 compact dimensions, X, and a vector bundle over them, V.
- Our job is to build the box...
- From the choice of (X,V), in principle we will get a prediction for all the particles/ symmetries of nature!
- The Standard Model alone has 25 free parameters (i.e. the mass of the electron, the strength of the forces, etc.)
- One geometric choice will fix them all!

String Geometry (X,V)



X



V

↓

Computation of
String Compactification
(Dimensional Reduction)

↓

"Real World" 4d Physics

We want a LOT...

Particle Physics:

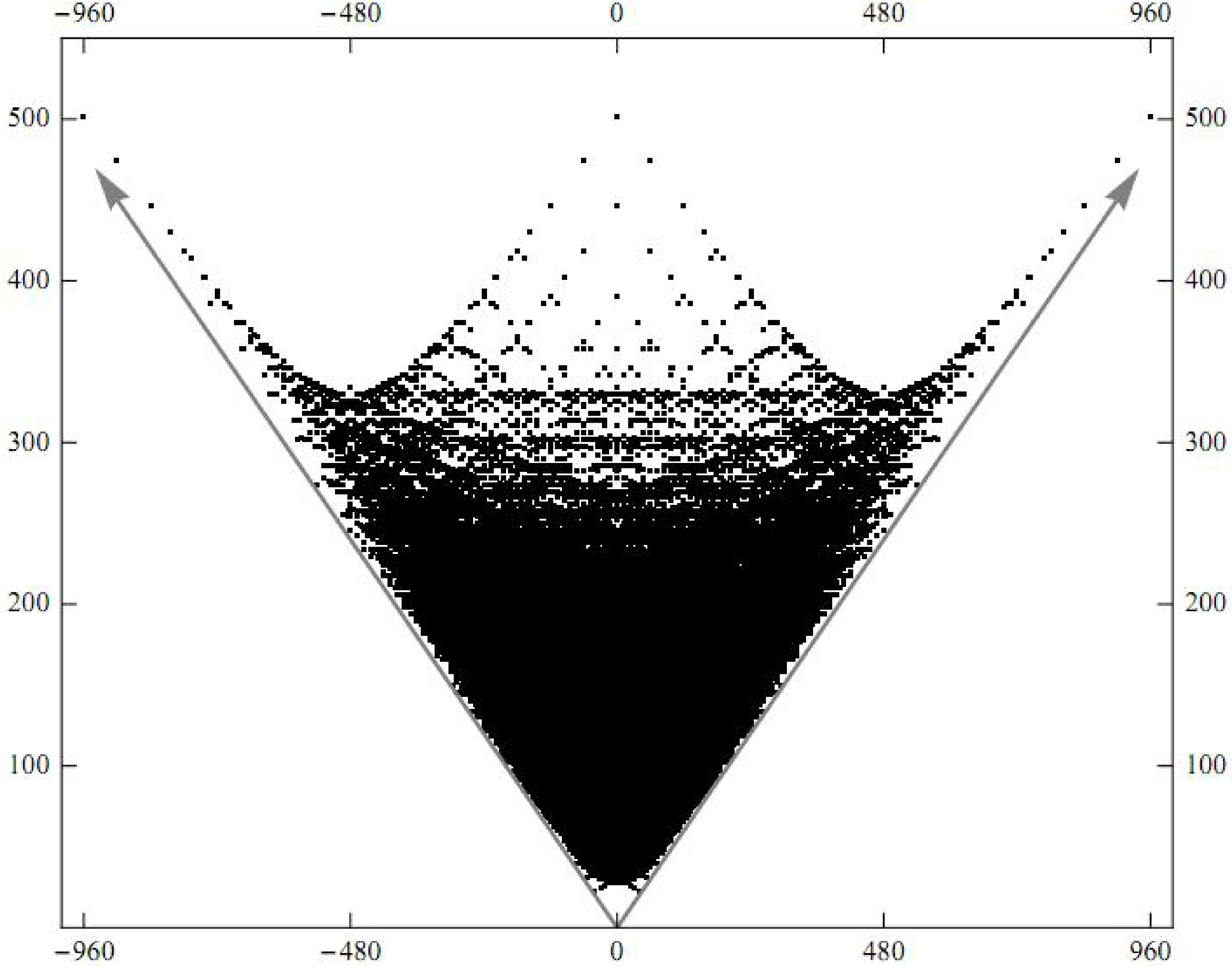
- Gauge and matter structure of the Standard Model
- Hierarchy of scales + masses (including Neutrinos)
- Flavor CKM, PMNS mixing, CP, no FCNC
- Hierarchy of gauge couplings (unification?)
- ‘Stable’ proton + baryogenesis
- Concrete, consistent predictions for “new” particles

Cosmology:

- Inflation or alternative for CMB fluctuations
- Dark matter (+avoid over-closing)
- Dark Radiation $N_{eff}>3$
- Dark Energy

Important: If any ONE of these doesn’t work, can rule out the model!

Half a billion Calabi-Yau manifolds and counting...



But there are challenges...

1) There are many consistent (though not realistic) geometries: (X,V) known. (Not even known if the list of all such things is finite)

2) Incredible mathematical complexity involved in working out the 4D physics. Many parts of the calculation of the string compactification (e.g. $g^{a\bar{b}}, A^{\bar{a}}$) unknown...

3) Most of these lead to the wrong particles and symmetries... How to find/classify the “good” ones?

4) Calabi-Yau and bundle moduli can be dangerous...

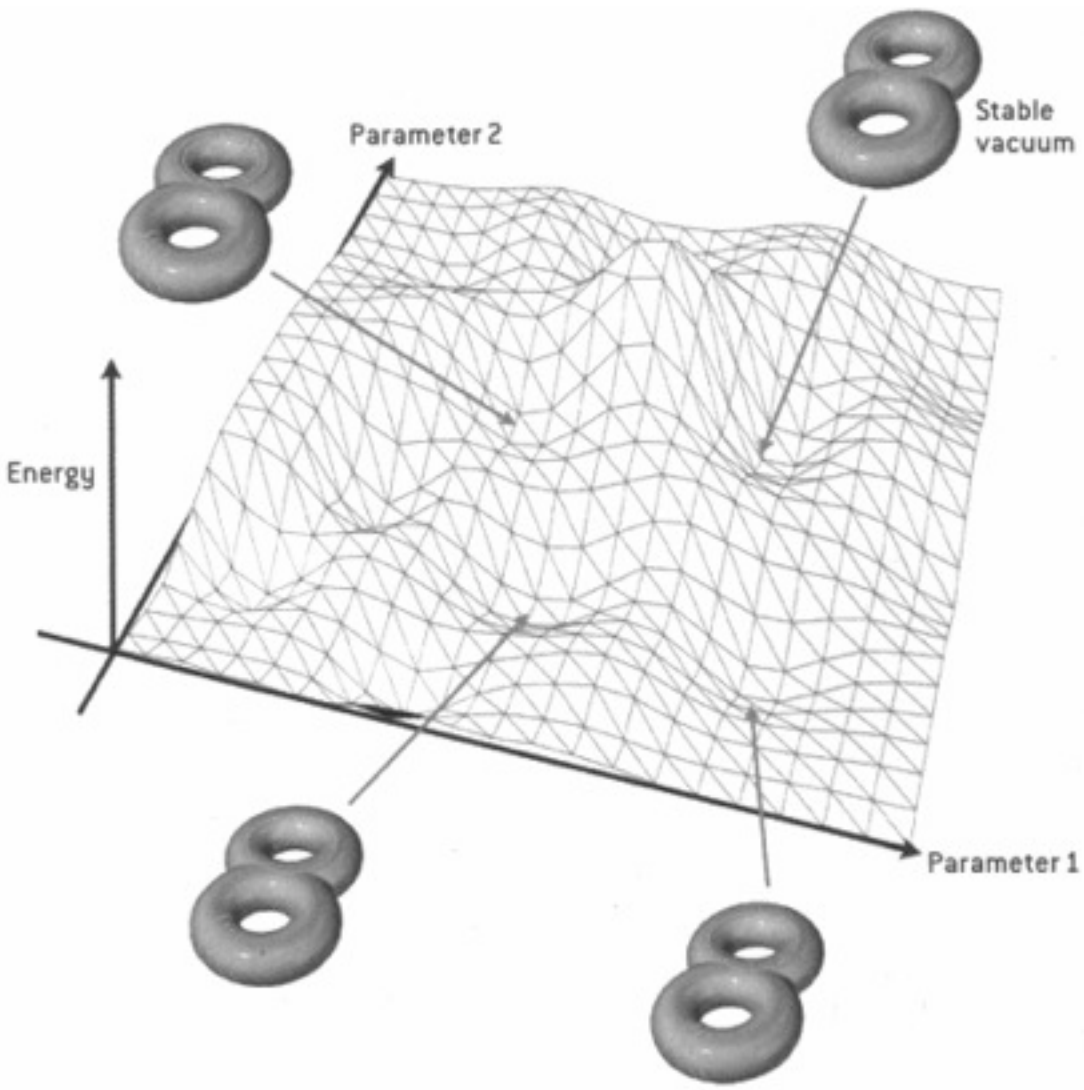
Moduli Problem

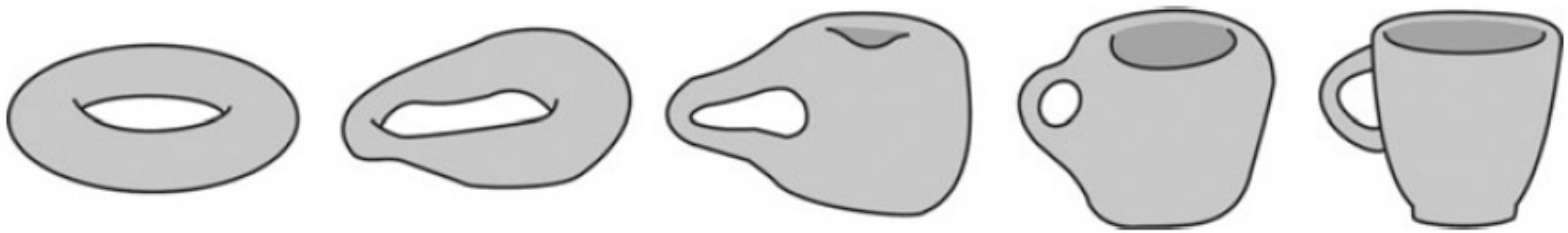
The 6-dimensional Calabi-Yau manifold has parameters (called "Moduli") associated to

1) Its size
2) Its shape

Order ~100 of such parameters and string theory does not tell us their values.

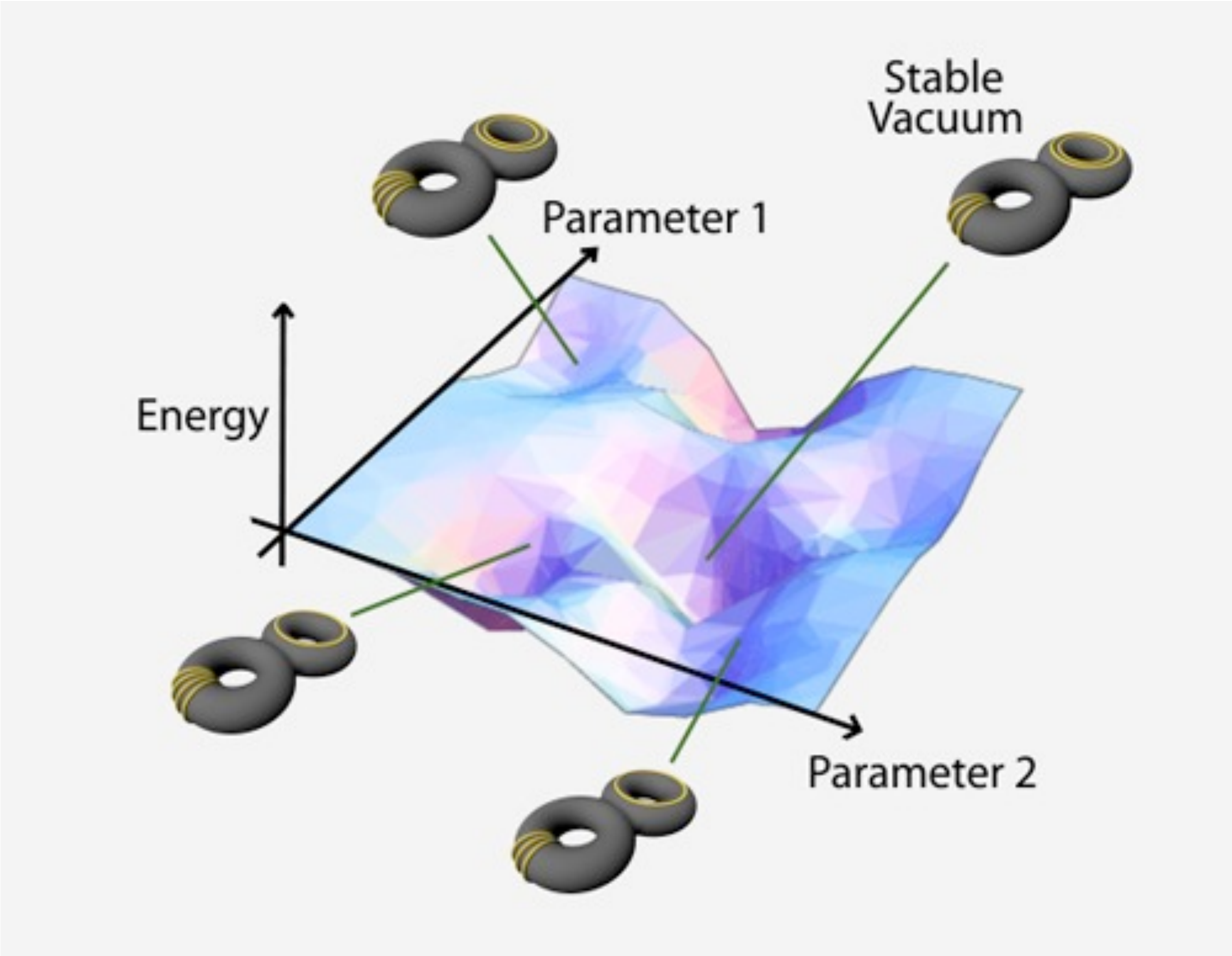
Unless we add something to the theory to fix the size/shape these lead to unphysical massless particles in 4d...





We're not here for the "Landscape"...

- String theory is a natural and powerful extension of Quantum Field Theory
- Once we add potentials to lift moduli there is the possibility of getting many vacua in the theory...
- 10^{500} vacua??
- But this counting was done for an entirely uninteresting set of geometries (none of those vacua have an electron!)



We've replaced the question

Which field theory?
with
Which Geometry?

Need to characterize "physically relevant geometries"

What’s taken so long?

- As I’ve just described it, heterotic string theory was first observed to be a rich arena for phenomenology 20 years ago.

- Great ideas for SM spectrum, Dark matter, SUSY breaking (gaugino condensation, etc)....

- So why hasn’t string pheno been “done” already?

<u>Three main problems:</u>

1) Very hard to work out the theory: Particle spectrum, couplings, etc.

E.g. It took until 2005 to write down models with even the correct SM spectrum (Only 2 such models, each took on order 5 years each to produce.).

2) Good Moduli Stabilization mechanisms hard to come by

3) Some parts of the theory (part of the matter field potential) completely unknown analytically. For example normalized particle masses depend on CY metric + bundle connection....

An Algorithmic Approach

Within the context of Heterotic string theory we're trying to resolve these problems:

1) Develop the technology to fully specify the effective 4d theory given string geometry.

2) Rather than attempting to engineer/tune a single "Real world" model, develop general techniques. Produce a large number (i.e. billions) of consistent models and then scan for those that match the Standard Model. Identify Patterns...

3) Remove ("stabilize") the free geometric moduli of heterotic string compactification.

Searching for the Standard Model...

With my collaborators, J. Gray (VT), A. Lukas (Oxford) and E. Palti (Heidelberg)...

1) We chose a simple "probe" construction of vector bundles over Calabi-Yau manifolds which allowed us to systematically produce large numbers of consistent string geometries.

The idea:

Observation: At special loci in moduli space, bundle structure groups can (and often do) "split", causing the low energy gauge group to enhance:

Bundle:

$$SU(5) \to S[U(4) \times U(1)]$$

4D symmetry:

$$SU(5) \to SU(5) \times U(1)$$

Maximal splitting:

$$SU(5) \to S[U(1)^5]$$

$$SU(5) \to SU(5) \times U(1)^4$$

Heterotic Line bundle models -> SM

- Gauge fields are Abelian and V is a sum of line bundles -> much easier to handle technically

$$V = \bigoplus_{i=1}^{5} L_i$$

- These Abelian sums of bundles reside in a larger non-Abelian bundle moduli space. Still carry many of the properties of the generic, non-Abelian bundles

- The enhanced U(1) symmetries are Green-Schwarz massive (with discrete global remnants) and most matter becomes charged under them

- These global symmetries restrict Yukawa couplings, mass terms, matter field Kahler potentials, etc. Can be both helpful and dangerously restrictive.

Searching for the Standard Model...

multiplet	$S(U(1)^5)$ charge	associated line bundle L	contained in
$\mathbf{10}_{\mathbf{e}_a}$	$\mathbf{e}_a$	L_a	V
$\overline{\mathbf{10}}_{-\mathbf{e}_a}$	$-\mathbf{e}_a$	L_a^*	V^*
$\bar{\mathbf{5}}_{\mathbf{e}_a+\mathbf{e}_b}$	$\mathbf{e}_a+\mathbf{e}_b$	$L_a \otimes L_b$	$\wedge^2 V$
$\mathbf{5}_{-\mathbf{e}_a-\mathbf{e}_b}$	$-\mathbf{e}_a-\mathbf{e}_b$	$L_a^* \otimes L_b^*$	$\wedge^2 V^*$
$\mathbf{1}_{\mathbf{e}_a-\mathbf{e}_b}$	$\mathbf{e}_a-\mathbf{e}_b$	$L_a \otimes L_b^*$	$V \otimes V^*$
$\mathbf{1}_{-\mathbf{e}_a+\mathbf{e}_b}$	$-\mathbf{e}_a+\mathbf{e}_b$	$L_a^* \otimes L_b$	

1) Scanned over 10^{40} string geometries to search for those that produced exactly the particle spectrum of the Standard Model

3) This is the largest and most systematic dataset of its kind in string theory

4) Scanned for coarse properties of the 4d physics consistent with experiment. Even “roughly” realistic string geometry is rare! We are working now to identify patterns...

SM Spectra	1 Higgs Pair	2 Higgs Pair	3 Higgs Pair	rk($Y^{(u)}$) >0	No proton decay	1 Higgs, No PD, rk($Y^{(u)}$)>0
407	262	77	63	45	198	13

New Techniques in Moduli Stabilization...

With collaborators, J. Gray (VT), A. Lukas (Oxford) and B. Ovrut (UPenn)

1) Discovered a powerful new tool to stabilize moduli in heterotic string compactifications

2) The basic idea: Some vector bundles prevent their base manifolds from deforming, possible to choose bundles which fix all the "shape" moduli of the compact dimensions

$$S_{partial} \sim \int_{M_{10}} \sqrt{-g}\{(F_{a\bar{b}}g^{a\bar{b}})^2 + (F_{ab}F_{\bar{a}\bar{b}}g^{a\bar{a}}g^{a\bar{a}})\} + \ldots$$

$$\delta(F_{\bar{a}\bar{b}}) = 0$$

$$\delta\mathfrak{z}^{a}_{[\bar{a}}F_{a\bar{b}]} + 2D_{[\bar{a}}\delta A_{\bar{b}]} = 0$$

3) An example of the importance of interdisciplinary work

i) Key mathematical results developed by Atiyah ~1950's

ii) Key physics question raised by Witten ~1980s

iii) Only now did we develop new techniques and the language/understanding of both to combine the two!

iv) And calculation only possible with modern computer resources/ computational algebraic geometry. First of its kind in both the physics and mathematics literature...

Vector bundles and Moduli Stabilization

- Structure of the scalar potential: $S_{partial} \sim \int_{M_{10}} \sqrt{-g}\{(F_{a\bar{b}}g^{a\bar{b}})^2 + (F_{ab}F_{\bar{a}\bar{b}}g^{a\bar{a}}g^{a\bar{a}})\} + \ldots$
- Recall, 10-dimensional E.O.M.: $g^{a\bar{b}}F_{a\bar{b}} = 0 \qquad F_{ab} = F_{\bar{a}\bar{b}} = 0$
- What happens as we vary the complex structure of the manifold? Must the E.O.M remain satisfied? No!
- In general, get constraints of the form: $\delta\mathfrak{z}^a_{[\bar{a}}F_{a\bar{b}]} + 2D_{[\bar{a}}\delta A_{\bar{b}]} = 0$
- (All) Complex structure (can be) fixed by: $F_{ab} = F_{\bar{a}\bar{b}} = 0$
- Kahler (shape) moduli constrained by $g^{a\bar{b}}F_{a\bar{b}} = 0$ (Except for the overall volume)
- Why wasn't this done 20 years ago? General story not applied and tough to compute!

How far can we get?

- Using improved observations about vector bundle geometry allows us to remove all complex structure (shape) moduli and most Kahler (size) moduli

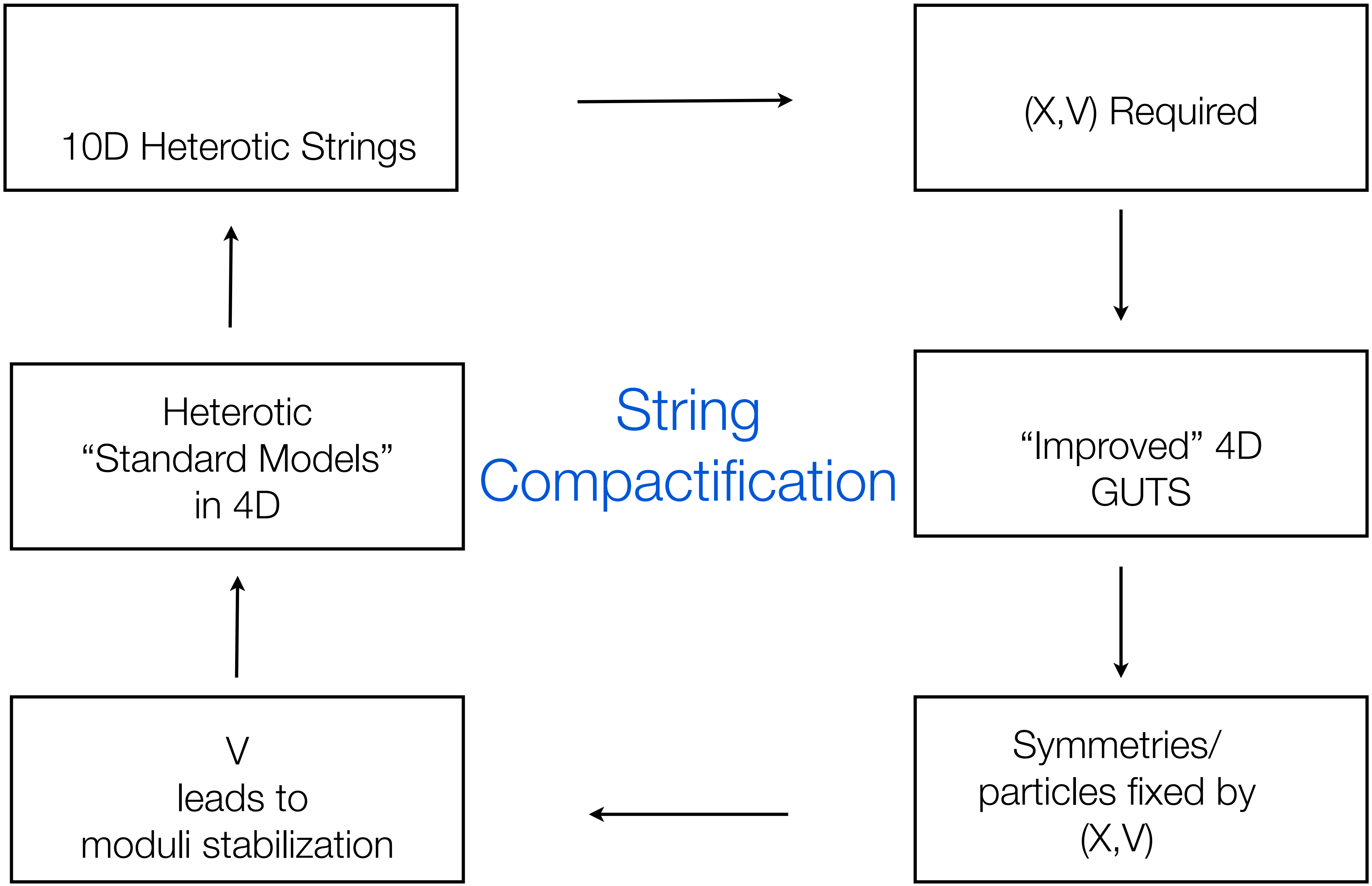
- But one linear combination of the string coupling (dilaton) and overall volume remains unconstrained

- Dream: Stabilize all moduli perturbatively and then add non-perturbative effects (world sheet instantons, gaugino condensation, etc) to break supersymmetry --> A small de Sitter vacuum?

- Reality: Hard to accomplish and existing tools not yet convincing. Ongoing work...

Lots of pieces fit together...

...but we have to go further



Summary and Conclusions

The work described here is only a part of a broader program...

Today: String compactifications = “Manifolds/Bundles for fun and profit”

Recent and substantial progress:

We’ve demonstrated that it is possible to combine new geometric tools in

1) Model building (Standard Model Particle Spectra)
2) Moduli stabilization
3) Computational tools (including numeric CY metrics)

To carry the determination/engineering of the effective theory further than ever before...

More to come...

String Phenomenology as a field is just beginning in earnest and there is much exciting work to be done... so stay tuned!