Towards a unified description of the nuclear electroweak response

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Outline

★ Motivations
★ Lessons from electron scattering
★ Neutrino-nucleus scattering
  ▶ High energy regime: can the models developed to describe electron scattering data be extended to the case of neutrino scattering?
  ▶ Low energy regime: can the relevant reaction mechanisms be consistently described within *ab initio* many body approaches?
★ Summary & Outlook
Motivation I: detection of neutrino oscillations

- Probability that a neutrino oscillate from flavor $\alpha$ to flavor $\beta$ after travelling a distance $L$

$$P_{\nu\alpha \rightarrow \nu\beta} = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E_\nu} \right)$$

- Addressing a number of fundamental issues, such as the mass hierarchy, leptonic CP violation and the existence of sterile neutrinos, will require precise measurements of neutrino and antineutrino oscillations.
Neutrino interactions are very weak

- Neutrino-nucleon scattering

\[ \nu + N \rightarrow l + X \]

\[ \sigma_{\nu N} \sim 10^{-38} \text{ cm}^2, \quad \sigma_{\nu N}/\sigma_{eN} \sim 10^{-6} \]

- Total cross section of the process

\[ \nu_{\mu} + n \rightarrow \mu^- + p \]
Detecting neutrinos requires big detectors

- The **SUPER-K** detector, in Japan, is filled with 12.5 million gallon of ultra-clean water
- The **MiniBooNE** detector, at FNAL, is filled with 800 tons of mineral oil
- The detected signal results from neutrino interactions with Oxygen and Carbon nuclei
- A quantitative understanding of their response to neutrino interactions is required for the interpretation of the measured cross sections

Relevant energy scale of accelerator-based experiments: $E_{\nu} \sim 1$ GeV
Motivation II: neutron star evolution

★ At much lower energies, \((E_\nu \lesssim 5\text{MeV})\), neutrino interactions with nuclear matter play a critical role in determining both the evolution of newly formed proto-neutron stars and cooling of aged stars.

★ The gravitational energy released in a supernova collapse is \(\sim 200\text{-}300\) times higher than that produced by the Sun over its entire lifetime.

★ \(\sim 99\%\) of it is radiated over a timescale of a few tens of seconds in the form of an immense flux of low-energy neutrinos.
Neutrino-nucleus x-section

- Differential cross section of the charged current process $\nu_\ell + A \rightarrow \ell^- + X$

\[
\frac{d\sigma_A}{d\Omega_{\ell^-}dE_{\ell^-}} \propto L^{\mu\nu}W_{\mu\nu}
\]

- $L^{\mu\nu}$ is fully specified by the lepton kinematical variables. Same as in scattering, but in this case the beam energy is not known.

- The calculation of the target response tensor

\[
W_{\mu\nu} = \sum_X \langle 0|J^\dagger_\mu|X\rangle \langle X|J_\nu|0\rangle \delta^{(4)}(P_0 + k_{\nu_\ell} - P_X - k_{\ell^-})
\]

requires a consistent description of the target internal dynamics, determining the initial and final states, as well as of the nuclear weak current

\[
J_\mu = J^V_\mu - J^A_\mu = \sum_i j_\mu(i) + \sum_{j>i} j_\mu(ij) + \ldots
\]
Information from electron scattering

- Vast supply of precise data available

\[ Q^2 = 4E_e E_e' \sin^2 \frac{\theta_e}{2}, \quad x = \frac{Q^2}{2M \omega} \]

- Carbon target

- Different reaction mechanisms contributing to the measured cross sections can be readily identified

\[ e + A \rightarrow e' + X \]

\[ E_e \sim 1 \text{ GeV} \]
**ab initio** calculations of the electromagnetic response

- The nucleus is seen as a collection of pointlike protons and neutrons interacting through the non relativistic hamiltonian

\[ H = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} U_{ijk} \]

- the potentials are determined by a fit to the properties of the *exactly solvable* two- and three-nucleon systems

- the nuclear electromagnetic current \( J_\mu \equiv (\rho_{\text{ch}}, J) \) is constructed in such a way as to fulfill the continuity equation

\[ \nabla \cdot J + i[H, \rho_{\text{ch}}] = 0 \]

- at low to moderate momentum transfer, typically \(|q| \lesssim 400 \text{ MeV}\), the non relativistic approach provides a set of electroweak charge and current operators consistent with the hamiltonian.
Predictions of the *ab initio* approach

- Non relativistic nuclear hamiltonians can be used to carry out *exact* Quantum Monte Carlo (QMC) calculations of the energies of the ground and low-lying excited states of nuclei with $A \leq 12$ using the Green’s Function Monte Carlo (GFMC) technique.

GFMC calculation of the nuclear response

- The GFMC technique is ideally suited for the calculations of the Euclidean responses

\[ E_{\mu\nu}(|q|, \tau) \propto \int_{\omega_{\text{thr}}}^{\infty} d\omega e^{-\omega \tau} W_{\mu\nu}(|q|, \omega) \]

- Contributions to the longitudinal (L) and transverse (T) channels

\[ E_L(|q|, \tau) = E_{00}(|q|, \tau), \quad E_T(|q|, \tau) = E_{11}(|q|, \tau) \]

- The inversion of the Euclidean responses of light nuclei has been recently obtained exploiting the maximum entropy method
Euclidean responses of carbon

A. Lovato et al, PRC 91 062501(R), (2015)
Inversion of the Euclidean responses of $^4$He

- Significant two-nucleon current contribution in the transverse channel

A. Lovato et al, PRC 91 062501(R), (2015)
High momentum transfer: the Impulse Approximation (IA)

- In the kinematical regime corresponding to $|q| \gtrsim 500$ MeV

$$\lambda \sim \frac{\pi}{|q|} \ll d$$

where $d$ is the average nucleon-nucleon distance

- Nuclear scattering reduces to the incoherent sum of elementary scattering processes involving individual nucleons
The IA amounts obviously implies the replacement
\[ J_\mu \rightarrow \sum_i j_\mu(i) \]

Assuming that Final State Interactions (FSI) between the struck nucleon and the spectators be negligible leads to the factorized final state
\[ |X\rangle \rightarrow |x, p_x\rangle \otimes |R, p_R\rangle \]

Nuclear dynamics and interaction vertex are decoupled
\[ d\sigma_A = \int d^3k dE \ d\sigma_N \ P(k, E) \]

- The electron-nucleon cross section \( d\sigma_N \) can be written in terms of structure functions extracted from electron-proton and electron-deuteron scattering data.
- The nuclear spectral function \( P(k, E) \), yielding the momentum and energy distribution of the knocked out nucleon, is an intrinsic property of the target nucleus, independent of momentum transfer, calculable within the \textit{ab initio} many-body approach.
Carbon quasi elastic cross section within IA

- FSI corrections included [A. Ankowski et al, PRD 91 033005, (2015)]
Inclusion of inelastic channels

\[ \frac{d\sigma}{d\omega d\Omega} \ [\mu b/\text{sr}/\text{GeV}] \]

\( ^3\text{He} \)

\( E=11. \text{ GeV} \)

\( \theta=8^\circ \)

\( Q^2 = 2.1 \text{ GeV}^2 \)

Generalized factorization ansatz

- GFMC calculations strongly suggest that the contributions arising from two-nucleon currents is important in the transverse channel
- Use of relativistic currents and a realistic description of the nuclear ground state requires the extension of the factorization ansatz underlying the IA

\[ |X\rangle \rightarrow |p, p'\rangle \otimes |m_{(A-2)}\rangle \]

\[ \langle X| j^\mu_{ij}|0\rangle \rightarrow \int d^3k d^3k' M_m(k, k') \langle pp'| j^\mu_{ij}|kk'\rangle \]

- The matrix elements of the two-nucleon current between states describing non interacting nucleons can be computed using the fully relativistic expression.
- The amplitude \( M_m(k, k') = \langle n_{(A-2)}| \otimes \langle k, k'\rangle|0\rangle \) is independent of \(|q|\) and can be obtained from non relativistic many-body theory
Results of the generalized factorization ansatz

- Transverse response of carbon, not corrected for FSI

\[ R_T(q,\omega) \text{ [MeV}^{-1}] \]

\[ 12^C \]

\[ q=570 \text{ MeV} \]

\[ 0.025 \]

\[ 0.020 \]

\[ 0.015 \]

\[ 0.010 \]

\[ 0.005 \]

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Comparison to measured carbon cross sections

N. Rocco et al, preliminary

\begin{align*}
\text{e} + ^{12}\text{C} \to \text{e}^\prime + \text{X} &
\quad \begin{array}{c}
\text{E}_e = 0.680 \text{ GeV} \\
\theta_e = 36 \text{ deg}
\end{array} \\
\text{e} + ^{12}\text{C} \to \text{e}^\prime + \text{X} &
\quad \begin{array}{c}
\text{E}_e = 0.560 \text{ GeV} \\
\theta_e = 36 \text{ deg}
\end{array} \\
\text{e} + ^{12}\text{C} \to \text{e}^\prime + \text{X} &
\quad \begin{array}{c}
\text{E}_e = 1.3 \text{ GeV} \\
\theta_e = 37.5 \text{ deg}
\end{array} \\
\text{e} + ^{12}\text{C} \to \text{e}^\prime + \text{X} &
\quad \begin{array}{c}
\text{E}_e = 0.961 \text{ GeV} \\
\theta_e = 37.5 \text{ deg}
\end{array}
\end{align*}
In neutrino experiments, the measured double differential cross section is averaged over the energy of the incoming neutrino, broadly distributed according to the flux $\Phi$

$$\frac{d\sigma_A}{dT_\mu d \cos \theta_\mu} = \frac{1}{N_\Phi} \int dE_\nu \Phi(E_\nu) \frac{d\sigma_A}{dE_\nu dT_\mu d \cos \theta_\mu}$$

In addition to $F_1$ and $F_2$, the QE electron-nucleon cross section is determined by the axial form factor $F_A$, assumed to be of dipole form and parametrized in terms of the axial mass $M_A$

According to the paradigm successfully employed to describe electron scattering data within the IA, $M_A$ must be determined from measurement carried out using a deuterium target. The resulting value is $M_A = 1.03$ GeV
Analysis of CCQE data

MiniBooNE flux

Theoretical calculations carried out setting $M_A = 1.03$

OB et al, PRL 105, 132301 (2010)

MiniBooNe CCQE data

CCQE $\langle E_\nu \rangle = 788$ MeV
$M_A = 1.03$ GeV
$37^\circ < \theta_\mu < 46^\circ$

$25^\circ < \theta_\mu < 37^\circ$

$\langle E_\nu \rangle = 788$ MeV
$M_A = 1.03$ GeV
$37^\circ < \theta_\mu < 46^\circ$

$25^\circ < \theta_\mu < 37^\circ$
“Flux averaged” electron-nucleus x-section

- Electron scattering x-sections off Carbon at $\theta_e = 37^\circ$ and different beam energies

Owing to flux average, a single bin of energy of the outgoing charged lepton picks up strength corresponding to different reaction mechanism

All relevant mechanisms must be included in a consistent fashion
The opacity of nuclear matter to neutrinos of energy $E_\nu \lesssim 10$ MeV, which plays a critical role in determining the evolution of compact stars, is parametrized in terms of neutrino mean free path

$$\lambda_\nu = \frac{1}{\sigma \rho}$$

with

$$\sigma \propto \int \frac{d^3 q}{(2\pi)^3} L_{\mu\nu} W^{\mu\nu}$$

Consider weak neutral current interactions. In the extreme non relativistic approximation, the vector and axial vector weak currents reduce to

$$\bar{\psi}_n \gamma^\mu \psi_n \rightarrow \psi_n^\dagger \psi_n \delta^\mu_o = O_F : \text{Fermi (F) operator}$$

$$\bar{\psi}_n \gamma^\mu \gamma^5 \psi_n \rightarrow \psi_n^\dagger \sigma^i \psi_n \delta^\mu_i = O_{GT}^i : \text{Gamow–Teller (GT) operator}$$
The nuclear cross section is computed from

\[ L_{\mu\nu} W^{\mu\nu} \propto [(1 + \cos \theta)S(q, \omega) + C_A^2 (3 - \cos \theta)S(q, \omega)] \]

with

\[ S(q, \omega) = \sum_n |\langle n|O_F|0\rangle|^2 \delta(\omega + E_0 - E_n) \quad \text{, density response} \]

\[ S(q, \omega) \sum_i \sum_n |\langle n|O^{i}_{GT}|0\rangle|^2 \delta(\omega + E_0 - E_n) \quad \text{, spin response} \]

The expression of the mean free path in terms of the response functions is

\[ \frac{1}{\lambda_{\nu}} = \frac{G_F^2}{4} \rho \int \frac{d^3 q}{(2\pi)^3} \left[ (1 + \cos \theta)S(q, \omega) + C_A^2 (3 - \cos \theta)S(q, \omega) \right] \]
Interaction effects in the low-energy regime

- In the absence of interactions, nuclear matter reduces to a degenerate Fermi gas, and the final state is a one particle-one hole state

\[
|n\rangle = |ph\rangle, \quad E_n - E_0 = e_p^0 - e_h^0 = \frac{|p|^2}{2m} - \frac{|h|^2}{2m}
\]

- Interactions can be included using an effective interaction, \( V_{\text{eff}} \), and consistently replacing the Fermi and Gamow-Teller operators with effective operators, \( \tilde{O}_F \) and \( \tilde{O}_{GT} \)

- Interaction lead to a modification of the spectrum

\[
e_k^0 \rightarrow e_k = \frac{k^2}{2m} + \sum_{k'} (kk'|V_{\text{eff}}|kk' - k'k)
\]

as well as to a quenching of the transition matrix elements \((ph|\tilde{O}_F|0)\) and \((ph|\tilde{O}_{GT}|0)\), arising from correlations, coupling between 1p1h states and more complex, npnh final states
Interaction effects on the neutron matter response

★ (A), (B), (C) $\rightarrow |q| = 0.3, 1.8, 3.0 \text{ fm}^{-1}$
Excitation of collective (phonon-like) modes

- At low neutrino energy $\lambda \sim \pi/|q| > d$, and the interaction process may involve many nucleons

- Propagation of particle-hole states produced at the interactions vertex, leading to the occurrence of collective excitations, must be taken into account replacing

$$|\text{ph}\rangle \rightarrow |n\rangle = \sum_{i=1}^{N} C_i |p_i h_i\rangle$$

★ The energy of the state $|n\rangle$ and the coefficients $C_i$ are obtained diagonalizing the $N \times N$ hamiltonian matrix

$$H_{ij} = (E_0 + e_{p_i} - e_{h_i})\delta_{ij} + (h_i p_i |V_{\text{eff}}| h_j p_j - p_j h_j)$$
The formalism of Correlated Basis Functions (CBF) and the cluster expansion technique can be exploited to obtain $V_{\text{eff}}$ from a realistic nuclear hamiltonian.

The effective interaction is defined through the relation

$$
\langle 0|H|0 \rangle = \frac{3}{5} \frac{k_F^2}{2m} + \sum_n (\Delta E)_n = \frac{3}{5} \frac{k_F^2}{2m} + \langle 0_{FG}|V_{\text{eff}}|0_{FG} \rangle
$$

where $|0_{FG}\rangle$ is the ground state of the non-interacting Fermi gas, and the ground state expectation value of the Hamiltonian is expanded in a series whose terms correspond to contributions of clusters containing an increasing number of particles.

The effective interaction constructed retaining two- and three-body cluster contributions includes the effects of three-nucleon interactions, which are known to be needed to explain saturation of isospin-symmetric nuclear matter.
• CBF effective interaction obtained from the cluster expansion of the relevant matrix elements, keeping two- or two- and three-body terms

![Graphs showing effective and bare potentials for different spin and isospin states.](image)

• Being well-behaved (unlike the bare nucleon-nucleon potential) the CBF effective interaction can be used to carry out perturbative calculations in the Femi gas basis
Comparison between the EOS obtained using the CBF effective interaction and those resulting from advanced many-body approaches.
Excitation of collective modes at low-momentum transfer

- Fermi (density, left) and Gamow-Teller (spin, right) contributions to the response of pure neutron matter at nuclear matter equilibrium density and momentum transfer $|q| = 0.1$ fm$^{-1}$

A. Lovato et al. PRC 89, 025804 (2014)

- the collective mode is only excited in the spin channel
Neutrino mean free path in neutron matter

A Lovato et al, arXiv 1310.0510 [nucl-th]

★ Both short and long range correlations important
Ab initio calculations based on nuclear many-body theory and the available experimental information on electron-nucleon interactions provide a remarkably accurate description of the nuclear cross sections in a broad kinematical range.

The generalization to neutrino-nucleus scattering, needed to reduce the systematic uncertainty of LBL neutrino oscillation experiments, while being feasible, involves additional difficulties, arising from the flux average, and requires a consistent description of all relevant reaction mechanisms.

The same formalism and dynamical input can be used to investigate neutrino interactions with nuclear matter in the low energy region, relevant to astrophysical applications.

The development of a unified treatment of the nuclear response to electroweak interactions at energies ranging from few MeV to few GeV is well under way.