Time-Modulated Bright Beam Squeezing and Non-Gaussian States of Light

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Outline

• Define squeezing
  • Frequency Domain Squeezing
  • Time Domain Squeezing
• Time-modulated squeezing
• Current Status
• Future Experiment
  • Quantum Tomography
Squeezing

Squeezed State is where the noise in one quadrature is reduced below the minimum uncertain of a coherent state

Quadrature of light is \( A_\theta = e^{-i\theta}a + e^{i\theta}a^\dagger \)

Noise of a quadrature is its variance

For a coherent state it is \((\Delta A_\theta)^2 = 1\)

The Heisenberg Uncertainty Principle is \( \Delta X \Delta P \geq 1 \)

\[
\Delta A_0 = \Delta X = 1 \\
\Delta A_\frac{\pi}{2} = \Delta P = 1
\]
Amplitude Squeezed State

- **Minimum Uncertainty Coherent State**
  - $\Delta X = \Delta P = 1$

- **Squeezed State**
  - $\Delta X < 1; \Delta P > 1$
Optical Parametric Oscillator

\[ \omega = \omega_1 + \omega_2 \]

\[ \omega = 2\pi \times 563.5 \text{ rad} \cdot \text{THz} \]

\[ \lambda = 532 \text{ nm} \]

\[ \omega_1 \approx \omega_2 \approx 2\pi \times 281.7 \text{ rad} \cdot \text{THz} \]

\[ \lambda_1 \approx \lambda_2 \approx 1064 \text{ nm} \]
Squeezing

Pump → OPO → PBS

Data Acquisition Device
\[ N_a = a^\dagger a \approx \alpha^2 + \alpha \delta X_a(t) \]
\[ N_b = b^\dagger b \approx \beta^2 + \beta \delta X_b(t) \]

\[ i_-(t) \propto N_a - N_b \]
\[ \propto \alpha^2 - \beta^2 + \alpha \delta X_a(t) - \beta \delta X_b(t) \]

\[ \tilde{i}_-(\Omega) \propto (\alpha^2 - \beta^2) \delta(\Omega) + \alpha \delta \tilde{X}_a(\Omega) - \beta \delta \tilde{X}_b(\Omega) \]

\[ \langle |\tilde{i}_-(\Omega)|^2 \rangle \propto (\alpha^2 - \beta^2)^2 \delta(\Omega) \]
\[ + \alpha^2 \langle |\delta \tilde{X}_a(\Omega)|^2 \rangle + \beta^2 \langle |\delta \tilde{X}_b(\Omega)|^2 \rangle \]
\[ - 2\alpha\beta \langle \delta \tilde{X}_a(\Omega) \delta \tilde{X}_b(\Omega) \rangle \]

\[ \langle \delta \tilde{X}_a(\Omega) \delta \tilde{X}_b(\Omega) \rangle \rightarrow 1 \]
Shot Noise
PBS

\[ N_a = a^\dagger a \approx \alpha^2 + \alpha \delta X_a(t) \]
\[ N_b = b^\dagger b \approx \beta^2 + \beta \delta X_b(t) \]

\[ \dot{i}_-(t) \propto N_a - N_b \]
\[ \propto \alpha^2 - \beta^2 + \alpha \delta X_a(t) - \beta \delta X_b(t) \]

\[ \tilde{i}_-(\Omega) \propto (\alpha^2 - \beta^2) \delta(\Omega) + \alpha \delta \tilde{X}_a(\Omega) - \beta \delta \tilde{X}_b(\Omega) \]

\[ \langle |\tilde{i}_-(\Omega)|^2 \rangle \propto (\alpha^2 - \beta^2)^2 \delta(\Omega) \]
\[ + \alpha^2 \langle |\delta \tilde{X}_a(\Omega)|^2 \rangle + \beta^2 \langle |\delta \tilde{X}_b(\Omega)|^2 \rangle \]
\[ - 2\alpha \beta \langle \delta \tilde{X}_a(\Omega) \delta \tilde{X}_b(\Omega) \rangle \]

\[ \langle \delta \tilde{X}_a(\Omega) \delta \tilde{X}_b(\Omega) \rangle = 0 \]
Frequency Domain Squeezing

Noise reduction of the signal at each frequency

\[
\langle |\tilde{i}_-(\Omega)|^2 \rangle = S(\Omega) = 1 - \frac{4d}{(1 + d)^2 + (2\pi\Omega)^2}
\]
Setup

- Pump
- OPO
- Spectrum Analyzer
- PBS

\[ \frac{\lambda}{2} \]
Frequency Domain Squeezing
Time-Domain Squeezing

Time-Domain Squeezing is the noise reduction in time.

Instead of $\langle |\tilde{i}(\Omega)|^2 \rangle \rightarrow \langle |i(t)|^2 \rangle$

This is a measure of total squeezing
Total Squeezing

• Total Squeezing is the noise reduction of light integrated over the bandwidth of the OPO

• The variance of the total squeezing has a limit of

\[ \overline{V(\mathcal{X}_i)} = 1/2 \]
Setup

- Pump
- OPO
- PBS
- Digital Scope
Time-Modulated Squeezing

- Time-modulated squeezing is squeezing from an OPO that has a modulated pump
- The modulation rate of the pump is equal to the bandwidth of the OPO
- This rate is in between continuous wave and pulsed limits
Why Modulate the Pump?

\[
\tilde{f}/f_{th} = \text{The pump strength normalized to the threshold}
\]

(1) Modulation power = 0
(2) Modulation power = 0.75 × pump
(3) Modulation power = 3.0 × pump

Setup

Pump → Modulator → OPO → PBS

\[ \frac{\lambda}{2} \]

Digital Scope
Current Status

- Measured Frequency Domain Squeezing of 3 dB
- Replaced the 1% Transmission Output Mirror with a 2% Transmission Mirror
- Squeezing is proportional to the Transmittivity
- Threshold is proportional to the square of the Transmittivity
- Ordered new Nonlinear crystal
Quantum Tomography

• Classical Tomography is finding the shape of an object from its shadow

• From balanced homodyne detection we can measure quasi-probability distribution in quantum phase space

• Each measurement is a slice of the quantum state
Gaussian States of Light

• A Gaussian state of light is a state which the quasi-probability distribution can be described by a Gaussian

• For a Coherent state, the probability in $\Delta X$ and $\Delta P$ are equal, so it is a symmetric Gaussian

• For a Squeezed state, $\Delta X$ and $\Delta P$ are not equal
Wigner Function

- Quasi-probability distribution
- For a Gaussian, it is never negative
- For a squeezed state the Wigner function is never negative, also

\[ \langle q | \hat{U}(\theta) \hat{\rho} \hat{U}^\dagger(\theta) | q \rangle = \int_{-\infty}^{\infty} W(q \cos \theta - p \sin \theta, q \sin \theta + p \cos \theta) dp \]
Wigner Function of Coherent and Squeezed Light

\[ W_s(q, p) = \frac{1}{\pi} \exp \left( -e^{2r} q^2 - e^{-2r} p^2 \right) \]
Negative Wigner Function

- A negative Wigner function is one that has a negative value at some position in phase space.
- An example is the Fock states

\[ W_n(q, p) = \frac{(-1)^n}{\pi} \exp\left(-q^2 - p^2\right) L_n \left(2q^2 + 2p^2\right) \]
Conclusion

- Explained the current limitation of an OPO
- Method to increase squeezing by modulating the pump
- Current Status
- Explained Future Experiment
- Any Questions
Setup

Power

OPO

PBS

\( \frac{\lambda}{2} \)

Data Acquisition Device
Possible Future Experiments

• Quantum Tomography of a non-gaussian state
• Quantum Teleportation of a Bright Squeezed state
Setup

Pump → OPO → PBS → LO

Digital Oscilloscope
Some Basic Experiments

- Frequency Domain Squeezing
- Time Domain Squeezing
Time Domain Squeezing
Time-Modulated Squeezing
Quantum Teleportation

In Quantum Teleportation, an unknown quantum state can be transferred from Alice to Bob with high fidelity.
Setup

\[ \omega = 2\pi \times 563.5 \text{ rad/s} \]
\[ \lambda = 532 \text{ nm} \]

\[ \lambda_1 = \lambda_2 = 1064 \text{ nm} \]
\[ \omega_1 = \omega_2 = 2\pi \times 281.7 \text{ rad/s} \]

\[ \int_{-\infty}^{\infty} W(q, p) dp = \langle q | \hat{\rho} | q \rangle \]

\[ \Delta X \]
\[ \Delta P \]
Compare with previous Results
Non-Gaussian States of Light

- Define Non-gaussian