Coherence and optical electron spin rotation in a quantum dot

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Part I
Background: QC with quantum dots, Λ system
Spontaneously generated coherence: theory
Experimental results

Part II
Background: Rabi oscillations, hyperbolic secant pulses
Single-qubit rotations
Quantum computing

**Requirements**
- **Qubit/Scalability** – Operations: arbitrary qubit-rotations and 2-qubit conditional operations
- **Initialization** – Readout – Qubit-specific measurement
- **Long coherence times**

**Two-level system**

Qubit candidates
- Electron spins in QDs
- Nuclear spins
- Atomic levels
- Superconducting qubits
- ...

**Bloch vector**
Two-level QM systems can be represented by a vector on/in a unit-radius sphere

$$|\Psi\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2) e^{i\phi} |1\rangle$$

$$\rho = \alpha^\prime |0\rangle\langle 0| + \beta^\prime |1\rangle\langle 1|$$

$$\alpha^\prime = \beta^\prime = \frac{1}{2}$$

Completely mixed
(Unpolarized)
Quantum dots

- Semiconductor nanostructures with 3D nanometer confinement for electrons/holes
- Atomic-like energy levels
- Fluctuation dots, SADs, gated dots
- Growth axis $\equiv z$

D. Gammon et al., PRL 76, 305 (1996)
QIP with optically controlled electron spins trapped in QDs

- Quantum dot with single excess electron
- e spin carries quantum information
- Operations: optically by Raman transitions via trion
- Trion: bound state of electron and exciton
- Inter-dot coupling:
  - With common cavity mode (Imamoglu et al. PRL ’99)
  - Optical RKKY (C. Piermarocchi et al. PRL ’02)
Energy levels & HH-LH splitting

Bulk

Bands of III-V compounds

| e, ±1/2 \rangle
\begin{array}{c}
J = \frac{1}{2}
\end{array}

| h, ±3/2 \rangle
\begin{array}{c}
J = \frac{3}{2}
\end{array}

| h, ±1/2 \rangle

Quantum dot

\begin{array}{c}
J = \frac{1}{2}
\end{array}

\begin{array}{c}
| e, ±1/2 \rangle
\end{array}

\begin{array}{c}
\begin{array}{c}
J = \frac{3}{2}
\end{array}
\end{array}

Confinement- induced H-L hole splitting
Lambda system in QD

Without B field, no Raman transitions possible: cannot implement qubit operations:

\[ B = 0 \]

\[ |3/2\rangle \quad |\pm 3/2\rangle \quad |1/2\rangle \quad |\pm 1/2\rangle \]

Perpendicular B field mixes spin states, enables Raman transitions:

\[ B_x \neq 0 \]

\[ |3/2\rangle \quad |\pm 3/2\rangle \quad |1/2\rangle \quad |\pm 1/2\rangle \]

Choosing eg \( \sigma^+ \) light yields a Lambda system

\[ |1/2\rangle \pm |\pm 1/2\rangle \equiv |x\rangle \]

\[ |1/2\rangle \pm |\pm 1/2\rangle \equiv |\pm x\rangle \]
Decay & decoherence

- Decay equations of *generic* system known from atomic physics

- Can be derived from a Master equation.  
  **Basic idea:**
  - Start with total system dynamics, ignore (trace out) the bath
  - End up with non unitary evolution for system
    Wavefunction $\rightarrow$ Density matrix

Decay & decoherence come from ignoring a part (‘bath’) of the total system
Example: Spontaneous emission of generic Λ system initially excited

$$|\Psi\rangle = |e\rangle$$

Finally: $$\rho = 0.5 |1\rangle\langle 1| + 0.5 |2\rangle\langle 2|$$

Common ‘wisdom’: spontaneous emission always produces decoherence.
Spontaneously generated coherence (SGC)

- Theoretically predicted in atoms: Spontaneous decay may result in superposition (coherence) of recipient states, i.e. a term $\frac{\partial_t \rho_{12}}{sp} = \Gamma \rho_{ee}$ (Javanainen’92)

- Has not been observed in atoms

Conditions
- $E_{12}$ small
- $d_1 \cdot d_2 \neq 0$
Features of the QD Λ-type system

- Small Zeeman splitting
- 2 transitions have same polarization
- Fluctuation QDs: HH trion splitting $\propto B^3 \rightarrow g_{x,\text{hh}} \approx 0$ (J. G. Tischler et al.) $\rightarrow$ trion does not precess!
- SGC requirements are fulfilled
Instead of energy eigenstates $|\pm x\rangle$ consider the $|\pm z\rangle$ states
\[ \Rightarrow \text{two-level system} \ (| - z\rangle \text{ decoupled by selection rules}) \]
**Experimental setup (theorist’s view)**

**Pump-probe experiment**

Differential transmission as fn of delay time $t_d = \tau_2 - \tau_1$

**Bloch sphere**
Experimental setup (experimentalist’s view)

- Picks up nonlinear response (DTS)
- For low excitation power → 3rd order dominant
- DTS(σ−) − DTS(σ+) ~ S_z

Dutt et al., PRL 94, 227403 (2005)
\[ \Delta T \propto A \cos(2\omega_L t_d - \phi) e^{-\gamma t_d} + B e^{-2\Gamma t_d} \]

Amplitude

\[ A \propto \sqrt{\frac{\gamma^2 + 4 \omega_L^2}{(2\Gamma - \gamma)^2 + 4 \omega_L^2}} \]

Phase

\[ \phi = -\arctan\left(\frac{2 \Gamma_c - \gamma}{2 \omega_L}\right) - \arctan\left(\frac{\gamma}{2 \omega_L}\right) \]

Economou, Liu, Sham and Steel, PRB 71, 195327 (2005)
Calculated & experimental results

Dutt, Cheng, Li, Xu, Li, Berman, Steel, Bracker, Gammon, Economou, Liu, and Sham, PRL 94, 227403 (2005)
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Review of proposals for optical spin rotations in QDs

- **Chen, Piermarocchi, Sham, Steel (PRB ’04):**
  - No explicit frequency selectivity, but $\omega_L \gg \Omega$ (weak pulses)
  - Adiabatically eliminate trion
    - *Implicitly* requires long pulses

- **Kis & Renzoni (PRA ’03):**
  - Stimulated Raman adiabatic passage
  - Requires auxiliary lower level

- **Calarco, Datta, Fedichev, Pazy, Zoller (PRA ’03):**
  - $\pi$ pulse to populate trion/wait/$\pi$ pulse to de-excite trion
  - Suffers from trion decay rate $z$ rotations only
Rabi oscillations

- Two-level system with energy splitting $\omega_o$
- Driven by laser with central frequency $\omega$
- Define detuning $\Delta = \omega_o - \omega$
- Laser can be
  - CW $\rightarrow$ Rabi oscillations in time
  - Pulsed $\rightarrow$ Rabi oscillations as fn of pulse area $A = 2 \int dt \Omega_R(t)$

$\Omega_R = d \cdot E_o$

A two-level system can be mapped onto a spin (pseudospin).
SU(2) dynamics $\rightarrow$ 2$\pi$ rotation: back to $-|g>$
Review of sech pulses in 2–level systems

\[ V_{ge} = \Omega \text{sech}(\beta t) \, e^{i\Delta t} \]

\[ \Delta = \text{detuning} \]
\[ \beta = \text{bandwidth} \]

- Exact solution (Rosen & Zener Phys. Rev. ’32)
- Pulse area can be defined for any \( \Delta \)
- When \( \Omega = \beta \rightarrow 2\pi \) pulse
- Population returns to \( |g> \) with an acquired phase:

\[ \phi = 2 \arctan \left( \frac{\beta}{\Delta} \right) \]

Global for 2lvl sys
Useful in presence of
A third level

Economou et al. PRB 74 (2006)
Use of $2\pi$ sech pulses for rotations: Strategy outline

- By choice of polarization, decouple different two level systems:
  
  $$|T_z\rangle \quad \quad \quad |T_\pi\rangle$$
  
  $$|z\rangle \quad wiggly \quad \quad |\bar{z}\rangle$$
  
  $$\sigma^+$$

- Each time the ground state is a spin state $\hat{n}$ along

- A phase is induced, which is a function of the detuning $\hat{n}$

- Phase $\phi$ on spin $\hat{\sigma}_z$ is a rotation about $\hat{n}$ by $\phi$

- By changing $\hat{n}$ we can span the whole space
I. Small Zeeman splitting: z rotations

Broadband $\sigma^+$ pulse means $\beta \gg \omega_e$

$\rightarrow$ Spin precession $\sim$ ‘frozen’ during pulse

$\rightarrow$ 2-level system + 2 uncoupled levels

$U \simeq \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi} & 0 \\ 0 & 0 & e^{i\phi} \end{bmatrix}$

$|T_z\rangle \quad |\bar{T}_z\rangle$

$\sigma^+$

$|z\rangle \quad |\bar{z}\rangle$

Ultra fast z rotations

$U_{\text{spin}} \simeq \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\phi} \end{bmatrix} = e^{-i\phi/2} \begin{bmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{bmatrix}$

(in the z basis)

Economou, Sham, Wu, Steel, PRB 74, 205415 (2006)
I. Small Zeeman splitting: x rotations

- Use of linearly polarized light decouples the 4-level system to two 2-level systems:
- Detunings for 2 transitions $\Delta_1, \Delta_2$
- Bandwidth $\beta_x$

$2\pi$ sech pulse induces a different phase in $|x\rangle$ and $|\bar{x}\rangle$. The difference of phases is angle of rotation

$$\phi_x = 2 \arctan \frac{\beta_x (\Delta_1 - \Delta_2)}{\Delta_1 \Delta_2 + \beta_x^2}$$

We have designed rotations about two axes, z and x. By combining them we can make any rotation!
Example: $\pi$ rotation about $y$ axis

Fidelity 99.28%

Parameters for InAs QDs used

Economou & Reinecke, cond-mat/070309
II. Large Zeeman splitting

- Above scheme requires large bandwidths for z rotations
- For QDs with large Zeeman splittings such lasers may not be available
- Modification of proposal

Use narrowband pulses to select a $\Lambda$ system
Total laser field

$$\vec{E} = E_x f_x(t) e^{i\omega_x t} \hat{x} + e^{i\alpha} E_y f_y(t) e^{i\omega_y t} \hat{y} + c.c.$$  

Choosing equal detuning and same $f(t)$ creates a coherently trapped state
Bright/dark states determined by phase and relative strength of two lasers
Energy eigenstates $|x\rangle$, $|\bar{x}\rangle$ are related to bright/dark states, by

$$\mathcal{T} = \begin{bmatrix}
\cos \vartheta & -e^{i\alpha} \sin \vartheta \\
e^{-i\alpha} \sin \vartheta & \cos \vartheta
\end{bmatrix}$$

where

$$\tan \vartheta = E_y / E_x$$

Bright state coupling to trion is

$$V_{B,T_x} = \Omega_o f(t) e^{i\Delta t}$$

where

$$\Omega_o = \sqrt{\Omega_x^2 + \Omega_y^2}$$

We want the total pulse acting on bright state to have area $2\pi$:

$$\Omega_o = \beta$$

$$\phi = 2 \arctan \left( \frac{\beta}{\Delta} \right)$$

$$\hat{n} = \left( \cos \vartheta, \sin \vartheta \sin \alpha, \sin \vartheta \cos \alpha \right)$$
Example: $\pi/2$ rotation about z axis

Fidelity 98.84%

Parameters for CdSe QDs used

Economou & Reinecke, cond-mat/0703098
Summary

- SGC has important effect on quantum beats in QDs
- First observation of SGC in QDs (not atoms)
Summary II

- 2π sech pulses to decouple 2 level systems
  \[ \phi = 2 \arctan\left( \frac{\beta}{\Delta} \right) \]

- Phase

- Small Zeeman splitting

- Large Zeeman splitting: CPT scheme

- Simple for experimental demonstration