Lattice hadron physics

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• Strong interactions and lattice QCD

• External field techniques in lattice QCD
  Two examples:

  1. Hadron polarisabilities

  2. EMC effect: nuclear modification of parton distributions
Strong interactions

- What is strongly interacting QCD useful for?
Strong interactions

- What is strongly interacting QCD useful for?
  - Searches for physics beyond the Standard Model
    - Proton decay
    - CP violation: eg $B^0 - \overline{B}^0$
    - DAs for hard processes
    - Neutron EDM
Strong interactions

- What is strongly interacting QCD useful for?
  - Searches for physics beyond the Standard Model
  - Hadron spectra and structure
    - Parton distributions/GPDs
    - Hadron polarisabilities
    - EM and weak form factors
    - Excited states, glueball and exotic spectra

[Most pretty pictures from JLab]
Strong interactions

• What is strongly interacting QCD useful for?
  • Searches for physics beyond the Standard Model
  • Hadron spectra and structure
  • Nuclear structure and interactions

• EMC effect
• Electroweak: $\bar{n}p \rightarrow d\gamma$, $d\nu \rightarrow np$
• $\pi\pi$, $NN$, $YN$, $BB$
Strong interactions

- What is strongly interacting QCD useful for?
  - Searches for physics beyond the Standard Model
  - Hadron spectra and structure
  - Nuclear structure and interactions
  - Sensitivity of nature to QCD parameters
    - $\varepsilon_d$ dependence on $m_q$
    - Fine tunings: stellar carbon production
Strong interactions

- What is strongly interacting QCD useful for?
  - Searches for physics beyond the Standard Model
  - Hadron spectra and structure
  - Nuclear structure and interactions
  - Sensitivity of nature to QCD parameters
  - New forms of matter
    - Non-zero $T$, $\mu$
Strong interactions

• Exotic hadrons within QCD and beyond
  • higher SU(3)_c representation quarks
  • hadrons containing gluinos (SUSY) \cite{Farrar-Fayet}
  • decays and signals at LHC

• Non-perturbative physics in theories beyond the SM: little Higgs models, “hidden valleys”,

• Low dimensional systems (eg graphene)
Graphene
Simple equations ...

- Field equations are easy to write down

\[ \mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_f \bar{\psi}_f (i\not{D} + m_f) \psi_f \]

\[ F^a_{\mu\nu} = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a + g f_{abc} A^\mu_b A^\nu_c \]

\[ \not{D} = \gamma^\mu (\partial^\mu + igA^\mu) \]

- ... but unsolved ... ($$1M$$ Clay Prize)
The running coupling

- Gross, Politzer and Wilczek showed that QCD has a negative $\beta$ function

$$\alpha_s(Q^2) = \frac{g^2}{4\pi} = \frac{4\pi}{(11 - \frac{2}{3} N_f) \log \left[ \frac{Q^2}{\Lambda_{QCD}^2} \right]}$$

- Two important consequences
  - Asymptotic freedom
  - Confinement

Gross, Politzer & Wilczek
2004
Asymptotic Freedom

Expansion in powers of $\alpha_s$ becomes better and better

[Bethke 02]
High energies

- We believe in QCD because high energy calculations beautifully reproduce data.
- Major experiments (TeVatron, RHIC, LHC) collide hadron beams at very high energy.
QCD vs Experiment
Confinement

and up and up ...

[Bethke 02]
Why lattice QCD?

- Typical energies outside of colliders, exploding stars and the Early Universe
  \[ E \sim M_{\text{proton}} \]

- Perturbation theory breaks down
  \[ \alpha_s (M_{\text{proton}}) \sim 1 \]

- Cannot calculate the proton mass

- Need a non-perturbative method
Lattice QCD

- Numerical solution of QCD field equations
- QCD partition function
  \[ Z \sim \int DA_\mu D\psi D\bar{\psi} e^{-S_{QCD}[A,\psi,\bar{\psi}]} \]
- Quark functional integral done exactly
- Observable
  \[ \langle O \rangle \sim \frac{1}{Z} \int DA_\mu \det[M[A]]O[A,M]e^{-S_{QCD}[A]} \]
- Euclidean space

[K Wilson]
Lattice QCD

- Discretise and compactify space-time
- Lattice spacing $a$, volume $L^3 \times T$
Lattice QCD Simplify

- **Quarks** live on lattice sites, **gluons** on links
- Functional integral is finite dimensional but still too many integrals ($>10^7$ !) to do
- Use **Monte Carlo** techniques to estimate
  - Configurations $\{\phi_i\}$ generated according to Boltzmann weight $\det[M] \exp(-S_{QCD})$
  - Observable: $\langle O \rangle \rightarrow \frac{1}{N} \sum_{i=1}^{N} O[\phi_i]$
  - Errors $\sim 1/\sqrt{N}$
Ex: energy spectrum

- Measure correlator \((\chi = \text{source with } q\# \text{ of hadron})\)

\[
G_2(p, t) = \sum_x e^{i p \cdot x} \langle 0 | \chi(x, t) \chi(0, 0) | 0 \rangle
\]

- Long times: only ground state survives

\[
t \to \infty \quad e^{-E_0(p)t} \langle 0 | \chi(0, 0) | E_0, p \rangle \langle E_0, p | \chi(0, 0) | 0 \rangle
\]
Lattice QCD in reality

- Light quark masses are numerically difficult
- Partially-quenched QCD: valence and sea quark masses different (sea quarks are more expensive)

\[
\langle O \rangle \sim \frac{1}{Z} \int \mathcal{D}A_\mu \det[M_s[A]] O[A, M_v] e^{-S_{QCD}[A]}
\]

- Has QCD as a limit
- Quenched QCD (ignore vacuum polarisation) has no connection to QCD
Extrapolations

• To get real world physics from the lattice calculations we need to take: PICTURE
  • Lattice spacing to zero
  • Lattice volume to infinity
  • Quark masses to their physical values
    • Calculations at the physical masses are too demanding for current computers
  • All can be addressed in chiral perturbation theory
External Fields

• The basic idea: compute lattice QCD correlation functions in the presence of controlled external sources (classical fields)

• The response to variation of the strength of the field determines the physics
External fields

\[ E + \delta E \]

Field Lines
Polarisabilities
Hadron polarisabilities

• Hadron polarisabilities describe the deformation of a particle in an external (EM) field

• Quadratic energy shifts from effective Hamiltonian:

\[ H = H_0 - \mu \cdot B - 2\pi \alpha |E|^2 - 2\pi \beta |H|^2 - 2\pi \gamma \sigma \cdot E \times E + \ldots \]

• Electric and magnetic polarisabilities: ability to align with or against the applied field

• Spin and higher order polarisabilities: more detailed view of EM structure
Compton scattering

- Experimentally measured in the low frequency limit of real Compton scattering

\[ T_{\gamma N} = f(\omega, \vec{q}, \vec{q}', \vec{e}, \vec{e}', \vec{\sigma}; Z, \mu, \alpha, \beta, \gamma_1, \ldots, 4) + \mathcal{O}(\omega^4) \]

- Thomson limit and Low–Gell-Mann–Goldberger LET determined by Born terms (charge and magnetic moment)

- Next order given in terms EM and spin polarisabilities
Experiment

- MAMI, Saskatoon, JLab, OOPS, ELSA, HIγS
- EM and 2 combinations of spin polarisabilities are measurable for the proton but difficult experiments
- Neutron accessed via (quasi-)elastic Compton scattering on the deuteron - even more difficult

\[ \alpha_p = 12.0(6), \quad \beta_p = 1.9(6), \quad \alpha_n = 13(2), \quad \beta_n = 3(2) \]
\[ \gamma^{(p)}_\pi = -39(2), \quad \gamma^{(p)}_0 = -1.0(1), \quad \gamma^{(n)}_\pi = 59(4), \]

[de Jaeger & Hyde-Wright 05]

- Sign and small size of polarisabilities indicates tightly bound diamagnetic system - hard to deform
Lattice approaches

1. Four point correlators: \( \langle 0 | \chi(x_1) J^\mu(y_1) J^\nu(y_2) \overline{\chi}(x_2) | 0 \rangle \)
   - Analogous to experimental measurement
   - Difficult - many disconnected contractions
   - Require \( \omega \rightarrow 0 \) extrapolation

2. Energy shifts in two point correlators in external \( U(1) \) field
   - *Quenched QCD*: external field can be added after gauge configurations are generated
   - *QCD*: external field must be known during gauge field generation - multipurpose but costly
Lattice approaches

1. Four point correlators: $\langle 0 | \chi(x_1) J^\mu(y_1) J^\nu(y_2) \bar{\chi}(x_2) | 0 \rangle$
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External field method

- Quenched external EM fields simple to apply:

\[ U^a_\mu(x) \rightarrow U^a_\mu(x) \cdot U^\text{ext}_\mu(x) \]

\[ U^{(a)}_\mu(x) = e^{i a g A^{(a)}_\mu(x)} \]

- E.g.: magnetic field \( \vec{B} = (0, 0, B) \)

\[ U^\text{ext}_0 = U^\text{ext}_2 = U^\text{ext}_3 = 1, \quad U_1(x) = e^{ieB x_2} \]

- Look for shift in energy quadratic in \(|B|\)

\[ C^{\uparrow\uparrow}(\tau, B) = \sum_{\vec{x}} \langle 0 | \chi^{\uparrow}(\vec{x}, t) \bar{\chi}^{\uparrow}(0) | 0 \rangle \]

\[ = \exp \left[ -(M - \mu |B| + 2\pi \beta |B|^2) \tau \right] + \mathcal{O}(|B|^3) \]

Also Landau level contribution

Quantised for periodic links

Magnetic polarisability

Magnetic moment

Also Landau level contribution
Field constraints

• Field values are restricted by a number of constraints
  • Perturbative in EFT: \(|eB|, |eE| < m^2_\pi\)
  • For periodicity of box: e.g. magnetic field

\[
U_\mu(x + L\hat{\nu}) = U_\mu(x)
\]

\[
a^2 |eB| = \frac{2\pi n}{L}, \quad n \in \mathbb{Z}
\]

• Landau levels well represented
External field method

- Can study more than energy shifts - hadronic correlator analysis $\equiv$ effective field theory
- E.g.: charged particle in constant electric field

$$C_{ss'}(\tau; E) = \sum_{\vec{x}} \langle 0 | \chi_s(\vec{x}, t) \bar{\chi}_{s'}(0) | 0 \rangle_E$$

$$= \delta_{s, s'} \exp \left[-(M + 2\pi\alpha|E|^2)\tau - \frac{q^2|E|^2}{6M} \tau^3 \right] + \ldots$$

- Valid for $L^{-1} < m_{\pi}$, $|eE| < m_{\pi}^2$
External field method

• All six polarisabilities can be calculated
  • utilise all information in hadron correlators including spin-flip matrix elements
  • spin polarisabilities require space/time varying U(1) fields: E.g. $\gamma E_1 E_1$

\[ U_{\mu}^{\text{ext}} = e^{i a e A_\mu(x)}, \quad A_\mu(x) = \left( -\frac{a_6 t^2}{2 a}, -i b_6 t, 0, 0 \right) \]

\[ \frac{C_{\uparrow\uparrow}(\vec{p'}, \tau; A)}{C_{\downarrow\downarrow}(\vec{p}, \tau; A)} = \exp \left[ \frac{2\pi}{a} a_6 b_6 \gamma E_1 E_1 \tau \right] + \ldots \]
Quenched polarisabilities

- External field calculations of magnetic moments and EM polarisabilities have a long history
  - Martinelli et al., Bernard et al.: $\mu$ for n, p, $\Delta$ [83]
  - Fiebig et al.: $\alpha$ for neutron [89]
  - Christensen et al.: $\alpha$ for uncharged particles [05]
  - Lee et al.: $\mu$ for baryons [05]
  - Lee et al.: $\beta$ for many baryons and mesons [05]
- Preliminary work on spin polarisabilities
Quenched magnetic polarisabilities

- Use eight “weak” field values

\[ a^2 B = 10^{-6} - 0.13 B - 23 B^2 + 177 B^3 \]
Quenched electric polarisabilities

- Calculations with four field values (pos/neg)
- Neutral particles $n, \Sigma^0, \Xi^0, \Delta^0, \Sigma^{*0}, \Xi^{*0}, \pi^0, K^0, \rho^0, K^{*0}$
Chiral perturbation theory

- Many studies of nucleon polarisabilities in the context of chiral perturbation theory
- Extended to partially-quenched $\chi$PT at finite volume
- Very IR sensitive
- Expect large FV effects

[WD, A Walker-Loud, B Tiburzi 06]
Unquenched calculations

- Full QCD results from unphysical calculations!
- PQQCD: ghost quarks cancel valence loops
- Electric charge matrix:
  \[
  \text{diag}\{q_u, q_d, q_s\}
  \]
  \[
  \text{ddiag}\{q_u q_d q_s q_f, q_f, 0, q_q, q_u q_d q_s\}
  \]
- Turn sea charges off (unphysical hadrons)
- Use chiral perturbation theory to reconstruct (some!!) physical hadrons (take QCD limit)
Lattice polarisabilities

- All EM and spin polarisabilities can be measured with external fields
- Preliminary lattice calculations underway for spin polarisabilities
- Large volume effects and strong mass dependence require large volumes and small masses!
- Higher order and generalised polarisabilities [(doubly)-virtual Compton scattering] are also measurable
- Unquenched? First calculations through USQCD this year
Nuclear structure: EMC effect
EMC effect

- **EMC 1983**: Modification of PDFs in nuclei

- Large effect was a surprise since $\frac{\epsilon}{M} \sim 1\%$

- Can be investigated using lattice QCD
EMC on the lattice

- Simplest manifestation:

\[
R^d(x, Q^2) = \frac{F_2^d(x, Q^2)}{F_2^p(x, Q^2) + F_2^n(x, Q^2)} \neq 1
\]

- LC OPE: DIS structure \(\Rightarrow\) twist two operators

\[
\langle H|\bar{\psi}\gamma^{\mu_1}D^{\mu_2}\ldots D^{\mu_n}\psi|H \rangle = \langle x^n \rangle_H p^{\mu_1}\ldots p^{\mu_n}
\]

\[
\langle x^n \rangle_H = \int_0^1 dx \, x^{n-1} q^H(x) \sim M_2^H(n)
\]

- Shift in moments from nuclear EFT \([JW \text{ Chen} & WD \ 04]\)

\[
\langle x^n \rangle_d = 2\langle x^n \rangle_N + \alpha_n \langle d|(N^\dagger N)^2|d \rangle + \ldots
\]
EMC on the lattice

- Lattice methods combined with EFT can be used to investigate the EMC effect
- Measure shifts in two-particle energy levels in external field coupled to twist-two operators
- Determines two body coefficient $\alpha_n$
- Leading medium modification of moments of PDFs
- Extend to larger $A$ using nuclear EFT

$$S_{QCD} \rightarrow S_{QCD} + \int d^4 x \frac{d}{\mu_1 \ldots \mu_n (x)} \overline{\psi}(x) \gamma^{\mu_1 D_{\mu_2} \ldots D_{\mu_n}} \psi(x)$$

[WD Phys Rev D71 054506]
Finite volume energies

- Maiani-Testa: *impossible to get Minkowski space S-matrix elements from infinite volume Euclidean space Monte-Carlo calculations*

- Lüscher [86]: *two-particle energy levels at finite volume related to scattering amplitude*
Finite volume energies

- Energies satisfy eigenvalue equation [Lüscher 86]

\[ p \cot \delta(p) - \frac{1}{\pi L} S \left( \frac{L^2 p^2}{4\pi^2} \right) = 0 \]

\[ S(x) = \sum_{\vec{n}} \frac{1}{|\vec{n}|^2 - x} - 4\pi \Lambda \]

- Eg: lowest energy level (zero rel. mom.)

\[ E_0 = \frac{4\pi a}{ML^3} \left[ 1 + c_1 \frac{a}{L} + c_2 \left( \frac{a}{L} \right)^2 + \ldots \right] \]

- Calculation of energy levels on the lattice determines scattering parameters
Two particle energies

\[ S(\tilde{\mathbf{p}}^2) = \frac{2}{\mathbf{p}^2} - \frac{1}{\mathbf{p}} \]
Energy levels in BF

- Background field modifies eigenvalue equation

\[ p \cot \delta - \frac{1}{\pi L} S \left( \frac{L^2}{4\pi^2} \left[ p^2 \pm eB \kappa_0 \right] \right) \mp \frac{eB}{2} (L_2 - r_3 \kappa_0) = 0 \]

- Asymptotic expansion of lowest scattering level

\[ E_0^{(m=\pm 1)} = \mp \frac{eB_0}{M} \kappa_0 + \frac{4\pi A_3}{ML^3} \left[ 1 - c_1 \frac{A_3}{L} + c_2 \left( \frac{A_3}{L} \right)^2 + \ldots \right] \]

where

\[ \frac{1}{A_3} = \frac{1}{a_3} \pm \frac{eB_0}{2} L_2 \]

[WD & MJ Savage Nucl Phys A 743, 170]
Energy levels in B field

Matching to lattice measurement determines $L_2$

EFT prediction for behaviour of $m=\pm 1$ energy levels
Energy levels in B field

Matching to lattice measurement determines $L_1$.

$|eB| = 1000$ MeV

NB: box is asymmetric: $4 \times 4 \times 40$ fm$^3$
Reality sets in

• Are these measurements feasible? ✓?

• NPLQCD Collab. 2006 measurement of lowest
  \[ \Delta E = E_{2N} - 2E_N \]
  in \(^3S_1\) and \(^1S_0\) NN channels:
  \[ PRL.97,012001 \]

  • roughly \[ \Delta E = 40 \pm 5 \text{ MeV} \]  (error 10x too large)

• Lighter quark masses ⇒ bigger volumes

• Technical advances: anisotropic lattices, Lüscher-Wolff, algorithms for multiple external fields? ...
EMC effect

• Can also consider polarised/transversity cases

• Modified gauge fields also give
  ➡ Moments of singlet quark PDFs/GPDs
  ➡ $J_q$ in the proton (Ji’s sum rule)

• Extension to $A>2$ on lattice should be possible

• Long term: combine with nuclear EFT to assess nuclear effects in NuTeV anomaly: $F_3^A(x)$
Lattice nuclear physics

• Similar techniques allow investigation of
  
  • Cross-section for neutrino breakup of the deuteron (relevant for calibration of SNO)
  
  • $\varepsilon_d$ vs quark masses (isovector can be done now)
  
  • Deuteron polarisabilities

• Far future: $0\nu-\beta\beta$ decay nuclear matrix elements
More external fields

- Electric dipole moments [Shintani et al. hep-lat/0611032]
- Hadronic parity violation in baryons
- PV NN interactions: $h_{\pi NN}$
- Anapole moments of nucleon [WD, D O’Connell in progress]
Summary

- **Strong interaction physics** is a vital field
- Lattice field theory is the tool of choice
- *External fields in lattice QCD provide a novel technique for investigating strongly interacting aspects of the SM and beyond*

- **Examples**
  - Structure of the proton: *polarisabilities*
  - Structure of the nucleus: *EMC effect*
The End.
Further polarisabilities

- Higher orders in the frequency expansion gives higher order polarisabilities [Holstein et al. ‘99]
- Virtual and doubly virtual Compton scattering leads to generalised polarisabilities [Guichon, Liu & Thomas ‘95]
To be more specific...

\[ T_{\gamma N} = A_1(\omega, \theta) \bar{e}' \cdot \bar{e} + A_2(\omega, \theta) \bar{e}' \cdot \hat{k} \bar{e} \cdot \hat{k}' + i A_3(\omega, \theta) \bar{\sigma} \cdot (\bar{e}' \times \bar{e}) + i A_4(\omega, \theta) \bar{\sigma} \cdot (\hat{k}' \times \hat{k}) \bar{e}' \cdot \bar{e} + i A_5(\omega, \theta) \bar{\sigma} \cdot \left[ (\bar{e}' \times \hat{k}) \bar{e} \cdot \hat{k}' - (\bar{e} \times \hat{k}') \bar{e}' \cdot \hat{k} \right] + i A_6(\omega, \theta) \bar{\sigma} \cdot \left[ (\bar{e}' \times \hat{k}') \bar{e} \cdot \hat{k}' - (\bar{e} \times \hat{k}) \bar{e}' \cdot \hat{k} \right] \]

\[ A_1(\omega, \theta) = -Z^2 \frac{e^2}{M_N} + \frac{e^2}{4M_N^3} \left( \mu^2(1 + \cos \theta) - Z^2 \right) (1 - \cos \theta) \omega^2 + 4\pi(\alpha + \beta \cos \theta)\omega^2 + O(\omega^4) \]

\[ A_2(\omega, \theta) = \frac{e^2}{4M_N^3} (\mu^2 - Z^2)\omega^2 \cos \theta - 4\pi \beta \omega^2 + O(\omega^4) \]

\[ A_3(\omega, \theta) = \frac{e^2 \omega}{2M_N^2} \left( Z(2\mu - Z) - \mu^2 \cos \theta \right) + 4\pi \omega^3 (\gamma_1 - (\gamma_2 + 2\gamma_4) \cos \theta) + O(\omega^5) \]

\[ A_4(\omega, \theta) = -\frac{e^2 \omega}{2M_N^2} \mu^2 + 4\pi \omega^3 \gamma_2 + O(\omega^5) \]

\[ A_5(\omega, \theta) = \frac{e^2 \omega}{2M_N^2} \mu^2 + 4\pi \omega^3 \gamma_4 + O(\omega^5) \]

\[ A_6(\omega, \theta) = -\frac{e^2 \omega}{2M_N^2} Z \mu + 4\pi \omega^3 \gamma_3 + O(\omega^5) \]
EFT correlators

- Pionless effective field theory: cutoff $p < m_\pi$
- Lagrangian

\[
\mathcal{L}_{\text{eff}}(\vec{x}, \tau; A) = \Psi^\dagger(\vec{x}, \tau) \left[ \left( \frac{\partial}{\partial \tau} + i q A_4 \right) + \frac{(-i \vec{\nabla} - q \vec{A})^2}{2M} - \mu \vec{\sigma} \cdot \vec{H} 
\right. \\
+ 2\pi \left( \alpha \vec{E}^2 - \beta \vec{H}^2 \right) - 2\pi i \left( -\gamma_{E_1E_1} \vec{\sigma} \cdot \vec{E} \times \dot{\vec{E}} 
+ \gamma_{M_1M_1} \vec{\sigma} \cdot \vec{H} \times \dot{\vec{H}} + \gamma_{M_1E_2} \sigma^i E^{ij} H^j + \gamma_{E_1M_2} \sigma^i H^{ij} E^j \right) \left. \right] \Psi(\vec{x}, \tau) + \ldots
\]

- Resum interactions with external field
Quenched magnetic polarisabilities

[Lee et al., hep-lat/0509065]

- Calculated for many hadrons

\[ n, p, \Sigma^{\pm,0}, \Xi^{0,-}, \Delta^{++,\pm,0} \Sigma^{*\pm,0}, \Xi^{*0,-}, \Omega, \pi^{\pm,0}, K^{\pm,0}, \rho^{\pm,0}, K^{*\pm,0} \]
Asymptotic expansions

- Other energy levels

\[ E_0 = \frac{4\pi a}{ML^3} \left[ 1 - c_1 \left( \frac{a}{L} \right) + c_2 \left( \frac{a}{L} \right)^2 + \ldots \right] \]

\[ E_1 = \frac{4\pi^2}{ML^2} - \frac{12 \tan \delta_0}{ML^2} \left[ 1 + c'_1 \tan \delta_0 + c'_2 \tan \delta_0 + \ldots \right] \]

\[ E^{(3S_1)}_{-1} = -\frac{\gamma_0^2}{M} \left[ 1 + \frac{12}{\gamma_0 L (1 - \gamma_0 r_3)} e^{-\gamma_0 L} + \ldots \right] \]

- Expansions also for L/a [Beane et al.]
Landau levels

- Infinite volume: transverse single-particle modes are LL

\[ \hat{H} = \frac{\hat{p}^2}{2M} + \frac{1}{2} M \omega^2 (\hat{x}^2 + \hat{y}^2) + \frac{eB_0}{2M} \hat{l}_z \quad \omega = \frac{|eB_0|}{2M} \]

\[ E_p^{(n)}(B_0) = M + \frac{|eB_0|}{M} \left( n + \frac{1}{2} \right) + \frac{p_z^2}{2M} + \mu_p B_0 + 4\pi \frac{\beta_p}{2} B_0^2 \]

- Finite volume: potential bounded & perturbative
- Require weak field: \( |eB_0| < \frac{8\sqrt{3\pi}}{L^2_\perp} \)
- Large z direction: low energy levels longitudinal

\[ \Rightarrow \text{Asymmetric boxes} \]
Asymmetric boxes

- Asymmetry suppresses non-plane-wave-arity
- Eigenvalue equation modified

\[ S(\tilde{p}^2) \rightarrow S(\tilde{p}^2; \eta_1, \eta_2) = \frac{1}{\eta_1 \eta_2} \sum_{\tilde{n}} \Lambda_n \frac{1}{|\tilde{n}|^2 - \tilde{p}^2} - 4\pi \Lambda_n \]

- Asymptotic expansion

\[ E_0 = \frac{4\pi a}{\eta_1 \eta_2 M L^3} \left[ 1 - c_1(\eta_1, \eta_2) \left( \frac{a}{L} \right) + c_2(\eta_1, \eta_2) \left( \frac{a}{L} \right)^2 + \ldots \right] \]

- Geometric coefficients

\[ c_1(\eta_1, \eta_2) = \frac{1}{\pi} \left( \frac{1}{\eta_1 \eta_2} \sum_{\tilde{n} \neq 0} \frac{1}{|\tilde{n}|^2 - 4\pi \Lambda_n} \right) \]
Magic boxes

- Asymmetries exist where subleading FV effects are suppressed: $c_i(\eta_1, \eta_2)=0$
\[ np \rightarrow d \gamma: {^3S_1} - {^1S_0} \quad m = 0 \]

Matching to lattice measurement determines \( L_1 \)

NB: box is asymmetric: 4x4x40 fm\(^3\)
\( \nu d \rightarrow np: \text{EW BF} \)

Matching to lattice measurement determines \( L_{1,A} \)

NB: box is asymmetric: 4x4x40 fm\(^3\)

\[ |g_W| = 6 \text{ MeV} \]
NuTeV anomaly: $\sin^2 \theta_W$

- Neutrino deep-inelastic scattering
- Paschos-Wolfenstein relation:
  \[
  R^- = \frac{\sigma_{\nu N}^{NC} - \sigma_{\bar{\nu} N}^{NC}}{\sigma_{\nu N}^{CC} - \sigma_{\bar{\nu} N}^{CC}} = \frac{1}{2} - \sin^2 \theta_W
  \]
- NuTeV measure CC and NC neutrino scattering on steel target at Fermilab
- Extract the weak mixing angle
NuTeV anomaly: $\sin^2 \theta_W$
NuTeV anomaly: $\sin^2 \theta_W$

- Corrected Paschos-Wolfenstein relation:

$$R_A^- = \frac{\sigma^{\nu A}_{NC} - \sigma^{\bar{\nu} A}_{NC}}{\sigma^{\nu A}_{CC} - \sigma^{\bar{\nu} A}_{CC}} = \frac{1}{2} - \sin^2 \theta_W + \epsilon_v + \epsilon_n + \epsilon_s + \epsilon_c$$

- NuTeV take some of this into account

- Many authors find significant reduction in NuTeV significance from hadronic physics
Nuclear structure: EMC effect
EMC effect

- **EMC 1983:** Modification of PDFs in nuclei

- Large effect was a surprise since $\epsilon/M \sim 1\%$

- Can be investigated using lattice QCD
Two nucleon states

- Brief introduction to two particles in LQCD
- Two particle scattering phase shift (ERE)

\[ S = e^{2i\delta(p)} \quad p \cot \delta(p) = -\frac{1}{a} + rp^2 + \ldots \]

- $^1S_0$ channel: $a_1 = -23.7 \text{ fm} \quad r_1 = 2.7 \text{ fm}$
- $^3S_1(^3D_1)$ channel: $a_3 = 5.4 \text{ fm} \quad r_3 = 1.8 \text{ fm}$
- Deuteron: $B = 2.2 \text{ MeV}$
NN in EFT

- Extension of $\chi$PT to two nucleon systems well developed [Weinberg 90, ...]
- For $|p| \ll m_\pi$, pions can be integrated out ➡ theory of contact interactions amongst nucleons (similar to eff. range expansion)
- Scattering amplitude given by bubble sum

Infinite volume

\[
A = \frac{\sum_n C_{2n} p^{2n}}{1 - I_0(p) \sum_n C_{2n} p^{2n}}
\]

[Diagram of bubble sum]

\[
I_0(p) = \mu^{4-d} \int \frac{d^{d-1}k}{E - |k|^2/M + i\epsilon} = -\frac{M}{4\pi} (\mu + i p)
\]
Maiani-Testa: impossible to get Minkowski space S-matrix elements from infinite volume Euclidean space Monte-Carlo calculations

Lüscher [86]: two-particle energy levels at finite volume related to scattering amplitude
Finite volume energies

\[ A(L) = \frac{\sum_n C_{2n}p^{2n}}{1 - I_0(L)\sum_n C_{2n}p^{2n}} \]

\[ p \cot \delta(p) = \frac{4\pi}{M} \frac{1}{\sum_n C_{2n}p^{2n}} + \mu \]

\[ 0 = A^{-1}(L) = p \cot \delta(p) - \frac{M\mu}{4\pi} - I_0^{pds}(L) \]

\[ I_0^{pds}(L) = \frac{1}{L^3} \sum_k \frac{1}{E - |k|^2/M} \]

\[ = \frac{1}{L^3} \sum_k^\Lambda \frac{1}{E - |k|^2/M} + \int^\Lambda \frac{d^3k/(2\pi)^3}{|k|^2/M} - \int_{pds} \frac{d^3k/(2\pi)^3}{|k|^2/M} \]

\[ = \frac{M}{4\pi} \left[ -\frac{1}{\pi L} \sum_n^\Lambda \frac{1}{|n|^2 - \frac{L^2EM}{4\pi^2}} - 4\frac{\Lambda}{L} - \mu \right] \]
Finite volume energies

- Energies satisfy eigenvalue equation \([\text{Lüscher 86}]\)

\[
p \cot \delta(p) - \frac{1}{\pi L} S \left( \frac{L^2 p^2}{4\pi^2} \right) = 0
\]

- Eg: lowest energy level (zero rel. mom.)

\[
S(x) = \sum_{\vec{n}} \frac{1}{|\vec{n}|^2 - x} - 4\pi\Lambda
\]

- Calculation of energy levels on the lattice determines scattering parameters

\[
E_0 = \frac{4\pi a}{ML^3} \left[ 1 + c_1 \frac{a}{L} + c_2 \left( \frac{a}{L} \right)^2 + \ldots \right]
\]

known coefficients
Two particle energies
**EMC on the lattice**

- **Simplest manifestation:**
  \[ R^d(x, Q^2) = \frac{F^d_2(x, Q^2)}{F^p_2(x, Q^2) + F^n_2(x, Q^2)} \neq 1 \]

- **LC OPE: DIS structure \( \Rightarrow \) twist two operators**
  \[
  \langle H | \bar{\psi} \gamma^{\{\mu_1} D^{\mu_2} \ldots D^{\mu_n}\} \psi | H \rangle = \langle x^n \rangle_H p^{\mu_1} \ldots p^{\mu_n} \\
  \langle x^n \rangle_H = \int_0^1 dx \, x^{n-1} q^H(x) \sim M^H_2(n)
  \]

- **Lattice methods combined with EFT can be used to investigate the EMC effect**
- **Look for shift of two particle energy levels in external field coupled to twist two operators**

[WD Phys Rev D71 054506]
Background fields

- Demonstrate technique with electromagnetic fields
- Magnetic moment of the deuteron...

\[ \mu_d = 0.8574382329(92) \mu_N \]
Background fields

- Constant magnetic fields shift spin-$1/2$ particle masses
  \[ M_{\uparrow\downarrow} = M_0 \pm \mu |B| + 4\pi \beta |B|^2 \]

- Two nucleon states
- Levels split and mix
- Landau levels: consider asymmetric boxes
- Similar for electro-weak fields and twist-two fields
EFT two-body currents

- Two-body contributions

\[ \langle d|\mathcal{O}|d \rangle = \left( \begin{array}{ccc} \ldots \end{array} \right) + \left( \begin{array}{ccc} \ldots \end{array} \right) + \ldots \]

- Magnetic moment: two body modification \( L_2 \)

\[ \mu_d = \frac{2}{1 - \gamma r_3} (\gamma L_2 + \kappa_0) \]

- Twist-two current: leading EMC effect \( \alpha_n \) (more complicated as necessary to include pions)

\[ \langle x^n \rangle_d = 2\langle x^n \rangle_N + \alpha_n \langle d| (N^\dagger N)^2 |d \rangle + \ldots \]

[JW Chen, & WD PLB 625,165]
Energy levels in BF

- Background field modifies eigenvalue equation

\[ p \ cot \delta - \frac{1}{\pi L} S \left( \frac{L^2}{4\pi^2} \left[ p^2 \pm eB\kappa_0 \right] \right) \mp \frac{eB}{2} (L_2 - r_3\kappa_0) = 0 \]

- Asymptotic expansion of lowest scattering level

\[ E_{0}^{(m=\pm1)} = \mp \frac{eB_0}{M} \kappa_0 + \frac{4\pi A_3}{ML^3} \left[ 1 - c_1 \frac{A_3}{L} + c_2 \left( \frac{A_3}{L} \right)^2 + \ldots \right] \]

where

\[ \frac{1}{A_3} = \frac{1}{a_3} \pm \frac{eB_0}{2} L_2 \]

[WD & MJ Savage Nucl Phys A 743, 170]
Energy levels in BF

- Background field modifies eigenvalue equation

\[ p \cot \delta - \frac{1}{\pi L} S' \left( \frac{L^2}{4\pi^2} \left[ p^2 \pm eB\kappa_0 \right] \right) \mp \frac{eB}{2} \left( L_2 - r_3\kappa_0 \right) = 0 \]

- Mixes \( ^1S_0 \) and \( ^3S_1 \) m=0 states (coupled channels)

\[
\begin{bmatrix}
    p \cot \delta_1 - \frac{S_+ + S_-}{2\pi L} \\
    p \cot \delta_3 - \frac{S_+ + S_-}{2\pi L}
\end{bmatrix}
\begin{bmatrix}
    p \cot \delta_1 - \frac{S_+ + S_-}{2\pi L} \\
    p \cot \delta_3 - \frac{S_+ + S_-}{2\pi L}
\end{bmatrix} = \left[ \frac{eBL_1}{2} + \frac{S_+ - S_-}{2\pi L} \right]^2
\]

where \( S_\pm = S \left( \frac{L^2}{4\pi^2} \left[ p^2 \pm eB\kappa_1 \right] \right) \)

[WD & MJ Savage Nucl Phys A 743, 170]
Energy levels in B field

Matching to lattice measurement determines $L_2$

EFT prediction for behaviour of $m=\pm 1$ energy levels

$|e B| = 1000 \text{ MeV}^2$

$|e B| = 1000 \text{ MeV}^2$

$1^{st}$ excited state, $m=\pm 1$

$2^{nd}$ excited state, $m=\pm 1$

Bound state, $m=\pm 1$
$np \rightarrow d\gamma: ^3S_1 - ^1S_0 \ m=0$

Matching to lattice measurement determines $L_1$. 

$|eB| = 1000 \text{ MeV}^2$

NB: box is asymmetric: 4x4x40 fm$^3$
EMC on the lattice

- Lattice methods combined with EFT can be used to investigate the EMC effect
- Measure shifts in two-particle energy levels in external field coupled to twist-two operators
- Determines two body coefficient $\alpha_n$
- Leading medium modification of moments of PDFs
- Extend to larger $A$ using nuclear EFT

$$S_{QCD} \rightarrow S_{QCD} + \int d^4x \frac{\Omega_{\mu_1...\mu_n}(x)}{S} \bar{\psi}(x) \gamma^{\{\mu_1 D^{\mu_2} \ldots D^{\mu_n}\}} \psi(x)$$

[WD Phys Rev D71 054506]
Reality sets in

- Are these measurements feasible? ✓?
- NPLQCD Collab. 2006 measurement of lowest \( \Delta E = E_{2N} - 2E_N \) in \( ^3S_1 \) and \( ^1S_0 \) NN channels: PRL.97,012001
  - roughly \( \Delta E = 40 \pm 5 \text{ MeV} \) (error 10x too large)
- Lighter quark masses* \( \Rightarrow \) bigger volumes
- Technical advances: anisotropic lattices, Lüscher-Wolff, *algorithms for multiple external fields?* ...
- Moore’s Law is a powerful thing
EMC effect

• Can also consider polarised/transversity cases

• Modified gauge fields also give

  ➡ Moments of singlet quark PDFs/GPDs

  ➡ Quark contribution to total angular momentum of the proton (Ji’s sum rule)

• Extension to $A>2$ on lattice should be possible

• Technical tools to develop for 3+ particles

• Long term: combine with nuclear EFT to assess nuclear effects in NuTeV anomaly: $F_3^A(x)$
Lattice nuclear physics

- Similar techniques allow investigation of
  - Cross-section for neutrino breakup of the deuteron (relevant for calibration of SNO)
  - $\varepsilon_d$ vs quark masses (isovector can be done now)
  - Deuteron polarisabilities
  - Far future: $0\nu-\beta\beta$ decay nuclear matrix elements
NN scattering lengths

- This work
- BBSvK - NLO
- Experiment
Stellar $^{12}\text{C}$ production

\[
\begin{align*}
E \quad [\text{MeV}] & \quad 7.275 \quad 7.3667 \quad 7.654 \\
3 \quad ^4\text{He} & \quad ^8\text{Be} + ^4\text{He} & \quad ^{12}\text{C}^* \\
4.4 & \quad ^{12}\text{C} & \quad 0 \\
0 & \quad ^{12}\text{C} & \quad ^{16}\text{O}
\end{align*}
\]

Helium  Beryllium  Carbon

\[
\begin{align*}
E \quad [\text{MeV}] & \quad 7.1616 \\
^{12}\text{C} + ^4\text{He} & \quad ^{16}\text{O} & \quad 7.1187 \\
0 & \quad ^{16}\text{O}
\end{align*}
\]
Stellar $^{12}\text{C}$ production

- Why are these levels in exactly the right places for carbon-based life?
- The strong interaction must explain
- How sensitive are they to fundamental parameters (quark masses, QCD scale)?
- Many other nuclear fine-tunings: deuteron binding energy, $M_n > M_p$, ...
EMC fits

\[ R^A(x) \]

For various values of \( A \): 4, 14, 63, 197, 7, 27, 107, 208, 9, 40, 118, 12, 56, 131.
EMC fits

\[ R^A(x) \]
Chiral perturbation theory

- Pattern of **chiral symmetry breaking** in QCD
  
  ![Pions](image)
  
  Pions (pseudo-Goldstone bosons) very light and dominate low energy observables

- **Effective field theory** of low energy dynamics
  
  - Integrate out modes above scale $\Lambda_\chi \sim 1 \text{ GeV}$

- **Lagrangian**: all operators consistent with $\chi$ symm. with short distance physics encoded in LECs

- Power counting: expansion in $\frac{m_\pi}{\Lambda_\chi}$, $\frac{q}{\Lambda_\chi}$

- Heavy baryon $\chi$PT: nucleons and $\Delta$ resonances
Chiral perturbation theory

- Low energy observables have chiral expansion
- $\mathcal{O} \sim \{\text{terms analytic in } m_q, p\} + \{\text{chiral loops}\}$
- Chiral loops
  - Long range pion dynamics
  - Non-analytic mass dependence
  - Known coefficients
- Analytic terms have unknown coefficients
  - Can be fit from experiment or lattice data
- Finite $V$ and finite $a$ effects can be included
Partially-quenched $\chi$PT

- Low energy effective theory for PQ-QCD
- Spontaneously broken (graded) symmetry

$SU(2)_L \times SU(2)_R \rightarrow SU(4|2)_L \times SU(4|2)_R$

- Valence, sea and ghost quarks lead to both bosonic and fermionic Goldstone mesons
- PQ$\chi$PT LECs are a superset of those in $\chi$PT

[Bernard & Golterman, Sharpe]
PQχPT contributions

Anomalous $\pi^0 \rightarrow \gamma\gamma$

$\Delta$-pole graphs

Loops
Wess-Zumino-Witten

- Chiral anomaly contributes through $\pi^0 \to \gamma \gamma$

$$L_{\pi^0 \gamma \gamma} = \frac{3e^2}{16\pi^2 f} \text{tr} [\Phi Q^2] \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

- Extension to partially quenched QCD non-trivial

$$L_{\pi^0 \gamma \gamma}^{PQ} = -\frac{3e^2}{16\pi^2 f} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \left[ \text{tr} [\Phi Q^2] + a_2 \text{tr} [\Phi Q] \text{tr} [Q] \right]$$

- No need to extend Witten’s global quantisation construction to graded Lie groups
Infinite volume results

- Proton electric polarisability

\[
\alpha_p = \frac{e^2}{4\pi f^2} \left[ \frac{5G_B}{192\pi} \frac{1}{m_\pi} + \frac{5G'_B}{192\pi} \frac{1}{m_{u,j}} + \frac{G_T}{72\pi^2} F_\alpha(m_\pi) + \frac{G'_T}{72\pi^2} F_\alpha(m_{u,j}) \right]
\]

- Results for other polarisabilities similar but also have contributions from anomaly and \(\Delta\) poles

\[
F_\alpha(m) = \frac{9\Delta}{\Delta^2 - m^2} - \frac{\Delta^2 - 10m^2}{2(\Delta^2 - m^2)^{3/2}} \ln \left[ \frac{\Delta - \sqrt{\Delta^2 - m^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m^2 + i\epsilon}} \right]
\]

Singular in chiral limit

Non-analytic function involving \(\Delta\) isobar

Involve axial couplings and quark charges
$\chi$PT at finite volume

- Volume dependence can be incorporated depending on pion mass and volume

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_\pi L \gg 1$</td>
<td>p-regime</td>
</tr>
<tr>
<td>$\mu_{\text{had}} L \gg 1$</td>
<td>$\varepsilon$-regime (pion zero modes become non-perturbative)</td>
</tr>
<tr>
<td>$\mu_{\text{had}} L \lesssim 1$</td>
<td>“Out of luck”-regime</td>
</tr>
</tbody>
</table>
Finite volume effects

- Polarisabilities are very sensitive to infrared scales
  
  $\Rightarrow$ Expect large FV effects in lattice calculations

- Easily included in EFT for large volumes

- Quantised momenta: $\mathbf{k} = \frac{2\pi}{L} \mathbf{n}$ for $n_i \in \mathbb{Z}$

- Momentum integrals $\Rightarrow$ mode sums

\[
\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_\pi^2 + i\epsilon} \rightarrow \int \frac{dk_0}{2\pi L^3} \sum \frac{1}{k^2_0 - |\mathbf{k}|^2 - m_\pi^2 + i\epsilon}
\]

\[
\rightarrow \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_\pi^2 + i\epsilon} + \frac{m_\pi^2}{4\pi^2} \sum_{n \neq 0} \frac{1}{|\mathbf{n}|L} K_1(|\mathbf{n}|m_\pi L)
\]

Poisson summation $\sum_{\mathbf{n}} \delta^{(3)}(\mathbf{y} - \mathbf{n}) = \sum_{\mathbf{m}} e^{2\pi i \mathbf{m} \cdot \mathbf{y}}$

$m_\pi L \rightarrow \infty \sqrt{m_\pi / 32\pi^3 L^3} \exp(-m_\pi L)$
Volume Dependence: Neutron

\[ \Delta \alpha_n \]
\[ \Delta \beta_n \]
\[ \Delta \gamma_{1n} \]
\[ \Delta \gamma_{2n} \]
\[ \alpha(L) = \frac{e^2}{1152\pi f^2} \int_0^\infty d\lambda \left[ 3 G_B \mathcal{F}_\alpha(M_{uu}) + 3 G_B' \mathcal{F}_\alpha(M_{uj}) + 8 G_T \mathcal{F}_\alpha(M_{uu}^\Delta) + 8 G_T' \mathcal{F}_\alpha(M_{uj}^\Delta) \right] \]

\[ M_{ab}^\Delta = \sqrt{m_{ab}^2 + 2\lambda\Delta + \lambda^2} \]

\[ \mathcal{F}_\alpha(m) = 180\lambda^2 \mathcal{I}_{\frac{7}{2}}(m) + 190 \mathcal{J}_{\frac{7}{2}}(m) - 280\lambda^2 \mathcal{J}_{\frac{9}{2}}(m) - 455 \mathcal{K}_{\frac{9}{2}}(m) + 315\lambda^2 \mathcal{K}_{\frac{11}{2}}(m) + 252 \mathcal{L}_{\frac{11}{2}}(m) \]

\[ \mathcal{I}_\beta(M) = \sum_{\vec{n}} \frac{E_{1-\beta}(|\vec{n}|^2 + x^2)}{L^3 \Gamma(\beta)} + \frac{\pi^\frac{3}{2}}{\Gamma(\beta)L^3} \int_0^1 dt t^{\beta-5/2} e^{-t x^2} \left[ \sum_{\vec{n}\neq 0} e^{-\frac{\pi^2|\vec{n}|^2}{t}} + 1 \right] \]

\[ \mathcal{L}_\beta(M) = \mathcal{I}_{\beta-3}(M) - 3M^2 \mathcal{I}_{\beta-2}(M) + 3M^4 \mathcal{I}_{\beta-1}(M) - M^6 \mathcal{I}_\beta(M) \]

\[ \mathcal{K}_\beta(M) = \mathcal{I}_{\beta-2}(M) - 2M^2 \mathcal{I}_{\beta-1}(M) + M^4 \mathcal{I}_\beta(M) \]

\[ \mathcal{J}_\beta(M) = \mathcal{I}_{\beta-1}(M) - M^2 \mathcal{I}_\beta(M) \]
Volume Dependence: Proton

\[ \Delta X = \frac{X(L) - X(\infty)}{X(\infty)} \]
Volume Dependence: Quenched

\[ \Delta \alpha_p \]

\[ \Delta \beta_p \]

\[ \Delta \gamma_{1p} \]

\[ \Delta \gamma_{2p} \]

\[ m_\pi = 500 \text{ MeV} \]
Thomson Limit \((\omega = 0)\)

- Thomson limit for photon-neutron scattering 
  \(\rightarrow\) Vanishes at infinite volume!
Hadronic parity violation
Hadronic PV

- Hadronic PV arises from W/Z exchange within the hadronic state
- CP violation in kaons, $B^0$, $B_s$
- Consequences for baryons:
  - P-odd NN interactions
  - P-odd anapole EM form-factor [Zeldovich 58]
Hadronic PV

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- Consequences for baryons:
  - P-odd NN interactions
  - P-odd anapole EM form-factor [Zeldovich 58]
Anapole Form Factor

- Decomposition of hadronic EM current

\[
\langle N(p')|J_{em}^\mu|N(p)\rangle = \bar{u}(p') \left[ \gamma^\mu F_1 + \frac{\sigma^{\mu\nu} q_\nu}{2M} F_2 \right. \\
+ \sigma^{\mu\nu} q_\nu \gamma_5 F_D + \left( q^2 \gamma^\mu - \frac{q\mu}{q} \right) \gamma_5 F_A \bigg] u(p)
\]

- PV: electric dipole and anapole form-factors

- Weak interactions produce AFF

- \( F_A(0) \) = Anapole Moment: magnetic field of toroidal winding carrying current

- Vanishes for real photons
Anapole Form Factor
Anapole Form Factor

- P-odd, T-even contribution to EM processes
- Measured in $^{133}\text{Cs}$ [Wood... 97]: $Z$ enhancement
- Significant source of uncertainty in PV ep scattering: SAMPLE, HAPPEX, G0
- Only estimate for AM from chiral PT: $A_1 = -0.11(44), A_0 = 0.02(26)$ [Zhu et al 00]
- Direct measurement in LQCD very useful
Anapole Form Factor

- Hadronic scales: integrate out W/Z
- Operator product expansion (effective H)

\[ \sum_i C_i \langle N | T J^\mu_{\text{em}} O_i | N \rangle \]
Anapole Form Factor

- Short distance physics encoded in Wilson coefficients $C_i(x, M_W, \mu)$
- Perturbatively calculable
- Leading operators are dimension-6 four quark operators (12 operators) eg:

\[
O_1 = (\overline{q} \gamma_\mu q)_{aa} (\overline{q} \gamma^\mu \gamma_5 \tau_3 q)_{bb}
\]

\[
O_4 = (\overline{q} \gamma_\mu \gamma_5 q)_{ab} (\overline{q} \gamma^\mu \tau_3 q)_{ab}
\]

\[
O_4^s = (\overline{s} \gamma_\mu \gamma_5 s)_{ab} (\overline{q} \gamma^\mu \tau_3 q)_{ab}
\]

- Other operators suppressed by $1/M_W$

[Kaplan & Savage 92]
Lattice AFF

- Computation of the anapole moment/FF very involved on the lattice
- $O(20)$ Wick contractions per four quark operator (x12): possible but unmanageable
- Many (doubly) "disconnected contributions": hard to determine
Lattice AFF

- Greatly simplified using external fields
- Two methods
  - 4Q matrix elements in external EM field
  - Current matrix element in “EW” field
Weak scales on the lattice

- Currently, lattice spacing \( a \sim 0.1 \) fm
- Physical \( M_W \) requires separations \( x \ll a \)
- In the OPE all short distance physics in Wilson coefficients (perturbation theory)
- We are not limited to physical W/Z exchange!

[WD & CJD Lin 05, Dawson et al 97]
Method

- Consider $S_{QCD} + \int_x \Omega^\mu(x) \bar{q}(x) \Gamma^\mu q(x)$ with

$$\Omega^\mu(x) = \omega \epsilon^\mu \exp \left[ -M_X \sqrt{x^2} \right]$$
Related stuff

- Technique also allows extraction of the PV pion-nucleon coupling $h_{\pi NN}$
- Similar to $K \rightarrow \pi\pi$ decays
- First place to start
- Electric dipole moment also accessed using external fields (electric) [Shintani et al. hep-lat/0611032]