Measurement of the $\pi^0$ Lifetime: 
QCD Axial Anomaly and Chiral Corrections

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• Symmetries of QCD and their fate
• Spontaneous Chiral symmetry breaking $\Rightarrow$ pions
  $\Rightarrow$ low energy theorems
• threshold electromagnetic pion production
• $\pi^0 \rightarrow \gamma \gamma$: Axial anomaly, chiral corrections $\sim m_d - m_u$
• Previous experiments
• Primex Experiment at Jefferson Lab
• Conclusions
\[
m_\pi^2 = B(m_u + m_d) \approx (140 \text{ MeV})^2 \rightarrow 0
\]

\[
m_u \approx 5 \text{ MeV}, \quad m_d \approx 9 \text{ MeV}
\]

\[
m_s \approx 140 \text{ MeV}
\]
A large range of physical phenomena
The experiments require a wide range of techniques
Spontaneous Breaking(Hiding) of Chiral Symmetry in QCD

• For massless particles \( h = \sigma \cdot p = \pm 1 \) is conserved
• For massless quarks \( L_{QC D} \) conserves chiral symmetry
• Therefore each state should have an opposite parity partner: Wigner - Weyl manifestation of the symmetry
• Since this is not observed in nature chiral symmetry has been spontaneously hidden
• The symmetry is exhibited by the appearance of massless, pseudoscalar (Nambu- Goldstone) Bosons
  • \( |p> 1/2^+ , |p> |\pi> 1/2^- \) are degenerate
\[ L_{QCD} = L_0 + L_m \]

chiral symmetry is explicitly broken by quark mass effects

Nambu-Goldstone Boson acquire mass

\[ L_m = A(m_u + m_d) + B(m_u - m_d) \]
\[ m_d / m_u \approx 1.8 \]

isospin broken by strong and EM interaction

strong int. effect \( \approx (m_d - m_u) / \Lambda_{(QCD)} \approx 2\% \)
Symmetry becomes Dynamics

• $\pi h$ system has to have gradient coupling due to the pseudoscalar nature of the pion
• weak in the s wave, generally strong in the p wave
• At low energies the interaction vanishes
• this can be systematically exploited $\Rightarrow$ effective field theory of QCD (ChPT)
- **low energy $\pi$-hadron scattering**
  
  pure Nambu-Goldstone Boson: $a(\pi h) = 0$

  strong interactions: $a(\pi h) \approx 1/m_\pi \approx 1$ fm

  **PCAC calculation by Weinberg (1966)**

  lowest order ChPT calculation

  $\Lambda =$chiral sym. breaking scale $\approx 4\pi F_\pi \approx 1$ GeV

  Expect chiral corrections of order $(m_\pi/\Lambda)^2 \approx 0.02$

  **threshold $\gamma^* N \rightarrow \pi N$**

  s and p wave production amplitudes

  there are ChPT formulas [Bernard, Meissner, Kaiser]

  the chiral limit for s wave amplitudes

  $A(\gamma^* N \rightarrow \pi^0 N) \rightarrow 0$

  $A(\gamma^* N \rightarrow \pi^\pm N') \neq 0$ and large (Kroll-Ruderman theorem)
Some Experimental Tests of Spontaneous Chiral Symmetry Hiding

- $\pi \pi$ scattering
- atomic $\pi^- p$ energies, widths
- $\pi N$ scattering
- $\gamma^* p \rightarrow \pi^0 p$
**Mainz**

**Experimental setup** $\bar{\gamma} + p \rightarrow p + \pi^0$

- Linearly polarized photons: $\vec{E} \perp$ and $\parallel$ to TAPS
  - $\rightarrow N_\perp$ and $N_\parallel$

- 6 blocks with 64 modules each

- Forward wall with 120 modules

- TAPS - setup
  - Array of 504 BaF$_2$ - detectors

- $E_0 = 405$ MeV

- $E_\gamma = E_0 - E_{e^-}$
  - $E_\gamma = 140 - 350$ MeV
  - $\Delta E_\gamma = 1$ MeV
\[ \gamma p \rightarrow \pi^0 p \quad \text{Mainz data, ChPT} \]
\[ \gamma \rightarrow \pi^0 p \]  
Mainz data

ChPT: $O(p^4)$

ChPT: $O(p^3)$

DR

\[ \Sigma \]

\[ \theta_{\pi^0}^{\text{cms}} / \text{deg} \]

MAMI/TAPS (2001)  
(A. Schmidt et al.)
Unitary Cusp \( \gamma p \rightarrow \pi^0 p \)

\[ \beta = E_{0+}(\gamma p \rightarrow \pi^+ n) a_{cex}(\pi^0 p \leftrightarrow \pi^+ n) \]

cusp sign and magnitude
Mainz A1 ep → e'p π⁰ Weis, Merkel,.. Q²=0.05 GeV²

\[ A_{TL} \Rightarrow \text{Im } L_{0+} \]
Anomaly

When a symmetry of the classical theory is not present in the quantized version

In QCD the anomaly is not anomalous
It is an essential part of the theory

It is responsible for the large \( \eta' \) mass
so that it is not the 9th Goldstone Boson

It is primarily responsible for the \( \pi^0 \rightarrow \gamma \gamma \) decay mode
Opportunity to perform a precision $\pi^0$ lifetime measurement

• $\pi^0$ is the lightest hadron
  spontaneous chiral symmetry hiding:
  $m(\pi) \approx 140$ MeV
  + EM: $m(\pi^\pm) - m(\pi^0) = 4.6$ MeV

• EM decay $\pi^0 \rightarrow \gamma \gamma$ (BR $=98.8 \pm 0.032\%$)
  Axial anomaly dominant
History: $\pi^0$ Lifetime Experiments

• 1947 $\pi^+$ discovered in cosmic rays
• 1950 $\pi^0$ discovered, cosmic rays, Berkeley cyclotron
  $\pi^0 \rightarrow \gamma \gamma$ decay mode observed
  lifetime too short for electronic measurement
• $\tau < 10^{-15}$ sec established by 1957
  $K^+ \rightarrow \pi^+ \pi^0$ emulsion experiment ($d < 0.5\mu$)
• 1951: Primakoff effect $\gamma \gamma \rightarrow \pi^0$ : experiments 1970-5
• PDB data base established by 1988
History: \( \pi^0 \) Lifetime Theory

- PCAC predicts \( A_{\pi\gamma\gamma} = 0 \) in the chiral limit

- 1968 Adler, Bell, Jackiw discover the axial anomaly

\[ A_{\pi\gamma\gamma} = \frac{\alpha}{\pi} F_{\pi} \]

\[ \Gamma(\pi^0 \rightarrow \gamma \gamma) = (m_{\pi^3}/64\pi)A_{\pi\gamma\gamma}^2 = 7.725 \text{ eV} \pm 0.5\% \]

\[ \tau(\pi^0) = 0.807 \times 10^{-16} \text{ sec} \]

\[ c \tau(\pi^0) = 0.0253 \mu \]
SPONTANEOUS breaking of \( U(3)_L \circledast U(3)_R \)

\( m_u = m_d = m_s = 0 \)

NINE PSEUDOSCALAR GOLDSTONE BOSONS

\[ \text{EXPLICIT breaking of SU}(3)_L \circledast SU(3)_R \]

\( m_u = m_d = 5 \text{ MeV} \)

\( m_s = 130 \text{ MeV} \)
Goity, AB, Holstein
$O(p^6), O(p^4 \cdot 1/Nc): \pi, \eta, \eta'$

Anomaly, $O(p^4)$

Ananathanarayan, Moussallam, $O(p^4 e^2): \pi, \eta, \eta'$

Ioffe, Oganiesian: $\pi, \eta$

PDB
PDB average

Direct(1985)

Primakoff

Projected error

axial anomaly

Chiral

Cornell 1974

DESY 1970

e^+e^- 1988

Tomsk 1970

PDB average

Primex

$\pi^0$-width-exp-theory
- $\pi^0$ photoproduction from Coulomb field of nucleus.
- Equivalent production ($\gamma\gamma^* \rightarrow \pi^0$) and decay ($\pi^0 \rightarrow \gamma\gamma$) mechanism implies Primakoff cross section proportional to $\pi^0$ lifetime.
- Primakoff $\pi^0$ produced at very forward angles.
**Full Cross Section Components**

\[
\frac{d\sigma_{\pi^0}}{d\Omega} = \frac{d\sigma_P}{d\Omega} + \frac{d\sigma_C}{d\Omega} + \frac{d\sigma_I}{d\Omega} + 2 \cdot \sqrt{\frac{d\sigma_P}{d\Omega} \cdot \frac{d\sigma_C}{d\Omega}} \cdot \cos(\phi)
\]

- Primakoff
- Nucl. Coherent
- Incoherent
- Interference

**Primakoff:**

Proportional to \(Z^2\),
peaked at \(\theta_{\pi^0} = m_{\pi^0}^2 / 2E_\gamma^2\)

**Nuclear Coherent:**

\[
\frac{d\sigma_C}{d\Omega} = C \cdot A^2 |F_N(Q)|^2 \sin^2 \theta_{\pi}
\]

**Nuclear Incoherent:**

\[
\frac{d\sigma_I}{d\Omega} = \xi A(1 - G(Q)) \frac{d\sigma_H}{d\Omega}
\]

**Interference:**
EXPERIMENTAL SETUP

- DATA TAKING FALL 2004

- C, Pb TARGETS

- JLAB HALL B PHOTON TAGGING FACILITY - HIGH INTENSITY, HIGH RESOLUTION DEVICE

- STATE-OF-THE-ART CALORIMETRY - HyCal

- PAIR SPECTROMETER
Experiment Overview

- Tagged photons of energy 4.9 - 5.5 GeV were used to measure the absolute cross section of small angle $\pi^0$ photoproduction from the coulomb field of two nuclei ($^{12}\text{C}$ and $^{208}\text{Pb}$).

- The invariant mass and production angle of the pion were reconstructed by detecting the two $\pi^0$ decay photons in a highly segmented calorimeter centered on the beamline.

- The number of tagged photons reaching the target was calibrated using a Total Absorption Counter (TAC) and monitored with an $e^+e^-$ pair spectrometer.
**Analysis Status:** $^{208}\text{Pb}: \gamma\gamma$ Invariant Mass

**Two $\gamma$ Invariant mass**

- **PbWO$_4$ crystal Calorimeter**
  - $\sigma_{m_{\gamma\gamma}} = 2.3\text{MeV}$

**PbWO$_4$ crystal Calorimeter + TAGGER**

- $\sigma_{m_{\gamma\gamma}} = 1.2\text{MeV}$
π⁰ Elasticity

π⁰ Yield/0.010

Eπ/k

C

Pb

All l_beam, Radiator B
θ_π: [1.20,1.30] deg
tdiff: [-7.4,0.6] ns (BC)

^{12}C, crystal only

^{206}Pb, crystal only

Eπ/k
Primakoff Interference

C

Pb

coherent

incoherent

Primakoff

coherent

incoherent

Interference
### Systematic Error Table

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\gamma\gamma}$ fits + inelastic bkgd corr.</td>
<td>±1.0</td>
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<tr>
<td>Inelastic bkgd shape uncert.</td>
<td>±0.75</td>
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<tr>
<td>Photon flux</td>
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<tr>
<td>Incoherent XS shape uncert.</td>
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<tr>
<td>Nuclear coh. XS energy dep.</td>
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<tr>
<td>Detection/Recon efficiency</td>
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<tr>
<td>Fiducial Acceptance</td>
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<tr>
<td>Event Selection</td>
<td>±1.0</td>
</tr>
<tr>
<td>Target thick. + branch ratio</td>
<td>±0.06</td>
</tr>
<tr>
<td>Tagged Photon Energy</td>
<td>±0.1</td>
</tr>
</tbody>
</table>

Total Systematic                                           ±2.4%
Preliminary
*analysis results from E. Clinton, I. Larin, and D. McNulty

Average = 7.93 eV ± 2.1%(stat) ± 2.5%(sys)
Primex preliminary
PRIMEX Collaboration

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Outlook for Experimental Tests of Spontaneous Chiral Symmetry Breaking Nambu $\Rightarrow$ Goldstone Boson

√ most low energy reactions involving pions:
there are a few problems (electroproduction, …)
quark mass difference effects are still unverified
extension to eta, kaon still in beginning stages

Axial Anomaly + chiral corrections($\sim m_d - m_u$)

√ for $\pi^0 \rightarrow \gamma \gamma$ at $\sim 3\%$
needs work for $\eta \rightarrow \gamma \gamma$, $\eta' \rightarrow \gamma \gamma$