Note on the Decay of the $\pi$-Meson

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Assuming the symmetric coupling scheme proposed by Wheeler and Tiomno, and others, we have calculated the ratio of the decay rate $\pi$-meson-electron-neutrino to the decay rate of $\pi$-meson-$\mu$-meson +neutrino. The electron-neutrino decay proceeds faster, in disagreement with experiment, unless the $\pi$-meson is pseudoscalar and the $\beta$-decay coupling is pseudovector. Hence if the symmetric coupling scheme is correct and no other direct couplings are introduced, the $\pi$-meson must be pseudoscalar and $\beta$-decay must be at least partially pseudovector. If symmetric coupling is not assumed, no conclusion of this kind can be drawn.

Although the $\pi$-meson appears to decay into a $\mu$-meson and a neutrino at least 100 times faster than into an electron and neutrino, the latter process is to be expected according to certain formulations of meson theory. We have therefore calculated the rate of the decay $\pi \rightarrow e^+ + \nu$, and wish to mention the possible bearing of our results on the nature of the $\pi$-meson and nuclear $\beta$-decay.

It is customary to assume that the $\pi$-meson is coupled directly to nucleons, in order to explain its production in nuclear collisions and to account, at least partially, for nuclear forces. Assuming further that the nucleons are Dirac particles, one then postulates a coupling of the form

$$G(\overline{\psi} \gamma^\mu O \psi)\psi, \quad (1)$$

or a similar coupling involving first derivatives of $\psi$, where $O$ is some Dirac operator. This interaction leads to the real capture process

$$N + \pi^+ \rightarrow P^+, \quad (2)$$

but it also permits the virtual decomposition of the $\pi$:

$$\pi^+ \rightarrow P^+ + N^-, \quad (2)$$

where $N^-$ signifies an anti-neutron.

On the other hand, to account for $\beta$-decay along the general lines of the Fermi or Gamow-Teller theories, one postulates the interaction

$$g(\overline{\psi} \gamma^\mu A \psi)(\overline{\psi} \gamma^\mu A \psi),$$

where $A$ is also a Dirac operator. This interaction leads to the observed $\beta$-process

$$N \rightarrow P^+ + e^- + \nu, \quad (3)$$

but it also leads one to expect the reaction

$$P^+ + N^- \rightarrow e^+ + \nu, \quad (3)$$

The virtual decomposition (2) followed by (3) leads to the real decay

$$\pi^+ \rightarrow e^+ + \nu. \quad (4)$$

It can then be concluded that any theory which couples $\pi$-mesons to nucleons also predicts the $\pi \rightarrow (e, \nu)$ decay. This argument depends not on the existence of real anti-neutrons, but only on the role of such particles in virtual processes.

Since (4) has not been observed experimentally, we have compared its rate with that of the observed decay

$$\pi^+ \rightarrow \mu^+ + \nu. \quad (5)$$

To do this, a further hypothesis must be introduced by postulating the nature of the field interaction responsible for (5). One may assume a direct coupling between the $\pi$- and $\mu$-fields, or one may assume that (5) goes indirectly. We have tested the symmetrical coupling scheme proposed by Wheeler and others, according to which (5) goes indirectly. According to this scheme the following three processes

$$\mu^+ \rightarrow \pi^+ + e^- + \nu, \quad (6a)$$

$$\beta$-decay $N \rightarrow P^+ + e^- + \nu, \quad (6b)$$

$$\mu^- \rightarrow e^- + \nu, \quad (6c)$$

result from the direct couplings

$$g_\pi(\overline{\psi} \gamma^\mu A \psi)(\overline{\psi} \gamma^\mu A \psi), \quad (7a)$$

$$g_\mu(\overline{\psi} \gamma^\mu B \psi)(\overline{\psi} \gamma^\mu B \psi), \quad (7b)$$

$$g_\mu(\overline{\psi} \gamma^\mu C \psi)(\overline{\psi} \gamma^\mu C \psi). \quad (7c)$$

All fields in (6) are spinor fields; $A, B,$ and $C$ are Dirac operators.

It has been found that $g_\pi \equiv g_\mu \equiv g_\epsilon$. We have assumed

$$g_\pi = g_\mu = g_\epsilon,$$

and

$$A = B = C. \quad (8)$$

These three couplings are thus assumed to be of the same nature and strength. According to the sym-
metrical scheme (5) is a second-order process
\[ \pi^+ \rightarrow P^+ + N^- , \]  
\[ P^+ + N^- \rightarrow \mu^+ + \nu , \]  
(9a)
(9b)
The matrix element for (9b) is contributed by (7a).
We have compared the rates of the two second-order decays (4) and (5) for various operators in the fundamental interaction (7) and several meson field couplings in (1). The decay may progress through either of two intermediate states: (a) A \( \pi \)-meson decays with the production of a pair of virtual nucleons which in turn are annihilated with the creation of a \( (\mu, \nu) \)-pair. (b) A vacuum fluctuation produces a \( (\mu, \nu) \)-pair and a nucleon pair. The latter disappear with the absorption of the \( \pi \)-meson. The transition probability is given by:
\[ \tau^{-1} = 2\pi |H|^2 p(E) = \sum_{\sigma \mu s} 2\pi \delta(E) \left[ \int dp \sum_{\sigma \mu s} \left( \frac{1}{2\mu_s} \right)^4 \right] \]
\[ \times \left( \varphi_{\sigma s} \langle u_{\sigma} \bar{u}_{\mu} O_{\mu} \rangle \langle u_{\sigma} A_{\mu} \rangle \langle u_{\sigma} \bar{u}_{\mu} \rangle \right) \]
\[ \times \left( \varphi_{\sigma s} \langle u_{\sigma} \bar{u}_{\mu} O_{\mu} \rangle \langle u_{\sigma} \bar{u}_{\mu} \rangle \right) \rho(E) \]
\[ \times \frac{2E_P + \mu_s}{2E_P - \mu_s} \]  
\[ \times \left( \frac{\mu_s^2 - \mu^2}{6\pi^3 \mu_s^3} \right) \left( 1 - \frac{c \rho}{E_p} \right) \]
\[ \times \left[ \ln(\theta + 1) + \sqrt{\frac{\theta}{\theta^2 + 1}} \right] \theta^2 , \]  
(11)
where \( \theta \) is the cut-off momentum in units of \( \mu_p C \). A covariant calculation using an invariant cut-off procedure of Feynman leads to the same expression with the bracket replaced by
\[ \left[ \ln \left( \frac{\lambda}{\mu_p} \right) + 1 - \frac{4(\mu_s^2 - \mu^2)}{\mu_s^2} \sin^{-1} \left( \frac{\mu_s}{2\mu_p} \right) \right] \]  
(11a)
where terms of order \( \mu_p/\lambda \) and higher have been dropped. \( \lambda \) is a cut-off with the dimensions of mass. For large cut-offs both expressions are essentially equal.
Although the expressions (11) are only logarithmically divergent, they are sensitive to the choice of cut-off. Choosing \( G^7/hc = 1/4 \) and \( g = 2 \times 10^{-49} \) erg-cm\(^4\), if \( \theta = 10, \tau = 9 \times 10^{-8} \) sec.; \( \theta = 10, \tau = 6 \times 10^{-8} \) sec.; \( \theta = 100, \tau = 1 \times 10^{-8} \) sec. Steinberger has also calculated the lifetime for this decay, and after cutting off with Pauli regulators, he finds \( 2 \times 10^{-8} \) sec. (pseudoscalar coupling).

In view of the sensitivity of the result to the cut-off procedure, and in the absence of any reliable theory, we believe that no conclusion, either for or against symmetric coupling, can be drawn from a consideration of the absolute rate. We emphasize, however, that the ratios in the table are independent of the divergent integrals. The above calculation is also open to the

Table I gives the ratio of \( \pi \rightarrow (e, \nu) \) to \( \pi \rightarrow (\mu, \nu) \)-decay for couplings (1) and (7). Table II gives this ratio when the coupling of (1) involves first derivatives of \( \psi \). \( f \) signifies a forbidden transition for both processes. In most instances the decay of the \( \pi \)-meson is forbidden. The symmetric coupling scheme is in agreement with experimental facts only if the \( \pi \)-meson is pseudoscalar (with either pseudoscalar or pseudovector coupling to nucleons) and the \( \beta \)-decay coupling contains a pseudovector term. According to the table the \( \beta \)-decay coupling may also contain arbitrary admixtures of scalar, vector, and tensor terms, since these do not contribute to the \( \pi \rightarrow \mu \) or \( \pi \rightarrow e \) decays. This result is in agreement with Wigner's conclusion\(^4\) that the evidence from nuclear \( \beta \)-decay requires a pseudovector coupling and does not exclude scalar or pseudoscalar terms.

### Table I. Ratio of \( \pi \rightarrow (e, \nu) \) to \( \pi \rightarrow (\mu, \nu) \)-decay for couplings (1) and (7).

<table>
<thead>
<tr>
<th>Type of ( \beta )-decay</th>
<th>Scalar</th>
<th>P-scalar</th>
<th>Vector</th>
<th>P-vector</th>
<th>Tensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meson</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scalar</td>
<td>5.1</td>
<td>( f )</td>
<td>5.1</td>
<td>( f )</td>
<td>( 1.0 \times 10^{-4} )</td>
</tr>
<tr>
<td>P-scalar</td>
<td>( f )</td>
<td></td>
<td>( f )</td>
<td></td>
<td>( f )</td>
</tr>
<tr>
<td>Vector</td>
<td>( f )</td>
<td></td>
<td>( f )</td>
<td></td>
<td>( f )</td>
</tr>
<tr>
<td>P-vector</td>
<td>( f )</td>
<td></td>
<td>( f )</td>
<td></td>
<td>( f )</td>
</tr>
</tbody>
</table>

### Table II. Ratio of \( \pi \rightarrow (e, \nu) \) to \( \pi \rightarrow (\mu, \nu) \)-decay when the coupling of (1) involves first derivatives of \( \psi \).

<table>
<thead>
<tr>
<th>Meson</th>
<th>Scalar</th>
<th>P-scalar</th>
<th>Vector</th>
<th>P-vector</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f )</td>
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<td></td>
<td>( f )</td>
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<td>( f )</td>
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</tr>
</tbody>
</table>

\(^4\) E. P. Wigner, Phys. Rev., to be published.
\(^6\) J. Steinberger, Phys. Rev., to be published.
\[ \pi^+ + P(q) \rightarrow W^+ + \overline{N}(\overline{q}) \rightarrow \overline{\ell} = \ell^+ \]
π⁺ → ℓ⁺ν

\[ M(\pi) = 139.56995 \text{ MeV} \]
\[ KE(\ell) = 70 \text{ MeV} \quad M(\ell) = 0.51099 \text{ MeV} \quad \gamma(\ell) = 141 \]
\[ KE(\mu) = 4 \text{ MeV} \quad M(\mu) = 106.65839 \text{ MeV} \quad \gamma(\mu) < 1.04 \]
\[ KE(\nu) > 29 \text{ MeV} \quad M(\nu) < 10^{-6} \text{ MeV} \quad \gamma(\nu) > 2.9 \times 10^6 \]
\[\pi^+ \rightarrow \mu^+ \nu \quad KE(\mu)=4 \text{ MeV}\]

\(\pi\) and \(\mu\) stop in target

\[\mu^+ \rightarrow e^+\nu\nu \quad KE_{\text{max}}(e)=52.5 \text{ MeV}\]
\(\pi \rightarrow e\nu\)

\(\pi \rightarrow \mu \rightarrow e\)

TIME (TAC–ADC Channels)
Experimental Search for the Beta-Decay of the $\pi^+$ Meson*

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The possibility that nuclear beta-decay is associated with the beta-decay of the meson, formed during an intermediate step, has long been of interest in the development of meson theories. To explain nuclear beta-decay, the beta-decay rate for the $\pi^+$-meson should be comparable to its $\mu$-decay rate. The present investigation is an attempt to detect other than $\pi^-\mu^-$ events for $\pi^+$ mesons which stop in G-S 400 and 600 micron emulsions. The result is one or zero $\pi^-\mu^-$ event compared to 1419 $\pi^-\mu^-$ events for mesons satisfying certain selection criteria.

A collimating exposure chamber was used in the fringing magnetic field inside the vacuum chamber of the Columbia 164-inch cyclotron to provide energy and direction selection of the mesons at the photographic plate. The expected total energy spread for any point $\pi$ along the plate was calculated to be about 1 Mev.

Only mesons were considered which entered the top surface of the emulsion, ended in the emulsion, and had directions and ranges within intervals including about 94 percent of the $\pi$-mesons. This procedure assured consideration of essentially all $\pi$-mesons but discriminated against $\mu$-mesons produced by decays in flight.

I. INTRODUCTION

YUKAWA's hypothesis1 of the meson as a particle exchanged between nucleons in order to explain nuclear forces seemed strikingly confirmed by the discovery of particles of intermediate mass in cosmic rays.2 The particles were observed to have a mass about 200 $m_e$, which agrees, in order of magnitude, with the value predicted from the range of nuclear forces. Also they spontaneously decay into an electron, as first predicted by Bhabha.3 These mesons are now called $\mu$-mesons. However, it was gradually found that quantitative agreement between the nuclear force meson and the $\mu$-meson of cosmic rays was lacking. Even if one adjusted the coupling constant with the nucleon field to account for nuclear forces and the coupling constant with the electron-neutrino field so as to obtain the correct results for nuclear beta-decay, then a mean lifetime of about $10^{-8}$ sec was predicted, whereas the observed lifetime is much larger ($2 \times 10^{-4}$ sec). Moreover, the experimental cross section for interaction of $\mu$-mesons with nucleons is smaller than that required for nuclear forces by many orders of magnitude.

On the other hand the $\pi$-mesons, discovered by Powell et al.4 are at least partly responsible for nuclear forces. They have been shown to interact with nucleons with about geometric cross section.5 However, whether or not the creation and annihilation of virtual $\pi$-mesons can be used to explain nuclear beta-decay, according to the scheme: $p^+ \rightarrow \pi^+ n', \pi^- n' + n + \nu$ is quite another question. The theoretical investigations of this subject are summarized in the Appendix. Experimentally, one wishes to decide whether the $\pi^+$-meson does undergo beta-decay

$$\pi^+ \rightarrow \pi^0 \nu$$

and if so, to determine the partial lifetime $\tau_\pi$ for beta-decay in terms of the known lifetime $\tau_\nu = 2.5 \times 10^{-4}$ sec for total decay (which is mainly, if not entirely, due to $\mu$-decay),

$$1/\tau = (1/\tau_\nu) + (1/\tau_\mu).$$

(Since $\pi^- \mu^-$ mesons stopped in matter are trapped in atomic orbits and absorbed by the nucleus in a time short compared to $10^{-4}$ sec, only $\pi^+$ mesons are suitable for such an investigation.)

When photographic plates were first used to detect $\pi^+$ mesons at Berkeley6 there was some indication that a few percent of the $\pi^+$ mesons which stopped in the C-2 emulsions used did not give rise to $\mu^+$. The difficulties inherent in any systematic investigation of the $\tau_\pi/\tau_\mu$ ratio are twofold: (1) One must, of course, use electron

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Search for the $\beta$-Decay of the Pion. (*)

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- Introduction.

11. *Theoretical.* - Normally the charged pion decays into a muon and a light neutral particle, usually assumed to be the neutrino. The possible counting decay into electron and neutrino is not without interest, and we recall some theoretical points:

a) *Connection with nuclear $\beta$-decay.* - Yukawa postulated the meson decay in a two step theory of nucleon $\beta$-decay. This hypothesis fails on the one hand because the transition rate of pion-electron decay if non-zero, is at a rate too small to account for the nuclear $\beta$-lifetimes, and on the other hand cause the observed properties of $\beta$-decay require Fermi couplings (*) which are not a consequence of a two step theory with pseudoscalar mesons.

The argument may be reversed, and it may be supposed that the pion can transform into an intermediate nucleon-antinucleon pair which annihilates through normal $\beta$-decay:

$$\pi^+ \to p + n \to e^+ + \nu.$$  

The pion being pseudoscalar, this transition is forbidden except in pseudoscalar and axial vector $\beta$-coupling theories. It may then be recalled that it is possible to account for the bulk of $\beta$-decay data using only scalar and tensor

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(*) This research was supported by the Joint Program of the Atomic Energy Commission and the Office of Naval Research.

(**) Lecture given by J. Steinberger.

electrons and \(\mu\)-decay electrons. This is accomplished by making the detector sensitive to the shorter lifetime and higher energy of the \(\pi\)-decay electron.

2. - Experimental Procedure.

The experimental arrangement is shown in Fig. 1. The 60 MeV \(\pi^+\) beam of the Columbia University Nevis Cyclotron is collimated and monitored by counters no. 1 and no. 2. Counter no. 1 is a plastic scintillator 41/2 inches in diameter and 2/3 inches thick and counter no. 2 is a stilbene crystal 21/2 inches in diameter and 1/3 inches thick. The beam is further collimated by a 2 inch diameter aperture in a 2 inch thick lead shield directly preceding counter no. 2 and is slowed by carbon absorbers inserted between no. 1 and no. 2.

The target is a 1/2 inch thick piece of polyethylene (1.7 g/cm\(^2\)) mounted at approximately 30° to the incident beam. Of the order of 500 pions stop in the target per second and this represents somewhat more than half of the 1-2 rate. Of these roughly 1 in 300 will decay with the charged decay product within the acceptance angle of the detector. This detector consists of four plastic scintillators, each 41/2 inches in diameter, the first three 3/8 inches thick and the last 1/4 inch thick. The counters no. 3,
and 

\[(.254 \pm .10) - (.089 \pm .02) - (.22 \pm .11) = (-.055 \pm .15)\]

per 10^8 monitor counts for run no. 2.

The fraction of π-mesons undergoing β-decay is

\[(MD_f/M)_{corrected} \cdot 1/K \cdot 1/E = f\]

\[f_1 = -.13 \pm 1.36 \cdot 10^{-4}\] for run no. 1,

\[f_2 = -.45 \pm 1.23 \cdot 10^{-4}\] for run no. 2.

Combining these two results, our experiment yields the ratio:

\[\frac{\pi \rightarrow 6}{\pi \rightarrow \mu} = f = (.3 \pm .9) \cdot 10^{-4}.\]

The quoted error is the standard deviation and includes the statistical uncertainty as well as an estimate of the error in the subtraction for the inverse photomeson production.

It is therefore not likely that the actual π → e decay fraction is greater than 6 \cdot 10^{-4} or one in 17,000. The experiment is approximately twenty times more sensitive than previous attempts to find this decay mode, but no positive evidence is obtained. It seems therefore improbable that the pion is coupled symmetrically to the muon.

APPENDIX


In this energy range the straggling is primarily due to radiation and multiple scattering. This problem has not been solved analytically, although the processes are well understood. We have solved the problem with an accuracy sufficient for our purposes by making Monte Carlo calculations for the radiation straggling and combining these with similar calculations (1) on the reduction in range due to the irregularity of the trajectory (multiple scattering) chiefly near its end.

The radiation straggling calculations were carried out at 6 energies: \(E = 25, 35, 50, 70, 85\) and 100 MeV. The Bethe-Heitler radiation loss formula is approximated by the form which corresponds to uniform energy loss over the spectrum:

\[\frac{dN(E)}{dx} = 1/EX.\]
Search for the Electronic Decay of the Positive Pion (*)

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(ricevuto il 24 Luglio 1957)

Summary. — A double focussing magnetic spectrometer of high transmission (1.8%) and good resolution (3.0%) was used in a search for the electronic decay of the positive pion. No evidence was found. The fraction of decays of the type \( \pi \rightarrow e + \gamma \) was found to be \( f = (-0.4 \pm 0.9) \times 10^{-6} \). The result appears to be statistically significant and thereby allows only a 1% probability that \( f \) could have a value greater than \( 2.1 \times 10^{-5} \). From a search for electrons of momentum 60.3 MeV/c we could conclude that the fraction of decays of the type \( \pi \rightarrow e + \gamma + \nu \) was \( f_\gamma = (-2.0 \pm 1.6) \times 10^{-4} \) assuming tensor interaction determines the spectrum. Much lower limits for this last process have been recently reported by CASSELS and by LOKANATHAN.

1. Introduction.

The normal decay of the charged pion is into a muon and a light neutral particle, presumed to be a neutrino. An alternative possibility, the decay into an electron instead of the muon, has never been observed. This seems pur-

(*) Experimental work carried out at The Enrico Fermi Institute for Nuclear Studies, The University of Chicago, under a joint program of the Office of Naval Research and the Atomic Energy Commission.

(+) Fulbright Lecturer and Guggenheim Fellow on leave from The University of Chicago.

(²) "On leave from Centro Brasileiro de Pesquisas Físicas" and from Universidade do Brasil."
Theory of the Fermi Interaction

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(Received September 16, 1957)

The representation of Fermi particles by two-component Pauli spinors satisfying a second order differential equation and the suggestion that in β decay these spinors act without gradient couplings leads to an essentially unique weak four-fermion coupling. It is equivalent to equal amounts of vector and axial vector coupling with two-component neutrinos and conservation of leptons. (The relative sign is not determined theoretically.) It is taken to be "universal"; the lifetime of the μ agrees to within the experimental errors of 2%. The vector part of the coupling is, by analogy with electric charge, assumed to be not renormalized by virtual mesons. This requires, for example, that pions are also "charged" in the sense that there is a direct interaction in which, say, a π⁺ goes to π⁻ and an electron goes to a neutrino. The weak decays of strange particles will result qualitatively if the universality is extended to include a coupling involving a Λ or Σ fermion. Parity is then not conserved even for those decays like K→ππ or Σ→ππ which involve no neutrinos. The theory is at variance with the measured angular correlation of electron and neutrino in He's, and with the fact that fewer than 10⁻⁴ pion decay into electron and neutrino.

The failure of the law of reflection symmetry for weak decays has prompted Salam, Landau, and Lee and Yang1 to propose that the neutrino be described by a two-component wave function. As a consequence neutrinos emitted in β decay are fully polarized along their direction of motion. The simplicity of this idea makes it very appealing, and considerable experimental evidence is in its favor. There still remains the question of the determination of the coefficients of the scalar, vector, etc., couplings.

There is another way to introduce a violation of parity into weak decays which also has a certain amount of theoretical raison d'être. It has to do with the number of components used to describe the electron in the Dirac equation,

$$-(i \nabla - A) \psi = m \psi.$$  (1)

Why must the wave function have four components? It is usually explained by pointing out that to describe the electron spin we must have two, and we must also represent the negative-energy states or positrons, requiring two more. Yet this argument is unsatisfactory. For a particle of spin zero we use a wave function of only one component. The sign of the energy is determined by how the wave function varies in space and time. The Klein-Gordon equation is second order and we need both the function and its time derivative to predict the future. So instead of two components for spin zero we use one, but it satisfies a second order equation. Initial states require specification of that one and its time derivative. Thus for the case of spin ½ we would expect to be able to use a simple two-component spinor for the wave function, but have it satisfy a second order differential equation. For example, the wave function for a free particle would look like $U \exp \left[ -i (E t - \mathbf{P} \cdot \mathbf{x}) \right]$, where $U$ has just the two components of a Pauli spinor and the particle refers to electron or positron depends on the sign of $E$ in the four-vector $p_\mu = (E, \mathbf{P})$.

In fact it is easy to do this. If we substitute

$$\psi = \frac{1}{m}(i \nabla - A - m) \chi$$  (2)

in the Dirac equation, we find that $\chi$ satisfies

$$-(i \nabla - A) \chi = \frac{1}{m} \left[ (i \nabla_\mu - A_\mu) \cdot (i \nabla_\mu - A_\mu) \right] \chi = m^2 \chi,$$  (3)

where $F_\mu = \partial A_\mu / \partial x_\nu - \partial A_\nu / \partial x_\mu$ and $\sigma_\mu = \frac{i}{2} (\gamma_\mu \gamma_5 - \gamma_5 \gamma_\mu)$. Now we have a second order equation, but $\chi$ still has four components and we have twice as many solutions as we want. But the operator $\gamma_5 = \gamma_5 \gamma_\mu \gamma_\nu \gamma_\tau$ commutes with $\sigma_\mu$; therefore there are solutions of (3) for which $i \gamma_5 \chi = \chi$ and solutions for $i \gamma_5 \chi = -\chi$. We may select, say, the first set. We always take

$$i \gamma_5 \chi = \chi.$$  (4)

Then we can put the solutions of (3) into one-to-one correspondence with the Dirac equation (1). For each $\psi$ there is a unique $\chi$; in fact we find

$$\chi = \frac{1}{2} (1 + i \gamma_5) \psi$$  (5)

by multiplying (2) by $1 + i \gamma_5$ and using (4). The function $\chi$ has really only two independent components. The conventional $\psi$ requires knowledge of both $\chi$ and its time derivative [see Eq. (2)]. Further, the six $\sigma_\mu$ in (3) can be reduced to just the three $\sigma_{23}, \sigma_{21}, \sigma_{13}$ since $\sigma_{21} = i \gamma_5 \sigma_{23} = -i \sigma_{23} \gamma_5$. Eq. (4) shows that $\sigma_{23}$ may be replaced by $i \sigma_{23}$ when operating on $\chi$ as it does in (3).

Let us use the representation

$$\gamma_i = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix}, \quad i \gamma_5 = -\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

where $\sigma_{x,y,z}$ are the Pauli matrices. If

$$\psi = \begin{pmatrix} a \\ b \end{pmatrix},$$

where $a$, $b$ are two-component spinors, we find from (5) that
\[ \chi = \begin{pmatrix} \varphi \\ -\varphi \end{pmatrix}, \]
where $\varphi = \frac{1}{2}(a - b)$. Our Eq. (3) for the two-component spinor $\varphi$ is
\[ \left[(\sqrt{B_0} - A_0) + \mathbf{a} \cdot \mathbf{B} + \mathbf{e} \right]\varphi = m^2 \varphi, \]
where $B_\alpha = F_{\alpha 2}$, $E_\alpha = F_{\alpha 3}$, etc., which is the equation we are looking for.

Rules of calculation for electrodynamics which involve only the algebra of the Pauli matrices can be worked out on the basis of (6). They, of course, give results exactly the same as those calculated with Dirac matrices. The details will perhaps be published later.

One of the authors has always had a predilection for this equation. If one tries to represent relativistic quantum mechanics by the method of path integrals, the Klein-Gordon equation is easily handled, but the Dirac equation is very hard to represent directly. Instead, one is led first to (3), or (6), and from there one must work back to (1).

For this reason let us imagine that (6) had been discovered first, and (1) only deduced from it later. It would make no difference for any problem in electrodynamics, where electrons are neither created nor destroyed (except with positrons). But what would we do if we were trying to describe $\beta$ decay, in which an electron is created? Would we use a field operator $\varphi$ directly in the Hamiltonian to represent the annihilation of an electron, or would we use $\varphi$? Now everything we can do one way, we can represent the other way. Thus if $\varphi$ were used it could be replaced by
\[ \frac{1}{m}(\beta - A + m) \begin{pmatrix} \varphi \\ -\varphi \end{pmatrix}, \]
while an expression in which $\varphi$ was used could be rewritten by substituting
\[ \frac{1}{2} (1 + i\gamma_3) \varphi. \]
If $\varphi$ were really fundamental, however, we might be prejudiced against (a) on the grounds that gradients are involved. That is, an expression for $\beta$ coupling which does not involve gradients from the point of view of $\varphi$, does from the point of view of $\varphi$. So we are led to suggest $\varphi$ as the field annihilation operator to be used in $\beta$ decay without gradients. If $\varphi$ is written as in (b), we see this does not conserve parity, but now we know that it is consistent with experiment.

For this reason one of us suggested the rule that the electron in $\beta$ decay is coupled directly through $\varphi$, or, what amounts to the same thing, in the usual four-particle coupling
\[ \sum C_i (\bar{\psi}_i O \psi_\beta) (\bar{\psi}_\beta O \psi_i), \]
we always replace $\psi_i$ by $\frac{1}{2}(1 + i\gamma_3) \psi_i$.

One direct consequence is that the electron emitted in $\beta$ decay will always be left-hand polarized (and the positron right) with polarization approaching 100% as $\beta \to \alpha$, irrespective of the kind of coupling. That is a direct consequence of the projection operator
\[ a = \frac{1}{2}(1 - i\gamma_3). \]

A priori we could equally well have made the other choice and used
\[ a = \frac{1}{2}(1 - i\gamma_3); \]
electrons emitted would then be polarized to the right. We appeal to experiment to determine the sign.

But now we go further, and suppose that the same rule applies to the wave functions of all the particles entering the interaction. We take for the $\beta$-decay interaction the form
\[ \sum C_i (\bar{\psi}_i O \psi_\beta) (\bar{\psi}_\beta O \psi_i), \]
and we should like to discuss the consequences of this hypothesis.

The coupling is now essentially completely determined. Since $\bar{\psi} = \bar{\psi}_a$, we have in each term expressions like $\partial O \partial$. Now for $S$, $T$, and $P$ we have $O_i$ commuting with $\gamma_3$, so that $\partial O \partial = O \partial a = 0$. For $A$ and $V$ we have $\partial O_a = O_a \partial = 0$ and the coupling survives. Furthermore, for axial vector $O_i = i\gamma_\mu \gamma_\sigma$, and since $i\gamma_\mu a = a$, we find $O_a = \gamma_\mu a$; thus $A$ leads to the same coupling as $V$:
\[ (8) G (\bar{\psi}_\mu \gamma_\nu \psi_\beta) (\bar{\psi}_\beta \gamma_\nu \psi_i), \]
the most general $\beta$-decay coupling possible with our hypothesis.

This coupling is not yet completely unique, because our hypothesis could be varied in one respect. Instead of dealing with the neutron and proton, we could have made use of the antineutron and antiproton, considering them as the “true particles.” Then it would be the wave function $\varphi_\beta$ of the antineutron that enters with the factor $a$. We would be led to
\[ (8) G (\bar{\psi}_\mu \gamma_\nu \psi_\beta) (\bar{\psi}_\beta \gamma_\nu \psi_a), \]
This amounts to the same thing as
\[ (8) G (\bar{\psi}_\mu \gamma_\nu \psi_\beta) (\bar{\psi}_\beta \gamma_\nu \psi_i), \]
and from the a priori theoretical standpoint is just as good a choice as (8).

We have assumed that the neutron and proton are

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3. See, for example, Boehm, Novey, Barnes, and Stech, Phys. Rev. 185, 1497 (1957).
4. A universal $V$, $A$ interaction has also been proposed by E. C. G. Sudarshan and R. E. Marshak (to be published).
either both “particles” or both “antiparticles.” We have defined the electron to be a “particle” and the neutrino must then be a particle too.

We shall further assume the interaction “universal,” so for example it is

\[(8)G(\bar{\psi}_\mu \gamma_\mu \psi)\langle \xi \gamma_\mu \psi \rangle \tag{10}\]

for \(\mu\) decay, as currently supposed; the \(\mu^-\) then has a particle. Here the other choice, that the \(\mu^-\) is an antiparticle, leads to \(8)G(\bar{\psi}_\mu \gamma_\mu \psi)\langle \bar{\xi} \gamma_\mu \psi \rangle\), which is excluded by experiment since it leads to a spectrum falling off at high energy (Michel’s \(\alpha=0\)).

Since the neutrino function always appears in the form \(\bar{\psi} \gamma_\mu \psi\), only neutrinos with left-hand spin can exist. That is, the two-component neutrino theory with conservation of leptons is valid. Our neutrinos spin oppositely to those of Lee and Yang.\(^6\) For example, a \(\beta\) particle is a lepton and spins to the left; emitted with it is an antineutrino which is an antilepton and spins to the right. In a transition with \(\Delta J=0\) they tend to go parallel to cancel angular momentum. This is the angular correlation typical of very weak coupling.

We have conservation of leptons and double \(\beta\) decay is excluded.

There is a symmetry in that the incoming particles can be exchanged without affecting the coupling. Thus if we define the symbol

\[\langle \bar{A} B \rangle \langle \bar{C} D \rangle = \langle \bar{\psi}_\Lambda \gamma_\mu \psi_D \rangle \langle \bar{\psi}_\gamma \gamma_\mu \psi \rangle \tag{11}\]

we have \(\langle \bar{A} B \rangle \langle \bar{C} D \rangle = \langle \bar{C} D \rangle \langle \bar{A} B \rangle\). (We have used anti-commuting \(\psi's\); for \(C\)-number \(\psi's\) the interchange gives a minus sign.) \(^7\)

The capture of muons by nucleons results from a coupling \((\bar{\mu} \gamma_\mu \mu)\). It is already known that this capture is fitted very well if the coupling constant and coupling are the same as in \(\beta\) decay.\(^8\)

If we postulate that the universality extends also to the strange particles, we may have couplings such as \((\bar{\Lambda} \mu) \langle \bar{\psi}_\mu \rangle\), \((\bar{\Xi} \mu) \langle \bar{\psi}_\mu \rangle\), and \((\bar{\Xi} \mu) \langle \bar{\psi}_\mu \rangle\). The \((\bar{\Lambda} \mu)\) might be replaced by \((\bar{\Xi} \mu)\), etc. At any rate the existence of such couplings would account qualitatively for the existence of all the weak decays. Consider, for example, the decay of the \(K^+\). It can go virtually into an anti-\(K^+\) and a proton by the fairly strong coupling of strange particle production. This by the weak decay \((\bar{\Xi} \mu) \langle \bar{\psi}_\mu \rangle\) becomes a virtual antineutrino and proton. These become, on annihilating, two or three pions. The parity is not conserved because of the

\(^6\) This is only because they used \(S\) and \(T\) couplings in \(\beta\) decay; had they used \(V\) and \(A\), their theory would be similar to ours, with left-handed neutrinos.

\(^7\) We can express \(\langle \bar{A} B \rangle \langle \bar{C} D \rangle\) directly in terms of the two-component spinors \(\psi:\ (\bar{A} B)\langle \bar{C} D \rangle = 4(\bar{\sigma} \cdot \sigma_D \psi_D)\langle \bar{\psi} \sigma \cdot \sigma_D \psi_D \rangle\). If we put \(\sigma = A^T\), etc., \(A\) and \(A^T\) are complex numbers, we obtain \(8(\bar{\sigma} A^* C^* - \bar{A}^* C*) (B_D D_T - B_D D_T)\) and the symmetry is evident.


\(a\) in front of the nucleons in the virtual transition. The theory in which only the neutrino carries the \(a\) cannot explain the parity failure for decays not involving neutrinos (the \(\tau-\bar{\tau}\) puzzle). Here we turn the argument around; both the lack of parity conservation for the \(K\) and the fact that neutrinos are always fully polarized are consequences of the same universal weak coupling.

For \(\beta\) decay the expression \(8)\) will be recognized as that for the two-component neutrino theory with couplings \(V\) and \(A\) with equal coefficients and opposite signs \[expression \(9)\) or \(9') makes the coupling \(V-A\).\] The coupling constant of the Fermi (\(V\)) part is equal to \(G\). This constant has been determined from the decay of \(O_{\mu}^5\) to be \((1.41\pm 0.01) \times 10^{-4}\) erg/cm\(^4\). In units where \(\hbar = c = 1\), and \(M\) is the mass of the proton, this is

\[G = (1.01 \pm 0.01) \times 10^{-4}/M^2.\]

At the present time several \(\beta\)-decay experiments seem to be in disagreement with one another. Limiting ourselves to those that are well established, we find that the most serious disagreement with our theory is the recoil experiment in \(He^4\) of Rustad and Ruby\(^9\) indicating that the \(T\) interaction is more likely than the \(A\). Further check on this is obviously very desirable. Any experiment indicating that the electron is not 100% left polarized as \(\tau-\bar{\tau}\) for any transition allowed or forbidden would mean that \(8)\) and \(9)\) are incorrect.

An interesting experiment is the angular distribution of electrons from polarized neutrons for here there is an interference between the \(V\) and \(A\) contributions such that if the coupling is \(V-A\) there is no asymmetry, while if it is \(V+A\) there is a maximal asymmetry. This would permit us to choose between the alternatives \(8)\) and \(9)\). The present experimental results\(^10\) agree with neither alternative.

We now look at the muon decay. The fact that the two neutrinos spin oppositely and the \(\rho\) parameter is \(\frac{1}{2}\) permitted us to decide that the \(\mu^-\) is a lepton if the electron is, and determines the order of \((\bar{\mu} \mu)\) which we write in \(10)\). But now we can predict the direction of the electron in the \(\pi^-\mu^-\bar{\nu}^e\) sequence. Since the muon comes out with an antineutrino which spins to the right, the muon must also be spinning to the right (all senses of spin are taken looking down the direction of motion of the particle in question). When the muon disintegrates with a high-energy electron the two neutrinos are emitted in the opposite direction. They have spins opposed. The electron emitted must spin to the left, but must carry off the angular momentum of the muon, so it must proceed in the direction opposite to that of the muon. This direction agrees with experiment. The proposal of Lee and Yang predicted


the electron spin here to be opposite to that in the case of $\beta$ decay. Our $\beta$-decay coupling is $V$, $A$ instead of $S$, $T$ and this reverses the sign. That the electron have the same spin polarization in all decays ($\beta$, muon, or strange particles) is a consequence of putting $2\nu \gamma$ in the coupling for this particle. It would be interesting to test this for the muon decay.

Finally we can calculate the lifetime of the muon, which comes out

$$\tau = 192\pi^2/G^2 \mu^2 = (2.26 \pm 0.04) \times 10^{-6} \text{ sec}$$

using the value (11) of $G$. This agrees with the experimental lifetime$^{12}$ (2.22$\pm$0.02)$\times 10^{-6}$ sec.

It might be asked why this agreement should be so good. Because nucleons can emit virtual pions there might be expected to be a renormalization of the effective coupling constant. On the other hand, if there is some truth in the idea of an interaction with a universal constant strength it may be that the other interactions are so arranged as not to destroy this constant. We have an example in electrodynamics. Here the coupling constant $e$ to the electromagnetic field is the same for all particles coupled. Yet the virtual mesons do not disturb the value of this coupling constant. Of course the distribution of charge is altered, so the coupling for high-energy fields is apparently reduced (as evidenced by the scattering of fast electrons by protons), but the coupling in the low-energy limit, which we call the total charge, is not changed.

Using this analogy to electrodynamics, we can see immediately how the Fermi part, at least, can be made to have no renormalization. For the sake of this discussion imagine that the interaction is due to some intermediate (electrically charged) vector meson of very high mass $M_0$. If this meson is coupled to the “current” $\langle \bar{\psi}_s \gamma^a \psi_n \rangle$ and $\langle \bar{\psi}_s \gamma^a \gamma^5 \psi_n \rangle$ by a coupling $(4\pi f^2)$, then the interaction of the two “currents” would result from the exchange of this “meson” if $4\pi f^2 M_0^2 \gamma = (8) G$. Now we must arrange that the total current

$$J_\mu = \langle \bar{\psi}_s \gamma^a \psi_n \rangle + \langle \bar{\psi}_s \gamma^a \gamma^5 \psi_n \rangle + \cdots$$  \hspace{1cm} (12)

be not renormalized. There are no known large interaction terms to renormalize the $(\bar{\nu} e)$ or $(\bar{\nu} \mu)$, so let us concentrate on the nucleon term. This current can be split into two: $J_\mu = \frac{1}{2}(J_\mu^V + J_\mu^A)$, where $J_\mu^V = \bar{\psi}_s \gamma^a \psi_n$ and $J_\mu^A = \bar{\psi}_s i\gamma^5 \gamma^a \psi_n$. The term $J_\mu^V = \bar{\psi}_s \gamma^a \gamma^5 \psi_n$, in isospin spin notation, is just like the electric current. The electric current is

$$J_\mu = \frac{1}{2} \bar{\psi}_s \gamma^a \gamma^5 \psi_n$$

The term $\frac{1}{2} \bar{\psi}_s \gamma^a \gamma^5 \psi_n$ is conserved, but the term $\bar{\psi}_s \gamma^a \gamma^5 \psi_n$ is not, unless we add the current of pions, $i\bar{\psi}_s \gamma^a T_\mu_\nu \nu \psi_n - (\bar{\nu} \gamma^a T_\mu_\nu \nu \psi_n)$, because the pions are charged. Likewise $\bar{\psi}_s \gamma^a \gamma^5 \psi_n$ is not conserved but the sum

$$J_\mu^V = \bar{\psi}_s \gamma^a \gamma^5 \psi_n + i[\bar{\psi}_s \gamma^a T_\mu \nu \nu \psi_n - (\bar{\nu} \gamma^a T_\mu \nu \nu \psi_n)]$$  \hspace{1cm} (13)

12 W. E. Bell and E. P. Hincks, Phys. Rev. 84, 1243 (1951). is conserved, and, like electricity, leads to a quantity whose value (for low-energy transitions) is unchanged by the interaction of pions and nucleons. If we include interactions with hyperons and $K$ particles, further terms must be added to obtain the conserved quantity.

We therefore suppose that this conserved quantity be substituted for the vector part of the first term in (12). Then the Fermi coupling constant will be strictly universal, except for small electromagnetic corrections. That is, the constant $G$ from the $\mu$ decay, which is accurately $V - A$, should be also the exact coupling constant for at least the vector part of the $\beta$ decay. (Since the energies involved are so low, the spread in space of $J_\mu^V$ due to the meson couplings is not important, only the total “charge.”) It is just this part which is determined by the experiment with $O^4$, and that is why the agreement should be so close.

The existence of the extra term in (13) means that other weak processes must be predicted. In this case there is, for example, a coupling

$$(8) G_i (\bar{\psi}_s \gamma^a T_\mu \nu \nu \psi_n - (\bar{\nu} \gamma^a T_\mu \nu \nu \psi_n) \langle \bar{\psi}_s \gamma^a \gamma^5 \psi_n \rangle)$$

by which a $\pi^-$ can go to a $\pi^0$ with emission of $\bar{\nu}$ and $e$. The amplitude is

$$4G (p^- + p^0) \langle \bar{\psi}_s \gamma^a \gamma^5 \psi_n \rangle$$

where $p^-, p^0$ are the four-momenta of $\pi^-$ and $\pi^0$. Because of the low energies involved, the probability of the disintegration is too low to be observable. To be sure, the process $\pi^- \rightarrow \pi^0 + e^+ + \bar{\nu}$ could be understood to be qualitatively necessary just from the existence of $\beta$ decay. For the $\pi^-$ may become virtually an anti-proton and neutron, the neutron decay virtually to a proton, $e$, and $\bar{\nu}$ by $\beta$ decay and the protons annihilate forming the $\pi^0$. But the point is that by our principle of a universal coupling whose vector part requires no renormalization we can calculate the rate directly without being involved in closed loops, strong couplings, and divergent intervals.

For any transition in which strangeness doesn’t change, the current $J_\mu^V$ is the total current density of isotopic spin $T_\mu$. Thus the vector part gives transitions $\Delta T = 0$ with square matrix element $T(T + 1) - T_\mu T_\mu'$ if we can neglect the energy release relative to the rest mass of the particle decaying. For the nucleon and $K^+ \rightarrow K^0 + e^- + \bar{\nu}$ the square of the matrix element is 1, for the pion and $\Sigma^- \rightarrow \Sigma^0 + e^- + \bar{\nu}$ it is 2. The axial coupling in the low-energy limit is zero between states of zero angular momentum like the $\pi$ meson or $O^4$, so for both of these we can compute the lifetime knowing only the vector part. Thus the $\pi^- \rightarrow \pi^0 + e^+ + \bar{\nu}$ decay should have the same $f^2$ value as $O^4$. Unfortunately because of the very small energies involved (because isotopic spin is such a good quantum number) none of these decays of mesons or hyperons are fast enough to observe in competition to other decay processes in which $T$ or strangeness changes.
This principle, that the vector part is not renormalized, may be useful in deducing some relations among the decays of the strange particles.

Now with present knowledge it is not so easy to say whether or not a pseudovector current like \( \bar{V}V \) can be arranged to be not renormalized. The present experiments in \( \beta \) decay indicate that the ratio of the coupling constant squared for Gamow-Teller and Fermi is about \( 1.3 \pm 0.1 \). This departure from 1 might be a renormalization effect. On the other hand, an interesting theoretical possibility is that it is exactly unity and that the various interactions in nature are so arranged that it need not be renormalized (just as for \( V \)). It might be profitable to try to work out a way of doing this. Experimentally it is not excluded. One would have to say that the \( f_3 \) value of 1220 \( \pm 150 \) measured for the neutron was really 1520, and some uncertain matrix elements in the \( \beta \) decay of the mirror nuclei were incorrectly estimated.

The decay of the \( \pi^- \) into a \( \mu^- \) and \( \bar{\nu}_e \) might be understood as a result of a virtual process in which the \( \pi \) becomes a nucleon loop which decays into the \( \mu^- \bar{\nu}_e \). In any event one would expect a decay into \( e^- \bar{\nu}_e \) also. The ratio of the rates of the two processes can be calculated without knowledge of the character of the closed loops. It is \( (m_\mu/m_\pi)^2(1-m_\pi^2/m_\pi^2)^{-2} = 1.6 \times 10^{-5} \). Experimentally it has been found that the ratio is less than 10\(^{-5}\). This is a very serious discrepancy. The authors have no idea how it can be resolved.

We adopt the point of view that the weak interactions all arise from the interaction of a current \( J_\mu \) with itself, possibly via an intermediate charged vector meson of high mass. This has the consequence that any term in the current must interact with all the rest of the terms and with itself. To account for \( \beta \) decay and \( \mu \) decay we have to introduce the terms in (12) into the current; the phenomenon of \( \mu \) capture must also occur. In addition, however, the pairs \( e\bar{\nu}_e, \mu\bar{\nu}_\mu \), and \( p\bar{n} \) must interact with themselves. In the case of the \( (\bar{\nu}_e)e \) coupling, experimental detection of electron-neutrino scattering might some day be possible if electron recoils are looked for in materials exposed to pile neutrinos; the cross section with our universal coupling is of the order of 10\(^{-10}\) cm\(^2\).

To account for all observed strange particle decays it is sufficient to add to the current a term like \( \langle \bar{p}A^0 \rangle \), \( \langle \bar{p}A^0 \rangle \), or \( \langle \bar{\Sigma}^+n \rangle \), in which strangeness is increased by one as charge is increased by one. For instance, \( \langle \bar{p}A^0 \rangle \) gives us the couplings \( \langle \bar{p}A^0 \rangle \bar{e} \), \( \langle \bar{p}A^0 \rangle \bar{\nu}_\mu \), and \( \langle \bar{p}A^0 \rangle \bar{\nu}_\mu \bar{\bar{\nu}}_e \). A direct consequence of the coupling \( \langle \bar{p}A^0 \rangle \bar{e} \) would be the reaction

\[
A^0 \rightarrow p + e^- + \bar{\nu}
\]

at a rate \( 3.5 \times 10^{-10} \text{ sec}^{-1} \), assuming no renormalization of the constants. Since the observed lifetime of the \( A^0 \) (for disintegration into other products, like \( p + \pi^- \), \( n + \pi^0 \)) is about \( 3 \times 10^{-10} \text{ sec} \), we should observe process (14) in about 1.6% of the disintegrations. This is not excluded by experiments. If a term like \( \langle \bar{\Sigma}^+n \rangle \) appears, the decay \( \Sigma^+ \rightarrow n + e^- + \nu \) is possible at a predicted rate \( 3.5 \times 10^{-10} \text{ sec}^{-1} \) and should occur (for \( \tau \Sigma^+ = 1.6 \times 10^{-10} \text{ sec} \) in about 5.6% of the disintegrations of the \( \Sigma^+ \). Decays with \( \mu \) replacing the electron are still less frequent. That such disintegrations actually occur at the above rates is not excluded by present experiments. It would be very interesting to look for them and to measure their rates.

These rates were calculated from the formula

\[
\text{Rate} = \frac{(2G_F^2W_0^2/3m_p^2)}{\text{with neglect of the electron mass. Here } W^2 = (M_\pi^2 - M_\pi^2)/2M_A \text{ is the maximum electron energy possible and } c \text{ is a correction factor for recoil. If } x = W/M_A \text{ it is}}
\]

\[
c = -\frac{1}{2}x^{-5}(1-2x)5\ln(1-2x)
\]

\[
= -\frac{5}{3}(1-x)(3-6x-2x^2)
\]

and equals 1 for small \( x \), about 1.25 for the \( \Sigma \) decay, and 2.5 for \( M_\mu = 0 \).

It should be noted that decays like \( \Sigma^+ \rightarrow n + e^- + \nu \) are forbidden if we add to the current only terms for which \( \Delta S = +1 \) when \( \Delta Q = +1 \). In order to cause such a decay, the current would have to contain a term with \( \Delta S = -1 \) when \( \Delta Q = +1 \), for example \( \langle \bar{\Sigma}^+n \rangle \). Such a term would then be coupled not only to \( (e\bar{\nu}_e) \), but also to all the others, including one like \( \langle \bar{p}A^0 \rangle \). But a coupling of the form \( \langle \bar{\Sigma}^+n \rangle \langle A^0 \bar{\nu}_e \rangle \) leads to strange particle decays with \( \Delta S = \pm 2 \), violating the proposed rule \( \Delta S = \pm 1 \). It is important to know whether this rule really holds; there is evidence for it in the apparent absence of the decay \( \Xi^- \rightarrow n + p \), but so few \( \Xi \) particles have been seen that this is not really conclusive. We are not sure, therefore, whether terms like \( \langle \bar{\Sigma}^+n \rangle \) are excluded from the current.

We deliberately ignore the possibility of a neutral current, containing terms like \( \langle e\bar{\nu}_e \rangle \), \( \langle (e\bar{\nu}_e) \rangle \), \( \langle (e\bar{\nu}_e) \rangle \), etc., and possibly coupled to a neutral intermediate field. No weak coupling is known that requires the existence of such an interaction. Moreover, some of these couplings, like \( \langle e\bar{\nu}_e \rangle \), leading to the decay of a muon into three electrons, are excluded by experiment.

It is amusing that this interaction satisfies simultaneously all the principles that have been
proposed on simple theoretical grounds to limit the possible $\beta$ couplings. It is universal, it is symmetric, it produces two-component neutrinos, it conserves leptons, it preserves invariance under $CP$ and $T$, and it is the simplest possibility from a certain point of view (that of two-component wave functions emphasized in this paper).

These theoretical arguments seem to the authors to be strong enough to suggest that the disagreement with the $\text{He}^8$ recoil experiment and with some other less accurate experiments indicates that these experiments are wrong. The $\pi \rightarrow e + \bar{\nu}$ problem may have a more subtle solution.

After all, the theory also has a number of successes. It yields the rate of $\mu$ decay to $2\%$ and the asymmetry in direction in the $\pi \rightarrow \mu \rightarrow e$ chain. For $\beta$ decay, it agrees with the recoil experiments\textsuperscript{18} in $A_{22}$ indicating a vector coupling, the absence of Fierz terms distorting the allowed spectra, and the more recent electron spin polarization\textsuperscript{19} measurements in $\beta$ decay.

\begin{footnote}{\textsuperscript{18} Herrmannsfeldt, Maxson, Stähelin, and Allen, Phys. Rev. 107, 641 (1957). \textsuperscript{19} Herrmannsfeldt, Maxson, Stähelin, and Allen, Phys. Rev. 107, 641 (1957).}

Besides the various experiments which this theory suggests be done or rechecked, there are a number of directions indicated for theoretical study. First it is suggested that all the various theories, such as meson theory, be recast in the form with the two-component wave functions to see if new possibilities of coupling, etc., are suggested. Second, it may be fruitful to analyze further the idea that the vector part of the weak coupling is not renormalized; to see if a set of couplings could be arranged so that the axial part is also not renormalized; and to study the meaning of the transformation groups which are involved. Finally, attempts to understand the strange particle decays should be made assuming that they are related to this universal interaction of definite form.

\section*{Acknowledgments}

The authors have profited by conversations with F. Boehm, A. H. Wapstra, and B. Stech. One of us (M. G. M.) would like to thank R. E. Marshak and E. C. G. Sudarshan for valuable discussions.

\begin{flushright}
\textbf{DISPERSION RELATIONS FOR DIRAC POTENTIAL SCATTERING}

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Dispersion relations for scattering of a Dirac particle by a potential are shown to hold for a broad class of potentials. In contrast to the field theoretic case, the derivation here makes no use of the concept of causality but is instead based directly on the analytic properties of the Fredholm solution of the scattering integral equation. It is shown that the scattering amplitude, considered as a function of energy and momentum transfer, can be extended to a function analytic in the complex energy plane, for real momentum transfer. The dispersion relations then follow in the standard way from Cauchy’s theorem. The final results involve one “subtraction.” It is also shown that the analytic continuation into the unphysical region for nonforward scattering can be carried out by means of a partial wave expansion.

\section*{I. Introduction}

It has recently been shown\textsuperscript{1} that, under certain broad conditions, dispersion relations of the type so much discussed for relativistic field theories\textsuperscript{2} also hold in ordinary nonrelativistic quantum mechanics for scattering of a particle by a potential. The treatment of this problem is quite straightforward and explicit; in contrast to the field theoretic case, one can show explicitly that the dispersion relations involve no “subtractions” and that the scattering amplitude can be analytically continued into the unphysical region for nonforward scattering by means of a partial wave expansion. In this sense, nonrelativistic quantum mechanics provides a complete and simple model of a system for which dispersion relations are valid. It has already been used as a basis for investigating to what extent the dispersion relations, taken together with the unitarity of the $S$-matrix, constitute a self-contained formulation of scattering theory.\textsuperscript{3}

In the present paper, the discussion of dispersion relations in ordinary quantum mechanics is extended to the case of scattering of a Dirac particle by a potential. Using arguments similar to those employed for the Schrödinger case,\textsuperscript{1} one again finds that dispersion relations hold for a broad class of potentials. The restrictions on the potentials are now somewhat more severe; and in the present case one finds that the dispersion

\begin{footnotesize}
\begin{enumerate}
\item N. N. Khuri, Phys. Rev. 107, 1148 (1957).
\item S. Gasiorowicz and M. Ruder (to be published).
\end{enumerate}
\end{footnotesize}
V. Telegdi:

“The F-G theory is no F…G…”
Optimal reactions for the search for $\pi^0 \rightarrow e^+ e^-$

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Abstract. We examine various reactions in which one might search for $\pi^0 \rightarrow e^+ e^-$; apart from the problem of the low branching ratio, the most serious observational difficulties arise from the competition of intermediate virtual photon states of close to the pion mass. We show that the obvious choice of $\pi^- p \rightarrow \pi^0 n$ at threshold is an unfavourable pion source but that the same reaction in the $\Delta(1236)$ resonance region seems currently to offer the best opportunity; we analyse it in detail. $K^+ n \rightarrow \pi^+ \pi^0$ seems the most hopeful alternative.

1. Introduction

Rare leptonic decay modes of the neutral pseudoscalar mesons are of great interest because of the information they may reveal about neutral currents (see eg Marshak et al 1969) and, more recently, the possibility of new CP violating channels of K decay (Clark et al 1971, Carithers et al 1973, Gjesdal et al 1973). Much experimental evidence has been compiled for $K \rightarrow l^+ l^-$ (Clark et al 1971, Carithers et al 1973, Gjesdal et al 1973, see Particle Data Group 1973 for a critical comparison and earlier references) and there have been two experiments on $\eta \rightarrow \mu^+ \mu^-$ (Hyams et al 1969, Wehmann et al 1968). However, no one has observed $\pi \rightarrow e^+ e^-$. From angular momentum, C and P conservation alone, the rate for direct lepton-pair decays of $0^-$ particles can be shown to be proportional to the square of the lepton mass. As a result, electron–positron pair decays are considerably more difficult to observe than the muon mode. Since $\pi^0 \rightarrow \mu^+ \mu^-$ is impossible, the lepton-pair mode of pion decay is particularly rare, so a high $\pi^0$ flux is required. To reduce background, an experiment should also identify and momentum analyse the lepton pair so as to reconstruct the $\pi^0$ mass. Kinematically indistinguishable background can arise from other $\pi^0$ decay modes, eg $\pi^0 \rightarrow \gamma e^+ e^-$ or from processes competing with $\pi^0$ production, particularly photoemission. Section 2 shows that the former is negligible, evaluates the latter for various incident beams and selects the optimal reaction among those considered. Section 3 analyses this reaction in detail.

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Why $\pi \rightarrow e/\pi \rightarrow \mu$ is so small

\[ \pi^+ \rightarrow l^+\nu \]

$M(\pi) = 139.56995$ MeV
KE(e) = 70 MeV $M(e) = 0.51099$ MeV $\gamma(e) = 141$
KE(\mu) = 4 MeV $M(\mu) = 106.65839$ MeV $\gamma(\mu) < 1.04$
KE(\nu) > 29 MeV $M(\nu) < 10^{-6}$ $\text{MeV} \quad \gamma(\nu) > 2.9 \times 10^6$
then $G_{\pi N} = 13.7$; $G_{\pi N}^{\text{int}} = 8.2$; $G_{NKA} = 2.3$; $G_{NK\Sigma} = 1.8$ ($\gamma_5$ in $K$ ints.), $G_{\pi N} = 13.7$; $G_{\pi N}^{\text{int}} = 8.2$; $G_{NKA} = 2.2$; $G_{NK\Sigma} = 1.8$ ($\gamma_5$ in $K$ ints.).

If the calculated results for the relative sign of $G_{\pi N}$ to $G_{\pi N}^{\text{int}}$ and $G_{NKA}$ to $G_{NK\Sigma}$ can be carried over to the $g$-coupling constants, the heavy-fermion mass degeneracy must be broken by the $\Sigma K (\Lambda, \Sigma)$ interactions. Possible Lagrangians are

$$
L = \frac{i}{2} g_{\pi^\mu} \left\{ \phi_\mu \gamma^\mu \gamma_5 \gamma_3 \gamma_5 \psi_{\psi} - \phi_{\bar{\psi}} \gamma_5 \gamma_3 \gamma_5 \psi_{\bar{\psi}} \right\} 
+ g_{K^\mu} \left\{ \phi_\mu \gamma_5 \gamma_3 \gamma_5 \gamma_3 \psi_{\psi} + \gamma_3 \gamma_5 \gamma_3 \gamma_5 \psi_{\bar{\psi}} \right\} 
- \phi_\mu \gamma_5 \gamma_3 \gamma_5 \gamma_3 \psi_{\psi} 
- \phi_{\bar{\psi}} \gamma_5 \gamma_3 \gamma_5 \gamma_3 \psi_{\bar{\psi}} 
$$

(3)

The $\gamma_5$ multiplies $(1 - \gamma_5 \gamma_2)/2$ instead of $(1 + \gamma_5 \gamma_2)/2$ for the case of "no $\gamma_5$ in $K$ ints." Another $K$ interaction,

$$
L_K = g_{K^\mu} \phi_\mu \left\{ \psi_{\bar{\psi}} + i \gamma_3 \gamma_5 \gamma_3 \gamma_5 \psi_{\bar{\psi}} \right\} \beta(\gamma_5, 1) \psi_{\bar{\psi}},
$$

with the $\pi$ interaction of (3) also splits the masses.

It must be mentioned that lowest order perturbation theory is unable to account for any polarization of the $\Lambda$ hyperon if the effective coupling constants are assumed real.\(^1\)

\(^2\) See reference 1 for notation.
\(^3\) Similar work was done by T. Ogimoto and T. Shimizu, Progr. Theoret. Phys. Japan 18, 213 (1957).

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**ELECTRON DECAY OF THE PION**

T. Fazzini, G. Fidecaro, A. W. Merrison, H. Paul, and A. V. Tollestrup

CERN, Geneva, Switzerland

(Received September 12, 1958)

It was predicted some years ago by Ruderman and Finkelstein\(^4\) that if the decay of the pion into an electron and a neutrino goes through an axial vector interaction, then it should occur at a rate $1.3 \times 10^{-4}$ of the normal decay into a muon and a neutrino. This conclusion has not been changed in the light of recent work on the nonconservation of parity in weak interactions.\(^5\) However, experiments by Lokanathan and Steinberger\(^3\) and by Anderson and Lattes\(^6\) failed to show the existence of the electron mode of decay. Interest has been revived in a search for this decay by the evidence for the validity of a universal Fermi interaction, which, with the single exception of the $\pi-e$ decay, is good.\(^5\) Theoretical attempts to remove this discrepancy have been made by a number of authors.\(^8\)

The experiment is made difficult by the presence of a large background of electrons from the $\pi - \mu - e$ chain of decay. However, these electrons can be distinguished in three ways. Firstly, they have a continuous spectrum with a maximum kinetic energy of 52.3 Mev, compared with the line spectrum of electrons from $\pi-e$ decay of 69.3 Mev. Secondly, the $\pi-e$ electrons should show a simple exponential decay with the mean life identical to that of the $\pi-\mu$ decay, while the $\pi-\mu-e$ electron time distribution would show a two-stage radioactivity decay, with a fast rise (approx. $\pi-\mu$ mean life) and a slow fall (approx. $\mu-e$ mean life). Lastly, three charged particles can be seen in the $\pi-\mu-e$ chain, while only two are shown in the $\pi-e$ decay.

We have used, like Lokanathan and Steinberger,\(^3\) a range telescope to search for the high energy $\pi-e$ electron. The apparatus is shown in Fig. 1. Pions from the CERN 600-Mev synchrocyclotron are incident upon the counter telescope $123\Sigma$ and stopped in counter 3. The electron telescope is formed of counters 5-12 in fast coincidence,

---

**FIG. 1.** Experimental layout, and (inset) typical $\pi-\mu-e$ and $\pi-e$ pulse.
with various amounts of high-density graphite inserted between the counters. These two telescopes are in coincidence so that all 5-12 coincidences which occur between 60 μsec before and 160 μsec after a stopped pion in the 1234 telescope are counted. If such a coincidence occurs, the pulses appearing in counter 3 and in counter 12 are photographed with a fast (~2000 Mc/sec) travelling-wave oscilloscope. Figure 1 (inset) shows two typical events corresponding to a π-μ-e and a π-e decay. There is a fixed delay between the e(3) pulse and the e(12) pulse. In this way all the time information associated with the decaying pion is recorded. At the same time we recorded on a slow oscilloscope the pulses from a large sodium iodide crystal backing the range telescope.

The events which we saw on the fast oscilloscope could be classified into various categories:

1. π-μ-e (see Fig. 1).
2. π-e (see Fig. 1). Included in this category will be false "π-e" events, where we could not resolve the muon pulse.
3. Prompt coincidences between the two fast telescopes. π-e events where the electron appeared very close to the pion are included here.
4. Randoms, i.e., events which have an improper time distribution; among them, for example, events where e(3) comes before the pion pulse.

The fraction of π-μ-e events where we could not resolve the muon was obtained from runs with no absorber in the electron telescope, where the number of genuine π-e events was negligible, and was about 0.23 of the total detected π-μ-e events. Figure 2 shows the electron range curves we obtained with different thicknesses of absorber. The π-μ-e and π-e events were selected directly from the film, and both curves are normalized to the same number of stopped pions. The π-μ-e curve shown does not, of course, represent the background in the experiment, because only the false "π-e" events provide a background there. It can be seen from the π-e range curve that within the errors the number of π-e events does not fall with increasing absorber thickness. The end-point of the μ-electron spectrum shown in Fig. 2 has been calculated using the (dE/dx) for positrons given by Rossi,8 corrected according to Sternheimer's calculation of the density effect.8

From the runs with absorber thicknesses of 30, 31, 32, and 34 g/cm² we obtained 40 π-e events in which the pion decayed later than 8.3 mμsec, from a total of 124 photographs. The rest of the events were made up of 16 π-μ-e, 27 prompt and 41 accidental. At the maximum absorber thickness we had a total of 17 events; of these 8 were π-e, 7 prompt, 2 accidental, and we observed no π-μ-e event. In the 40 π-e events there were 16×0.23 ± 4 false π-e events.

An integral decay curve of the 40 π-e events is shown in Fig. 3(a). The straight line corresponds to the π-μ mean life given by Crowe. The mean life calculated from these events, after subtraction of background, is

\[ \tau_{π-e} = 22 ± 4 \text{ μsec} \]

From the runs with no absorber we selected the false π-e events and an integral decay curve for these events is also shown in Fig. 3(a). It can be seen that this distribution is linear in time and is quite different from the exponential distribution of the true π-e events. A further check on these results is an integral decay curve for the π-μ decay from the π-μ-e events corrected for the resolution time between the two telescopes. This is shown in Fig. 3(b).

FIG. 2. Range curves for the π-μ-e and π-e events.
We also observed that in the sodium iodide crystal there was in general no pulse, or at most a very small pulse, associated with each \( \pi - \mu - e \) event. On the other hand, most of the pulses associated with \( \pi - e \) events were large. We did not make any systematic use of this information, as the number of \( \pi - e \) events seen in each absorber run was too small.

The above results seem good evidence for the existence of the electron decay mode of the pion. We have not so far determined experimentally the efficiency and effective solid angle of the electron telescope. If we assume the merely geometric solid angle of 0.8% of 4\( \pi \) and an efficiency of 100% we get as a lower limit for the branching ratio

\[
\frac{\pi - e + \nu}{\pi - \mu + \nu} > 4 \times 10^{-5}.
\]

This number has been calculated from our 40 events, from which we have subtracted 4 events for the false "\( \pi - e \)" events which must be there, and we have added 37% for the early \( \pi - e \) decays occurring in the first 8.3 ms. The correction for secondaries was evaluated from the number of events which show on the fast scope as time inverted \( \pi - e \) decays (e-\( e \) events). This correction turned out to be zero.

This value for the branching ratio must be considered as a lower limit because we know that a large correction must be made for the efficiency of the telescope. To give an idea of the order of magnitude of this correction, we recall that Lakanathan and Steinberger calculated an efficiency of 50% for their telescope which was similar to ours. As a conclusion we can say that the value we obtain for the branching ratio is not in disagreement with that predicted by Ruderman and Finkelstein. The experiment is still in progress and we hope soon to have a more accurate numerical result.

We should like to thank Professor G. Bernardini for many discussions and his continuous encouragement. Mr. M. Fell and Mr. M. Reneve built most of the apparatus with great skill and care, and Mr. F. Blythe was responsible for all the drawing work involved. We should like to thank Mr. O. Fredriksson and the members of the cyclotron crew for their efficient and helpful running of the machine. Dr. I. Halpern aided us considerably in the running of the experiment. In particular, we should like to thank Dr. J. Ashkin for his friendly collaboration in the later stages of this experiment.

\^National Science Foundation Fellow on leave from the California Institute of Technology, Pasadena, California.


\^H. L. Anderson and C. M. G. Lattes, Nuovo cimento 4, 1356 (1957).


\Beta Decay of the Pion\^*


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(Received September 15, 1958)

The electron-neutrino decay mode of the pion has been the object of several unsuccessful searches.\^*\^\^* The latest and most sensitive of these\^*\^\^* puts an upper limit of about \( 10^{-8} \) for the relative frequency of this process.

On the other hand, there has recently been
spectacular progress in the understanding of nuclear $\beta$ decay and of $\mu$ decay in terms of a "universal" $V-A$ theory.\textsuperscript{4-6} It has been emphasized, especially by Feynman,\textsuperscript{7} that in a universal $V-A$ theory the ratio of pion $\beta$ decay to $\mu$ decay should be $1/8000$, and that deviations from this value are difficult and awkward to accommodate theoretically.

There is, therefore, considerable interest in the announcement of the CERN group\textsuperscript{6} that positive evidence for the $\pi-e$ decay in the theoretically expected order of magnitude has been found. In view of the disagreement between these results and those of Anderson and Lattes,\textsuperscript{3} there may be some interest in the progress of our experiment on this decay.

A liquid $\text{H}_2$ bubble chamber 12 in. diameter, 6 in. deep in a field of 8800 gauss was exposed to slow $\pi^+\pi^0$ mesons. On the average, 10 mesons stop per picture. We look for events in which the stopped meson emits a minimum-ionizing secondary with no visible intermediate $\mu$ meson; and we measure the secondary momentum. One-forth of the stopping events are of this type and are chiefly examples of $\mu-e$ decay. Separation of the $\pi-e$ from these $\mu-e$ is achieved by means of the momentum measurement; the $\mu$ spectrum extends to a maximum energy of 53 MeV/$c$, and the electron from $\pi-e$ decay must have 70 MeV/$c$. The error of momentum measurement, including multiple scattering error and possible distortion in the liquid, is about 3%.

To the present, we have analyzed an effective sample of 65000 meson decays and have found 6 clear examples of $\pi-e$ decay. One of these is reproduced in Fig. 1.

We wish to examine here the possibility that these events have some other origin. We begin with the possibility that they are really $\mu-e$ decays with electrons of abnormally large measurement error. To answer this question the observed energy spectrum of all 1766 measured events is reproduced in Fig. 2. We have drawn in the theoretical curve for $\rho = 0.75$ with the

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2.png}
\caption{Histogram of the momenta of the secondaries of all events in which the incoming stopping track apparently decays directly into an electron.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure3.png}
\caption{Histogram of the electron momenta of 2997 events of normal $\pi-\mu-e$ decay, type "B" of Fig. 1.}
\end{figure}
3% resolution folded in. It is clear that the tail of the $\mu$ spectrum follows very closely, and the probability for finding an event in the $\pi-e$ region is very small. This is further demonstrated in Fig. 3. Figure 3 exhibits the spectrum of 3000 events measured for the purpose of a $p$-value determination. They are events with the normal $\pi-\mu-e$ sequence (see Fig. 3), and therefore, no $\pi-e$ decays are expected. The fact that no high-energy events are found in this larger sample demonstrates that the $\mu-e$ contamination in the high-energy region is negligible.

We have also considered the possibility that the events are decays in flight of $\mu$ mesons, or perhaps $\pi-\mu$ decays in flight. These possibilities can be ruled out on kinematic grounds for each of the 6 events. We conclude that each event is a clear example of $\pi-e$ decay.

The average energy for the 6 events is $72.9\pm1.5$ Mev. This is higher than the expected energy of 69.6 Mev. We have taken great care in the systematics of the energy measurement and are quite certain that this cannot be the cause. We are perplexed by the discrepancy and tentatively ascribe it to a statistical fluctuation in the measurement errors.

The relative rate of $\pi-e$ decay is $6/65000$ or $1/10800 \pm 40\%$. This is unfortunately statistically poor, but serves to indicate that the decay is at least roughly as expected theoretically.

The method does not yield a precise measurement of the branching ratio and cannot reasonably be extended to do so. However, the results here offer a very convincing proof of the existence of this decay mode, and show that the relative rate is close to that expected theoretically.

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NEUTRON THRESHOLD MEASUREMENTS USING THE CHALK RIVER TANDEM VAN DE GRAAFF ACCELERATOR

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(Received September 2, 1958)

As part of the program to test the operating characteristics of the first Tandem Van de Graaff accelerator, a number of neutron threshold measurements were made in the proton energy range from 1.8 to 10.5 Mev. Three of these, $\text{Pb}^{20}(p,n), \text{Ni}^{60}(p,n)$, and $\text{Ni}^{58}(p,n)$, all above 6.4 Mev, had not been measured previously. Over the complete energy range proton beams of 0.5 microampere or greater were available and the accelerator operated in a stable and reproducible fashion.

A momentum calibration of the $90^\circ$ energy analyzing magnet was obtained by measuring a number of well known neutron thresholds using the conventional counter ratio technique. The two neutron detectors were enriched $\text{B}^{10}$ loaded scintillators optically coupled by Lucite light pipes to 3 in. diameter photomultipliers. A cylinder of paraffin of suitable geometry surrounded the second detector to ensure uniform response to fast neutrons.

In a series of measurements carried out prior to final accelerator adjustments, the $\text{Li}^{7}(p,n)$, $\text{Cu}^{65}(p,n)$, $\text{B}^{11}(p,n)$, and $\text{Al}^{27}(p,n)$ thresholds were used to calibrate the magnet momentum scale. With this calibration, thresholds for the $\text{Pb}^{20}(p,n)$ and $\text{Ni}^{60}(p,n)$ reactions were measured.

The first three lines of Table I list the $(p,n)$ reactions used to calibrate the magnet in a second series of measurements carried out under improved operating conditions. To calibrate the magnet for particle momenta equivalent to higher proton energies, oxygen gas was supplied to the ion source. The threshold for the reaction $\text{H}^{2}(\text{O}^{16},n)^{17}$ was observed by bombarding a deuterated zirconium target with $\text{O}^{16}$ ions of charge +4 and +5. This threshold was computed to be $14.750 \pm 0.024$ Mev using the
decreases rapidly as the energy increases due to the decrease of $\gamma_\rho$. At about 20 Mev $\gamma_\rho$ and $\gamma_1$ are of the same order of magnitude and the forward maximum has totally disappeared. As the energy increases still more, $\gamma_1$ becomes larger than $\gamma_\rho$ and at the same time $\gamma_2$ comes into play. We get again a forward maximum. The forward polarization, however, is now positive. The backward maximum decreases slightly and shifts to somewhat smaller angles as the energy increases. In Fig. 2 the polarization of the proton as well as that of the neutron is given for 80.4-Mev $\gamma$ rays.

The difference between the proton and neutron polarization at lower energies is mainly due to the M1-E2 interference and at higher energies mainly to the E1-E2 interference. The neutron polarization is more negative (less positive) at small angles and less negative at large angles. The maximum difference occurs near the maxima of the polarization and is 2% at 9.3 Mev, 5% at 22.4 Mev, and 7% at 80.4 Mev.

We would like to thank Dr. P. S. Signell, Dr. P. J. Eberlein, and Mr. R. A. Bryan for their help with the numerical computations. J. J. deS. wishes to thank Professor R. E. Marshak for his guidance and stimulation in many discussions during the course of this work, and W. C. wishes to thank Professor N. Bohr and Professor A. Bohr for hospitality at the Institute for Theoretical Physics, University of Copenhagen, Denmark.

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J. J. de Swart, Physica (to be published).


J. J. de Swart and R. E. Marshak (to be published).


This convention differs from that of reference 1 by the change of sign of $\hat{r}$, thus of $P(\hat{r})$.


The signature of $P(\hat{r})$ is opposite to the present one if the convention $\hat{r} = \vec{r} \times \hat{r} / |\vec{r} \times \hat{r}|$ of reference 1 is used.

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**ELECTRON DECAY OF THE POSITIVE PION**

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(Received December 22, 1958)

In a previous communication from this laboratory an unsuccessful search for the electron decay mode of the pion was reported which made it seem unlikely that the branching ratio $f = (\pi^- e^- + \nu)/(\pi^- \mu^- + \nu)$ could be much larger than 2x10^{-5}. The remarkable success of the A-V theory in accounting correctly for many related phenomena had raised doubt about our failure to observe this process. Accordingly, we decided to make a new attempt, and this effort was accelerated when we learned of the successful observation of the electronic mode by the CERN group and by Steenbergen.

Our new work confirms the existence of the $\pi^- e^-$ mode in an amount not very different from the value 1.28x10^{-4} predicted by the universal A-V theory (without radiation correction). We also know that additional confirmations have been obtained from experiments similar to ours which are in progress at Stanford and Berkeley. We
a \( \pi^-e \) event. These are displayed in Fig. 3. The smooth curve is the best fit calculated by folding into a \( \delta \)-function at \( E = E_\pi^- \) the known energy loss (including bremsstrahlung effects) in the source, as well as the resolution function of the spectrometer, known from \( \alpha \)-particle calibrations, in the manner described previously. A constant background entered as a parameter in the fit. This accounted for \( 39 \pm 15 \) of our events. We could show from timing measurements that it was likely that \( 20 \pm 5 \) of these were due to \( \pi^-\mu \) events in counter 3, in accidental coincidence with spurious counts in (4, 5). We supposed that the remainder were due to \( \mu^- \)-electrons which managed, by scattering, to give counts (3, 4, 5) in accidental coincidence with pion counts in (2, 3). Taking account of the time distribution to be expected from such a composition of background events the mean life of the pion obtained from our \( \pi^-e \) events turned out to be \( 25 \pm 3 \) m\( \mu \)sec, close to the accepted value.

In recording the \( \mu^- \)-electrons at the lower current settings of the spectrometer, we removed the requirement of a delayed pion coincidence, using simply the coincidence (2, 3, 5) with one of the counters 4. The data shown in Fig. 4 are from channel b (others were hardly different). It was fitted by adjusting the amplitude, the Michel parameter \( \rho \), and the end-point energy \( E_{\pi^-} \). From all the data we found \( E_{\pi^-}/E_{\mu^-} = 1.32 \pm 0.01 \) as expected, and \( \rho = 0.74 \pm 0.03 \).

The ratio of the integrals was found to be \( (3.49 \pm 0.56) \times 10^{-3} \). This must be multiplied by a factor 2.57 to take into account the pions which decayed outside the measured time interval, a factor 1.13 \( \pm 0.01 \) for the \( \pi^-e \) events missed by our selection criteria, a factor 1.04 \( \pm 0.02 \) for losses due to bremsstrahlung, and a factor 0.98 \( \pm 0.02 \) for the muons which escaped from the source. The branching ratio calculated from these data is \( f = (1.03 \pm 0.20) \times 10^{-3} \). We have augmented the stated error a small amount to reflect our own uncertainty in the manner of dealing with the background.

Note: See ADDENDUM on page 64.

*Research supported by a joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

1H. L. Anderson and C. M. G. Lattes, Nuovo cimento 6, 1556 (1957).


6The original calculation of M. Ruderman and R. Finkelstein, Phys. Rev. 76, 1458 (1949), for a pseudoscalar pion with axial vector \( \beta \) coupling is appli-
pion-nucleon 600-Mev scattering resonance and hence occurs in the $I = \frac{1}{2}$ state. Since $\sigma'$ represents only the partial cross section for magnetic dipole photons leading to $j = \frac{1}{2}$, $I = \frac{1}{2}$ states, it is certainly reasonable to assume that $\sigma' < 100 \mu$b. Hence we assume $W_1' / W_2' < 2$. In order to estimate $R_2$, we note that at 450 Mev lab energy the total inelastic cross section for an incident $\pi^+$-proton state ($I = \frac{3}{2}$ state) has been measured to be less than 2.5 millibarns.\(^8\) Hence the partial cross section $\sigma_2^{\text{in}}$ must also be less than 2.5 mb. Since the maximum possible partial inelastic cross section $\sigma_2^{\text{max}}$ at this energy is 17.5 mb, it is seen that $R_2 < 0.15$. By definition $W_2$ satisfies the inequality $W_2 < R_2$. We conclude that $R_1$, $R_2$, $W_2$, and $W_1' / W_1' < 2$ satisfy the relations $R_1 = 0$, $R_2 < 0.15$, $W_2 < R_2$, and $W_1' / W_1' < 2$. From these relations and Eq. (2) it can be shown that $\cos(2\theta_{12}) > 0.84$. Therefore, the phase $\phi_2$ of the amplitude $T_{13}$ must satisfy one of the two inequalities,

$$|\phi_{12} - \delta_2| < 17^\circ \quad \text{or} \quad |\phi_{12} - \delta_2 - \pi| < 17^\circ,$$

where the small photon scattering phase $\delta_2$ has been neglected. Thus the phase relation characteristic of the two-channel case is nearly satisfied here, even though several channels are important.

The amplitudes for production of a $p - \pi^0$ state are linear combinations of the amplitudes for production of $I = \frac{1}{2}$ and $I = \frac{3}{2}$ pion-nucleon states. In the model of references 4 and 5 however, the $I = \frac{1}{2}, j = \frac{1}{2}$ magnetic dipole state is neglected, so that the phase $\phi_{12}$ of Eq. (3) above is equal to the phase of the $j = \frac{1}{2}$, magnetic dipole amplitude for the process $\gamma + p - p + \pi^0$. Similar results may be obtained for the phases of other angular momentum and parity states. In this manner polarization and angular distribution measurements of photoproduction may be related to similar measurements of pion-nucleon scattering.

An interesting discussion concerning this subject was had with Ronald F. Peierls.

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*Supported by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.


\(^3\)The choice is definite except that the phase of any state (and hence of all nondiagonal elements of $T$ involving this state) may be increased by $\pi$ without destroying the symmetry of $T$. See reference 2.


\(^6\)P. C. Stein (to be published). Other experimental references are listed in reference 4.

\(^7\)Sellen, Cocconi, Cocconi, and Hart, Phys. Rev. 110, 779 (1958).


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RADIATIVE CORRECTIONS TO $\pi-e$ DECAY*

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(Received May 11, 1959)

As is well known, the ratio of probabilities for $\pi-e$ and $\pi-\mu$ decays is given by\(^4\)

$$R_\pi = \left( \frac{m_\pi}{m_e} \right)^2 \left( \frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2} \right)^2 = 1.28 \times 10^{-4},$$

(1)

neglecting electromagnetic corrections, if the decay interaction is assumed to be

$$i g^4 \vec{\gamma} \not\! \mu \not\! \nu (\partial \phi / \partial x),$$

(2)

where $a = (1 + i \gamma_5) / 2$ and $l$ represents either muon or electron. This interaction is consistent\(^2\) with the hypothesis of universal $V-A$ interaction of Fermi couplings.\(^3\) Recent experiments\(^4\) support these assumptions strongly. The new measurements are becoming sufficiently accurate to justify a calculation of the effect of radiative corrections. This problem has recently been studied by Berman,\(^5\) who has found surprisingly large corrections to $\pi-e$ decay. In this note it is attempted to understand the reason why the radiative corrections are so large. We are also interested to see whether the pion decay agrees with the recently conjectured "theorem" that the radiative correction to the total probability of a decay process is finite in the limit where the
mass of the secondary electron is assumed to be arbitrarily small, although corrections to partial probabilities may be divergent in such a limit.

We first calculated the probability for the inner bremsstrahlung (IB) process in which the pion disintegrates into electron, neutrino, and photon.\(^7\) We integrate it over all neutrino and photon momenta to find the energy spectrum of the electron.\(^8\) Integrating it further over the electron energy, the probability of observing any electron whose energy is less that \(E_{\text{max}} - \Delta E\) is found to be

\[
\frac{\Delta P_{\text{IB}}}{P_0} = \frac{\alpha}{\pi} \left\{ b(\mu) \left[ \ln \left( \frac{m_\pi}{\Delta E} \right) + 2 \ln(1 - \mu^2) - \frac{1}{2} \right] \right.
\]
\[-\frac{\mu^2 (10 - 7\mu^2)}{2(1 - \mu^2)^3} \ln \mu + \frac{2(1 + \mu^2)}{1 - \mu^2} L(1 - \mu^2)
\]
\[+ \frac{15 - 21 \mu^2}{8(1 - \mu^2)} \right\},
\]

where \(P_0\) is the uncorrected rate of decay,

\[
L(\alpha) = \int_0^\alpha \int_0^\infty (1 - \alpha) d\alpha d\gamma,
\]

\[\mu = m_e/m_\pi,
\]

and \(\Delta E\) is assumed to be small compared with the maximum energy \(E_{\text{max}} = m_\pi (1 + \mu^2)/2\). The total probability of inner bremsstrahlung is given by

\[
\frac{\Delta P_{\text{IB}}}{P_0} = \frac{\alpha}{\pi} \left\{ b(\mu) \left[ \ln \left( \frac{\lambda}{m_\pi} \right) - \ln(1 - \mu^2) - \frac{1}{2} \ln \mu + \frac{1}{2} \right] \right.
\]
\[-\frac{\mu^2 (10 - 7\mu^2)}{2(1 - \mu^2)^3} \ln \mu + \frac{2(1 + \mu^2)}{1 - \mu^2} L(1 - \mu^2)
\]
\[+ \frac{15 - 21 \mu^2}{8(1 - \mu^2)} \right\}.
\]

This contains an infrared divergence since photons of very low energy (with infinitesimal mass \(\lambda_{\text{min}}\)) are emitted near the maximum electron energy.

To find the correction due to virtual emission and reabsorption of photons, let us note that the interaction (2), or more precisely

\[
g \bar{\psi} \gamma^\mu a^\mu \gamma^\nu (i \partial \phi_{\pi}/\partial \mu - eA_{\mu} \phi_{\pi}),
\]

is equivalent to

\[
gm_{\pi} \bar{\psi} \gamma^\nu a^\mu \gamma^\nu \phi_{\pi},
\]

in the lowest order in \(g\) and to any order in \(e\),

where \(m_\pi^0\) is the bare mass of the lepton.\(^9,10\)

Thus the effect of virtual photons may be regarded as consisting of two parts: (A) the correction to the operator \(\bar{\psi} a^\mu \gamma^\nu \phi_{\pi}\) due to the dynamical effects of virtual emission of photons, and (B) the correction to the coefficient \(g \bar{\psi} a^\mu \gamma^\nu \phi_{\pi}\) that arises when one tries to express it in terms of the observed mass \(m_\pi\).

The correction \(A\) is found to be

\[
\frac{\Delta P_A}{P_0} = \frac{\alpha}{\pi} \left[ \frac{3}{4} \ln \left( \frac{\lambda}{m_\pi} \right) - b(\mu) \left[ \ln \left( \frac{\lambda_{\text{min}}}{m_\pi} \right) - \frac{1}{2} \ln \mu + \frac{1}{2} \right] \right.
\]
\[+ \frac{\mu^2}{1 - \mu^2} \ln \mu + \frac{1}{2} \right],
\]

where \(\lambda\) is the ultraviolet cutoff. Thus, if the correction \(B\) is disregarded for the moment, the corrected rate of \(\pi^\pm\) decay is given by

\[
P = P_0 (1 + \eta),
\]

where

\[
\eta = \frac{\alpha}{\pi} \left[ \frac{3}{4} \ln \left( \frac{\lambda}{m_\pi} \right) - b(\mu) \ln(1 - \mu^2) - \frac{\mu^2 (8 - 5\mu^2)}{2(1 - \mu^2)^3} \ln \mu
\]
\[+ \frac{2(1 + \mu^2)}{1 - \mu^2} L(1 - \mu^2) + \frac{19 - 25 \mu^2}{8(1 - \mu^2)} \right].
\]

from (4) to (7). As a matter of fact, it is not necessary to calculate the correction \(B\) since it is already included in (8) if one remembers that the factor \((m_\pi^0)^2\) which appears in \(P_0\) is (bare mass)\(^2\) even when the radiative correction is taken into account.

The total decay rate \(P\) depends logarithmically on the cutoff \(\lambda\), but the ratio \(R\) of total decay rates for \(\pi^\pm\) and \(\mu^0\) is independent of the cutoff if it is taken to be the same for both.\(^11\) Under this assumption, the radiative corrections to \(R_0\) of (1) can be expressed as

\[
R = R_0 (1 + \delta)(1 + \epsilon),
\]

where \(1 + \delta = [(m_e^0/m_\pi)(m_\mu/m_\pi)]^2\) comes from correction \(B\) and \(1 + \epsilon\) from correction \(A\) and inner bremsstrahlung. Using the electromagnetic mass of order \(\alpha\), one obtains

\[
\delta = -3(3\alpha/\pi) \ln(m_\mu/m_e) = -16.0(\alpha/\pi).
\]

It is seen from (9) that \(\epsilon = -0.92(\alpha/\pi)\) to order \(\alpha\).

Note that there is no physical distinction between correction \(A\) and correction \(B\), their only role being to modify the decay coupling constant.

It is therefore misleading to talk of them se-
parately. As is seen from the smallness of $\epsilon$, however, the sum of correction $A$ and inner bremsstrahlung is quite insensitive to whether the decay particle is muon or electron. Thus the radiative correction to $R_\beta$ is mostly due to the correction $B$. In this sense one might say that, when one measures $R$, one is actually observing a finite difference of electron and muon self-energies.

Ordinarily the experiment looking for $\pi^-e$ decay excludes electrons whose energy is too low, in order to distinguish them from those of $\pi^-\mu^-e$ decays. Thus we should like to have the ratio $R(\Delta E) = (\text{number of } \pi^-e \text{ decays for which the energy of the electron is within } \Delta E \text{ of the maximum energy})/(\text{number of } \pi^-\mu^- \text{ decays}).$ This is obtained from (3) and (8). As is easily seen, $R(\Delta E)$ can be written in the same form as (10) where $\delta$ is still given by (11) but $\epsilon$ now depends on $\Delta E$. The result agrees with Eq. (2) of reference 5.

Numerically, the radiative correction to the ratio $R_\beta$ is $-3.9\%$ if all decay electrons are counted. Of this value, $-3.7\%$ is due to the mass correction (11). Only $-0.2\%$ comes from the inner bremsstrahlung and the virtual photon correction $A$. If only those electrons are observed whose energy is larger than $E_{\text{max}} - \Delta E$, the correction is $-7.8\%$ for $\Delta E \sim 10m_e$ and $-14\%$ for $\Delta E \sim 0.5m_e$. Of these, $-3.7\%$ are always due to the mass correction $\delta$. The large negative corrections that still remain are nearly equal in magnitude to the positive probabilities of inner bremsstrahlung (3) which are $3.9\%$ for $\Delta E \sim 10m_e$ and $10\%$ for $\Delta E \sim 0.5m_e$. This may be understood qualitatively if one imagines that the probability of finding high-energy electrons is reduced simply because some of them have been shifted to the low-energy side of the spectrum by inner bremsstrahlung. Thus, putting aside the bare mass correction which is energy independent, the large radiative correction found by Berman in $R(\Delta E)$ may be regarded as a consequence of the high efficiency of the $\pi^-e$ system as an emitter of hard photons.

The results of this paper can be qualitatively understood by making use of the “theorem” mentioned at the beginning. At first sight, the radiative correction to $\pi^-e$ decay, although it does not contradict the “theorem,” supports it only in a trivial fashion, since not only the total rate but any partial rate of $\pi^-e$ decay vanishes for $m_e^-0$ because the interaction (5) is proportional to $m_e^-$. The fact that the correction $\delta$ of (11) diverges logarithmically for $m_e^-0$ gives no trouble since $R_\beta$ tends to zero at the same time. This is in fact expected from our “theorem.” It is interesting to note however that the correction $\epsilon$ does not diverge in this limit. This can be easily explained by our “theorem,” too. For this purpose, we have only to point out that $\epsilon$ would be the total radiative correction for pion decay if the interaction were given not by (5) but by

$$f\vec{\alpha}_\mu^\nu \phi_{\pi}^\nu,$$

(12)

where $f$ is a coupling constant independent of $m_\pi$. Since $P_\beta$ is now finite for $m_e^-0$, the correction $\epsilon$ cannot afford to diverge if the “theorem” should hold. But (9) is in fact finite for $m_e^-0$ whereas (3), (4), and (7) diverge. Thus the pion decay gives additional support for the general validity of this conjectured “theorem.”

The author would like to thank Professor R. P. Feynman who contributed greatly to this work. He also wishes to thank Dr. S. M. Berman for checking our calculation including the total decay rate.

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6Supported in part by the joint program of the Office of Naval Research and the U.S. Atomic Energy Commission.


6T. Kinoshita and A. Sirlin, 113, 1652 (1959). This “theorem” is conjectured on the basis of our study of radiative corrections to muon decay and beta decay. An attempt is now being made to prove or disprove it for the general case.

7The inner bremsstrahlung accompanying pion decay was studied some years ago in connection with observations of anomalously short muon tracks in pion decay. See W. F. Fry, Phys. Rev. 56, 418 (1932). The latest experimental result is reported by C. Castagnoli and M. Muchnik, Phys. Rev. 112, 1779 (1959). For theoretical work, see H. Primakoff, Phys. Rev. 84, 1255 (1951); Nakano, Nishimura, and Yamaguchi, Progr. Theoret. Phys. (Kyoto) 5, 1028 (1951); T. Eguchi, Phys. Rev. 85, 943 (1952).

8If the electron mass is neglected in comparison with its energy whenever this approximation does not lead to spurious divergences, the differential spectrum of
Theorem on $\pi_{12}$ Decays and Electron-Muon Universality*

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We show that the coefficient of the logarithmic lepton-mass singularity in the radiative corrections of order $\alpha$ to the total $\pi_{12}$ decay probability is not affected by the strong interactions and can therefore be rigorously computed. The phenomenological implications of this result and its connection with electron-muon universality are briefly discussed.

For a long time the study of $\pi_{12}$ decays has been one of the cornerstones of weak-interaction theory. Roughly at the time of the discovery of the electron decay mode, Berman and Kinoshita published their pioneering calculations of the order-$\alpha$ corrections to these processes, neglecting the effects of the strong interactions.1,2 In particular Kinoshita calculated the corrections to the total decay probability and found a surprisingly large result. More recently, the calculation was repeated on the basis of a renormalizable model in which the $\pi$ is treated as a member of a Higgs multiplet and essentially the same answer was obtained.3 To this date it has remained unclear whether the results of the early calculations are merely properties of the peculiar models assumed or whether they apply to the real world. In this paper we answer this question to a considerable extent by means of the following theorem: “The coefficient of the logarithmic lepton-mass singularity (lms) in the radiative corrections of order $\alpha$ to the total $\pi_{12}$ decay probability is not affected by the strong interactions and can therefore be rigorously computed.”4 In proving this theorem we assume that the weak and electromagnetic interactions are described by a renormalizable gauge theory in which $e-\mu$ universality is natural and we neglect corrections of order $G_F^2$. As we will see, the theorem does not hold in general for partial decay probabilities.

In the gauge theories under consideration, the interaction of the $W$ mesons with leptons and hadrons is described by

$$L_W = -\frac{g}{\sqrt{2}}(J^\lambda + L^\lambda)W^\dagger + H.c.,$$

where $L^\lambda = \bar{\nu}\gamma^\lambda a\mu + \ldots$ and $J^\lambda = \frac{1}{2} \cos \theta_W \times (V^\lambda - A^\lambda) + \ldots$ are the leptonic and hadronic currents, $a = \frac{1}{2}(1 - \gamma_5)$, and $\ldots$ indicates the possible contributions of heavy leptons and currents which carry strangeness, charm, and other possible flavors.

Virtual corrections.—We recall that the lms can arise because in the limit of zero lepton mass the invariants $k^2$ and $l-k$ ($k$ and $l$ are the photon and lepton four-momenta) may vanish for quanta of nonzero frequency. A moment’s thought tells us that aside from the usual field renormalization of the lepton, the only other virtual diagram that contributes to the lms is the one depicted in Fig. 1(a). (In nonphoton diagrams,

![Diagrams involving the strong interactions which contribute to lms](image-url)
\[ l = m_\pi^2 + m_e^2 \] appropriate to the two-body decay, and introducing the new integration variable \( s = m_\pi^2 (1 - x) \) in place of \( x \), we find

\[
M_a(H_2, H_0) = -M_0 (\alpha / 2\pi) \left\{ \ln (m_\pi / m_1) (1 / f_\pi) \int_0^{m_\pi^2} ds (1 - s / m_\pi^2)^2 [H_\pi(0, s) - H_\eta(0, s)] + \ldots \right\} .
\]  
(7)

Combining Eq. (7) with the contribution of the first two terms in Eq. (4) and adding the field renormalization of the lepton and the photon-infrared divergent part of the \( \pi \) field renormalization we finally obtain for the virtual corrections to the transition probability

\[
\Delta P_f = P_0 \frac{\alpha}{\pi} \left\{ 1 + \frac{1}{\mu^2} \left[ 2 \ln \mu \ln \left( \frac{m_\pi}{\mu_{\min}} \right) + \ln \mu \right] + 2 \ln \left( \frac{m_\pi}{\mu_{\min}} \right) - \frac{3}{2} \ln \mu \right\} + \ln \mu \int_0^{m_\pi^2} ds \left( 1 - \frac{s}{m_\pi^2} \right)^2 \left[ H_\pi(0, s) - H_\eta(0, s) \right] + \ldots \right\} ,
\]  
(8)

where \( \mu = m_1 / m_\pi \) and \( P_0 \) is the uncorrected decay rate. The terms involving \( H_\pi \) and \( H_\eta \) in Eq. (8) represent the structure-dependent contributions to Ims induced by the strong interactions. The remainder agrees with Kinoshita’s result.²

**Inner bremsstrahlung.**—We consider the contribution of diagram 1(b) and the additional graph in which the photon is emitted by the lepton. The total amplitude can be written as \( M_{1, B} = M_{1, B}^{(1)} + M_{1, B}^{(2)} \), where \( M_{1, B}^{(1)} \) is independent of the strong interaction form factors and \( M_{1, B}^{(2)} \) involves only terms proportional to \( H_\pi \) and \( H_\eta \). The contribution of \( |M_{1, B}^{(1)}|^2 \) to the total transition probability is given in Eq. (4) of Ref. 2. The only additional Ims arises from the interference of \( M_{1, B}^{(1)} \) and \( M_{1, B}^{(2)} \). This can be readily computed by summing over photon and lepton polarizations and integrating over the neutrino momenta, photon-lepton angle, and electron energy, in that order. For this structure-dependent contribution we find

\[
\Delta P_{1, B,(H_\pi, H_\eta)} = -P_0 (\alpha / \pi) \ln \mu (1 / f_\pi) \int_0^{m_\pi^2} ds (1 - s / m_\pi^2)^2 [H_\pi(0, s) - H_\eta(0, s)] + \ldots .
\]  
(9)

Compare Eqs. (8) and (9): The structure-dependent terms cancel in the total decay probability thus establishing the theorem! To obtain the model-independent answer we add Eq. (8), Eq. (9), and Kinoshita’s calculation of the contributions from \( |M_{1, B}^{(1)}|^2 \)² and find that all the Ims cancel in the total decay probability except for the term \( 3 (\alpha / \pi) \ln \mu \). This surviving contribution arises from the presence of the \( f_\pi g_{\mu \nu} \) term in Eq. (4): a consequence of gauge invariance! [See Eq. 3(b)]. As pointed out by Kinoshita, because \( M_0 \) is proportional to \( m_1 \), this result does not contradict the theorem on the cancellation of mass singularities.⁵

The cancellation of structure-dependent terms does not generally occur in partial decay probabilities. For example, if the lepton energy is restricted to the configuration \( E_m - \Delta E = E \ll E_m \) with \( \Delta E \ll E_m \) (\( E_m \) is the lepton energy in the two-body decay), the interference of \( M_{1, B}^{(1)} \) and \( M_{1, B}^{(2)} \) can be neglected and we are left with the structure-dependent terms of Eq. (8).

The above theorem has interesting phenomenological implications. In fact, aside from the contribution of \( |M_{1, B}^{(3)}|^2 \) which will be discussed later, the theoretical expression for the fractional correction \( \Delta P / P_0 \) to the total \( \pi \gamma \) decay rate can be expanded in a power series in \( m_1 \) and \( \ln m_1 \). We have shown that the terms of order \( \ln m_1 \) are structure independent. Their contribution to the ratio \( R \) of the total decay probabilities for \( \pi^- \rightarrow e^- + \nu_e \) and \( \pi^- \rightarrow \mu^- + \nu_\mu \) is \( -3 (\alpha / \pi) \ln (m_\mu / m_\pi) \times R_\pi \) which amounts to a very large correction: \(-3.7\%\). Aside from this, the strong interactions may affect the terms of zeroth order in \( m_1 \) and of order \( m_\pi^2 \ln m_1^2 m_1^2 \), etc. The terms of zeroth order in \( m_1 \) cancel in the ratio \( R \) while the contributions of order \( m_\pi^2 \) can be potentially significant only for the muon decay mode. As structure-dependent contributions induced by the strong interactions are expected to be of relative order \( m_\mu^2 / m_\pi^2 \), where \( m \) is a typical hadronic mass, and there are no large logarithms available, it is difficult to see how such terms can give rise to large unaccounted corrections. In addition to these effects, one must also consider the contribution of \( |M_{1, B}^{(3)}|^2 \) which is potentially large for the electron decay mode because, unlike \( P_0 \) and the other contributions to \( \Delta P \), it is not suppressed by a factor \( m_\pi^2 \). Fortunately, these terms can be analyzed by a combination of theoretical and experimental arguments: Neglecting the slight energy dependence of \( H_\pi(0, s) \)
New Measurement of the $\pi^{\rightarrow e\nu}$ Branching Ratio

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A new measurement of the $\pi^{\rightarrow e\nu}$ branching ratio yields $\Gamma(\pi^{\rightarrow e\nu}+\pi^{\rightarrow e\nu})/\Gamma(\pi^{\rightarrow \mu\nu}) = (1.218 \pm 0.014) \times 10^{-4}$. The measured value is in good agreement with the standard-model prediction incorporating electron-muon universality.

PACS numbers: 13.20.Cz, 14.40.Aq, 14.60.-z

A major puzzle in particle physics involves the existence of multiple seemingly redundant generations of fundamental particles. The lepton generations apparently have identical electromagnetic interactions as evidenced by the agreement of experiments and theory on the values of electron and muon anomalous magnetic moments. The hypothesis of universality of the weak interaction is tested most stringently by using the branching ratio for the electronic and muonic decay modes of the pion. The theoretical value of the branching ratio including radiative corrections,

$$R = \frac{\Gamma(\pi^{\rightarrow e\nu}+\pi^{\rightarrow e\nu})}{\Gamma(\pi^{\rightarrow \mu\nu})} = 1.233(f_{\pi}^{\rightarrow \mu}/f_{\pi}^{\rightarrow e})^{2} \times 10^{-4},$$

was determined by Kinoshita in an early $(V,A)$ phenomenological calculation, where $f_{\pi}^{\rightarrow e}$ is the pion decay constant and $f_{\pi}^{\rightarrow e}=f_{\pi}^{\rightarrow \mu}$ if universality holds. This result was reaffirmed to the level $\pm 0.5\%$ in the context of the standard gauge theory of weak and electromagnetic interactions by the theorem on radiative corrections in $\pi^{\rightarrow 3\pi}$ decay proved by Marciano and Sirlin and by specific model calculations by Goldberg and Wilson. An early measurement by Anderson et al. found $R = (1.21 \pm 0.07) \times 10^{-4}$ in agreement with Eq. (1).

Subsequently, an experiment by Di Capua et al. obtained $R = (1.274 \pm 0.024) \times 10^{-4}$, which differs from the theoretical prediction by $3.3 \pm 1.9\%$. In this Letter, a new measurement of the branching ratio is reported.

The experiment was performed in a pion beam at the Tri-University Meson Facility (TRIUMF) in Vancouver, Canada. The setup is shown in Fig. 1. Positive pions with initial momentum $p_{\pi} = 77 \pm 1$ MeV/c were degraded and stopped at a rate of $2 \times 10^{9}$ s$^{-1}$ in the inner three layers of a five-layer scintillation-counter target designed to absorb all muons from $\pi^{\rightarrow \mu\nu}$ decay. 70-MeV positrons from $\pi^{\rightarrow e\nu}$ decay and 0–53 MeV positrons from the $\pi^{\rightarrow \mu\nu}$ decay chain ($\pi^{+} \rightarrow \mu^{+}\nu_{\mu}$ followed by $\mu^{+} \rightarrow e^{+}\nu_{e}\overline{\nu}_{\mu}$) were detected by a three-element scintillation-counter telescope (T1, T2, T3) which preceded the 46-cm-diam x 51-cm NaI(Tl) crystal TINA. T3 limited the positron solid-angle acceptance to $\Delta\omega/4\pi = 0.7\%$ so that the measurements were confined to the central portion of the crystal for best energy resolution and to avoid edge effects. Three multiwire proportional chambers (MWPC 1–3) were used to test for position-dependent systematic effects.

Since NaI detectors are sensitive to both charged particles and $\gamma$ rays, the energy measurement in-
incluced internal bremsstrahlung photons which are emitted generally in the direction of the positrons in $\pi_\mu$ and $\mu$ decays. The response function of the NaI crystal with its characteristic low-energy tail was measured with the positron-beam component of the pion channel in the momentum range 20–80 MeV/c. At 70 MeV/c the observed resolution was $\Delta E/E \approx 3.5\%$ (full width at half maximum).

In addition to the energy measurement, the time of decay into T3 within ±200 ns of the pion stop was recorded for each event along with energy losses $(dE/dx)$ in T1–T3 and several pulse pileup times. Pileup of additional beam particles within ±5 μs of the pion stop and charged-particle pileup in the counter T2 within ±5 μs of the decay event were recorded. These measurements facilitated study of backgrounds and time-dependent systematic effects.

The branching ratio was determined with two procedures chosen to minimize systematic uncertainties. In the method of Di Capua et al., 9 positrons were detected during two identical 25-ns-long time intervals, bin 1 starting at $t_0 = 3$ ns after the arrival of the pion and bin 2 beginning $t_2 = 173.5$ ns, or 6.7 pion lifetimes, later. Because the pion lifetime $\tau_\pi = 26$ ns is short compared with the muon lifetime $\tau_\mu = 2200$ ns, bin 2 contains essentially only positrons from the $\pi^{-}\mu^{-}e^{-}$ chain $[N(2)_{\pi^{\mu}e}]$ whereas bin 1 contains events from both $\pi^{-}\mu^{-}e^{-}$ $[N(1)_{\pi^{\mu}e}]$ and $\pi^{-}\nu$ ($N_{\pi\nu}$) origins. The branching ratio can then be expressed as

$$R = \frac{N_{\mu} - \lambda_{\mu} N_{\mu e}}{N_{\pi} - \lambda_{\pi} N_{\pi e}} \left\{ 1 - \exp\left[ -\lambda_{\pi} t_2 - \lambda_{\mu} t_2\right] \right\}$$

where $\lambda_{\pi}$ and $\lambda_{\mu}$ are the pion and muon decay rates, respectively. This method of determining $R$ is independent of several important sources of possible uncertainties including the displacement of the first interval from the arrival time of the pion $t_0$, the positron-detector solid angle, the absolute width of the two time bins (as long as they are identical), and the fraction of muons in the beam or the contribution from decays of muons left in the target by previous pion stops.

Figure 2(a) shows the positron energy spectrum for bin 1 including the $\pi^{+} - e^{+}\nu_\mu$ peak and $\mu^+ - e^{+}\nu_\mu\bar{\nu}_\mu$ spectrum which extends to zero observed energy since the NaI pulse was not required in the event logic. The two low-energy peaks are due to the zero-energy pedestal and the 511-keV positron annihilation line. Figure 2(b) gives the pure $\mu^+ - e^{+}\nu_\mu\bar{\nu}_\mu$ spectrum from bin 2. Figure 3 shows the resultant $\pi^{+} - e^{+}\nu_\mu$ spectrum in the peak region obtained by subtracting the normalized bin-2 muon decay distribution from that of bin 1. The solid line in Fig. 3 is a fit to the data with the Monte Carlo-generated $\pi^{+} - e^{+}\nu_\mu$ line shape (see below). The chi-squared of the fit is 1.1 per degree of freedom. The measured peak width is $\Delta E/E \sim 5.5\%$, in good agreement with the Monte Carlo calculation. There are $N_{\pi e} = 3.2 \times 10^4$ counts in the peak with $E > 51$ MeV.

The second method for calculation of the branching ratio used the two spectra in Figs. 4(a) and 4(b) which show the time distributions for events in the $\pi^{-}\mu^{-}e^{-}$ region and the $\pi^{-}\nu$ peak region, respectively. Events which occur prior to the arrival of the pion ($t = 0$) are due to decays of muons left in the target by previous pion stops. The
timing spectra were fitted (solid lines) for the amplitudes of the $\pi^- e^-$, $\pi^- \mu^- e^-$, and pure $\mu^- e^-$ decay contributions $A_{\pi e}$, $A_{\pi \mu e}$, and $A_{\mu e}$. Small additional terms were included to account for effects of pileup and pulse-pair resolution of the scintillators. The branching ratio is given by

$$ R = \frac{A_{\pi e}}{A_{\pi \mu e}}. $$  

A chi-squared of 1.2 per degree of freedom was obtained in both fits shown as solid lines in Figs. 4(a) and 4(b). When the pion lifetime $\tau_\pi$ was also a parameter in the fit, it was found that $\tau_\pi = 26.10 \pm 0.13$ ns which agrees with the current value $\tau_\pi = 26.030 \pm 0.023$ ns. This method for determining $R$ makes use of data in a wider time range than just bins 1 and 2. However, it may be subject to additional uncertainties arising, for example, from nonlinearities in the time-measuring system.

To determine the branching ratio with the two methods described, several corrections are required. Table I lists the correction factors for Eq. (2). A similar set applies to Eq. (3). The $\pi^- e^-\nu$ tail correction accounts for $\pi^- e^-\nu$ events lost below the 51-MeV cutoff energy. It was determined by Monte Carlo calculation of the $\pi^- e^-\nu$ line shape including the effects of the intrinsic NaI response function, radiative corrections, Bhabha scattering, Landau straggling, and the beam-target-detector geometry (see Fig. 3).

Monte Carlo calculations were also done to determine the NaI response function for $\pi^- \mu^- e^-$ chain events and to obtain other corrections including those applying to the energy dependence of processes such as multiple Coulomb scatter-

FIG. 4. Timing spectra and fits (solid lines). (a) $\pi^- \mu^- e^-$ events in the range 0–53 MeV; (b) $\pi^- e^-\nu$ events in the range > 53 MeV. A pileup background due to $\pi^- \mu^- e^-$ events was also present.

ing and positron annihilation. Small nonuniformities in the application and efficiency of pileup cuts were determined from the data itself. The data set was tested under a series of cuts resulting in the estimated uncertainties given in Table I. The final value of the branching ratio obtained with Eq. (2) is

$$ R = (1.218 \pm 0.014) \times 10^{-4}. $$

A consistent result $R = (1.219 \pm 0.014) \times 10^{-4}$ is found from the analysis method using Eq. (3).

The result Eq. (4) is in substantial agreement with the predictions of the standard model of weak and electromagnetic interactions with the assump-

![Image of graph showing counts/energy and timing spectra](image)

**Table I. Multiplicative corrections to $\pi^- e^-\nu$ branching ratio.**

<table>
<thead>
<tr>
<th>Correction</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^- e^-\nu$ tail</td>
<td>1.0147 ± 0.0075</td>
</tr>
<tr>
<td>Low-energy $\mu^- e^-\nu$</td>
<td>0.9982 ± 0.0005</td>
</tr>
<tr>
<td>Multiple Coulomb scattering</td>
<td>0.9977 ± 0.0040</td>
</tr>
<tr>
<td>Positron annihilation</td>
<td>0.9959 ± 0.0010</td>
</tr>
<tr>
<td>$\mu^+$ losses from target</td>
<td>1.0002 ± 0.0010</td>
</tr>
<tr>
<td>Bin 1 and bin 2 equality</td>
<td>0.9989 ± 0.0004</td>
</tr>
<tr>
<td>Pulse pileup efficiency</td>
<td>0.9931 ± 0.0029</td>
</tr>
<tr>
<td>Bin separation $t_s$</td>
<td>1.0000 ± 0.0000</td>
</tr>
<tr>
<td>Pion lifetime</td>
<td>1.0000 ± 0.0009</td>
</tr>
<tr>
<td>Other</td>
<td>1.0004 ± 0.0020</td>
</tr>
</tbody>
</table>
Measurement of the $\pi^+ \rightarrow e^+\nu$ Branching Ratio

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A new measurement of the $\pi^+ \rightarrow e^+\nu$ branching ratio gives $R_{ee} = \Gamma(\pi \rightarrow e\nu + \pi \rightarrow e\nu\gamma) / \Gamma(\pi \rightarrow \mu\nu + \pi \rightarrow \mu\nu\gamma) = 1.2265 \pm 0.0034\text{(stat)} \pm 0.0044\text{(syst)} \times 10^{-4}$. This result is in agreement with standard model calculations and confirms the hypothesis of electron-muon universality at the 0.2% level.

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The electroweak couplings of the three lepton generations—the electron, muon, and tau—are equal in the standard model (SM). This lepton universality is studied with $\pi$, $\tau$, and $W$ leptonic decays; in particular, $e\mu$ universality is tested precisely by a measurement of the branching ratio of the helicity-suppressed decay $\pi \rightarrow e\nu$ with respect to the common decay $\pi \rightarrow \mu\nu$. Within the context of the SM, radiative corrections to the ratio [1] are nearly independent of strong interaction effects and are explicitly calculable as discussed by Marciano and Sirlin [2]. The calculated value [3] of the $\pi \rightarrow e\nu$ branching ratio $R_{ee} = \Gamma(\pi \rightarrow e\nu + \pi \rightarrow e\nu\gamma) / \Gamma(\pi \rightarrow \mu\nu + \pi \rightarrow \mu\nu\gamma)$ is $R_{ee} = (1.234 \pm 0.001) \times 10^{-4}$, where the uncertainty arises from uncalculated but bounded pion structure effects. A measurement in disagreement with this value could imply a deviation from universality, or indicate the existence of other physics beyond the SM. A relatively straightforward extension of the SM would involve the admixture of non-zero mass eigenstates in the neutrino [4].

The existence of new hypothetical particles such as massless Majorons [5] or charged Higgs scalars arising from extended symmetries [6] could also influence the observed branching ratio. A previous experiment found [7] $R_{ee} = (1.218 \pm 0.014) \times 10^{-4}$, consistent with the calculation at the 1% level. In this Letter we report on a new measurement of $R_{ee}$ with improved precision.

The experiment [8] was carried out on the M13 channel at TRIUMF in a $\pi^+$ beam of momentum $P = 83$ MeV/c and $\Delta P/P = 1\%$ using the setup shown schematically in Fig. 1. The incoming beam was detected in scintillators B1 and stopped near the downstream side of the target counter B3 at a rate of $7 \times 10^4$ s$^{-1}$. Veto scintillators VL and VR confined the stopping region to the central 30% of B3 in order to contain the muons from $\pi \rightarrow \mu\nu$ decay, and V0 rejected penetrating particles. The LL, LR, and B4 counters also served a similar purpose. Positrons from stopped-pion decay in B3 were detected at 90° to the beam, passing through two planar 203 mm $\times$ 203 mm wire chambers (WC) for position measurement, and trigger scintillators T1–T4, before being energy analyzed in a 460-mm-diam $\times$ 510-mm-long NaI(Tl) crystal "TINA." The solid-angle acceptance of 2.9% was determined by the 152-mm-diam counter T4. The positron energy spectrum consisted of a peak at 70 MeV with $\sim 1.2 \times 10^3 \pi \rightarrow e\nu$ decays and a distribution from 0 to 53 MeV from the decay $\mu \rightarrow e\nu\gamma$ following $\pi \rightarrow \mu\nu$ decay (the $\pi \rightarrow \mu\nu$ chain) as shown in Fig. 2(a) [9]. Timing of the incoming pion was obtained from scintillator B2, and decay-event time was defined by T4. Event time was measured using a time-to-amplitude converter (TAC) feeding an analog-to-digital converter (ADC). All events occurring within 30 ns following a pion stop, or having energy > 50 MeV deposited in TINA, were recorded in order to favor $\pi \rightarrow e\nu$ events. The sample of $\pi \rightarrow \mu\nu\rightarrow e$ and background events was obtained from (1:16)-prescaled triggers in the time range $-120$ to $+300$ ns with respect to a pion stop.

The measurement required the determination of the ratio of the positrons in the $\pi \rightarrow e\nu$ peak to those from the $\pi \rightarrow \mu\nu$ chain. The largest potential source of systematic uncertainty arose from the low-energy tail of the $\pi \rightarrow e\nu$ peak which extended under the $\pi \rightarrow \mu\nu$ distribution. The tail was due mostly to the response function of TINA; the component due to radiative processes was small (about 0.4%) since NaI is also sensitive to the forward-peak bremsstrahlung gamma rays emitted in $\pi \rightarrow e\nu\gamma$ decay.

FIG. 1. Schematic view of the experimental setup.
Radiative Corrections to $\pi_{12}$ Decays

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Radiative corrections to $\pi_{12}$ ($l = e$ or $\mu$) decays are examined. Higher order electroweak leading logarithms, short-distance QCD corrections, and structure dependent effects are incorporated. The results are employed to (1) test $e-\mu$ universality in $\Gamma(\pi \to e\bar{\nu}_e(x)) / \Gamma(\pi \to \mu\bar{\nu}_\mu(x))$, (2) extract an $f_x$, which is used to check the Goldberger-Treiman relation and PCAC-anomaly prediction for $\Gamma(\pi^0 \to \gamma\gamma)$, and (3) determine the tau partial decay rate $\Gamma(\tau \to \pi\nu_\tau(x))$.

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Calculations of electroweak radiative corrections and reliable estimates of their underlying theoretical uncertainties are crucial ingredients for precision tests of the standard model. An important case is provided by $\pi_{12}$ decays, $\pi \to l\bar{\nu}_l$, where $l = e$ or $\mu$. Recently, experiments at TRIUMF [1] and PSI [2] have reported

$$R_{e/\mu} \equiv \frac{\Gamma(\pi \to e\bar{\nu}_e + \pi \to e\bar{\nu}_e)}{\Gamma(\pi \to \mu\bar{\nu}_\mu + \pi \to \mu\bar{\nu}_\mu)} = 1.2265 \pm 0.0034 \pm 0.0044 \times 10^{-4} \text{ (TRIUMF)},$$

$$R_{e/\mu} = 1.2346 \pm 0.0035 \pm 0.0036 \times 10^{-4} \text{ (PSI)},$$

for the ratio of radiative inclusive decay rates. Those results represent about a factor of 3 (error) improvement when compared with the previous experimental value [3]

$$R_{e/\mu} = (1.218 \pm 0.014) \times 10^{-4}.$$}

Future measurements are expected to further reduce the uncertainty in $R_{e/\mu}$. However, already at the level in (1), $e-\mu$ universality is well tested and “new physics” scenarios are very constrained [4].

To fully utilize the results in (1), the theoretical prediction for $R_{e/\mu}$ must be known to at least the same level of precision and preferably much better. That entails the inclusion of electroweak radiative corrections which in the case of $R_{e/\mu}$ have long been known from the pioneering work of Berman [5] and Kinoshita [6] to be large, $\sim -4\%$. The main purpose of this Letter is to scrutinize the $O(\alpha)$ radiative corrections to $\pi_{12}$, incorporate higher order effects, and most importantly, argue that the underlying theoretical uncertainties give rise to less than a $\pm 0.05\%$ error in the standard model prediction for $R_{e/\mu}$.

Radiative corrections are also important for the extraction and application of electroweak parameters. In the case of $\pi_{12}$ decays, one obtains the pion decay constant $f_\pi$, defined by the weak axial-current matrix element

$$\langle 0|A_\mu(0)|\pi(p)\rangle = if_{\pi}\gamma\mu,$$

by comparing the experimental rate [7]

$$\Gamma(\pi \to \mu\bar{\nu}_\mu(x)) = (2.5284 \pm 0.0023) \times 10^{-14} \text{ MeV}$$

with theory. However, electroweak radiative corrections must be properly accounted for in extracting $f_x$ [8,9].

After determining $f_x$, one can test the Goldberger-Treiman relation [10]

$$f_{\pi\pi\mu\nu} = \frac{1}{\sqrt{2}}(m_\pi + m_\mu) g_A,$$

and the PCAC (partially conserved axial-vector current) anomaly [11] prediction

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{a^2 f^2}{32\pi^3 f^2},$$

both of which are expected to hold up to the $(1-2)\%$ level. In addition, one can employ $f_\pi$ to predict the tau partial decay rate [12,13]

$$\Gamma(\tau \to \pi\nu_\tau(x)) = \frac{G^2 f^2}{16\pi} |V_{ud}|^2 \mu_\tau^3 \left[ 1 - \frac{m_\tau^2}{m_\mu^2} \right]^2 \left[ 1 + O(\alpha) \right].$$

Of course, the full $O(\alpha)$ corrections to the decay $\tau \to \pi\nu_\tau(x)$ as well as the parameters in (4) and (5) should be included for precise confrontations [8, 14,15].

Extensive studies of the $O(\alpha)$ radiative corrections to $\pi_{12}$ decays already exist [5,6,8,16-19]. Here, we summarize those calculations, describe how they should be utilized, and assess their level of theoretical uncertainty.

Combining the known short- and long-distance radiative corrections for the inclusive decays $\pi \to l\bar{\nu}_l(x) = \pi \to l\bar{\nu}_l + \pi \to l\bar{\nu}_l \gamma$, ignoring for now pure structure dependent bremsstrahlung, we find

$$\Gamma(\pi \to l\bar{\nu}_l(x)) = \frac{G^2 f^2 |V_{ud}|^2}{8\pi} \left[ 1 + \frac{m_\tau^2}{m_\mu^2} \right]^2 \left[ 1 + \frac{2\alpha}{\pi} \ln \left( \frac{m_\tau^2}{m_\mu^2} \right) \right] \times \left[ 1 - \frac{a}{\pi} \left( \frac{3}{2} \ln \left( \frac{m_\tau^2}{m_\mu^2} \right) + C_1 + C_2 \frac{m_\tau^2}{m_\mu^2} \ln \frac{m_\tau^2}{m_\mu^2} + C_3 \frac{m_\tau^2}{m_\mu^2} + \cdots \right) \right] \left[ 1 + \frac{a}{\pi} F(x) \right].$$

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could be absorbed into $f_{\eta}$; however, we choose not to do so because similar corrections were separated out in the extraction of $|V_{ud}|$ from superallowed $\beta$ decays. Other unknown two-loop short-distance corrections are presumably much smaller and can be safely neglected.

In the case of higher order long-distance corrections of the form $[(a/\pi)\ln(m_\mu/m_\pi)]^n$, $n \geq 2$, we can also estimate their effect using the renormalization group. However, such terms are only important for $l=\mu$; so, we will examine them separately in our discussion of $R_{\mu\mu}$. (Corrections of the form $[(a/\pi)\ln(m_\mu/m_\pi)]^n$, $n \geq 2$ are small $\approx 0.00002$ and can be lumped into the unknown constant $C_1$.)

Taking the ratio of $e$ and $\mu$ decay rates in (7), the short-distance corrections and most uncertainties cancel. One finds (still neglecting pure structure dependent bremsstrahlung)

$$R_{e\mu} = R_0 \left\{ 1 + \frac{a}{\pi} \left[ F \left( \frac{m_\pi}{m_e} \right) - F \left( \frac{m_\mu}{m_e} \right) + C_2 \frac{m_\mu}{m_\pi} \ln \frac{m_\mu^2}{m_\pi^2} + C_3 \frac{m_\pi^2}{m_\mu^2} \right] \right\},$$

where

$$R_0 = \frac{m_\mu^2}{m_e^2} \left( \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 - m_e^2} \right)^2 = 1.28347 \times 10^{-4}, \quad C_2 = 3.1,$$

and $F(x)$ is defined in (7b). Inserting mass values into those expressions, the ratio is reduced to $R_{e\mu} = (1.2337 + 0.00005 C_3) \times 10^{-4}$ [6].

There are pure structure dependent (SD) bremsstrahlung corrections to $\pi \rightarrow l\nu\gamma$ which are not helicity suppressed by $m_l^2/m_\pi^2$ and, therefore, potentially important for $l=\mu$. Such effects are suppressed by $m_\pi^4/m_\mu^4$ and hence small. Calling those contributions $\Delta R_{e\mu}$, one finds [16,24]

$$\frac{\Delta R_{SD}}{R_{e\mu}} = 5.4 \times 10^{-4} (1 + \gamma^2),$$

Employing [7] $\gamma_{\exp} = 0.45$ leads to

$$\Delta R_{SD} = 8 \times 10^{-8},$$

The final contributions to $R_{e\mu}$ that we need consider are corrections of the form $[(a/\pi)\ln(m_\mu/m_\pi)]^n$, $n \geq 2$. Indeed, since the $-3(a/\pi)\ln(m_\mu/m_\pi)$ correction dominates the $O(a)$ terms in (14), one expects its higher order counterparts to similarly dominate their respective orders. Summing all such logs via the renormalization group gives the enhancement

$$\left( 1 - \frac{(2a/3\pi)\ln(m_\mu/m_\pi)^{3/2}}{1 - (3a/\pi)\ln(m_\mu/m_\pi)} - 1.00055, \right.$$

which increases (14) to $1.2344 \times 10^{-4}$. Including pure SD bremsstrahlung in (17),

$$R_{\mu\mu}^{\text{theory}} = (1.2352 \pm 0.00055) \times 10^{-4},$$

where a conservative range of $C_1 = 0 \pm 10$ has been employed for the hadronic structure uncertainties.

Comparing the theoretical prediction in (19) with the experimental results in (1), we find

$$\frac{R_{\mu\mu}^{\text{exp}}}{R_{\mu\mu}^{\text{theory}}} = 0.9930 \pm 0.0045 \pm 0.0004 \quad \text{(TRIUMF)}, \quad (20a)$$

$$\frac{R_{\mu\mu}^{\text{exp}}}{R_{\mu\mu}^{\text{theory}}} = 0.9995 \pm 0.0041 \pm 0.0004 \quad \text{(PSI)}. \quad (20b)$$

The level of agreement between theory and experiment is impressive. It constrains all sorts of new physics scenarios [4]. A further significant reduction in the experimental uncertainties would provide a stringent test of the standard model and could unveil, rather than merely restrict, new physics.

The radiative corrections in (7) are also necessary for extracting $f_\pi$ from $\pi\nu_2$ decays. Including the short-distance enhancement in (12), we find by comparing (7) with (3)

$$f_\pi = 130.7 \pm 0.1 \pm 0.15 C_1 \text{ MeV}. \quad (21)$$

The uncertainty in (21) comes from $|V_{ud}|$ while the second term illustrates the dependence of $f_\pi$ on $C_1$. For applications of $f_\pi$ we allow $C_1 = 0 \pm 2.4$ as suggested by (9), and then an additional $\pm 0.28\%$ uncertainty is implied. That uncertainty is not particularly large. Nevertheless, one would like to see a calculation of $C_1$ in a model of hadronic structure.

As our first application of $f_\pi$, we consider the Goldberger-Treiman relation [10] in (4), which should be exact in the chiral limit $m_\pi = m_d = 0$ (modulo radiative corrections). Employing [7] $g_A = 1.257 \pm 0.003$ and [25] $g_{\pi\pi} = 13.04 \pm 0.06$, one finds [15]

$$\Delta_\pi = 1 - \frac{(m_\pi^2 + m_\rho^2)g_A}{\sqrt{2}g_{\pi\pi}g_{\pi\pi}} = 0.021 \pm 0.005 + 0.0011 C_1,$$

where the $\pm 0.005$ uncertainty stems mainly from $g_{\pi\pi}$. The effect of $C_1$ is not very significant, unless $C_1$ is well outside the range in (9). The deviation from zero in (22) is in accord with theoretical expectations [26] (if $C_1 = 0$ or not too large), which roughly suggests $|\Delta_\pi| \approx (m_\mu + m_d)/2m_\pi \approx 1\%$. We note, however, that an earlier [27] $g_{\pi\pi} = 13.4 \pm 0.1$ value gives a less acceptable 4.7\% deviation in (22). The situation regarding the value of $g_{\pi\pi}$ is still not completely settled and deserves continued scrutiny. (A small discrepancy also exists between the direct-
ly measured $g_A$ we employ and the value $g_A = 1.264$ implied by the neutron lifetime and superallowed Fermi transitions.) In addition, the effect of $O(a)$ radiative corrections on $g_{\pi n}$ should be examined. If $g_{\pi n}$ should return to its former value, it might be suggestive of a large negative $C_1 = - (20-30)$. However, as we shall see, such a range could be inconsistent with other tests of $f_{\pi}$.

Another test of $f_{\pi}$ is provided by the PCAC-anomaly prediction [11] for $\pi^0 \rightarrow \gamma \gamma$ in (5). Employing (21),

$$\Gamma(\pi^0 \rightarrow \gamma \gamma) = 7.73 \pm 0.01 - 0.018 C_1 \text{ eV}.$$  \hspace{1cm} (23)

That prediction is to be compared with the particle data group value $\Gamma(\pi^0 \rightarrow \gamma \gamma)_{\text{exp}} = 7.74 \pm 0.55 \text{ eV}$ where the error has been scaled by a factor of 3 due to experimental inconsistencies [7]. The good agreement is consistent with chiral symmetry breaking which could easily accommodate a 1% or 2% difference [14]. We note, however, that the single best $\pi^0$ lifetime experiment [28] suggests $\Gamma(\pi^0 \rightarrow \gamma \gamma) = 7.25 \pm 0.23$ which implies a much harder to explain [14] (6.3 \pm 0.3 - 0.23C_1)\% deviation from (23). That potential discrepancy also needs an additional experimental study. If confirmed, a very large positive $C_1 \approx + 20$ would bring theory and experiment together, but the expense of weakening the Goldberger-Treiman relation (particularly if $g_{\pi n}$ reverts back towards its earlier value). It therefore seems that at present $C_1 = 0$ is a good central value in applications of $f_{\pi}$, but not well tested.

As a final application of $f_{\pi}$, we consider the decay $\tau \rightarrow \pi \nu_\tau(\gamma)$. Including only the leading short-distance radiative corrections [13] gives

$$\Gamma(\tau \rightarrow \pi \nu_\tau(\gamma)) = \frac{G_F^2 |V_{ud}|^2 f_\pi^2}{16 \pi} m_\tau \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \times \left(1 + \frac{2a}{\pi} \ln \frac{m_\tau}{m_\tau} + \cdots \right),$$  \hspace{1cm} (24)

where $\cdots$ now represent uncalculated $O(a)$ corrections. The full $O(a)$ corrections depend on structure dependent and independent contributions. Employing $f_{\pi} |V_{ud}| = 127.44 \text{ MeV}$, $m_\tau = 1777 \text{ MeV}$, and including leading logs and short-distance QCD corrections [18] to (24) we find

$$\Gamma(\tau \rightarrow \pi \nu_\tau(\gamma)) = (2.48 \pm 0.025) \times 10^{-13} \text{ GeV},$$  \hspace{1cm} (25)

or normalizing in terms of the $\tau$ lifetime

$$B(\tau \rightarrow \pi \nu_\tau(\gamma)) = 0.1113 \pm 0.0011 \frac{\tau_{\text{tau}}}{2.95 \times 10^{-13} \text{ s}}.$$  \hspace{1cm} (26)

The unknown $O(a)$ corrections have been crudely estimated [29] to give a $\pm 1\%$ uncertainty in (25) and (26). At present, the Particle Data Group gives [7]

$B(\tau \rightarrow \pi \nu_\tau(\gamma)) = 0.116 \pm 0.004$ which is in rough accord with (26). An interesting confrontation between theory and experiment will be realized when the experimental error on $B(\tau \rightarrow \pi \nu_\tau(\gamma))$ reaches the $\pm 0.001$ level. At point that, the full $O(a)$ corrections must be included.

In summary, we have argued that the theoretical uncertainty in $R_{\ell \mu}$ is less than $\pm 0.05\%$ and hence presently negligible in the comparison of theory and experiment. Experiments could be pushed another order of magnitude before further theoretical refinements become necessary. We also found $f_{\pi} = 130.7 \pm 0.1 + 0.15 C_1 \text{ MeV}$ and then employed $C_1 = 0 \pm 2.4$, in applications of $f_{\pi}$. There are, however, at present no precise tests of that uncertainty range. In the future, continued scrutiny of $g_{\pi n}$, $\Gamma(\pi^0 \rightarrow \gamma \gamma)$, and particularly $B(\tau \rightarrow \pi \nu_\tau(\gamma))$ should provide consistency checks on $C_1$ and tests of the standard model.

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[21] We have appended the lepton mass renormalization $m_\ell = m_\ell - \delta m_\ell$ with $\delta m_\ell = (3a/2\pi) m_\ell \ln m_\ell m_\tau^2$ to Kinoshita’s Eq. (8) in Ref. [6].
Figure 5.2: Schematics of the PIENU detector. Red lines near the target denote silicon stip pairs. Two pairs are located before the target and one pair after. Color scheme: scintillators are blue; wire chambers are green; Aluminum flanges of the NaI crystal are black; NaI crystal is cyan; CsI crystal array is red.
Figure 5.3: Overall (top) and side (bottom) view of the PIENU-I detector.
Figure 5.4: Overall view of the PIENU-II detector. The CsI crystal array is shown as a “see-through” element in order to reveal details and make CsI crystal visible.
Figure 5.7: BINA with some of the photo tubes removed.

Figure 5.8: BINA with all the tubes and their bases in place.

Figure 5.9: BINA from the top.
Who is This?