Effect of order parameter fluctuations on spectral density in d-wave superconductors

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Outline

1. Motivation
2. Phase fluctuations and Spectral Density
3. Fermi Surface reconstruction
4. Conclusions
Motivation: ARPES in cuprates

Energy

Momentum \((\theta, \phi)\)

\(V_{\text{in}}\)

\(V_{\text{out}}\)

\(R_{\text{in}}\)

\(R_{\text{out}}\)

\(\theta\)

\(\phi\)

CCD
Motivation: ARPES (cont)
Motivation: ARPES (cont)

Nodal region Excited States at $T>T_c$ ($T = 140K$)

$T_c = 91$ K
(Optimal doping)

“Arc”

$T_c = 65$ K
(under doping)

Area=doping!

“Pocket”
Motivation: ARPES (cont)

Superconducting State

$T < T_c$

d-wave symmetry

Order parameter
What is measured?

\[ I(k', \omega') \propto A(k, \omega) f(\omega) \otimes R(k', \omega'|k, \omega) \]

\[ A(k, \omega) = -\frac{1}{\pi} \text{Im} \ G^R(k, \omega) \]

BCS: \[ A(k, \omega) = u_k^2 \delta(\omega - E_k) + v_k^2 \delta(\omega + E_k) \]
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Model

Quasi-particles:
- Anisotropic FS
- Effectively 1D

Fluctuations:
- Isotropic
- Classical
Model (cont)

Nambu-spinor:
\[ \chi = (\psi_{\uparrow,1}, \psi_{\downarrow,2})^T \]

Classical phase fluctuations

\[ \mathcal{L} = \bar{\chi}_\omega \left[ -i \omega_n \hat{1}_2 - i v \partial_x \sigma^z + \Delta \sigma^+ + \Delta^* \sigma^- \right] \chi_\omega + F_\phi \]

\[ \frac{F_\phi}{T} = \frac{\rho_s}{2T} \int dx dy \left[ (\partial_x \phi)^2 + (\partial_y \phi)^2 \right] \]
Impurity Model

Nambu-type transformation:

\[ [\psi_{\uparrow}, \psi_{\downarrow}^\dagger] = [\Psi_{\uparrow}, -i\Psi_{\downarrow}^\dagger] \]

(Nambu) particle number conservation

Effective Impurity model

\[ \tau^a = \Psi^+ \tau^a \Psi \]

\[ H = v^{-1} \{ (i\omega - 0)\tau^3 + \Delta_q [\tau^+ \exp(-i\varphi) + \tau^- \exp(i\varphi)] \} + H_{\text{bulk}} \]
\[ \varphi = \varphi(x,0) \]

\[ H_{\text{bulk}}[\varphi] = \frac{2\pi}{d} \int dy [\Pi^2 + (\partial_x \varphi)^2 + \text{vortices}] \]

\[ [\varphi(y), \Pi(y')] = i \delta(y - y') \]

\[ d = T / 8T_{\text{BKT}} \]

\[ < e^{i\varphi(r)} e^{-i\varphi(r')} > = (a/\xi)^{2d} F(|\vec{r} - \vec{r}'|/\xi) \]

\[ F(z << 1) = z^{-2d}, \]

\[ F(z > 1) \sim K_0(z) \]
Results: \((T<T_c)\)

\[ A_\omega(k) \]

\[ G_\omega(k) = \left[ \omega - k - \frac{\Delta^2(q) a^{2d}}{(\omega + k)^{1-2d}} (1 - i\pi d) \right]^{-1} \]

\[ T_K = \Delta(q) \left[ G_\omega(k) \approx \frac{1}{2\omega} \right] \frac{1 + \frac{\Delta^2(q)e^{-i\pi d}}{2\omega \left( \omega + k - \frac{\Delta^2(q)e^{-\pi i d}}{(2\omega)^{1-2d}} \right)^{1-2d}}}{2\omega \left( \omega + k - \frac{\Delta^2(q)e^{-\pi i d}}{(2\omega)^{1-2d}} \right)^{1-2d}} \]
Results: \((T<T_c)\)

Mass Shell: \(\omega - v k \ll \omega\)

\[\begin{array}{c}
\text{Lorentzian} \\
\text{exponents fuse}
\end{array}\]

\(\omega + v k \ll \omega\)

\[\begin{array}{c}
\text{Lorentzian} \\
+ \\
\text{Power Law}
\end{array}\]

Orthogonality Catastrophe
Results (T>Tc)

\[
\Sigma^{(2)}(q, k, \omega) = \Delta^2(q) \xi(\xi/\alpha)^{-2d} (1 + (\xi(\omega + k))^2)^{-1/2}
\times \left[ \exp \left\{ -i\pi d + 2d \sinh^{-1}[\xi(\omega + k)] \right\} - (1 + (\xi(\omega + k))^2)^{-1/2} \right]
\]

\(d = 0.2\)

\(d = 0.4\)

FS
Results (T>Tc), (cont)

\[ d = 0.05 \]

\[ d = 0.15 \]
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Fermi Surface Reconstruction

YBCO!
Fermi Surface Reconstruction (cont)

Shadow band

$Q = (\pi, \pi)$

Main band

Pockets
\[ H = \sum_{k\alpha} \xi(k) \psi_{k\alpha}^\dagger \psi_{k\alpha} + J \sum_k S \psi_{k+Q\alpha}^\dagger \sigma_{\alpha\beta} \psi_{k\beta} \]
Fermi Surface Reconstruction (cont)

\[ \mathcal{H} = K.E. + H_{coupl} + H\begin{bmatrix} S \end{bmatrix} \]

\[ \xi_k \rightarrow v_{1,2}k \]
Fermi Surface Reconstruction (cont)

\[ H[S] = \frac{2\pi}{d} \int d^2 r \left[ \nabla \phi(r) \right]^2 \]

\[ d = T / 4\pi \rho_s \]

\[ H_{\text{coupl}} = J \sum_{\alpha, \beta} e^{i\phi} \bar{\psi}_{1, \alpha} \sigma_{\alpha \beta}^{-} \psi_{2, \beta} \]

\[ \rho_s \rightarrow \infty \rightarrow \text{Mean field} \]

\[ v^2 k_x k_y - J^2 = 0 \]
Higher $T$
Results

\[ G^{-1} = \omega - k_x + \frac{iJ^2 a^{2d}}{(-i(\omega - k_y))^{1-2d}} \]

\[ G = \frac{J^2 e^{i\pi d}}{(\omega - k_x)^2} \left[ \omega - k_y + \frac{iJ^2 a^{2d}}{(-i(\omega - k_x))^{1-2d}} \right]^{-1+2d} \]
Conclusions

1. We have studied the spectral density in the presence of strong phase fluctuations.
2. Closed expressions is obtained below Tc. Interpolation expression in the close proximity of the transition were derived as well.
3. The Fermi Surface topology changes with the doping. We have elucidated the role played by critical fluctuations of the order parameter in formation of shadow bands.
Thank you!
\[ \Delta E = 0.0095 \pm 0.0006 \text{ meV} \]

\[ 2 \text{Im} \Sigma_k = \Gamma_k \]

\[ \tilde{A}(k, \omega) = \frac{1}{\pi} \frac{f(\omega) Z_k \text{Im} \Sigma_k(\omega)}{(\omega - \varepsilon_k - \text{Re} \Sigma_k(\omega))^2 + \text{Im} \Sigma_k(\omega)^2} \]
The spectral function along FS (k=0).

\[ \Sigma(q,k,\omega) = \Delta^2(q)(\xi/a)^{-2d} e^{-i\pi d} \frac{\left[ (k+\omega/v) \xi + \sqrt{(k+\omega/v)^2 \xi^2 + 1} \right]^{2d} - \left[ (k+\omega/v)^2 \xi^2 + 1 \right]^{1/2}}{\left[ (k+\omega/v)^2 + \xi^2 \right]^{1/2}} \]
Away from the singularity we employ the extrapolation formula

$$\Sigma(q,k,\omega) = \Delta^2(q) (\xi/\alpha)^{-2d} e^{-i\pi d} \frac{[(k+\omega/v)\xi+\sqrt{(k+\omega/v)^2 \xi^2 +1}]^{2d} -[(k+\omega/v)^2 \xi^2 +1]^{1/2}}{[(k+\omega/v)^2 + \xi^2]^{1/2}}$$

which reproduces both limits

$$\xi = \infty \quad \text{and} \quad \xi(\omega/v + k) \ll 1$$
Effective Impurity model.

\[ H = v^{-1} \{ (i \omega - 0) \tau^3 + (a \Delta_q )\left[ \tau^+ \exp(-i \varphi) + \tau^- \exp(i \varphi) \right] \} + H_{\text{bulk}} \]

\( \tau^a = \Psi^a \tau^a \Psi, \)

\( \Psi = (\psi^+ \downarrow, \psi^+ \uparrow)^T \)

\[ \sum_\sigma \psi_\sigma^+ \psi_\sigma = 1 \]

X is dual to k and plays a role of Matsubara time, real frequency is like an imaginary magnetic field.

\( \varphi = \varphi(x,0) \)

\[ H_{\text{bulk}}[\varphi] = \frac{2 \pi}{d} \int dy [ \Pi^2 + (\partial_x \varphi)^2 + \text{vortices} ] \]

\[ [\varphi(y), \Pi(y')] = i \delta(y - y') \]

\( d = T / 8T_{\text{BKT}} \)

The occupation number is fixed because the fermions are integrated out and hence there are no fermionic loops.
\[
< e^{i\varphi(r)} e^{-i\varphi(r')} >= \left( \frac{a}{\xi} \right)^{2d} F\left( |\vec{r} - \vec{r}'| / \xi \right)
\]

\[
F(z << 1) = z^{-2d},
\]
\[
F(z > 1) \sim K_0(z)
\]

\[d = \frac{T}{8T_{\text{BKT}}}\]

Below \(T_{\text{BKT}}\) the correlation length = and the problem \(\sim\) to anisotropic Kondo model in imaginary magnetic field
High Tc Phase Diagram

Doped Mott Insulator!

2/18/2010

SBU Thesis Defence
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2. Formulation of the model
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4. Fermi Surface reconstruction
5. Spin Density Wave order parameter fluctuations
6. Results and Conclusions