Flavor-branes in gauge/string duality and M-theory

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Flavor branes in gauge/string duality and M-theory

Historical introduction

• AdS/CFT correspondence
  (Maldacena ’97; Gubser, Klebanov, Polyakov; Witten, ’98)

• Extensions to NonAdS/NonCFT and reduced supersymmetry
  (Itzhaki, Maldacena, Sonnenschein, Yankielowicz ’98; Girardello, Petrini, Porrati, Zafferoni ’98; Klebanov, Witten/Strassler/Tseytlin ’98 and ’00; Polchinski, Strassler ’00; Maldacena, Núñez ’01)

• Flavor-branes
  (Karch, Katz ’02)

• Backreacting flavors
  (Klebanov, Maldacena ’04)

• Smearing
  (Bigazzi, Casero, Cotrone, Kiritsis, Paredes and Casero, Núñez, Paredes ’06)

• Flavor-branes and generalized calibrations
  (Gaillard, Schmude ’08)

• Kaluza-Klein monopole condensation and M-theory with torsion
  (Gaillard, Schmude ’09)
Flavor branes in gauge/string duality and M-theory

Outline

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   1.1. Essential gauge/string duality
   1.2. The flavoring problem

2. The geometry of backreacting flavors
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Introduction
Introduction

Essential string theory

Two perspectives on string theory:
- Open & closed strings, D-branes
- Supergravity and brane actions

In type IIA/B:
- There are always closed strings (gravity)
- D-branes add an open string sector (Yang-Mills)

\[
S_{\text{IIA}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} [e^{-2\Phi} (R + 4\partial_{\mu} \Phi \partial^{\mu} \Phi)] - \frac{1}{4} F_{(2)} \wedge \ast F_{(2)} + \mathcal{O}(H_{(3)}, F_{(4)}, \Psi)
\]
Introduction

Gauge/string duality

• Duality between

\[ \mathcal{N} = 4, SU(N_c) \leftrightarrow \text{IIB } AdS_5 \times S^5 \]

• Extended to less SUSY or no conformal symmetry:
  • Deformations
    \[ \mathcal{N} = 0^*, \mathcal{N} = 1^*, \mathcal{N} = 2^* \]
  • Branes at singularities
  • Wrapped branes
**Introduction**

**The flavoring problem**

- Local symmetry in the bulk provides global symmetry in the gauge theory.
- For the ‘flavor gauge group’ to decouple, flavor branes must wrap non-compact cycles.

\[
S_p = -T_p \int_{\mathcal{M}_{p+1}} d^{p+1}\xi e^{-\Phi} \sqrt{-\det(g_{\text{ind}} + 2\pi\alpha'\mathcal{F})} + T_p \int_{\mathcal{M}_{p+1}} (X^* C') \wedge e^{2\pi\alpha'\mathcal{F}}
\]

- Quark mass related to string length

\[
r'_{\text{min}} \sim m_f
\]

- Probe approximation

\[
N_c \gg N_f
\]

- Need to consider backreaction to go beyond that limit

\[
S = S_{\text{IIA/B}} + S_{\text{flav}}
\]
Introduction

Backreaction - The physics

- 't Hooft limit
  \[ \lambda = g_{YM}^2 N_c = \text{const.} \quad g_{YM}^2 \to 0 \quad N_c \to \infty \quad N_f = \text{const.} \]

- Veneziano limit
  \[ \lambda = g_{YM}^2 N_c = \text{const.} \quad g_{YM}^2 \to 0 \quad N_c \to \infty \quad N_f \to \infty \quad \frac{N_c}{N_f} = \text{const.} \]

\[ A \sim 1 \quad B \sim \frac{N_f}{N_c} \quad C \sim \frac{1}{N_c} \]

Hoyos, Núñez, Papadimitriou
The geometry of backreacting flavors
The geometry of backreacting flavors

Brane actions

- The standard brane action
  $$S_{\text{flavor}} = \sum_{N_f} -T_6 \int_{\mathcal{M}_7} d^{6+1} \xi e^{-\Phi} \sqrt{-g_{\text{ind}}} + T_6 \int_{\mathcal{M}_7} X^* C_7$$

- Supersymmetric branes satisfy a calibration condition (equivalent to kappa symmetry)
  $$X^* \phi_{D6} = d^{6+1} \xi \sqrt{-g_{\text{ind}}}$$
  $$\phi_{D6} = (\bar{\epsilon} \Gamma_{a_0...a_6} \epsilon) e^{a_0...e_6}$$

- We can express the action using forms defined on space time
  $$S_{\text{flavor}} = \sum_{N_f} -T_6 \int_{\mathcal{M}_7} X^* (e^{-\Phi} \phi_{D6} - C_7)$$

- And finally as a ten-dim integral
  $$S_{\text{flavor}} = -T_6 \int_{\mathcal{M}_{10}} (e^{-\Phi} \phi_{D6} - C_7) \wedge \Xi$$

- Where we have defined a distribution density (smearing form)
  $$\Xi$$

- Supersymmetry requires
  $$*d(e^{-\Phi} \phi_{D6}) = F$$
The geometry of backreacting flavors

Backreaction - The mathematics

\[
S = S_{IIA} + S_{flav} = \int \cdots - \frac{1}{8\kappa_{10}^2} F_{(2)} \wedge *F_{(2)} + T_p C_{(7)} \wedge \Xi + \ldots
\]

• Modified type IIA/B equations of motion (D6-branes in type IIA)

\[
\begin{align*}
\text{d}F_{(2)} &= - (2\kappa_{10}^2 T_6) \Xi_{(3)} \\
0 &= \text{d} *_{10} F_{(2)} \\
0 &= D + \mathcal{O}(\Xi) \\
0 &= E_{\mu\nu} + \mathcal{O}(\Xi)
\end{align*}
\]
The geometry of backreacting flavors

Integrability

\[ \delta \epsilon \psi_\mu = D_\mu \epsilon + \frac{1}{64} e^{\frac{3}{4} \Phi} F_{\mu_1 \mu_2} (\Gamma_{\mu_1}^{\mu_2} - 14 \delta_{\mu}^{\mu_1} \Gamma^{\mu_2}) \Gamma^{11} \epsilon = D_\mu \epsilon \]

\[ \tilde{\delta} \epsilon \lambda = \frac{\sqrt{2}}{4} \partial_\mu \Phi \Gamma^{11} + \frac{3}{16} \sqrt{2} e^{\frac{3}{4} \Phi} F_{\mu_1 \mu_2} \Gamma^{\mu_1 \mu_2} \epsilon \]

• Supersymmetry together with Bianchi identities and form e.o.m. implies Einstein and Dilaton e.o.m. (under certain, mild assumptions)

\[ 2[D_\mu, D_\nu] \epsilon = \left( \frac{1}{4} R_{\mu \nu \rho \sigma} \Gamma^{\rho \sigma} + \ldots \right) \epsilon \]

• Modified Bianchi identities and form e.o.m. are the signature of the presence of brane sources.

IIA: Lüst, Tsimpis - IIB: Gauntlett, Martelli, Sparks, Waldram - With sources: Koerber, Tsimpis
The geometry of backreacting flavors

Smearing I

\[ \Xi \sim N_f \delta^3(y) d^3y \]

\[ \Xi \sim \sum_{i=1}^{N_f} \delta^3(y - y_m) d^3y \]

\[ \Xi \sim N_f \text{vol}_3 \]

- Smearing approximates the brane density as continuous.
- Valid for sufficiently large number of branes

\[ \frac{N_f}{\text{VOL}_3} \geq \frac{1}{l_s^3} \]

- Smearing restores R-symmetries associated with transverse cycles.
The geometry of backreacting flavors

Smearing II - Distribution densities and the smearing form

• Supersymmetry and equations of motion conspire...
  \[ *d(e^{-\Phi} \phi_{D6}) = F \quad dF = -(2\kappa_{10}^2 T_6)\Xi \]
• ...to fix the most general distribution density, the smearing form.
  \[ d * d(e^{-\Phi} \phi_{D6}) = -(2\kappa_{10}^2 T_6)\Xi \]
• The smearing form depends on the ansatz for the background.

• It often imposes symmetries of the original background onto the additional flavor branes.

• Technically it is not necessary to search for flavor brane embeddings.
The geometry of backreacting flavors

An example - D6-branes on the conifold

\[ N_c g_{YM}^2 \ll 1 \]

D6 branes wrapping \( S^3 \)
Deformed conifold

\[ N_c g_{YM}^2 \gg 1 \]

Flux on \( S^2 \)
Resolved conifold

\[
\begin{align*}
\mathcal{L} &= e^{\frac{2}{3} \Phi} \left\{ dx_{1,3}^2 + E^2 d\rho^2 + A^2 (\sigma_1^2 + \sigma_2^2) + C^2 [(\Sigma_1 - f\sigma_1)^2 + \Sigma_2 - f\sigma_2)^2] \\
&\quad + D^2 \sin^2 \alpha (d\psi + \cos \theta d\phi - \cos \tilde{\theta} d\tilde{\phi})^2 \right\} \\
A^2 &= B^2 = \rho^2 \\
C^2 &= D^2 = \frac{\rho^3}{9} (1 - \frac{a^3}{\rho^3}) \\
E^2 &= 12 (1 - \frac{a^3}{\rho^3})^{-1} \\
\sin^2 \alpha &= \frac{B^2}{B^2 + (1 - g)^2 D^2}
\end{align*}
\]

Gopakumar, Vafa
The geometry of backreacting flavors

An example - D6-branes on the conifold

• After a choice of frame
\[ \Xi = e^{-5\Phi/3} \left[ \Xi_1 e^{\rho_{34}} + \Xi_2 (e^{\rho_{23}} + e^{\rho_{14}}) + \Xi_3 e^{\rho_{12}} + \Xi_4 (e^{135} + e^{245}) \right] \]
\[ \Xi_i = \Xi_i [A, B, C, D, E, f, g](\rho) \]

• Subject to
\[ \Xi_3 = -\Xi_1 - \frac{2\Xi_2}{\tan \alpha} \quad \Xi_4 = \frac{F_{34}}{2\kappa_{10}^2 T_6 D \sin^2 \alpha} \quad \Xi_1 = \Xi_2 \tan \alpha \]

• Color-flux quantization
\[ \int_{S^2} F = 2\pi N_c \]
The geometry of backreacting flavors

An example - D6-branes on the conifold

• Assuming

$$\Xi_1 = \Xi_2 = \Xi_3 = 0 \quad \partial_\rho F_{\mu\nu} = 0$$

• We have

$$\Xi = \Xi_4(e^{135} + e^{245})$$

• And can solve the first-order system

$$F = N_f [\sin \psi (d\theta \wedge d\tilde{\theta} + \sin \theta \sin \tilde{\theta} d\phi \wedge d\tilde{\phi}) + \cos \psi (\sin \tilde{\theta} d\theta \wedge d\tilde{\phi} + \sin \theta d\tilde{\theta} \wedge d\phi)]$$

$$- N_c (\sin \theta d\theta \wedge d\phi + \sin \tilde{\theta} d\tilde{\theta} \wedge d\tilde{\phi})$$

$$d\Omega_{\text{int}}^2 = \frac{1}{12} (d\theta^2 + \sin^2 \theta d\phi^2) + \frac{1}{12 (1 - f^2)} [(\omega_1 - fd\theta)^2 + (\omega_2 - f \sin \theta d\phi)^2]$$

$$+ \frac{1}{16 (1 - f^2)} (\omega_3 - \cos \theta d\phi)^2$$

$$\Phi(r) = \frac{3}{2} \log \left( \frac{f}{4N_c (1 - f^2) r} \right)$$
The geometry of backreacting flavors

An example - D6-branes on the conifold

- We find that the additional sources modify the fibration between the spheres

\[ N_f = \pm N_c \frac{4f^2 - 1}{3f} \]

- Graph showing the relationship between \( N_f/N_c \) and \( f \)

- \( f \to 0 \quad \Leftrightarrow \quad \frac{N_f}{N_c} \to -\infty \)
- \( f = \frac{1}{2} \quad \Leftrightarrow \quad N_f = 0 \)
- \( f = 1 \quad \Leftrightarrow \quad N_f = N_c \)
D6-sources in M-theory
D6-sources in M-theory

M-theory and type IIA supergravity

\[ d=11 \quad \text{M-theory - M2, M5} \]

\[
S_M = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{g} R + O(G, \Psi)
\]

\[
ds_{11}^2 = e^{-\frac{2}{3}\Phi} ds_{10}^2 + e^{\frac{4}{3}\Phi} (C_\mu dx^\mu + d\psi)^2
\]

\[ d=10 \quad \text{Type IIA - D0, F1, D2, D4, NS5, D6} \]

\[
S_{\text{IIA}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g}[e^{-2\Phi} (R + 4\partial_\mu \Phi \partial^\mu \Phi)] - \frac{1}{4} dC \wedge *dC + O(H_3, F_4, \Psi)
\]
**D6-sources in M-theory**

**Kaluza-Klein monopoles**

\[ ds^2 = dx^2_{1,6} + H dy^2_3 + H^{-1} (d\psi + A_i dy^i)^2, \]
\[ \nabla \times A = -\nabla H, \quad H = 1 + \sum_{i=1}^{n+1} \frac{2|N_i|}{|r - r_i|} \]

- **Euclidean Taub-NUT**
- **Brane world-volume**
- **Size of M-theory circle**
- **NUT charge**
- **Position of branes**

- M-theory circle quenches at D6 positions
- Periodicity related to charge
- Coincident branes give modding by \( \mathbb{Z}_n \Rightarrow A_{n-1} \)

- In general, D6-branes lift to pure gravity
  \[ R_{\mu\nu} = 0 \quad \delta_\epsilon \psi_\mu = D_\mu \epsilon \]
D6-sources in M-theory

M-theory on $G_2$ holonomy manifolds

World-volume

\[
    ds^2 = dx_{1,3}^2 + \left(1 - \frac{a^3}{\rho^3}\right)^{-1} d\rho^2 + \frac{\rho^2}{12} \tilde{w}^2 + \frac{\rho^2}{9} \left(1 - \frac{a^3}{\rho^3}\right) \left(w - \frac{1}{2} \tilde{w}\right)^2
\]

\[SU(2)_L \times \tilde{SU}(2)_L \times SU(2)_D\]

- Flop transition
- Cone over $S^3 \times \tilde{S}^3 \sim T^* S^3$
- Holonomy/supersymmetry

\[
    G_2 \Leftrightarrow \mathcal{N} = 1 \quad 1 \oplus 7 = 8
\]

Atiyah, Maldacena, Vafa
Edelstein, Núñez
Brandhuber, Gomis, Gubser, Gukov
Cvetic, Gibbons, Lu, Pope
Arbitrary D6-sources in M-theory
Arbitrary D6-sources in M-theory

The problem

• Kaluza-Klein reduction works in terms of
\[ ds_{11}^2 = e^{-\frac{2}{3}\Phi} ds_{10}^2 + e^{\frac{4}{3}\Phi}(C_\mu dx^\mu + d\psi)^2 \quad \Rightarrow \quad dF = d^2 C = 0 \]

• Yet the signature for D6 sources is
\[ dF = -(2\kappa_{10}^2 T_6)\Xi \]

• SUSY transformations and e.o.m.s, when derived from M-theory, are in terms of
\[ dC \]

• We will partially solve this problem at the level of the supergravities.

Possible solutions:
• Kaluza-Klein vortices, Kaluza-Klein domain walls
• Additional M-theory fields (Four-form field strength)
• Kaluza-Klein monopole actions
\[ S_{KK7} = -T_{KK7} \int_{M_7} d^7 \xi K^2 \sqrt{-\det \partial_i X^\mu \partial_j X^\nu \Pi_{\mu\nu}} \]
\[ \Pi_{\mu\nu} = g_{\mu\nu} - K^{-1} K_\mu K_\nu \]
Arbitrary D6-sources in M-theory

The argument

M-theory, Ricci flatness, $G_2$ holonomy \quad M-theory, $G_2$ structure, torsion

\begin{align*}
\downarrow & \quad \downarrow \\
\text{Type IIA, no sources, SU(3) structure} & \quad \text{Type IIA, sources, modified SU(3) structure} \\
& \quad \quad \quad \quad \quad dA \rightarrow F
\end{align*}
Initial considerations

Prior to flavoring: Ricci flatness and supersymmetry caused special holonomy
\[ d\phi_{G_2} = d(e^\Phi \Psi) + dJ \wedge (C + d\psi) + J \wedge dC = 0 \]
\[ d*7\phi_{G_2} = -\frac{1}{2} d(e^{-\frac{4}{3}\Phi} J \wedge J) + d(e^{-\frac{\Phi}{3}} *_6 \Psi) \wedge (C + d\psi) - e^{-\frac{\Phi}{3}} (*_6 \Psi) \wedge dC = 0 \]

The SU(3) structure is intimately linked to the presence of calibrated sources
\[ \phi_{D6} = -e^{x^0 x^1 x^2 x^3} \wedge \Psi \]

Now we expect:
- Sources carry energy-momentum and cause loss of Ricci flatness
- Therefore $G_2$ structure instead of $G_2$ holonomy
\[ d\phi_{G_2} = -J \wedge (F - dC) \]
\[ d*7\phi_{G_2} = e^{-\frac{\Phi}{3}} (*_6 \Psi) \wedge (F - dC) \]
- We are led to consider geometries with torsion.
Arbitrary D6-sources in M-theory

Supergravity variations

• SUSY variations in M-theory (with torsion)

\[ \delta_\epsilon \hat{\psi}_M = \partial_M \hat{\epsilon} + \frac{1}{4} \hat{\omega}_{MAB} \hat{\Gamma}^{AB} \hat{\epsilon} + \frac{1}{4} \hat{\tau}_{MAB} \hat{\Gamma}^{AB} \hat{\epsilon} \]

• Reduction gives the desired type IIA equations

\[ \delta_\epsilon \psi_\mu = D_\mu \epsilon + \frac{1}{64} e^{\frac{3}{4}} \Phi F_{\mu_1 \mu_2} (\Gamma_\mu_{\mu_1 \mu_2} - 14 \delta_{\mu_1}^{\mu_2} \Gamma^{\mu_2}) \Gamma^{11} \epsilon = D_\mu \epsilon \]

\[ \delta_\epsilon \lambda = \frac{\sqrt{2}}{4} \partial_\mu \Phi \Gamma^{\mu} \Gamma_{11}^{11} + \frac{3}{16} \frac{1}{\sqrt{2}} e^{\frac{3}{4}} \Phi F_{\mu_1 \mu_2} \Gamma^{\mu_1 \mu_2} \epsilon \]

• If and only if we make a suitable choice for the torsion

\[ \hat{\tau} \sim F - dC \]

• We obtain a torsion modified covariant derivative

\[ \nabla_M^{(\tau)} \phi G_2 = 0 \quad \nabla_M^{(\tau)} (*7 \phi G_2) = 0 \]
Integrability and equations of motion

• To find the relevant equations of motion, we reverse the integrability argument.

\[ 0 = [\nabla_K^{(\tau)}, \nabla_L^{(\tau)}] \phi_{G^2 MNP} \]

• So supersymmetry implies the following second order equation

\[ 0 = 2R_{KL}^{(\tau)} + R_{MNP}^{(\tau)}(\ast 7\phi_{G_2})_K^{MNP} \]
Arbitrary D6-sources in M-theory

The argument

M-theory, Ricci flatness, $G_2$ holonomy

M-theory, $G_2$ structure, torsion

Type IIA, no sources, SU(3) structure

Type IIA, sources, modified SU(3) structure

$dA \to F$
Flavor branes in gauge/string duality and M-theory

Conclusions

• Calibrated geometry provides a powerful tool in constructing flavored string duals.
• Use of calibrations makes the search for embeddings technically redundant.
• The more formal perspective allows explanations of characteristics of brane embeddings.

• Smeared D6-sources cannot be accommodated for by standard Kaluza-Klein mechanism.
• Issue can be resolve by adding torsion terms to eleven-dimensional supergravity.
• Present explanation is not fully general. It relies on supersymmetry and topology.
• Possible implications for monopole condensation.

• There is need to further study the physics of flavored solutions.
• Up to now that is mainly done on a case-by-case basis.
• Calibrated flavors may provide unifying formalism to do so.
Thank you for your attention.
Additional material
The modified SU(3) structure

\[ 0 = dJ \]
\[ 0 = d(e^{-\Phi/3} \ast_6 \Psi) \]
\[ 0 = d(e^\Phi \Psi) + J \wedge dC \]
\[ 0 = -\frac{1}{2} d(e^{-4\Phi/3} J \wedge J) - e^{-\Phi/3}(\ast_6 \Psi) \wedge dC \]

\[ 0 = dJ \]
\[ 0 = d(e^{-\Phi/3} \ast_6 \Psi) \]
\[ 0 = d(e^\Phi \Psi) + J \wedge F \]
\[ 0 = -\frac{1}{2} d(e^{-4\Phi/3} J \wedge J) - e^{-\Phi/3}(\ast_6 \Psi) \wedge F \]
Additional material

**Torsion**

\[ \delta_\epsilon \hat{\psi}_M = \partial_M \hat{\epsilon} + \frac{1}{4} \hat{\omega}_{MAB} \hat{\Gamma}^{AB} \hat{\epsilon} + \frac{1}{4} \hat{\tau}_{MAB} \hat{\Gamma}^{AB} \hat{\epsilon} \]

\[ \hat{\tau}_{\psi a \psi} = 0 \quad \hat{\tau}_{\psi b c} = \frac{e^{3/2} \Phi}{2T_6 \kappa_{10}^2} (F - dC)_{bc} \]

\[ \hat{\tau}_{\mu b c} = \frac{e^{3/2} \Phi}{2T_6 \kappa_{10}^2} C_\mu (F - dC)_{bc} \quad \hat{\tau}_{\mu b c} = -\frac{e^{3/4} \Phi}{2T_6 \kappa_{10}^2} (F - dC)_{\mu b} \]

\[ \delta_\epsilon \psi_\mu = D_\mu \epsilon + \frac{1}{64} e^{3/4} \Phi F_{\mu_1 \mu_2} (\Gamma_\mu^{\mu_1 \mu_2} - 14 \delta^{\mu_1}_\mu \Gamma^{\mu_2}) \Gamma^{11} \epsilon = D_\mu \epsilon \]

\[ \delta_\epsilon \lambda = \frac{\sqrt{2}}{4} \partial_\mu \Phi \Gamma^{\mu} \Gamma^{11} + \frac{3}{16} \frac{1}{\sqrt{2}} e^{3/4} \Phi F_{\mu_1 \mu_2} \Gamma^{\mu_1 \mu_2} \epsilon \]
Additional material

The Wilson loop

- Graphic shows the quark-antiquark potential calculated with the “standard methods”.
- There is actually need for a cutoff-brane as the length of the string is otherwise divergent.