Test of the Equivalence Principle in the Laboratory

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Mass does not add up
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Proton + Electron = $^1\text{H}$
Mass does not add up

Proton

Electron

$^1$H

$938 \, 272 \, 013 \text{ eV}/c^2$

$510 \, 999 \text{ eV}/c^2$

$938 \, 782 \, 999 \text{ eV}/c^2$
Mass does not add up

Proton

Electron

$^1\text{H}$

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510 999 eV/c$^2$

938 783 012 eV/c$^2$

Mass defect = binding Energy

Binding Energy = $\Delta E_{\text{pot}} - \Delta E_{\text{kin}}$

13.6 eV/c$^2$
The non-linearity is rather small but occurs also in gravitationally bound systems.

Mass defect for 1 kg of Earth: $\Delta m = 0.46 \, \mu g$

Mass defect for 1 kg of Moon: $\Delta m = 0.02 \, \mu g$
The mass of an object

\[ m = \sum m_c + \sum E_{\text{kin}} - \sum E_{\text{pot}} \]
Newton’s Principia (1689)

Newton’s 2\textsuperscript{nd} Law

\( F_i = m_i \, a \)

Gravitational Law

\( F_G = G \, m_{g1} \, m_{g2} / r^2 \)
Newton’s Principia (1689)

Newton’s 2\textsuperscript{nd} Law
\[ F_i = m_i a \]

Gravitational Law
\[ F_G = G \frac{m_{g1} m_{g2}}{r^2} \]

Equivalence Principle (EP): \[ m_i = m_g \]
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Equivalence Principle (EP):
\[ m_i = m_g \]

Is the Equivalence Principle valid for all contributions to the mass?
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Gravitational Law

\[ F_G = G \ m_{g1} \ m_{g2} / r^2 \]

Equivalence Principle (EP):

\[ m_i = m_g \]

Is the Equivalence Principle valid for all contributions to the mass?

Weak Equivalence Principle: Gravitational binding energy is excluded.

Strong Equivalence Principle: Includes all 4 fundamental interactions.
In General Relativity

Gravitational field $g$

Acceleration $a$

Inertial mass = gravitational mass, $m_i = m_G$ for all bodies
The known unknowns
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- GR is not a quantum theory.
The known unknowns

• GR is not a quantum theory.
• Cosmological constant problem.
The known unknowns

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- Dark matter.
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The unknown unknowns
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The unknown unknowns

• Is there another force (5\textsuperscript{th} force)?
• …
The known unknowns

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The unknown unknowns

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• ...

EP-Tests provide a big bang for the buck!
Searching for new interactions

\[ V(r) = \alpha G \left( \frac{q_1}{\mu_1} \right) \left( \frac{q_2}{\mu_2} \right) \frac{m_1 m_2}{r_{12}} e^{-r_{12}/\lambda} \]

- **Source** and **Test mass**
- **Strength relative to gravity**
- **Interaction range**
Searching for new interactions

\[ V(r) = \alpha G \left( \frac{q_1}{\mu_1} \right) \left( \frac{q_2}{\mu_2} \right) \frac{m_1 m_2}{r_{12}} e^{-r_{12}/\lambda} \]

Strength relative to gravity

Source

Test mass

Interaction range

Assumed charge | Be q/µ | Ti q/µ | Al q/µ | Be-Ti (x10^{-2}) | Be-Al (x10^{-2})
--- | --- | --- | --- | --- | ---
N | 0.554 80 | 0.541 47 | 0.518 87 | 1.33 | 3.59
Z | 0.443 84 | 0.459 61 | 0.481 81 | -1.58 | -3.80

and any linear combinations:

\[ q(\Psi) = Z \cos \Psi + N \sin \Psi \]
\[
\begin{array}{|c|c|c|c|}
\hline
 & \text{Mass (u)} & q=B & q/\mu \\
\hline
^{1}_{1}p & 1.0073 & 1 & 0.9928 \\
\hline
^{1}_{0}n & 1.0087 & 1 & 0.9914 \\
\hline
^{9}_{4}\text{Be} & 9.1012 & 9 & 0.9987 \\
\hline
^{48}_{22}\text{Ti} & 47.9479 & 48 & 1.0011 \\
\hline
\end{array}
\]
$B/\mu$ varies, because
1st Tests of the Equivalence Principle

\[ F = m_G g \]
\[ a = \frac{m_G}{m_I} g \]
\[ F = m_I a \]

Time \( t \) to fall from \( h \):
\[ t = \sqrt{\frac{2h}{m_G / m_I g}} \]

1600 Galileo:
\[ \eta = \frac{a_1 - a_2}{\frac{1}{2}(a_1 + a_2)} \approx 0.1 \]
2nd Generation Tests

Measurement of the swing periods of pendula:

\[ T = 2\pi \sqrt{\frac{L}{g} \frac{m_I}{m_G}} \]

Newton (1686), Bessel (1830), Potter (1923)

\[ \eta \approx 2 \times 10^{-5} \]
Eötvös Experiments

\[ F_I = m_I \omega^2 r \cos \theta \]

\[ F_G = m_G g \]

\[ \epsilon = \frac{m_I}{m_G} \frac{\omega^2 r}{2g} \sin(2\theta) \]
The torsion balance

- A violation of the EP would yield to different plumb-line for different materials.
- A torsion balance can be used to measure the difference in plumb-lines:

Torsion fiber hangs like the average plumb line.

Difference in plumb lines produces a torque on the beam.

- \( \eta \approx 5 \times 10^{-9} \)

Eötvös (1922)
Dicke’s idea

Using the Sun as a source

\[ \eta \approx 1 \times 10^{-11} \]
\[ \eta = \frac{|a_1 - a_2|}{\frac{1}{2} (a_1 + a_2)} \]

Historical overview

- Galileo
- Newton
- Bessel
- Eötvös
- Potter
- Dicke
- Braginsky
Historical overview

\[ \eta = \frac{|a_1 - a_2|}{\frac{1}{2} (a_1 + a_2)} \]

- Galileo
- Newton
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- Eötvös
- Potter
- Dicke
- Braginsky

Type of experiment:
- drop
- pendula
- torsion balance
- modulated torsion balance
Principle of our experiment

Source Mass

Composition dipole pendulum
(Be-Ti)

Rotation
1 rev./ 20min

$\alpha_{Be}$

$\alpha_{Ti}$

EP-Violating signal

Autocollimator (=optical readout)

13.3min
Principle of our experiment

Composition dipole pendulum
(Be-Ti)

EP-Violating signal

Rotation 1 rev./ 20min

<table>
<thead>
<tr>
<th>source mass</th>
<th>$\lambda$ (m)</th>
<th>horizontal acc. (ms$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>local masses (hill)</td>
<td>$1 - 10^4$</td>
<td>$&lt; 7 \times 10^{-5}$ Northwest</td>
</tr>
<tr>
<td>entire earth</td>
<td>$10^6 - 10^7$</td>
<td>$1.7 \times 10^{-3}$ North</td>
</tr>
<tr>
<td>Sun</td>
<td>$10^{11} - \infty$</td>
<td>$5.9 \times 10^{-3}$ modulated</td>
</tr>
<tr>
<td>Milky Way (incl. DM)</td>
<td>$10^{20} - \infty$</td>
<td>$1.9 \times 10^{-10}$ modulated</td>
</tr>
</tbody>
</table>
The torsion pendulum

- 20 μm diameter tungsten fiber (length: 108 cm) \( \kappa = 2.36 \) nNm
- 8 test masses (4 Be & 4 Ti) 4.84 g each (within 0.1 mg) (can be interchanged)
- 4 mirrors
- Tuning screws for adjusting tiny asymmetries

Data:
- Torsional period: 800 s
- Quality factor: 4000
- Decay time: 11d 19 hrs
- Machining tolerance: 5 μm
- Total mass: 70 g
Signal + free Oscillation

\[ \omega_{\text{SIG}} = \frac{2}{3} \quad \omega_{\text{PEND}} = \frac{2\pi}{1200 \text{ s}} \]
Filtered $F(t) = \Theta(t - T/4) + \Theta(t + T/4)$

This data was taken with an asymmetric pendulum to illustrate the data analysis.
Segmented & fitted

- **West**:
  - $\cos$ function
  - Data points show a cosine-like pattern.

- **North**:
  - $\sin$ function
  - Data points show a sine-like pattern.

- **F (μrad)**:
  - Measurement data decreases sharply in the initial phase and then stabilizes.

*ϕ* Turntable angle (deg):

- Range from 0 to 3240 degrees.

The graph illustrates the segmented and fitted data over the turntable angle, with distinct phases for cosine and sine functions, along with measurement data for $F (\mu rad)$. The fitted data aligns well with the expected periodic behavior for cosine and sine functions.
A day of data

\[ \sigma \approx 3 \text{ nrad} \sqrt{\text{day}} \]
Conversion of angle to differential acceleration

\[ \Delta a = \frac{\kappa}{4md} \phi \]

Can be measured with an uncertainty of 3 nrad per day

\[ \Delta a \text{ can be measured to } 20 \text{ fm/s}^2 \]

| \( \kappa \) | \( 2.36 \times 10^{-9} \text{ Nm} \) |
| \( m \)   | \( 4.84 \times 10^{-3} \text{ kg} \) |
| \( d \)   | \( 1.9 \times 10^{-2} \text{ m} \) |
After a 2 months of data taking and systematic checks we physically invert the dipole on the pendulum and put it back into the apparatus.

These data points have been corrected for systematic effects.

Only statistical uncertainties shown.

Offset is caused by the magnetic damper
## Corrected result

<table>
<thead>
<tr>
<th></th>
<th>$\Delta a_{N,Be-Ti}$  \hspace{0.5cm} (10^{-15} \text{ ms}^{-2})</th>
<th>$\Delta a_{W,Be-Ti}$ \hspace{0.5cm} (10^{-15} \text{ ms}^{-2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>measured</td>
<td>3.3 ± 2.5</td>
<td>-2.4 ± 2.4</td>
</tr>
<tr>
<td>gravity gradients</td>
<td>1.6 ± 0.2</td>
<td>0.3 ± 1.7</td>
</tr>
<tr>
<td>tilt</td>
<td>1.2 ± 0.6</td>
<td>-0.2 ± 0.7</td>
</tr>
<tr>
<td>temperature gradients</td>
<td>0 ± 1.7</td>
<td>0 ± 1.7</td>
</tr>
<tr>
<td>magnetic coupling</td>
<td>0 ± 0.3</td>
<td>0 ± 0.3</td>
</tr>
<tr>
<td>corrected result</td>
<td>0.6 ± 3.1</td>
<td>-2.5 ± 3.5</td>
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Tilt coupling
Tilt coupling

rotation axis

Tilt of the suspension point + anisotropies of the fiber → will rotate
Tilt coupling

1. Thicker fiber at the top provides more torsional stiffness

Tilt of the suspension point + anisotropies of the fiber will rotate
Tilt coupling

1. Thicker fiber at the top provides more torsional stiffness.

2. Feedback minimizes tilt of TT.

Tilt of the suspension point + anisotropies of the fiber will rotate.
The tilt matrix

No signal!

1.70m

0.23m

53 nrad

14 nrad

60 nrad

Remaining tilt uncertainty:

0.6 fm/s² N

0.7 fm/s² W
Thermal

Effect on the applied gradient on the signal (measurement was done on one mirror):

-4.6 nrad/K
-4.1 nrad/K

50 nrad Signal
ΔT = 7 K
During data taking
ΔT ≈ 0.053 K
⇒ 0.38 nrad

1.7 fm/s² N
1.7 fm/s² W

Effect of a 7 K gradient on two different mirrors.
### Results

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<td>-1.8 ± 2.8</td>
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<tr>
<td>$a_{Gal}$(Be) - $a_{Gal}$(Ti)</td>
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\[ \Delta \vec{a} = \alpha \Delta \left( \frac{q}{\mu} \right)_p \vec{I} \]

\[ \vec{I} = G \int d^3 r' \rho(r') \left( \frac{q}{\mu} \right)_S \left( \frac{1}{r'} + \frac{1}{\lambda} \right) e^{-r'/\lambda} \frac{\hat{r}'}{r'} \]

- **Local topography**
- **Preliminary Earth Reference Model**

\[ \lambda (m) \text{ range of the yukawa potential} \]
Source integration

\[ \Delta \vec{\alpha} = \alpha \Delta \left( \frac{q}{\mu} \right)_p \vec{I} \]

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Local topography

Preliminary Earth Reference Model

Difficult and tedious

\[ \lambda(m) \text{ range of the yukawa potential} \]
\[ \alpha - \lambda \text{ plot for } q = N \ (\psi = \pi/2) \]
$\psi - \alpha$ plot for $q=N$ (acc. towards the Sun)
Acceleration to the center of our galaxy

1825 h of data taken over 220 days

Differential acceleration to the center of the galaxy:

\((-2.1 \pm 3.1) \times 10^{-15} \text{ m/s}^2\)

In quadrature:

\((2.7 \pm 3.1) \times 10^{-15} \text{ m/s}^2\)

Hypothetical signal:

(7 times larger)

\(20 \times 10^{-15} \text{ m/s}^2\)
Galactic dark matter

Earth

Milky Way
Galactic dark matter

Earth

Milky Way

Halo of dark matter
Galactic dark matter

Dark matter inside the Earth’s galactic orbit causes 25-30% of the total acceleration.

Halo of dark matter
Galactic dark matter

Dark matter inside the Earth’s galactic orbit causes 25-30% of the total acceleration.

Halo of dark matter

Our acceleration toward the galactic center is:

\[ a_{\text{gal}} = a_{\text{dark}} + a_{\text{ordinary}} = 1.9 \times 10^{-10} \text{ m/s}^2 \Rightarrow a_{\text{dark}} = 5 \times 10^{-11} \text{ m/s}^2 \]
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Measured differential acceleration toward the galactic center:

\[ \Delta a_{\text{gal}} = (-2.1 \pm 3.1) \times 10^{-15} \text{ m/s}^2 \Rightarrow \eta_{\text{dark}} = |\Delta a_{\text{gal}}|/a_{\text{dark}} = (-4 \pm 7) \times 10^{-5} \]
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Halo of dark matter

Milky Way

Earth

Halo of dark matter

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The acceleration of Be and Ti towards dark matter does not differ by more than 150 ppm (with 95 % confidence).
Limits on the acceleration of neutral $H$ towards dark matter
Summary

- The test of the equivalence principle is a sensitive probe for fundamental physics.
- Principle of the measurement.
- Main systematic effects.
- Results
  - Earth (North): $a_{Be} - a_{Ti} = (0.6 \pm 3.1) \times 10^{-15} \text{ m/s}^2$.
  - $\eta = (0.3 \pm 1.8) \times 10^{-13}$.
  - Towards Galaxy: $a_{Be} - a_{Ti} = (-2.1 \pm 3.1) \times 10^{-15} \text{ m/s}^2$.
  - $\eta_{DM} = (-4 \pm 7) \times 10^{-5}$.

- 10x improved limits on a long range interaction.
Thank you

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