Quantum information processing with prethreshold superconducting qubits

J. M. Martinis (UCSB)
C. Benjamin (UGA), A. Galiautdinov (UCR),
and E. J. Pritchett (IQC Waterloo)

work supported by the NSF EAGER & CDI

www.physast.uga.edu/~mgeller/group.htm
**SES computing**

$n$ qubits

- full Hilbert space
  - $\psi \rightarrow U\psi$
  - $2^n$ variables
  - exponential storage capacity

- single-excitation subspace (SES)
  - $\psi \rightarrow U\psi$
  - but $2^{2n}$ gates
  - natural Hamiltonians are one- and two-body
  - efficient algorithms poly$(n)$

- number of gates is large
- error correction required

- requires fully connected qubits ($\sim n^2$ JJs)
- can only run for time $\tau$

- this is the bottleneck for superconducting architectures

- more qubits required, but this is OK

- 1 shot!
  - full controllability of projected $H$
single-excitation subspace

\[ n = 3 \quad \{ |000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle \} \]
SES computing

$n$ qubits

full Hilbert space

single-excitation subspace (SES)

$\psi \rightarrow U\psi$

$2^n$ variables
exponential storage
capacity

but $2^{2n}$ gates
natural Hamiltonians
are one- and two-body
efficient algorithms poly(n)

$n$ variables

1 shot!
full controllability
of projected $H$

applications:

1. factoring $n \sim 2^{100s}$ (not possible)
2. general purpose simulation

polynomial speedup for class of simulations
quantum simulation

Feynman, 1982
Lloyd, 1996

digital (gate-based)

Aspuru et al., 2005

analog

Buluta & Nori, 2009
Greiner et al., 2002
quantum simulation

- Feynman, 1982
- Lloyd, 1996

General-purpose simulator (UGA/UCSB)
- digital (gate-based)
- analog

- Aspuru et al., 2005
- Greiner et al., 2002
- Buluta & Nori, 2009

• tied to UCSB phase qubit architecture
• simulates arbitrary time-dependent $H$’s
• uses and interfaces with conventional QC
mapping

\[ H = \frac{p_1^2}{2m_1} + \frac{P_1^2}{2M_1} - \frac{Z_1 e^2}{r_1 - R_1} + \frac{p_2^2}{2m_2} + \frac{P_2^2}{2M_2} - \frac{Z_2 e^2}{r_2 - R_2} + \frac{Z_1 Z_2 e^2}{R_1 - R_2} + \frac{e^2}{|r_1 - r_2|} - \frac{Z_2 e^2}{|r_1 - R_2|} - \frac{Z_1 e^2}{|r_2 - R_1|} \]

\[ H_{ucsb} = \sum_{i=1}^{n} \epsilon_i c_i^\dagger c_i + \frac{1}{2} \sum_{i,j=1 \atop i \neq j}^{n} g_{ij} K^{\alpha \beta} \sigma_i^\alpha \otimes \sigma_j^\beta \]
what we solve

given a d-dimensional Hilbert space

\[ \{|m \rangle \}_{m=1}^{d}, \quad \langle m | m' \rangle = \delta_{mm'} \]

and time-dependent Hamiltonian

\[ H_{mm'}(t), \quad t \in (t_i, t_f) \] (real?)

time-evolution operator

\[ U \equiv T e^{-\frac{i}{\hbar} \int_{t_i}^{t_f} H(t) \, dt} \]

\[ = \lim_{\Delta t \to 0} e^{-\frac{i}{\hbar} H(t_f) \Delta t} \times \cdots \times e^{-\frac{i}{\hbar} H(t_i + \Delta t) \Delta t} e^{-\frac{i}{\hbar} H(t_i) \Delta t} \]

\[ |\psi(t_f)\rangle = U |\psi(t_i)\rangle \]

we compute U
n-qubit simulator

UCSB phase qubits with tunable inductive coupling

\[ H_{\text{ucsb}} = \sum_{i=1}^{n} \epsilon_i(t) c_i^\dagger c_i + \frac{1}{2} \sum_{i,j=1}^{n} g_{ij}(t) K^{\alpha\beta} \sigma_i^\alpha \otimes \sigma_j^\beta \quad (g \ll \epsilon) \]

\[ \text{dim}(H_{\text{ucsb}}) = 2^n \quad \text{not controllable} \]

R. C. Bialczak et al, arXiv:1007.2209
single-excitation subspace

\[ |m\rangle \equiv c_m^* |00 \cdots 0\rangle = |00 \cdots 1_m \cdots 0\rangle \]

this has dimension \(n\)
(number of qubits)

complete control over real Hamiltonians
in this subspace
energy/time rescaling

\[ U = e^{-(i/\hbar)Ht} = e^{-(i/\hbar)\left(\frac{H}{\lambda}\right)(\lambda t)} = e^{-(i/\hbar)H_{qc}t_{qc}} \]

\[ H_{qc} \equiv \frac{H}{\lambda}, \quad t_{qc} \equiv \lambda t \]

molecular collisions \( \lambda \sim 10^6 \)

\sim \text{GHz}

coherence limits length of process simulated

we will have to emulate \( H_{qc} \)

(can also subtract time-dependent c-number)
continuous rescaling
continuous rescaling

\[ U = T e^{-(i/\hbar) \int_{t_i}^{t_f} H(t) \, dt} \]

\( t_{qc}(t) \) is nonlinear

\[ \lambda \equiv \frac{dt_{qc}}{dt} > 0 \]

smaller \( \lambda \) is better

\[ H_{qc}(t_{qc}) \equiv \left. \frac{H(t)}{\lambda(t)} \right|_{t=t(t_{qc})} \]

exact symmetry of \( U \)
main error: leakage

leakage prob \sim \left( \frac{g}{\varepsilon} \right)^2 \approx 10^{-4} \text{ (UCSB)}

\[ |111\rangle, |011\rangle, |101\rangle, |110\rangle, |001\rangle, |010\rangle, |100\rangle, |000\rangle \]
3-channel molecular collision

uses classically computed potential energy surface

Na(3s) + He → Na(m) + He \quad (m = 1,2,3)
\[ t_{qc}(t) = \int_{t_i}^{t} \lambda(t') dt' \]
\[ t_{qc}(t_f) = 42 \, \text{ns} \]

3 qubit quantum simulation

\[ \text{Na}(3s) + \text{He} \rightarrow \text{Na}(3p) + \text{He} \]

3 qubit quantum simulation

Na(3s) + He \rightarrow Na(3p) + He
scaling

Hilbert space dimension d

classical (single fast processor)

- time slice \( t = 2d^3 \text{ ns} \) (large \( d \))
- \( t_{\text{cl}} = 100 \times 2d^3 \text{ ns} \)

unfavorable dimension dependence

(unrelated to exponentially large \( d \) problem)

- \( d > 1000 \) rare

quantum simulator

- each 100ns run \( t = 1 \mu s \)
- \( t_{\text{qu}} = 100 \times 1 \mu s = 10^5 \text{ ns} \)

independent of \( d \)!
scaling (continued)

100 qubit machine is 1000 times faster
1000 qubit machine is $10^6$ times faster!
a practical QC?

E. J. Pritchett et al., arXiv:1008.0701