Having your cake and seeing it too -

In Situ Observation of Incompressible Mott Domains in Ultracold Atomic Gases

Cheng Chin

James Franck institute
Physics Department
University of Chicago

Funding:
Cake Recipe
(Oth order approximation)

1. Mix flour and baking powder
2. Heat it to 350 F in an oven
3. Bake for 30~40 minutes

Voila!

Our “Ice Cream” Cake Recipe

1. Prepare some atoms in a bowl
2. Cool it to 10 nano-K
3. “Bake” for 200ms

Voila!
Comments

• We are not the first one to make the cake, but the first one to see it.

• We can see it because we make only one cake.

• All physics occurs in the baking.
Cake Terminology

1st tier

2nd tier

Cream frosting (not shown)

n=1 Mott insulator

n=2 Mott insulator

Superfluid
How does cake form in the lab?

(*Bose-Hubbard process*)

Lattice potential
(baking cup)

Density $n$

Chemical potential
Bose-Hubbard model \((Fisher\ et\ al.,\ PRB\ 1989,\ Greiner\ et\ al.,\ Nature\ 2002)\)

\[
\hat{H} = -t \sum_{i,j} (\hat{b}_i^+ \hat{b}_j + \hat{b}_j^+ \hat{b}_i) + U \sum_i \frac{\hat{n}_i(\hat{n}_i - 1)}{2} - \sum_i \mu_i \hat{n}_i
\]

Tunneling  Interaction  Trap potential

**Density distribution**

\(t >> U\)

Superfluid

compressible

\(U >> t\)

Mott insulator

incompressible

**Number statistics**

\[\rho_n\]

\[0\ 1\ 2\ 3\ 4\ 5\]

SF

MI

100%
Phase transition and quantum phase transition

Phase diagram and AdS4-CFT3 duality: Sachdev, Nature physics (2007)
Synopsis

How do we see the wedding cake?

Science Project 1:
- Properties of SF and Mott insulator
- Quantum phase transition

Science Project 2:
- Quantum microscopy and quantum information
In situ Imaging a single layer of 2D gas

**Microscope objective**

**Trap/lattice beams**

**Vertical compress. beams**

Cesium atoms in 2D

\[ 15\hbar\omega_r \sim k_B T \text{ and } \mu \sim \frac{\hbar\omega_z}{10} \]

Determined from the density profile.

**Imaging beam**


See also: (Harvard) W.S. Bakr et al., *Nature* 462 (2009)

What is an optical lattice?

Atomic polarization \( P = \varepsilon E \)

Atom-photon interaction \( V = -\int P \cdot dE = -\frac{1}{2} \varepsilon E^2 = -\frac{1}{2} \alpha_{AC} I \)

Optical lattice potential \( V(x) = V_0 \sin^2 k_x x \)

\( \alpha_{AC} \) : AC polarizability
Optical lattice toolbox

Hopping: \[-t \sum_{<i,j>} (\psi_i^+ \psi_j + \psi_j^+ \psi_i)\]

Interaction: \[\frac{U}{2} \sum_i n_i (n_i - 1)\]

Trapping potential: \[\sum_i n_i (\mu - V_i)\]

Controlled by:
- Lattice potential
- Feshbach tuning
- Trap potential
**Feshbach resonance in ultracold gases**

SF-MI transition in a single layer of 2D lattice
Why is it difficult to see the cake?

Conventional approach: 3D cake

Light Intensity $I$

Transmission $T(x,y) \ll 1$
~ 30 cakes of different sizes

Our approach: 2D cake

Light Intensity $I$

Transmission $T(x,y) \sim 0.5$
A closer look...

$N = 10000$

Resolution: 1.3 µm
Pixel: 0.6 µm
Lattice const.: 0.53 µm
Density fluctuations and correlations

Noise of $A \equiv A - \langle A \rangle$
Observation of the Mott domain (single tier cake)

Plateau density:
Theory: \( \frac{1}{d^2} = 3.53 \, \mu m^2 \)
Experiment: 3.5(3) \, \mu m^2

Emerging of Mott plateau

Weak Lattice
↓↓
Strong Lattice
Compressibility $\kappa$

**Definition**

$$
\kappa = \frac{\partial n}{\partial \mu} = \frac{1}{m\omega_r^2} \frac{n'(r)}{r}
$$

**Superfluid**

**Mott**

Radial lattice index

Density (atom/site)
Origin of the cake structure

Phase diagram at $T=0$

$\Delta$

MI plateau appears near $<n>=1$

Fisher et al., PRB 40, 546 (1989)

Quantum Monte Carlo calculation
(R. Scalettar, UC Davis)
Universal scaling regimes in 2D optical lattices

No lattice
(2D gas)

Strong lattice
Scaling symmetry

2D gas with **constant** contact interaction  

\[ H = \sum_i \frac{p_i^2}{2m} + \sum_{ij} g \delta^2(r_{ij}) \]


\[ H\psi(\lambda x) = \lambda^{-2} H\psi(\lambda x) \]

Another example: resonant Fermi gas.

\[ E_k \rightarrow \frac{E_k}{\lambda^2} \]
\[ gn \rightarrow g \frac{n}{\lambda^2} \]
Classical critical phenomena

Droplets in vapor

Bubbles in water

water

Vapor

M.C. Escher

YouTube: Optical Opalescence
Quantum phase transition in optical lattices

\[ H = \sum_i \frac{p_i^2}{2m} + \sum_{ij} g \delta^2(r_{ij}) + V(\sin^2 kx + \sin^2 ky) \]

No lattice

(2D gas)

strong lattice
Only the ratio of the length scale matters \( \Psi(x) = \lambda^x \Psi(\lambda x) \)

Examples:
Equation of state \( n(\mu, T) = \lambda_{dB}^{-2} F(\tilde{\mu}) \) \[ \tilde{\mu} = \frac{\mu}{kT} \]

Corollary: Grand potential can be fully determined by EoS.
\[ d\Omega = - s(\mu, T) dT - n(\mu, T) d\mu \]
\[ d\Omega = [ -2 \int F(x) dx - F(\tilde{\mu}) ] T^2 d\tilde{\mu} \]
\[ \tilde{n} = n \lambda_{dB}^2 \]
Scale invariance and universality in 2D gases
Temperature $T$ and chemical Potential $\mu$

LDA: Local Density Approximation

\[ \mu = \mu_0 - \frac{1}{2} m \omega^2 r^2 \]

\[ n(\mu, T) = -\lambda_{dB}^{-2} \ln [1 - \exp(\mu/k_B T - gn\lambda_{dB}^2/\pi)] \]

Rath et. al., PRA 82, 013609 (2010)
Scale Invariance - Density

\[ \tilde{n} = n \lambda_{dB}^2 = F \left( \frac{\mu}{kT} \right) \]

\[ n(\mu, T) = -\lambda_{dB}^{-2} \ln \left[ 1 - \exp \left( \frac{\mu}{k_B T} - gn \lambda_{dB}^2 / \pi \right) \right] \]

\[ \tilde{n} = 2\pi \tilde{\mu} / g \quad \text{(Thomas Fermi approximation)} \]
Universality – Density

\[ \tilde{n} - \tilde{n}_c = H\left(\frac{\mu - \mu_c}{kTg}\right) \]

\[ g = 0.26, 0.19, 0.13, 0.05 \]

\[ \tilde{n}_c = \ln\left(\frac{\xi}{g}\right), \quad \xi = 380 \]

\[ \tilde{\mu}_c = \left(\frac{g}{\pi}\right) \ln\left(\frac{\xi \mu}{g}\right), \quad \xi_\mu = 13.2 \]

△ Holzmann et. al., PRA 81, 043622 (2010)
○ Prokof’ev et. al., PRA 66, 043608 (2002)

Prokof’ev et. al., PRL 87, 270402 (2001)
Quantum phase transition in 2D lattice at $V=7 \ E_R$

$$\frac{n}{T^{d/z+1-1/v_z}} = G(\frac{\mu - \mu_{c,0}}{T^{1/v_z}})$$

Finite temp. effect

Quantum scaling law

Classical scaling law

Vanishing $T_c$ at $\mu_{c,0}$
1. Universal scaling.

\[ \tilde{N} = \frac{N_t}{k_B T} \]

\[ \tilde{\mu} = \frac{\mu - \mu_{c,0}}{k_B T} \]
1. Universal scaling.

\[ \tilde{N} = \frac{N}{k_B T} \]

\[ \tilde{\mu} = \frac{\mu - \mu_{c,0}}{k_B T} \]
1. Universal scaling.

\[ \tilde{N} = \frac{Nt}{k_B T} \]

\[ \tilde{\mu} = \left( \mu - \mu_{c,0} \right) / k_B T \]
1. Universal scaling.

![Graph showing scaled occupation number and scaled chemical potential with annotations for 6 to 15 nK, 24 nK, and 31 nK. Tunneling=3nk is also noted.]
2. Critical exponents \( z \) and \( \nu \):

\[
\frac{n - n_c}{T^{d/z + 1 - 1/\nu z}} = h\left(\frac{\mu - \mu_c}{T^{1/\nu z}}\right)
\]

Theory:
\( z = 2 \)
\( \nu = 1/2 \)

Experiment:
\( z = 2.0 \ (3) \)
\( \nu = 0.53 \ (5) \)
3. Quantum Critical Point

$$\mu_{c,0} = -4.2(6)t$$  \hspace{1em} (theory: $-4t$)

\[ \frac{n - n_c}{T^{d/z+1-1/\nu z}} = h\left(\frac{\mu - \mu_c}{T^{1/\nu z}}\right) \]

scaled occupation number

\[ \tilde{N} = \frac{N t}{k_B T} \]

chemical potential $\mu/t$
Conclusion

Scale invariance and universality in 2D

- In situ imaging: Cake structure  
  *Nature (2008)*
- Scale invariance and universality  
  *Nature (2010)*
- Quantum criticality: QPT in optical lattices  
  ...

New experiments:

- *Quantum fluctuations and correlations*  
  *NJP (2011)*
- *Quantum critical transport*  
  *PRL (2010), NJP (2011)*
Cold atom experiments at Univ. of Chicago

Group members
- Graduate Chen-Lung Hung
- Graduate Xibo Zhang
- New Student Harry Ha

Postdocs
- Dr. Shih-Kuang Tung
- Dr. Colin Parker

Theory Collaborators
- 2D gas:
  - N. Prokofev, B. Svistunov
- Criticality:
- Cold molecules:
  - P. Julienne, E. Tiesinga

Former group members
- Prof. Nathan Gemelke (Penn State Univ.)
- Dr. Kathy-Anne Soderberg (Booz allen Hamilton)
Dynamics across the SF-MI transition

Near-equilibrium dynamics

\[
\begin{bmatrix}
J_m \\
J_s
\end{bmatrix} = -\begin{bmatrix}
A_{00} & A_{01} \\
A_{10} & A_{11}
\end{bmatrix} \begin{bmatrix}
\nabla \mu \\
\nabla T
\end{bmatrix}
\]
Extracting density-density correlations from in situ images of 2D quantum gases

Interaction strength $g$
Discrete Fourier Transform (FFT) → Density noise power spectrum

$|\text{FFT}(\cdots)|^2$

→ Static structure factor $S(k)$ after removing imaging systematics

$$S(k) = \frac{1}{n} \int \langle \delta n(0) \delta n(r) \rangle e^{-ik \cdot r} d^2r = \frac{\langle |\delta n(k)|^2 \rangle}{N}$$

$k$: spatial frequency
Static structure factor

Universal scale behaviors regarding scale invariance and dilute Bose gas universality in 2D
Critical fluctuations and correlations

Classical superfluid phase transition at finite temperatures

Quantum phase transitions near absolute zero temperature
Phase transition and quantum phase transition

Phase diagram and AdS4-CFT3 duality: Sachdev, Nature physics (2007)
Reduced density fluctuations in a Mott Insulator

Mean Density

Density fluctuations (10nK)

Density fluctuation (atom/site)

Fluctuation-dissipation theorem

\[ \sum_j \langle \Delta n_i \Delta n_j \rangle = k_B T \kappa_i \]

compressibility
Quantum Simulation

Atoms and molecules

Other applications:
quantum chemistry, quantum information, precision metrology

Nuclear physics

Efimov physics
Quark-gluon plasma
...

Condensed matter physics

BEC-BCS crossover
Quantum magnetism
...

Illustration of scaling symmetry
Scale Invariance - Density

\[ \tilde{n} = n \lambda_{dB}^2 = F \left( \frac{\mu}{kT} \right) \]

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\[ n(\mu, T) = -\lambda_{dB}^{-2} \ln \left[ 1 - \exp \left( \frac{\mu}{k_B T} - gn \lambda_{dB}^2 / \pi \right) \right] \]

\[ \tilde{n} = 2\pi \tilde{\mu} / g + \ln \left( \frac{2\tilde{n}g}{\pi} - 2\tilde{\mu} \right) \]

Prokof'ev and Svistunov, PRA 66, 043608 (2002)