Superconducting quarks:
Condensed Matter in the Heavens

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Outline

I Quarks at high density
   Confined, quark-gluon plasma, color superconducting

II Color superconductivity
   Color-flavor locking (CFL), and beyond

III Compact stars
   Transport properties, equation of state

IV Looking to the future

A. Schmitt, arXiv:1001.3294
I. Quarks at high density

Quarks: Building blocks of matter

atom: nucleus & electrons

Quarks have color and flavor ("up" or "down")
proton: uud, uud, uud
neutron: udd, udd, udd

neutron/proton:
3 quarks bound by color force, Quantum Chromo Dynamics (QCD)
Phase Transitions

When you heat up or compress matter, the atoms reconfigure themselves. You get phase transitions between solid, liquid, and gas.

At super-high temperatures or densities, when the nuclei are constantly bashed around or remorselessly crushed together, do quarks reconfigure themselves?

\[ T \sim 150 \text{ MeV} \sim 10^{12} \text{ K} \]
\[ \rho \sim 300 \text{ MeV/fm}^3 \sim 10^{17} \text{ kg/m}^3 \]

At such a density, a oil super-tanker is 1mm\(^3\) in size.

Where might this occur?
- supernovas, neutron stars;
- Brookhaven (AGS, RHIC); CERN (SPS, LHC)
Interactions between Quarks

Dominant interaction between quarks is the strong interaction, described by $SU(3)$ “color” non-Abelian gauge theory (QCD).

Properties of QCD

▶ Short distances, $r \ll 1$ fm, asymptotically free: gauge coupling $g \ll 1$, single gluon exchange dominates, the theory is analytically tractable.

▶ Long distances $r > 1$ fm, QCD confines: color electric fields form flux tubes, only color-neutral states, baryons and mesons, exist.

▶ At low temperature ($T \lesssim 170$ MeV), Chiral (left-right) symmetry is broken: color force can’t turn a LH quark to RH, but our vacuum is full of $\bar{q}_L q_R$ pairs.
Conjectured QCD phase diagram

heavy ion collisions: chiral critical point and first-order line
compact stars: color superconducting quark matter core
At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.

\[ \mu = E_F \]

\[ F = E - \mu N \]

\[ \frac{dF}{dN} = 0 \]

But quarks have attractive QCD interactions.

Any attractive quark-quark interaction causes pairing instability of the Fermi surface: BCS mechanism of superconductivity.

What is a condensate of Cooper pairs?

\[ |\phi_0\rangle = \prod_p \left( \cos(\theta_{Ap}) + \sin(\theta_{Ap}) a^\dagger(p) a^\dagger(-p) \right) \left( \cos(\theta_{Bp}) + \sin(\theta_{Bp}) b^\dagger(p) b^\dagger(-p) \right) \times |\text{Fermi sea}\rangle \]

\[ |\phi_0\rangle, \text{ not } |\text{Fermi sea}\rangle, \text{ is the ground state.} \]
Physical consequences of Cooper pairing

Changes low energy excitations, affecting \textit{transport properties}.

- spontaneous breaking of global symmetries: \textit{Goldstone bosons}, massless degrees of freedom that dominate low energy behavior.
- spontaneous breaking of local (gauged) symmetries: massive gauge bosons, exclusion of magnetic fields (\textit{Meissner effect}).
- create a \textit{gap in fermion spectrum}.

Adding a fermion of momentum $\vec{p}$ near the Fermi surface disrupts the condensate in that mode:

$$ a_p^\dagger (\cos \theta + \sin \theta a_p^\dagger a_{-p}^\dagger) = \cos \theta a_p^\dagger $$

This kills that mode’s contribution to the binding energy of the condensate, i.e “breaks a Cooper pair”, costing energy $\Delta$. 
Handling QCD at high density

**Lattice**: “Sign problem”—negative probabilities

**SUSY**: Statistics crucial to quark Fermi surface

**large N**: Large corrections

**pert**: Applicable far beyond nuclear density. Neglects confinement and instantons.

**NJL**: Model, applicable at low density. Follows from instanton liquid model.

**EFT**: Effective field theory for lightest degrees of freedom. “Parameterization of our ignorance”: assume a phase, guess coefficients of interaction terms (or match to pert theory), obtain phenomenology.
High-density QCD calculations

Guess a color-flavor-spin pairing pattern $P$; to obtain gap $\Delta_P$, minimize free energy $\Omega$ with respect to $\Delta_P$ and impose color and electric neutrality

$$\frac{\partial \Omega}{\partial \Delta_P} = 0 \quad \frac{\partial \Omega}{\partial \mu_i} = 0$$

The pattern with the lowest $\Omega(\Delta_P)$ wins!

1. **Weak-coupling methods.** First-principles calculations direct from QCD Lagrangian, valid in the asymptotic regime, currently $\mu \gtrsim 10^6$ MeV.

2. **Nambu–Jona-Lasinio models,** ie quarks with four-fermion coupling based on instanton vertex, single gluon exchange, etc. This is a semi-quantitative guide to physics in the compact star regime $\mu \sim 400$ MeV, not a systematic approximation to QCD.

NJL gives $\Delta \sim 10–100$ MeV at $\mu \sim 400$ MeV.
Gap equation in a simple NJL model

Minimize free energy wrt $\Delta$:

$$1 = \frac{8K}{\pi^2} \int_0^\Lambda p^2 dp \left\{ \frac{1}{\sqrt{\Delta^2 + (p - \mu)^2}} \right\}$$

Note BCS divergence as $\Delta \to 0$: there is always a solution, for any interaction strength $K$ and chemical potential $\mu$. Roughly,

$$1 \sim K\mu^2 \ln \left( \frac{\Lambda}{\Delta} \right)$$

$$\Rightarrow \Delta \sim \Lambda \exp \left( -\frac{1}{K\mu^2} \right)$$

Superconducting gap is non-perturbative.
Color superconducting phases

Quark Cooper pair: $\langle q^\alpha_{ia} q^\beta_{jb} \rangle$

| color $\alpha, \beta$ = r, g, b |
| flavor $i, j$ = u, d, s |
| spin $a, b$ = $\uparrow, \downarrow$ |

There is a $9 \times 9$ matrix of possible BCS pairing patterns!

The attractive channel is:  

- color antisymmetric 
- spin antisymmetric 

$\Rightarrow$ flavor antisymmetric

So pairing between different flavors is favored.

Let’s start with the most symmetric case, where all three flavors are massless.
Color supercond. in 3 flavor quark matter: Color-flavor locking (CFL)

Equal number of colors and flavors gives a special pairing pattern (Alford, Rajagopal, Wilczek, hep-ph/9804403)

\[ \langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta n} \epsilon_{ijn} \]

color \( \alpha, \beta \), flavor \( i, j \)

This is invariant under equal and opposite rotations of color and (vector) flavor

\[
SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{C+L+R} \times \mathbb{Z}_2
\]

\[ \supset U(1)_Q \]

\[ \supset U(1)_{\bar{Q}} \]

- Breaks chiral symmetry, but not by a \( \langle \bar{q}q \rangle \) condensate.
- There need be no phase transition between the low and high density phases: ("quark-hadron continuity")
- Unbroken "rotated" electromagnetism, \( \bar{Q} \), photon-gluon mixture.
Color-flavor-locked ("CFL") quark pairing

\[\begin{array}{c|cccccccc}
\tilde{Q} & 0 & 0 & 0 & -1 & +1 & -1 & +1 & 0 & 0 \\
\hline
u & d & s & d & u & s & u & s & d \\
\hline
u & \Delta & \Delta \\
d & \Delta & \Delta \\
s & \Delta & \Delta \\
d & \Delta & \Delta & -\Delta \\
u & \Delta \\
s & \Delta \\
u & \Delta \\
s & \Delta \\
d & \Delta & \Delta & -\Delta \\
\end{array}\]
Where in the universe is color-superconducting quark matter most likely to exist? In compact stars.

A quick history of a compact star.

A star of mass $M \gtrsim 10 M_{\odot}$ burns Hydrogen by fusion, ending up with an Iron core. Core grows to Chandrasekhar mass, collapses $\Rightarrow$ supernova. Remnant is a compact star:

<table>
<thead>
<tr>
<th>mass</th>
<th>radius</th>
<th>density</th>
<th>initial temp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sim 1.4 M_{\odot}$</td>
<td>$\mathcal{O}(10 \text{ km})$</td>
<td>$\gtrsim \rho_{\text{nuclear}}$</td>
<td>$\sim 30 \text{ MeV}$</td>
</tr>
</tbody>
</table>

The star cools by neutrino emission for the first million years.
In the real world there are three complications to the simple account given so far.

1. **Strange quark mass** is not infinite nor zero, but intermediate. It depends on density, and ranges between about 500 MeV in the vacuum and about 100 MeV at high density.

2. **Neutrality requirement.** Bulk quark matter must be neutral with respect to all gauge charges: color and electromagnetism.

3. **Weak interaction equilibration.** In a compact star there is time for weak interactions to proceed: neutrinos escape and flavor is not conserved.

So quark matter in a compact star might be CFL, or something else: gapless CFL; kaon-condensed CFL, 2SC, 1SC, crystalline, . . .
Cooper pairing vs. the strange quark mass

\[ \langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta N} \epsilon_{ijN} \]

**CFL:** Color-flavor-locked phase, favored at the highest densities.

**2SC:** Two-flavor pairing phase. May occur at intermediate densities.

\[ \langle q_i^\alpha q_j^\beta \rangle \sim \epsilon^{\alpha\beta 3} \epsilon_{ij3} \sim (rg - gr)(ud - du) \]

**or:** Exotic non-BCS pairing: LOFF (crystalline phase), \( p \)-wave meson condensates, single-flavor pairing (color-spin locking, \( \sim \text{liq} \ ^3\text{He-B}) \).
Phases of quark matter, again

But there are also non-uniform phases, such as the crystalline ("LOFF"/"FFLO") phase. (Alford, Bowers, Rajagopal, hep-ph/0008208)
Crystalline (LOFF) superconductivity

When the Fermi momenta are such that one flavor of quark is just barely excluded from pairing with another, it may be favorable to make pairs with a net momentum, so each flavor can be close to its Fermi surface.

\[ q \quad p \]

Every quark pair in the condensate has the same nonzero total momentum \( 2q \) (single plane wave LOFF).
Free energy comparison of phases

Assuming $\Delta_{CFL} = 25$ MeV.

Curves for CubeX and 2Cube45z use G-L approx far from its area of validity: favored phase at $M_s^2 \sim 4\mu\Delta$ remains uncertain.
Signatures of color superconductivity in compact stars

Pairing energy \{ affects Equation of state. Hard to detect. \\
(Alford, Braby, Paris, Reddy, nucl-th/0411016)

Gaps in quark spectra and Goldstone bosons \{ affect Transport properties:
emissivity, heat capacity, viscosity (shear, bulk), conductivity (electrical, thermal).

1. Cooling by neutrino emission, neutrino pulse at birth
2. Glitches and crystalline (“LOFF”) pairing
3. Gravitational waves: r-mode instability, shear and bulk viscosity
An $r$-mode is a quadrupole flow that emits gravitational radiation. It becomes unstable (i.e. arises spontaneously) when a star spins fast enough, and if the shear and bulk viscosity are low enough.

Andersson gr-qc/9706075
Friedman and Morsink gr-qc/9706073
Regions above curves are “forbidden” because viscosity is too low to hold back the $r$-modes.

Data for accreting pulsars in binary systems (LMXBs) vs instability curves for nuclear and hybrid stars.

(Schwenzer arXiv:1212.5242)
IV. Looking to the future

- Neutron-star phenomenology of color superconducting quark matter:
  - mass-radius and equation of state
  - analysis of r-mode spindown vs data
  - elimination/evaluation of other r-mode damping mechanisms
  - neutrino emissivity and cooling
  - structure: nuclear-quark interface (gravitational waves?)
  - color supercond. crystalline phase (glitches) (gravitational waves?)
  - CFL: vortices but no flux tubes; stability of vortices...

- More general questions:
  - instability of gapless phases; better treatment of LOFF
  - better weak-coupling calculations
  - role of large magnetic fields
  - solve the sign problem and do lattice QCD at high density.