Exotic Zero Energy Modes at Topological Defects in Crystalline Superconductors

Jeffrey C.Y. Teo

University of Illinois at Urbana-Champaign

Collaborators:
Taylor Hughes
Wladimir A. Benalcazar
Kam Tuen Law
Noah Fan Qi Yuan


To appear soon
Outline

• **Topology and exotic particles**
  - Quantum Hall effect
  - Topological insulators
  - Bulk topology => exotic boundary excitations

• **Topological defects**
  - Vortices in unconventional 2D superconductors
  - Ising anyons (Majorana zero modes)

• **Majorana zero modes in crystalline superconductors**
  - Broken translation and rotation lattice symmetries
  - Crystalline defects: dislocation, disclinations
  - Majorana zero modes at
    - Strontium Ruthenate
    - Graphene and silicene
    - MoS2
TOPOLOGY AND EXOTIC EXCITATIONS
Quantum Hall Effect
Quantum Hall Effect

\[ V_x = R_{xy} I_y \]

\[ R_{xy} = \frac{B}{\rho} \]

\[ \frac{h}{e^2} = 25812.807557(18) \Omega \]

von Klitzing, et al. 80’
Quantum Hall Effect

Why so accurate?

\[ V_x = R_{xy} I_y \]
\[ R_{xy} = \frac{1}{n} \frac{h}{e^2} \]

\[ \frac{h}{e^2} = 25812.807557(18) \Omega \]

von Klitzing, et.al. 80'
Topology of electronic structure

$g = 0$

Gauss-Bonnet theorem

$$2 - 2g = \frac{1}{2\pi} \int_{\Sigma} R dA$$

Kubo formula

$$\frac{1}{R_{xy}} = \frac{e^2}{\hbar} \int \frac{dk_x dk_y}{2\pi} 2\text{Im} \langle \partial_{k_x} u(k) | \partial_{k_y} u(k) \rangle$$

Thouless, *et.al.* 82

$g = 1$

Chern number

Gaussian curvature

Berry curvature
Topologically protected excitations

Chiral edge mode of a Landau level

**Bulk-boundary correspondence**
- Single direction electronic channel
- No backscattering
- Dissipationless transport

**NOT realizable in any 1D systems**
Topologically protected excitations

Helical edge mode of a quantum spin Hall insulator

Kane, Mele 05 $\mathbb{Z}_2$ bulk topology

Bulk-boundary correspondence

Bulk – spin-orbit coupled time reversal symmetric 2D insulator
Boundary – gapless helical edge mode
Backscattering prohibited by time reversal symmetry

NOT realizable in any time reversal symmetric 1D systems

Konig, et.al. 07
Topologically protected excitations

Surface Dirac cone of a 3D topological insulator

$\mathbb{Z}_2$ bulk topology

Bulk-boundary correspondence

Bulk – spin-orbit coupled time reversal symmetric 3D insulator
Boundary – gapless surface Dirac cone
Weak anti-localization

Fu, Kane, Mele, 07
Moore, Balents, 07
Roy, 07
Hsieh, et.al. 08, 09

NOT realizable in any time reversal symmetric 2D systems
Topology of bulk electronic band structure

Exotic boundary excitations
TOPOLOGICAL DEFECTS
Boundary = Domain wall

topological insulator

vacuum (trivial insulator)

\[ m < 0 \]

\[ m > 0 \]

\[ H = \nu (k_x \sigma_x + k_y \sigma_y) \tau_z + m \tau_x \]
Topological point defects

Winding number = +1

Order parameter

Winding number = -1
Topological point defects

Vortices in conventional superconductors

BCS ground state \[ \left| GS \right\rangle \sim a_0 |0\rangle + a_2 \hat{c}_a^\dagger \hat{c}_b^\dagger |0\rangle + a_4 \hat{c}_a^\dagger \hat{c}_b^\dagger \hat{c}_c^\dagger \hat{c}_d^\dagger |0\rangle + \ldots \]

Broken charge conservation \( U(1) \)-symmetry

\[ \left\langle \hat{c}_\uparrow^\dagger (\mathbf{k}) \hat{c}_\downarrow^\dagger (-\mathbf{k}) \right\rangle \sim \Delta = |\Delta| e^{i\phi} \]

Flux vortex

Superconductor pairing phase

Feynman, 55
Abrikosov, 57
Topological point defects

Vortices in **unconventional** superconductors

Topological defects

- Winding of classical order parameter in real space

Topological superconductors

- Winding of quantum states in momentum space

Something really amazing !!!
Topological point defects

Vortices in chiral $p+ip$ superconductors

Pairing order: time reversal breaking
odd parity, spin-triplet

$$\langle c^{\dagger}(k)c^{\dagger}(-k) \rangle \sim \Delta(k_x + ik_y) = |\Delta|e^{i\phi}(k_x + ik_y)$$

**Bulk-boundary correspondence**

Quantum Hall effect

Chiral $p$-superconductor

Chiral charged complex (Dirac) fermion

Chiral neutral real (Majorana) fermion
Topological point defects

Vortices in chiral $p+ip$ superconductors

Pairing order: time reversal breaking
odd parity, spin-triplet

$$\langle c^\dagger(k)c^\dagger(-k) \rangle \sim \Delta(k_x + ik_y) = |\Delta|e^{i\phi}(k_x + ik_y)$$

Bulk-boundary correspondence

Chiral $p$-superconductor

Chiral neutral real (Majorana) fermion
Topological point defects

Vortices in chiral \( p+ip \) superconductors

Discrete chiral Majorana modes

Chiral \( p \)-superconductor with a flux vortex

Periodic boundary condition:

\[
360 \text{-deg rotation of fermion } \Rightarrow \quad -1
\]

\[
\pi \text{ Berry phase from flux vortex}
\]

\[
e^{2\pi i k \theta R} = +1
\]

Ising anyons

Zero energy Majorana mode

Flux vortex

Zero energy Majorana (real) fermion operators

\[ \hat{c} = \frac{\gamma_1 + i\gamma_2}{2} \]

\[ |0\rangle \]

\[ |1\rangle = \hat{c}^\dagger |0\rangle \]

Non-local Quantum Information Storage

Qubits

$|0\rangle$ + $|1\rangle$

$|0\rangle$ + $|1\rangle$

$|0\rangle$ + $|1\rangle$

$\dim = 2^{n/2} - 1$

$\sigma$ = $\sqrt{2}$

Robust against local perturbation, accidental measurement, quantum decoherence

Qubits
Non-Abelian braiding operations

Ground states: \( |00\rangle, \quad |11\rangle \)

\[
\hat{T}_1 \propto \exp \left( i \frac{\pi}{4} \sigma_z \right) \quad \hat{T}_2 \propto \exp \left( i \frac{\pi}{4} \sigma_x \right)
\]

Entangled state: \( \hat{T}_2 |00\rangle = \frac{|00\rangle + i|11\rangle}{\sqrt{2}} \)

Topological Quantum Computing
Majoranas at proximity interfaces

Fu and Kane, 09

Sau, Lutchyn, Tewari, Das Sarma, 10

• **What is the problem?**
  - Moore-Read \( v=5/2 \) FQH state requires very low temperature, high mobility and high magnetic field
  - Is \( \text{Sr}_2\text{RuO}_4 \) chiral? (Raghu, Kapitulnik, Kivelson, 2010)
  - TI-SC-FM heterostructures require smooth interface
  - Majoranas in non-chiral homogeneous materials in reasonable temperature without external magnetic field?
  - Non-abelian objects from abelian systems?

• **Defect related topics**
  - Disclinations in topological crystalline superconductors
  - Fractional vortices in liquid crystal SC
    - Gopalakrishnan, JT, Hughes; PRL 111, 025304 (2013)
  - Twists in bilayer FQH
  - Ising quasiparticles in 3D
    - JT, Kane; PRL 104, 046401 (2010)
MAJORANA ZERO MODES IN TOPOLOGICAL CRYSTALLINE SUPERCONDUCTORS

Topological point defects

Vortices in chiral $p+ip$ superconductors

Broken charge conservation $U(1)$-symmetry

Bulk topology + Winding of order parameter = Majorana zero mode

Are there any broken symmetries?
Topological crystalline defects

Crystalline order
  - Broken translation and rotation symmetry

Dislocation (torsion singularity)

Burgers’ vector
Topological crystalline defects

**Crystalline order**
- Broken translation and rotation symmetry

**Disclination** *(curvature singularity)*

Frank angle
Topological crystalline defects

Lattice symmetry protect bulk topology + Topological crystalline defects

Majorana zero mode
Strontium Ruthenate

- Layered perovskite structure, quasi-2D, fourfold rotation symmetry
- Unconventional SC (Kidwingira, et.al., 2004)
  - Spin-triplet $p$-wave, breaks time reversal, odd parity
- Chiral $p_x + ip_y$?
  - Weak edge current (Stone, Roy, 2004; Kirtley, et.al., 2007)
Strontium Ruthenate

- Layered perovskite structure, quasi-2D, fourfold rotation symmetry
- Unconventional SC \((\text{Kidwingira, et.al., 2004})\)
  - Spin-triplet \(p\)-wave, breaks time reversal and parity
- Chiral \(p_x + ip_y\)?
  - Weak edge current \((\text{Stone, Roy, 2004; Kirtley, et.al., 2007})\)
  - \(d_{xy} \rightarrow \) chiral \(p+ip\) pairing
  - Quasi-1D nature
    - from \(d_{xz}, d_{yz}\) bands? \((\text{Raghu, et.al., 2010})\)
  - STM on superconducting DOS \((\text{Firmo, et.al., 2013})\)
  - Incommensurate antiferromagnetic fluctuations \((\text{K. Iida, et.al., 2011})\)
Strontium Ruthenate

- Quasi-1D model on $d_{xz}$, $d_{yz}$ bands \textit{(Raghu, et.al., 2010)}

\begin{align*}
    d_{yz} \text{ band} & \rightarrow 1D \text{ metal} \\
    1D \text{ spin-triplet superconductor}
\end{align*}

- Majorana tight binding model

\begin{align*}
    \text{BdG Hamiltonian of SC } d_{xz}, d_{yz} \text{ bands} \\
    H &= [(t \cos k_x - \mu)\tau_z + \Delta \sin k_x\tau_y] \\
    &\oplus [(t \cos k_y - \mu)\tau_z + \Delta \sin k_y\tau_y]
\end{align*}

\text{Real space Majorana lattice representing SC } d_{xz}, d_{yz} \text{ bands}

Strontium Ruthenate

- Quasi-1D model on $d_{xz}$, $d_{yz}$ bands (Raghu, et.al., 2010)
- $d_{yz}$ band $\rightarrow$ 1D metal
- 1D spin-triplet superconductor
- Majorana tight binding model

Real space Majorana lattice representing SC $d_{xz}$, $d_{yz}$ bands

Majorana lattice of two layers

Strontium Ruthenate

- Is Sr$_2$RuO$_4$ topologically trivial?
  
  **Hidden translation symmetry protected topology**
  
  Weak $\mathbb{Z}_2$ topology $ G_\nu = (1, 1) $

- Majorana tight binding model
  
  Real space Majorana lattice representing SC $d_{xz}, \ d_{yz}$ bands
  
Strontium Ruthenate

- Is $\text{Sr}_2\text{RuO}_4$ topologically trivial?

**Hidden rotation symmetry protected topology**

<table>
<thead>
<tr>
<th></th>
<th>$C_4$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma$</td>
<td>$\pm e^{-i\pi/4}$</td>
<td>$-i$</td>
</tr>
<tr>
<td>$M$</td>
<td>$\pm e^{+i\pi/4}$</td>
<td>$+i$</td>
</tr>
<tr>
<td>$X$</td>
<td>$\pm i$</td>
<td></td>
</tr>
</tbody>
</table>

- Majorana tight binding model

**How to get Majorana bound states?**

Real space Majorana lattice representing SC $d_{xz}$, $d_{yz}$ bands

Majorana Bound States at Disclinations

- Kitaev’s 1D superconducting chain

\[ H = (t \cos k_x - \mu)\tau_z + \Delta \sin k_x \tau_y \]

- Trivial limit \( t < \mu \)

- Topological limit \( t > \mu \)

\[ t = \Delta = 0 \]
\[ c_r = (\gamma_r^a + i\gamma_r^b)/2 \]
\[ H = i\mu \sum_r \gamma_r^a \gamma_r^b \]
\[ \mu = 0, \quad t = \Delta \]
\[ H = i\Delta \sum_r \gamma_r^a \gamma_{r+1}^b \]
Majorana Bound States at Disclinations

- The tight binding models of Sr$_2$RuO$_4$
  2D version of Kitaev’s chain

Corner MBS in Strontium Ruthenate

#MBS at disclination = #MBS at corners + #MBS along edges (mod 2)

Due to weak topology

Non-chiral Majorana mode

Gapped out by on site hybridization

Sr$_2$RuO$_4$

2 × Sr$_2$RuO$_4$

Gapped out by perturbation

Non-chirality is actually good!!

Quasi-1D $Sr_2RuO_4$

Gapped out by perturbation

Chiral $p+ip$

$Sr_2RuO_4$

Robust against perturbations

Doped Graphene / Silicene

- Filling $\nu \sim 5/8$ (or $3/8$)
- Fermi energy at $M$ saddle points, van Hove singularity in DOS

$\nu = 5/8$
$\nu = 1/2$
$\nu = 3/8$

- Chiral $d_{\uparrow \downarrow xy} + id_{\downarrow \uparrow x^2-y^2} \; - y^2$ spin-singlet superconductivity

$\Delta = (\Delta_1, \Delta_2, \Delta_3)$

$\Delta_1(c_{0\uparrow}^\dagger c_{1\downarrow}^\dagger - c_{0\downarrow}^\dagger c_{1\uparrow}^\dagger) + h.c.$

$\Delta_2(c_{0\uparrow}^\dagger c_{2\downarrow}^\dagger - c_{0\downarrow}^\dagger c_{2\uparrow}^\dagger) + h.c.$

$\Delta_3(c_{0\uparrow}^\dagger c_{3\downarrow}^\dagger - c_{0\downarrow}^\dagger c_{3\uparrow}^\dagger) + h.c.$

$\Delta_s \propto (1, 1, 1)$

$\Delta_{d_{xy}} \propto (0, 1, -1)$

$\Delta_{d_{x^2-y^2}} \propto (2, -1, -1)$

$\Delta_{d_{xy}} + i\Delta_{d_{x^2-y^2}}$

Nandkishore, et.al., 2012
Black-Schaffer, et.al., 2007
Doped Graphene / Silicene

- Broken threefold rotation and time reversal symmetry

\[ \Delta d_{xy} + i \Delta d_{x^2 - y^2} \]

\[
\begin{array}{c|c}
\text{ch} & 2 \\
\hline
C_2 & \pm i \\
\hline
\Gamma & \pm i \\
M & \pm i \\
\end{array}
\]

Zigzag edge

How to get protected Majorana bound states?

Even number of Majorana zero modes

\[ \Rightarrow \text{Not protected} \]

Majoranas at Conical Defects in Graphene

- MBS at 180-deg disclination

\[ \Theta = \frac{1}{2\pi} \mathbf{T} \cdot \mathbf{G}_\nu + \frac{\Omega}{2\pi} (ch + [M]) \]

- \( 180 = 60 + 60 + 60 \) \( \Rightarrow \) MBS at 60-deg disclination

Majoranas at Dislocations in MoS$_2$

• Drawback of doped graphene / silicene
  - No observed superconductivity
  - Need huge dopant amount or gate voltage $\rightarrow$ alter band structure

• Superconducting MoS$_2$

Terrones, et.al. 2013
Majoranas at Dislocations in MoS$_2$

- **Drawback of doped graphene / Silicene**
  - No observed superconductivity
  - Need huge dopant amount or gate voltage → alter band structure

- **Superconducting MoS$_2$**
  - Possible $d+id$ superconducting phase
  - Breaks 3-fold symmetry
  - Weak topology protected by translation symmetry
  - Majorana zero mode at dislocation


Noah F.Q. Yuan, JT, K.T. Law, to appear soon
Majorana chain along grain boundaries

Sau, Lutchyn, Tewari, Das Sarma, 10

Kitaev, 01
Majorana chain along grain boundaries

• Advantages
  - No heterostructures or proximity induced superconductivity
  - Clean system
  - Possible STM experiments
Conclusion

Topological defects + Topological superconductors

Something really amazing !!!

• Majorana bound states in Topological crystalline SC
  ▪ Dislocation and disclination
  ▪ Crystalline symmetry protected topology
  ▪ Strontium Ruthenate
  ▪ Doped graphene and silicene
  ▪ MoS$_2$

• What’s next?
  ▪ Revisit materials with overlooked topology
Fractional Vortices

Say $p$-wave pairing order
How about a non-tight binding limit?

- **Chiral p+ip model**

\[
H_\alpha = \Delta (\sin k_x \tau_x + \sin k_y \tau_y) + u_1 (\cos k_x + \cos k_y) \tau_z + 2u_2 \cos k_x \cos k_y \tau_z
\]

\[|u_1| > |u_2| > 0\]

Exponentially localized zero modes

\[
H_\alpha u_1 < 0
\]

Probability amplitude for wave functions of Majorana zero modes

No zero mode for

\[
H_\alpha u_1 > 0
\]

*JT, Hughes; Phys. Rev. Lett. 111, 047006 (2013)*
How about a non-tight binding limit?

- **Chiral p+ip model**

\[ H_a = \Delta (\sin k_x \tau_x + \sin k_y \tau_y) + u_1 (\cos k_x + \cos k_y) \tau_z + 2u_2 \cos k_x \cos k_y \tau_z \]

\[ |u_1| > |u_2| > 0 \]

**No MBS at disclination**

\[ H_a^{u_1 > 0} \]

\[ ch = 1 \quad G_\nu = (0, 0) \]

**Single MBS at disclination**

\[ H_a^{u_1 < 0} \]

\[ ch = 1 \quad G_\nu = (1, 1) \]

*JT , Hughes; Phys. Rev. Lett. 111, 047006 (2013)*
How about a non-tight binding limit?

- **Chiral p+ip model**

  \[ H_a = \Delta (\sin k_x \tau_x + \sin k_y \tau_y) + u_1 (\cos k_x + \cos k_y) \tau_z + 2u_2 \cos k_x \cos k_y \tau_z \]

  \[ |u_1| > |u_2| > 0 \]

  

  **No MBS at disclination**

  \[ H_{a_1} > 0 \]

  \[ ch = 1 \quad G_\nu = (0, 0) \]

  \[
  \begin{array}{ccc}
  & C_4 & C_2 \\
  \Gamma & e^{-i\pi/4} & -i \\
  M & e^{+i\pi/4} & +i \\
  X & +i &
  \end{array}
  \]

  

  **Single MBS at disclination**

  \[ H_{a_1} < 0 \]

  \[ ch = 1 \quad G_\nu = (1, 1) \]

  \[
  \begin{array}{ccc}
  & C_4 & C_2 \\
  \Gamma & e^{+i\pi/4} & +i \\
  M & e^{-i\pi/4} & -i \\
  X & +i &
  \end{array}
  \]

  

  *JT, Hughes; Phys. Rev. Lett. 111, 047006 (2013)*
How do I know there is a MBS in general?

Product between some defect quantities - bulk topological invariants

Existence of helical mode

\[ \nu = \frac{1}{2\pi} B \cdot G_c \]

Weak \( \mathbb{Z}_2 \) indices
How do I know there is a MBS in general?

Product between some
- defect quantities
- bulk topological invariants

**Number of Majoranas at disclination**

\[
\Theta^{(4)} = \frac{1}{2\pi} T \cdot G_{\nu} + \frac{\Omega}{2\pi} \left( ch + 2[X] + [M_1] + 3[M_2] \right)
\]

- Even/odd translations around defect
- Frank angle
- Weak invariant
  \[ G_{\nu} = [X] + [M_1] + [M_2] \]
- Rotation eigenvalues

**Chern invariant**

\[
ch = \frac{i}{2\pi} \int_{BZ} \text{Tr}(dA)
\]

*JT, Hughes; Phys. Rev. Lett. 111, 047006 (2013)*
Topological Classification of TCS

- Stable topological invariants (all T-breaking)
  - Chern invariant
  - Rotation invariants

4-fold momenta rotation eigenvalues at $\Pi = \Gamma, M$

- $\Pi_5 = e^{-i\pi/4}$, $\Pi_6 = e^{i\pi/4}$
- $\Pi_7 = e^{i3\pi/4}$, $\Pi_8 = e^{-i3\pi/4}$

2-fold momenta rotation eigenvalues

- $X_3 = i$, $X_4 = -i$

Rotation spectra discrepancies in *valence* bands

$$[X] = \#X_4 - \#\Gamma_5 - \#\Gamma_7$$

$$[M_1] = \#M_6 - \#\Gamma_6$$

$$[M_2] = \#M_7 - \#\Gamma_7$$

*JT, Hughes; Phys. Rev. Lett. 111, 047006 (2013)*
Corner MBS in Strontium Ruthenate

#MBS at disclination = #MBS at corners + #MBS along edges \ (mod 2)

Due to weak topology

Non-chiral Majorana mode

Gapped out by perturbation

Gapped out by on site hybridization

\( 2 \times \text{Sr}_2\text{RuO}_4 \)

Non-chirality is actually good!!

Quasi-1D Sr$_2$RuO$_4$

Gapped out by perturbation

Chiral $p+ip$ Sr$_2$RuO$_4$

Robust against perturbations

Melting Phases of Liquid Crystal Superfluid

Non-abelian Twist Defects

- Topological order intertwined with classical order

\[
\begin{align*}
SL(2; \mathbb{Z}) &= \left\langle S, T \right| C^2 = 1, SCS^{-1} = TCT^{-1} = C \right\rangle \\
\Gamma_0(2) &= \left\langle S = t_y, T = t_x^2 \left| (ST^{-1})^2 = C, C^2 = 1, SCS^{-1} = TCT^{-1} = C \right\rangle
\end{align*}
\]

\text{JT, Roy, Chen; arXiv:1308.5984 (2013)}
\text{JT, Roy, Chen; arXiv:1306.1538 (2013)}
Conclusion

• **Majorana bound states in Topological crystalline SC**
  - Lattice symmetry protected topological superconductor
  - Topological Classification of TCS
  - Revisit classification of dislocation-disclination composite
  - Lattice defect MBS - Homogeneous environment
    - No external field
    - No proximity interfaces
  - \( \text{Sr}_2\text{RuO}_4 \) corners
  - 5-7 defects in doped graphene

• **What’s next?**
  - Revisit materials with overlooked topology
  - Melting phases with intertwined topological – classical order
Classification of Disclinations

\[ r(e_x)^3 r(e_x)^3 r(e_x)^3 = (-3e_x)r^{-1} \]

Classification:

\[ C_4 \times \mathbb{Z}_2 \]

Evenness / oddness of number of translations

Dislocation = Disclination dipole

**Overall odd dislocation**

**Overall even dislocation**


Majorana Bound states at Dislocations

\[ G_\nu = \nu_1 b_1 + \nu_2 b_2 \]
\[ \nu_i \equiv \frac{i}{\pi} \oint_{C_i} \text{Tr}(A) \mod 2 \]

number of Majorana zero mode

\[ \equiv \frac{1}{2\pi} \mathbf{B} \cdot \mathbf{G}_\nu \mod 2 \]

JT, Kane, 2010
Abelian bosonic bilayer FQH states

- Kitaev toric code

\[ K = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \]

1 = (0, 0) \quad e = (1, 0)
m = (0, 1) \quad \psi = e \times m = (1, 1)

\[ \theta_\psi = \psi = -1 \]

- Second hierarchy state

\[ K = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \quad \text{SU}(3)_1 \]

1 = \psi^3 = (0, 0) \quad \psi = (1, 0)
\psi^2 = (2, 0) \equiv (0, -1)

\[ \theta_\psi = \psi = e^{2\pi i/3} \]

F.D.M. Haldane, PRL 51, 605 (1983)