Inflation, Gravity Waves, and Dark Matter

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Feb 2015
University of Virginia
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Units

- $\hbar = c = k_B = 1$
- $M_P = 1.2 \times 10^{19}$ GeV ($m_P = 2.4 \times 10^{18}$ GeV)
- $l_P = 1.6 \times 10^{-33}$ cm, $t_P = 5.4 \times 10^{-44}$ sec
- $G = \text{Newton’s constant} = M_P^{-2}$
- $\text{GeV}^{-1} = 10^{-14}$ cm $= 10^{-24}$ sec
- $1 \text{ MeV} = 10^{10}$ K
Λ stands for Dark Energy with Einstein’s cosmological constant being the leading candidate

\( P_Λ = w_Λ \rho_Λ \), with \( w_Λ = -1 \)

\[ ρ_{Total} = ρ_Λ + ρ_{CDM} + ρ_M ≈ ρ_c \]

\[ ρ_Λ ≈ 10^{-120} m_p^4 \] ← Fine tuning?

CDM denotes ‘cold dark matter’ (particle have tiny velocities)

Where does ΛCDM come from?

Image courtesy of NASA / WMAP Science Team
## Four Fundamental Forces

<table>
<thead>
<tr>
<th>Force</th>
<th>Strength</th>
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<tr>
<td>Strong</td>
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<td>Electromagnetic</td>
<td>~$10^{-2}$</td>
</tr>
<tr>
<td>Weak</td>
<td>~$10^{-5}$</td>
</tr>
<tr>
<td>Gravity</td>
<td>~$10^{-38}$</td>
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</tbody>
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### Standard Model of High Energy Physics

### General Relativity
STANDARD MODEL OF HE PHYSICS

Provides excellent description of strong, weak and electromagnetic interactions.

Based on local gauge symmetry

\[ SU(3)_c \times SU(2)_L \times U(1)_Y \]

QCD - strong interactions involving ‘colored’ quarks & gluons

Electromagnetic and weak interactions mediated by \( W^\pm, Z^0 \) bosons and \( \gamma \), which have been found

Only ‘color neutral’ states exist in nature
Two Key features:
Color confinement;
Asymptotic freedom;
Higgs Boson

- Spin zero particle from spontaneous breaking of electroweak symmetry:
  \[ SU(2)_L \times U(1)_Y \langle \phi \rangle \rightarrow U(1)_{EM} \]

  \[ \langle \phi \rangle \sim 10^2 \text{ GeV} (t \sim 10^{-10} \text{ sec}) \]

  \[ m_h \approx 125 \text{ GeV} \ (\text{Huge discovery announced by ATLAS and CMS on July 4, 2012}) \]

- Compare: Superconductor (Cooper pairs \( \leftrightarrow \langle \phi \rangle \))
IS THERE “NEW” PHYSICS BEYOND THE STANDARD MODEL?

Most Likely Yes!

1) Neutrino Oscillations (solar & atmospheric):
These require non-zero (albeit ‘tiny’) neutrino masses
$\sim 10^{-1} – 10^{-2}$ eV.
In the SM, neutrinos have zero mass.

2) Dark Matter (non-baryonic)
SM has no plausible DM candidate
Oscillation Data

- $\sin^2(2\theta_{12}) = 0.846 \pm 0.021$
- $\Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5}\text{eV}^2$
- $\sin^2(2\theta_{23}) = 0.999^{+0.001}_{-0.018}$ (normal mass hierarchy)
  $\sin^2(2\theta_{23}) = 1.000^{+0.000}_{-0.017}$ (inverted mass hierarchy)
- $\Delta m_{32}^2 = (2.44 \pm 0.06) \times 10^{-3}\text{eV}^2$ (normal mass hierarchy)
  $\Delta m_{32}^2 = (2.52 \pm 0.07) \times 10^{-3}\text{eV}^2$ (inverted mass hierarchy)
- $\sin^2(2\theta_{13}) = (9.3 \pm 0.8) \times 10^{-2}$
- Tiny masses (compared to quarks and charged leptons)
- Mixing angles $\rightarrow$ large (compared to quark sector)
Dark Matter in The Universe

- Zwicky (~1930)
  Galaxies in the Coma cluster seem to be moving too rapidly to be held together by the gravitational attraction of the visible matter.

- Rotation curves of velocity versus radial distance for stars and gas provide indirect evidence for the existence of ‘missing’ non-luminous mass.

- \( \frac{\delta \rho}{\rho} \sim 10^{-5} \implies \text{structure formation (galaxies, clusters) hard without non-baryonic dark matter.} \)
Hot Big Bang Cosmology

- Comes from combining Standard Model (SM) of high energy physics with Einstein’ general relativity, and the assumption that on sufficiently large scales, the universe is isotropic and homogeneous.

Three remarkable predictions (Consequences):
1. Expanding Universe
2. Cosmic Microwave Background Radiation (CMB)
3. Nucleosynthesis
H_0 = 67.8\pm 0.9 \text{ (km/s)/Mpc} \\
\frac{1}{H_0} = 13.813\pm 0.038 \text{ Gyr} \\
(\text{Planck, arXiv:1502.01589}) \\
\text{In natural units, } H_0 \approx 10^{-33} \text{ eV!}
A homogeneous and isotropic universe is described by the Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right],$$

where $r$, $\phi$ and $\theta$ are ‘comoving’ polar coordinates, which remain fixed for objects that follow the general cosmological expansion.

$k$ is the scalar curvature of 3-space, with $k = 0, +1, -1$ describing a flat, closed and open universe respectively.
Geometry of the Universe

• Friedmann Equation

\[ \Omega \equiv \frac{\rho}{\rho_c} = 1 + \frac{k}{(aH)^2}, \]

where \( \rho_c = \frac{3H^2}{8\pi G} \) = critical density

• Closed (\( \Omega > 1 \) or \( k = 1 \))

• Open (\( \Omega < 1 \) or \( k = -1 \))

• Flat (\( \Omega = 1 \) or \( k = 0 \))

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Image courtesy of NASA:
http://map.gsfc.nasa.gov/universe/uni_shape.html
Solving Friedmann Equations:

- For flat universe

\[ H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 \propto \rho \]

\[ \rho_m \propto a^{-3} \Rightarrow a(t) \propto t^{2/3} \]

- Matter \( \rho_m = \frac{NM}{V} \)

- Radiation \( \rho_r = \frac{Nh_0 c}{V\dot{\lambda}} \)

\[ \rho_r \propto a^{-4} \Rightarrow a(t) \propto t^{1/2} \]

- Vacuum \( \rho_\Lambda = \text{const.} \)

\[ \rho_\Lambda \propto a^0 \Rightarrow a(t) \propto e^{Ht} \]
Cosmological Problems

• Flatness Problem

Present energy density of the universe is determined to be equal to its critical value corresponding to a flat universe. This means that in the early universe

$$\Omega - 1 = \frac{k}{(aH)^2} \propto t$$ (for a radiation dominated universe)

$$\Rightarrow |\Omega_{BBN} - 1| \leq 10^{-16} \quad (\Omega_{GUT} - 1| \leq 10^{-55})$$

How does this come about?
Horizon Problem

Why the CMB is so uniform on large scales?
• Origin of **primordial density fluctuation** which lead to Large Scale Structure and also explain

$$\delta T/T \sim 10^{-5}$$

observed by COBE/WMAP and other experiments?

• Origin of **baryon asymmetry** \((n_b/n_\gamma \sim 10^{-10})\)?
Successful Primordial Inflation should:

- Explain flatness, isotropy;
- Provide origin of $\frac{\delta T}{T}$;
- Offer testable predictions for $n_s$, $r$, $dn_s/d\ln k$;
- Recover Hot Big Bang Cosmology;
- Explain the observed baryon asymmetry;
- Offer plausible CDM candidate;

Physics Beyond the SM?
Cosmic Inflation

- Inflation can be defined as:

\[ \frac{d}{dt} \left( \frac{1}{aH} \right) < 0, \]

a decreasing comoving horizon

\[ \ddot{a} > 0, \]

an accelerated expansion

\[ P < -\rho / 3, \]

a negative pressure → repulsive gravity
drives inflation

- Consider a scalar field \( \phi \)

\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) \approx V, \]

\[ a(t) \approx e^{Ht} \rightarrow \text{inflation} \]

Slow rolling scalar field acts as an inflaton
Cosmic Inflation

Tiny patch $\sim 10^{-28}$ cm $\rightarrow$ $> 1$ cm after 60 e-foldings
(time constant $\sim 10^{-38}$ sec)

Inflation over radiation dominated universe (hot big bang)

Quantum fluctuations of inflation field give rise to nearly scale invariant, adiabatic, Gaussian density perturbations

Seed for forming large scale structure
\[ \ln(R_{RH}/R_1) \]

\[ \ln(R_{RH}/R_g) \]

\[ H^{-1} \sim t \sim R^{1/n} \]

(No Inflation)

Image courtesy of Kolb & Turner
• Solution to the Flatness Problem

\[
|\Omega_f - 1| = |\Omega_i - 1| e^{-2N} \to 0,
\]

where \( N = H \Delta t \geq 50 \)

• Solution to the Horizon Problem

Image courtesy of W. Kinney
Slow-roll Inflation

- Inflation is driven by some potential $V(\phi)$:
- Slow-roll parameters:
  \[ \epsilon = \frac{m_p^2}{2} \left( \frac{V'}{V} \right)^2, \; \eta = m_p^2 \left( \frac{V''}{V} \right). \]
- The spectral index $n_s$ and the tensor to scalar ratio $r$ are given by
  \[ n_s - 1 \equiv \frac{d \ln \Delta^2_R}{d \ln k}, \; r \equiv \frac{\Delta^2_h}{\Delta^2_R}, \]
  where $\Delta^2_h$ and $\Delta^2_R$ are the spectra of primordial gravity waves and curvature perturbation respectively.
- Assuming slow-roll approximation (i.e. $(\epsilon, |\eta|) \ll 1$), the spectral index $n_s$ and the tensor to scalar ratio $r$ are given by
  \[ n_s \simeq 1 - 6\epsilon + 2\eta, \; r \simeq 16\epsilon. \]
Slow-roll Inflation

- The tensor to scalar ratio $r$ can be related to the energy scale of inflation via

$$V(\phi_0)^{1/4} = 3.3 \times 10^{16} r^{1/4} \text{ GeV}.$$  

- The amplitude of the curvature perturbation is given by

$$\Delta^2_{\mathcal{R}} = \frac{1}{24\pi^2} \left( \frac{V}{m_p^4} \frac{\epsilon}{\epsilon^2} \right)_{\phi=\phi_0} = 2.43 \times 10^{-9} \text{ (WMAP7 normalization)}.$$  

- The spectrum of the tensor perturbation is given by

$$\Delta^2_h = \frac{2}{3\pi^2} \left( \frac{V}{m_p^4} \right)_{\phi=\phi_0}.$$  

- The number of $e$-folds after the comoving scale $l_0 = 2\pi/k_0$ has crossed the horizon is given by

$$N_0 = \frac{1}{m_p^2} \int_{\phi_e}^{\phi_0} \left( \frac{V}{V'} \right) d\phi.$$  

Inflation ends when $\max[\epsilon(\phi_e), |\eta(\phi_e)|] = 1.$
Scalar and Tensor Perturbations

During inflation, the universe contains a uniform scalar (inflaton) field and a uniform background metric.

There are quantum mechanical fluctuations about this zero-order scheme. According to inflationary cosmology, this generates $\delta \rho / \rho$ as well as gravity waves (from tensor fluctuations in the gravitational metric).
(1) $V = m^2 \phi^2$

$$\implies \frac{\delta \rho}{\rho} \sim \frac{m}{M_P} \implies m \sim 10^{13} \text{ GeV}$$

(2) $V = \lambda \phi^4$

$$\implies \frac{\delta \rho}{\rho} \propto \sqrt{\lambda} \implies \lambda \sim 10^{-12}$$

(tiny quartic coupling)

Can Standard Model Higgs field drive inflation?
Planck (2013), arXiv:1303.5075
Gravity Waves from Inflation

Inflation also generates tensor fluctuations in the gravitational metric which correspond to gravity waves. They induce fluctuations in the CMB and provide a unique signature of inflation. Their discovery would have far reaching implications for inflationary cosmology. The PLANCK satellite now in orbit has an excellent chance to ‘detect’ gravity waves if inflation is ‘driven’ by a grand unified theory with a characteristic energy scale $\sim 10^{16}$ GeV.

(note LHC cm energy $\sim 10^4$ GeV!)
CMB Polarization

• CMB radiation is expected to be polarized from Compton scattering during (matter-radiation) decoupling.
• To produce polarized radiation the incoming radiation must have a non-zero quadrupole. One expects the polarization signal to be small.
Polarization is generated by both scalar and tensor perturbations.

E modes
(varies in strength in the same direction as its orientation)

B modes
(varies in strength in a direction different from that in which it is pointing)
BICEP 2 Result

- BICEP 2 a few months ago surprised many people with their results that $r \sim 0.2$ (0.16).

- Some tension with the Planck upper bound $r < 0.11$.

- Somewhat earlier WMAP 9 stated that $r < 0.13$. 
Big Bang breakthrough announced; gravitational waves detected

By Elizabeth Landau, CNN

Updated 10:37 AM ET, Tue March 18, 2014
Speck of Interstellar Dust Obscures Glimpse of Big Bang

By DENNIS OVERBYE   JAN. 30, 2015
WMAP nine year data
Radiatively Corrected $\phi^2$ Potential:

$n_s$ vs. $r$ for radiatively corrected $\phi^2$ potential. The dashed portions are for $\kappa < 0$. The one loop radiative correction is larger than the tree level potential in the portions displayed in gray. $N$ is taken as 50 (left curves) and 60 (right curves).
Update of RGE analysis (@ 3-loop level) 

Figure 2. Upper: RG evolution of $\lambda$ (left) and of $\beta_\lambda$ (right) varying $M_t$, $\alpha_3(M_Z)$, $M_h$ by $\pm 3\sigma$. Lower: same as above, with more "physical" normalisations. The Higgs quartic coupling is compared with the top Yukawa and weak gauge coupling through the ratios $\text{sign}(\lambda) \sqrt{4|\lambda|}/y_t$ and $\text{sign}(\lambda) \sqrt{8|\lambda|}/g_2$, which correspond to the ratios of running masses $m_h/m_t$ and $m_h/m_W$, respectively (left). The Higgs quartic $\beta$-function is shown in units of its top contribution, $\beta_\lambda$ (top contribution) = $-3y_t^4/8\pi^2$ (right).

Indeed, $\lambda$ is the only SM coupling that is allowed to change sign during the RG evolution because it is not multiplicatively renormalised. For all other SM couplings, the $\beta$-functions are proportional to their respective couplings and crossing zero is not possible. This corresponds to the fact that $\lambda = 0$ is not a point of enhanced symmetry.
Consider the following Higgs Potential:

\[ V(\phi) = V_0 \left[ 1 - \left( \frac{\phi}{M} \right)^2 \right]^2 \quad \text{(tree level)} \]

Here \( \phi \) is a gauge singlet field.

- WMAP/Planck data favors BV inflation
Supersymmetry

\[ \alpha_1^{-1} \]
\[ \alpha_2^{-1} \]
\[ \alpha_3^{-1} \]

\( \text{MSSM} \)

\( \text{Log}_{10}[\Lambda/\text{GeV}] \)
Supersymmetric Higgs (Hybrid) Inflation

- Attractive scenario in which inflation can be associated with symmetry breaking $G \rightarrow H$

- Tree Level Potential

$$V_F = \kappa^2 (M^2 - |\Phi|^2)^2 + 2\kappa^2 |S|^2 |\Phi|^2$$

- Ground State

$$|\langle \Phi \rangle | = M, \quad \langle S \rangle = 0$$

Cf: Superconductor, $\langle \Phi \rangle \rightarrow$ cooper pair, $\langle S \rangle \rightarrow$ temperature

- To realize inflation

$S \gg M$ in early universe ($T \gg T_c$)

$\Rightarrow$ At tree level, $V \approx \kappa^2 M^4 \quad \Rightarrow$ exponential expansion
Tree Level plus radiative corrections:

\[ n_s \approx 1 - \frac{1}{N_0} \approx 0.98 \]

\[ \delta T/T \propto (M/M_P)^2 \sim 10^{-5} \rightarrow \text{attractive scenario (} M \sim M_G \text{)} \]
More complete analysis:
Candidates includes:

- **WIMP** (weakly interaction massive \((10^2 – 10^3 \text{ GeV})\) particle)
- **Axions** – very light \((\sim 10^{-5} \text{ eV})\), very weakly interaction particle
- **Wimpzilla** – very massive \((10^{12} \text{ GeV})\), perhaps not entirely stable, particle
- **Gravitino** – keV mass partner of graviton; behaves as ‘warm’ dark matter?
WIMP Candidates ($10^2 - 10^3$ GeV in mass)

- **Neutralino** (neutral, spin $\frac{1}{2}$, stable, light supersymmetric particle)

- **Lightest neutral Kaluza Klein particle**
  (E.g. KK excitation of some suitable known particle)

- **Dark (mirror/hidden universe) baryons**
## SUSY DM CANDIDATES

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<tr>
<th>Spin</th>
<th>U(1) ( M_1 )</th>
<th>SU(2) ( M_2 )</th>
<th>Up-type ( \mu )</th>
<th>Down-type ( \mu )</th>
<th>( m_\tilde{\nu} )</th>
<th>( m_{3/2} )</th>
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<td>( H_u )</td>
<td>( H_d )</td>
<td>( \tilde{\nu} )</td>
<td>sneutrino</td>
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Indirect Search

- IceCube Lab
- IceTop
  - 80 stations, each with
  - 2 IceTop Cherenkov detector tanks
  - 2 optical sensors per tank
  - 320 optical sensors
- IceCube Array
  - 26 strings including 6 DeepCore strings
  - 60 optical sensors on each string
  - 5160 optical sensors
- Amanda II Array
  - (precursor to IceCube)
- DeepCore
  - 9 strings-spacing optimized for lower energies
  - 360 optical sensors
- Eiffel Tower
  - 324 m

- 2010: 79 strings in operation
- 2011: Project completion, 86 strings

Bedrock
The predictions of $r$ (primordial gravity waves) for various inflation models:

1. Gauge Singlet Higgs Inflation:
   \[ r \geq 0.02 \text{ for } n_s \geq 0.96 \]

2. SM Higgs Inflation:
   \[ r \sim 0.003, \ n_s \sim 0.968 \]

3. Non-Minimal $\phi^4$ Inflation:
   \[ r \geq 0.002 \text{ for } n_s \geq 0.96 \]

4. Dark Matter Inflation:
   \[ 0.003 \leq r \leq 0.007 \]

5. MSSM Inflation:
   \[ r \sim 10^{-16} \text{ with } 0.93 \leq n_s \leq 1 \]

6. Susy Higgs (Hybrid) Inflation:
   \[ r \leq 10^{-4} \text{ (minimal), } r \leq 0.03 \text{ (non-minimal)} \]

Planck (2015) says that $r < 0.09$
Many Challenges:

- Dark Matter
- Supersymmetry
- Gravity Waves
- Neutrino Physics
- Proton Decay
- Dark Energy