High-precision measurements of the Rb87 D-line tune-out wavelength

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Outline

- Motivation
  - Dipole matrix elements
  - Atomic parity violation
- Stark effect and “tune-out” wavelengths
  - Polarization control
  - Results
- Vector polarizability
Dipole Matrix Elements

- Excited state energies known very well from spectroscopy $(E_f - E_i)$
- Need dipole matrix elements also
  - $|d_{if}| = \langle n' P_{J'} | d | n S_J \rangle$
  - Lifetime
  - Oscillator Strength
  - Einstein $A$ coefficient
- Difficult to measure directly in general
  - 0.2% error for lowest lying
    - Lifetime measurements
    - Better than typical
- Good enough for many applications
Dipole Matrix Elements

- Even more precision needed in some areas
- Atomic parity violation
- Atomic clocks
  - Precision limited by blackbody shift from environment
- Theoretical benchmark
  - Computational techniques
  - Phenomenological input
- Feshbach resonances
Tabletop atomic experiment to test fundamental particle physics theory

- Weak charge $Q_W$

- Competitive precision in low energy test of standard model

From usual selection rules, $S \rightarrow P$ allowed, $S \rightarrow S$ forbidden

APV - Nonzero $S \rightarrow S$ transition probability

Very small effect

- 4.5 a.u. vs. $10^{-11}$ a.u.!

Cs APV experiment (1997)

- Achieved 0.35% experimental uncertainty
- To convert to measurement of weak charge, need dipole matrix elements

$$Q^\text{SM}_W = -73.23(2)$$

$$Q^\text{Atomic}_W = -72.58(29)_{Exp}^{32(32)}_{Theory}$$

- 0.4% theoretical uncertainty from conversion

No reason for new experiments until theory catches up
Atomic Parity Violation

- Precise knowledge of alkali atom matrix elements $|d_{if}|$ needed in analysis
- Infinite number of such matrix elements - all contribute
- Direct measurements not possible in general to high enough precision
  - Lowest known through lifetime measurements
- Beginning to develop framework to reduce uncertainties through measurements of tune-out wavelengths
Bose-Einstein Condensate

- Create BEC in standard fashion
  - MOT
  - Magnetic quadrupole trap
  - rf evaporation
- Load atoms into “wave-guide”
  - $2\pi \times (5.1, 1.1, 3.2)$ Hz
  - Interferometer along weakest direction

www.bec.nist.gov
Bose-Einstein Condensate

$2\pi \times (5.1, 1.1, 3.2) \text{ Hz}$
Interferometry

- Split source and propagate along two paths
- Difference in phase at output - constructive vs. destructive interference

\[ \vec{E} = \vec{E}_0 e^{i\phi} = \vec{E}_0 e^{i(\omega t - kz)} \]
Interferometry

- Split source and propagate along two paths
- Difference in phase at output - constructive vs. destructive interference
  \[ \vec{E} = \vec{E}_0 e^{i\phi} = \vec{E}_0 e^{i(\omega t - k z)} \]
- Light interferometers measure time (optical path length) differences
Atom Interferometry

\[ \phi = \frac{S}{\hbar} = \frac{\int E \, dt}{\hbar} \]

- Analogous to light interferometer
- Atoms sensitive to many more phenomena - electromagnetic fields, gravity, accelerations, inter-atomic interactions, etc.
- Colder (slower) atoms = longer interrogation times

BEC before split
Atom Interferometry

\[ \phi = \frac{S}{\hbar} = \int E dt \]

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Atom Interferometry

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After recombination

Phase determined by ratio of atoms at rest to total, \( \frac{N_0}{N} \)
Stark Effect

\[ U = -\frac{1}{2} \alpha \langle \mathcal{E}^2 \rangle = -\frac{\alpha I}{2\epsilon_0 c} \]

- Energy shift due to applied electric field
  - Static or AC
  - Dynamic polarizability
- Difficult to calibrate intensity
  - Atoms inside vacuum chamber
- Want polarizability \( \alpha \)
  - Dependence on dipole matrix elements
Polarizability

\[ \alpha_i(\omega) = \frac{1}{\hbar} \sum_f \frac{2\omega_{if}}{\omega_{if}^2 - \omega^2} |d_{if}|^2 + \alpha_c + \alpha_{cv} \]

- \( \alpha_c = \) core contribution
- \( \alpha_{cv} = \) core-valence correction
- 5P states dominate

\[ \alpha(\omega) = \frac{2}{\hbar} \frac{\omega_{5P_{1/2}}}{\omega_{1/2}^2 - \omega^2} |d_{1/2}|^2 + \frac{2}{\hbar} \frac{\omega_{5P_{3/2}}}{\omega_{3/2}^2 - \omega^2} |d_{3/2}|^2 + \alpha_{\text{tail}} + \alpha_c + \alpha_{cv} \]

- \( \alpha_{\text{tail}} = \) valence contributions > 5P
- Difficult to calculate
  - Infinite number of matrix elements
  - Calculated up to \( n = 12 \)
  - Uncertainty in tail same scale as value
Previous Measurement (2008)

\[ U = -\frac{1}{2} \alpha \langle \mathcal{E}^2 \rangle = -\frac{\alpha I}{2\epsilon_0 c} \]

- Measured by former student Ben Deissler (PhD 2008)
  - \( \alpha(780.23 \text{ nm}) = \frac{4\pi\epsilon_0}{10^{25}} \times (8.37 \pm 0.24) \text{ m}^3 \)
  - \( \alpha(808.37 \text{ nm}) = \frac{4\pi\epsilon_0}{10^{28}} \times (9.48 \pm 0.25) \text{ m}^3 \)

- Deviations from predicted values about 3%
  - Attributed primarily to intensity calibration

- Need way to reduce dependence on intensity calibration
Tune-out wavelength

\[ \alpha_i(\omega) = \frac{1}{\hbar} \sum_f \frac{2\omega_{if}}{\omega_{if}^2 - \omega^2} |d_{if}|^2 + \alpha_c + \alpha_{cv} \]

- Zero in polarizability between resonances
- Extract info on $|d_{if}|$, $\alpha_c$, $\alpha_{cv}$
  - Mainly depends on
    \[ R = \frac{|d_{3/2}|^2}{|d_{1/2}|^2} \]
- Sensitive to polarization of light
  - Switch to spherical tensor form

Polarizability and Tune-out wavelength near 5P resonances

\[ \alpha^{(0)} \text{ (arb.)} \]

Wavelength (nm)
Spherical tensors

\[ U = -\frac{\langle \mathcal{E}^2 \rangle}{2} \left\{ \alpha^{(0)} - \frac{1}{2} \mathcal{V} \cos \chi \alpha^{(1)} + \left[ \frac{3 \cos^2(\xi) - 1}{2} \right] \alpha^{(2)} \right\} \]

- Dependence on polarization more obvious
- \( \mathcal{V} \) - fourth Stoke’s parameter
  - \( \pm 1 \) for \( \sigma^\pm \)
- \( \cos \chi = \hat{k} \cdot \hat{b} \)
- \( \cos \xi = \hat{\epsilon} \cdot \hat{b} \)
  - Angle of linear polarization w.r.t. magnetic field
- Near tune-out wavelength
  - \( \alpha^{(1)} = 25000 \text{ au} \)
  - \( d\alpha^{(0)}/d\lambda = -2500 \text{ au/nm} \)
  - Want sub-picometer uncertainty
Spherical tensors

\[ U = -\frac{\langle \mathcal{E}^2 \rangle}{2} \left\{ \alpha^{(0)} - \frac{1}{2} \mathcal{V} \cos \chi \alpha^{(1)} + \left[ \frac{3 \cos^2(\xi) - 1}{2} \right] \alpha^{(2)} \right\} \]

- Need to control \( |\mathcal{V} \cos \chi| \) to better than \( 10^{-5} \)
  - Better than typically maintained through vacuum window
  - Stress-induced birefringence
- Remove tensor polarizability later to report zero in \( \alpha^{(0)} \)
  - \( \lambda^{(0)} \)
- Use atoms to linearize light
Stark Interferometer

- Allow one packet of interferometer to pass through Stark beam (twice)
- Vary intensity to measure rate of phase buildup
- Make measurements at different wavelengths around tune-out
Polarization Control

\[ U = -\frac{\langle \mathcal{E}^2 \rangle}{2} \left\{ \alpha^{(0)} - \frac{1}{2} \mathcal{V} \cos \chi \alpha^{(1)} + \left[ \frac{3 \cos^2(\xi) - 1}{2} \right] \alpha^{(2)} \right\} \]

- Atoms held in Time Orbiting Potential (TOP) magnetic trap
  - Magnetic bias rotates at 12 kHz
  - Stark light aligned in plane of rotation
- Constant reversal of \( \sigma^+ \) and \( \sigma^- \)
  - \( \langle \cos \chi \rangle = 0 \)
- Time averaging alone not enough
Polarization Control

- Also want $\langle \mathcal{N} \rangle = 0$
- Interferometer run with Stark light pulsing
  - On for half of rotating bias period
- Phase buildup asymmetric when imbalance of $\sigma^+$ and $\sigma^-$
- Adjust external waveplate to correct imbalance
  - QWP at 780 nm
  - Need 0.1° precision - $\mathcal{N} \approx 2 \times 10^{-3}$
- Properly set when phase symmetric and small
A total of 21 tune-out measurements made over 2 months

Upper figure 1 hour
  - One point on lower figure

Lower figure 1 day

Check polarization before and after $\alpha$ measurement to assess drift
  - Typical drift over day 60 fm
  - Likely due to thermal fluctuations
  - Taken as polarization uncertainty for measurement
Tensor Polarizability

\[
U = -\frac{\langle \mathcal{E}^2 \rangle}{2} \left\{ \alpha^{(0)} + \left[ \frac{3 \cos^2(\xi) - 1}{2} \right] \alpha^{(2)} \right\}
\]

- \( \langle \mathcal{V} \cos \chi \rangle = 0 \)
- Depends on angle of linear polarization w.r.t. plane of rotating magnetic field
- Introduces shift in tune-out wavelength
- Couldn’t accurately set \( \xi \) prior to taking data
  - Difficult to determine plane of bias rotation
  - Several measurements at different angles of linear polarization
- Remove \( \alpha^{(2)} \) term to get \( \lambda^{(0)} \)
Tune-out Wavelength

\[ \lambda_0(\theta) = \lambda^{(0)} - \frac{\alpha^{(2)}}{d\alpha^{(0)}/d\lambda} \left( \frac{3}{4} \cos^2 \theta - \frac{1}{2} \right) \]

- Measurements at different polarization angles
- Tensor polarizability well resolved
- \( \lambda^{(0)} \) zero in scalar polarizability
- \( \frac{\alpha^{(2)}}{d\alpha^{(0)}/d\lambda} = 538.5(4) \text{ fm} \)
  - Straightforward to calculate from theory due to strong dependence on \( D_1 \) and \( D_2 \)
- Zero in scalar polarizability at \( \lambda_0 = 790.032388(32) \text{ nm} \)
Several other measurements in $^{87}$Rb
- Referenced to $F = 2$ groundstate
- Tune-outs also measured in $K$, $Na$, $He$
- Larger uncertainties for now

$\lambda_0$ (nm)
Tune-out Wavelength

\[ \lambda_0 (\text{nm}) \]

87Rb tune-out measurements to date

Lamporesi et al., Schmidt et al., Leonard et al., Theory
Ratios of Matrix Elements and Benchmarks

\[ \alpha^{(0)} = A + |d_{1/2}|^2 \left( \frac{\alpha_{5P_{1/2}}^{(0)}}{|d_{1/2}|^2} + \frac{\alpha_{5P_{3/2}}^{(0)}}{|d_{3/2}|^2} R \right) \]

- \( R = \frac{|d_{3/2}|}{|d_{1/2}|} \)
- \( A \) includes contributions from \( \alpha_c \), \( \alpha_{cv} \), and valence terms above 5\( P \)
  - \( A = 10.70(12) \) au from theory
  - \( d_{1/2} = 4.233(4) \) au from direct measurements

- From direct measurements, \( R = 1.995(7) \)
- From tune-out wavelength, \( R = 1.99221(3) \)
Contributions from Theory

M. Safronova

Experiment accuracy = 0.1 au
Implications for Theory

- From tune-out wavelength, $R = 1.99221(3)$
- Benchmark for theory
  - From M. Safronova, $R = 1.9919(5)$
  - Need to include additional effects
- Breit Interaction
  - Relativistic correction to Coulomb interaction
- QED effects
  - Radiative corrections
- Both effects 5x smaller than theoretical uncertainty
  - Come in at $5 \times 10^{-5}$ level
  - Compare to $3 \times 10^{-5}$ from tune-out measurement
  - Need more precise calculations
Other Tune-out Wavelengths

- Tune-out between any two resonances
- More ratios to determine higher lying matrix elements
- Begin to separate out various contributions
- Will also measure vector polarizability
  - $\langle V \cos \chi \rangle \neq 0$
Vector Polarizability

\[ U = -\frac{\langle \mathcal{E}^2 \rangle}{2} \left\{ \alpha^{(0)} - \frac{1}{2} \gamma \cos \chi \alpha^{(1)} \right\} \]

- Intentionally introduce circular polarization in controlled manner
- Measure ratio \( \alpha^{(1)}/\alpha^{(0)} \) at several points around tune-out wavelength
- What does vector polarizability get us?
Vector Polarizability

Scalar: \( \alpha^{(0)} = \frac{1}{3\hbar} \sum_f |D_{if}|^2 \frac{\omega_{if}}{\omega_{if}^2 - \omega^2} + \alpha_c + \alpha_{cv}^{(0)} \)

Vector: \( \alpha^{(1)} = \frac{1}{3\hbar} \sum_f C_{J'} |D_{if}|^2 \frac{\omega}{\omega_{if}^2 - \omega^2} + \alpha_{cv}^{(1)} \)

- Look at one pair of \( n' \) states

\[ \alpha_{n'}^{(0)} = \frac{\omega_{n'3/2}}{\omega_{n'3/2}^2 - \omega^2} |d_{n'3/2}|^2 + \frac{\omega_{n'1/2}}{\omega_{n'1/2}^2 - \omega^2} |d_{n'1/2}|^2 \]

\[ \alpha_{n'}^{(1)} = \frac{\omega}{\omega_{n'3/2}^2 - \omega^2} |d_{n'3/2}|^2 - 2 \frac{\omega}{\omega_{n'1/2}^2 - \omega^2} |d_{n'1/2}|^2 \]

- Contributions from \( n'P_{1/2} \) and \( n'P_{3/2} \) can be isolated
Vector Polarizability Polarization Control

- Pulse for $t \ll \tau_{TOP}$ and adjust relative phase
  - Fine control over $\langle \cos \chi \rangle$
- Need new method to determine polarization
  - Want $V = +1$
- Use $\sigma^+$ light tuned to D1 resonance
  - No resonant transition
  - Minimize scattering rate using external waveplate

Energy Level Diagram

$5S_{1/2} \rightarrow 5P_{1/2}$

$5P_{1/2}$

$5S_{1/2}$

$\sigma^+$

$F,m_F = 2,2$
Vector Polarizability

\[ \alpha \approx \frac{1}{6} \left( \frac{|D_1|^2(1 - 2\nu)}{\omega_{1/2} - \omega} + \frac{|D_2|^2(1 + \nu)}{\omega_{3/2} - \omega} \right) \]

- Circular polarization shifts tune-out location
- \( \nu \) can vary from -1/2 to +1/2
- Possible to prevent tune-out wavelength altogether
- Tune over 2/3 range between \( D_1 \) and \( D_2 \)
  - 785 nm to 795 nm
- Make measurements of shifted tune-out wavelength
Vector Polarizability

- Adjust $\langle V \cos \chi \rangle$ in controlled manner
  - $d\lambda_0/d\nu$ almost linear over full range
  - Deviation from linear is of interest
- Deviations on picometer scale
  - Compare to 32 fm uncertainty in tune-out measurement
Current theoretical values and their uncertainties

- $\alpha_c = 9.08(5) \text{ au}$
- $\alpha_{cv}^{(0)} = -0.37(4) \text{ au}$
- $\alpha_{cv}^{(1)} \sim -0.04(4) \text{ au}$
- $T_{1/2} = 0.022(22) \text{ au}$
- $T_{3/2} = 0.075(75) \text{ au}$

- $\alpha_{cv}^{(1)}$ approximated from $\alpha_{cv}^{(0)}$

- Tail terms have $n' > 12$
Vector Polarizability

- Polarization and frequency dependence ultimately allow separation of the contributions
- Simulated 3 tune-out and multiple vector polarizability measurements

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model Estimate</th>
<th>Model Error</th>
<th>Fit Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{5,3/2}$</td>
<td>1.99221</td>
<td>$3 \times 10^{-5}$</td>
<td>$3 \times 10^{-5}$</td>
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<td>$R_{6,1/2}$</td>
<td>0.00584</td>
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<td>$2 \times 10^{-6}$</td>
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<td>$R_{6,3/2}$</td>
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<td>$\alpha_c + \alpha_{cv}^{(0)}$</td>
<td>8.71</td>
<td>$9 \times 10^{-2}$</td>
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<tr>
<td>$\alpha_{cv}^{(1)}$</td>
<td>-0.04</td>
<td>$4 \times 10^{-2}$</td>
<td>$9 \times 10^{-3}$</td>
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<tr>
<td>$</td>
<td>t_{1/2}</td>
<td>^2$</td>
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<tr>
<td>$</td>
<td>t_{3/2}</td>
<td>^2$</td>
<td>0.03</td>
</tr>
</tbody>
</table>

- Model error based on 790 nm tune-out measurement
In the process of setting polarization
- $\sigma^+$ light tuned to $D_1$ resonance
- Correcting for chamber birefringence, stray fields, etc.

Acquired Babinet-Soleil Compensator
- Calibrate waveplates at different wavelengths
- Accurately change $\sigma^+ \rightarrow \sigma^-$ along with field reversal to test polarization and field corrections
Conclusions

- Measured longest tune-out wavelength in $^{87}\text{Rb}$
  - $\lambda_0 = 790.032388(32) \text{ nm}$
  - $R = 1.99221(3)$

- Used $R$ as a benchmark for theory
  - $R_{theory} = 1.9919(5)$

- Measurements of other tune-out wavelengths
  - Near 420 nm and 360 nm

- Measurements of vector polarizability
  - Separate out contributions beyond what tune-out wavelengths alone can do

Acknowledgments

Cass Sackett, Advisor
Bob Leonard
Oat Arpornthip
Eddie Moan
Wavemeter Calibration

- Wavemeter specced to $10^{-6}$ - Not good enough
- Calibrated it using well known lines in several atomic species
  - $^{39}$K $D_1$
  - $^{87}$Rb $D_2$
  - $^{85}$Rb $D_1$
  - $^{133}$Cs $D_2$

Add what we used as correction with error bars
Including Hyperfine Structure

\[ \alpha_{5P}^{(0)} = \frac{10}{\hbar \sqrt{15}} \sum_{J', F'} \frac{|d_{J'}|^2 \omega'}{\omega''^2 - \omega^2} (-1)^{1 + F'} (2F' + 1) \]
\[ x \left\{ \begin{array}{ccc} 2 & 1 & F' \\ 1 & 2 & 0 \end{array} \right\} \left\{ \begin{array}{ccc} F' & 3/2 & J' \end{array} \right\}^2 \]

\[ \alpha_{5P}^{(1)} = \frac{10}{\hbar \sqrt{15}} \sum_{J', F'} \frac{|d_{J'}|^2 \omega}{\omega''^2 - \omega^2} (-1)^{1 + F'} (2F' + 1) \]
\[ x \left\{ \begin{array}{ccc} 2 & 1 & F' \\ 1 & 2 & 1 \end{array} \right\} \left\{ \begin{array}{ccc} F' & 3/2 & J' \end{array} \right\}^2 \]

\[ \alpha_{5P}^{(2)} = \frac{20}{\hbar \sqrt{15}} \sum_{J', F'} \frac{|d_{J'}|^2 \omega'}{\omega''^2 - \omega^2} (-1)^{F'} (2F' + 1) \]
\[ x \left\{ \begin{array}{ccc} 2 & 1 & F' \\ 1 & 2 & 2 \end{array} \right\} \left\{ \begin{array}{ccc} F' & 3/2 & J' \end{array} \right\}^2 \]
Parity Non Conservation

\[ E_{PNC}^{Theory} = \]

\[ \sum_{n'=6}^{\infty} \left( \frac{\langle 7S|\mathbf{d}|n'P_{1/2}\rangle\langle n'P_{1/2}|H_{PNC}|6S\rangle}{E_{6S} - E_{n'P_{1/2}}} + \frac{\langle 7S|H_{PNC}|n'P_{1/2}\rangle\langle n'P_{1/2}|\mathbf{d}|6S\rangle}{E_{7S} - E_{n'P_{1/2}}} \right) \]

- \( Q_{W}^{SM} = -73.23(2) \)
- \( Q_{W}^{Atomic} = -72.58(29)_{Exp}(32)_{Theory} \)
- Differ by 1.5\( \sigma \)
Parity Non Conservation

\[ \frac{\text{Im}(E_{PNC})}{\beta} = i \frac{Q_W}{\beta N} k_{PNC} \]

- \( k_{PNC} \) contains all relevant parity conserving and PNC matrix elements
- Mixing of \( S_{1/2} \) and \( P_{1/2} \) states
- Weak interaction not parity conserving