On neutrino masses and phenomenological implications: $\mu \rightarrow e\gamma$ and $\mu$-e conversion

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HEP Seminar


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1. Introduction

2. Review
   - Seesaw Mechanism
   - Motivations to the EW-$\nu_R$ Model
   - Minimal EW-$\nu_R$ Model
   - Discrete Symmetry $A_4$

3. Model of neutrino masses

4. Phenomenological Implications
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   - $l_i \rightarrow l_j \gamma$
   - $\mu$-e Conversion

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Introduction
Neutrino in the Standard Model
Neutrino in the Standard Model
Neutrino in the Standard Model

In the Standard Model (SM)

- neutrinos have exactly zero masses
- there are exactly three neutrinos belonging to three lepton families \((e, \nu_e), (\mu, \nu_\mu), (\tau, \nu_\tau)\); lepton number is conserved
- neutrinos and antineutrinos are distinct
- all neutrinos are left-handed, and all antineutrinos are right-handed.
Neutrino Oscillation’s Evidences

2 problems
Neutrino Oscillation’s Evidences

2 problems

Solar neutrino problem
Neutrino Oscillation’s Evidences

2 problems

- Solar neutrino problem
- Atmospheric Neutrino Anomaly
Solar neutrino problem

First crack - Ray Davis

Cl 37 solar $\nu$ experiments: $\Phi_{\nu} (\text{observed}) = \frac{1}{3} \Phi_{\nu} (\text{SSM})$

Late 1980s, Kamiokande-II observed 46(±15)% $\Phi_{\text{expected}}$

GALLEX and SAGE saw about 62(±10)% of SSM prediction
First crack - Ray Davis $Cl^{37}$ solar $\nu$ experiments: $\Phi_{\nu_e}(\text{observed}) = 1/3 \Phi_{\nu_e} (\text{SSM})$
Solar neutrino problem

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Late 1980s, Kamiokande-II observed $46(\pm 15)\% \Phi_{\nu_e}^{\text{expected}}$

GALLEX and SAGE saw about $62(\pm 10)\%$ of SSM prediction
Atmospheric Neutrino Anomaly

Prediction

$N_{\nu_\mu} : N_{\nu_e} \simeq 2 : 1$

Observation

$N_{\nu_\mu} : N_{\nu_e} = 1.3 / 1$ by Super-Kamiokande in 1998 → discovery of neutrino oscillations

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Atmospheric Neutrino Anomaly

**Prediction**

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**Observation**

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by Super-Kamiokande in 1998

→ discovery of neutrino oscillations
Neutrino Oscillation

Neutrino oscillation arises from a mixture between the flavor and mass eigenstates of neutrinos.

\[ |\nu_\alpha \rangle = \sum_i U_{\alpha i}^{\text{lepton}} |\nu_i \rangle \]

where

- \( |\nu_\alpha \rangle \) is flavor eigenstate. \( \alpha = e, \mu, \tau \)
- \( |\nu_i \rangle \) is mass eigenstate. \( i = 1, 2, 3 \)
- \( U_{\alpha i}^{\text{lepton}} \) is lepton mixing matrix or Pontecorvo-Maki-Nakagawa-Sakata (\( U_{PMNS} \)) matrix
Motivation

The discovery of neutrino oscillations

- has revealed many valuable information concerning the mixing matrix $U_{PMNS}$ and the $\Delta m^2$ in the neutrino sector.
- first evidence of physics beyond the Standard Model (BSM).

Puzzled questions

- The origin of neutrino masses?
- Why is the mass of neutrino so tiny ($m_\nu < O(eV)$)?
- Can we access experimentally the physics that are responsible for the tininess of the neutrino masses and their mixings?
- Why is the leptonic mixing matrix $U_{PMNS}$ so different from $V_{CKM}$ of the quark sector?
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Motivation

New Physics Search in Charged Lepton Flavor

For instance, $\mu \rightarrow e\gamma$, $\mu$-e conversion

- Observing these will remove a hurdle to understand why particles in the same category (family) decay from heavy to lighter, more stable mass states.
- Physicists have searched for these since the 1940s.
- Discovering them is central to understand what physics lies beyond the SM.
Review
Neutrino masses

Dirac mass

Dirac neutrino masses are the neutrino analogues of the SM quark and charged lepton masses. They come from Yukawa coupling to the SM Higgs field $\tilde{\Phi}$

$$g_\nu \bar{\nu}_L \tilde{\Phi} \nu_R + h.c. \Rightarrow g_\nu \langle \tilde{\Phi} \rangle \bar{\nu}_L \nu_R + h.c. \equiv m_D^\nu (\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R)$$

Neutrinos have a phase of $e^{-i\phi}$ and antineutrinos have a phase of $e^{i\phi}$. Therefore, in the Dirac mass term, these phases cancel out $\rightarrow$ lepton number is conserved
Majorana mass of $\nu_R$

$$M_R \nu_R^T \sigma_2 \nu_R$$

In the Majorana mass term, the phase of $\nu_R^T \nu_R$ is not zero $\rightarrow$ lepton number is violated
Seesaw Mechanism

A generic model used to understand the observed neutrino masses ($\sim O(\text{eV})$), compared to those of quarks and charged leptons which are much much heavier.
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A generic model used to understand the observed neutrino masses ($\sim O(\text{eV})$), compared to those of quarks and charged leptons which are much much heavier.

With $\chi \equiv \sigma_2 \nu_R^*$ and $\nu \equiv \nu_L$ the mass terms can be written as

$$
\begin{pmatrix}
\nu^T & \chi^T
\end{pmatrix}
\begin{pmatrix}
0 & m_D^\nu \\
m_D^\nu & M_R
\end{pmatrix}
\sigma_2
\begin{pmatrix}
\nu \\
\chi
\end{pmatrix}
$$
Seesaw Mechanism

With the assumption: $m^D_\nu \ll M_R$, diagonalizing the matrix $M$ gives eigenvalues

$$m_\nu \approx \frac{(m^D_\nu)^2}{M_R} \quad \text{and} \quad M_R$$
Experimental neutrino mass

- **Cosmological constraints**: \( \sum m_\nu < 0.23 \text{eV} \)

- **Neutrino oscillation experiments**: the largest \( \Delta m^2 \) is \( \Delta m_{\text{atm}}^2 \approx 2.4 \times 10^{-3} \text{ eV}^2 \) \( \Rightarrow \) the heaviest \( m_\nu \gtrsim 4.9 \times 10^{-2} \text{ eV} \).

Cosmology + Oscillation: \( 4.9 \times 10^{-2} \text{ eV} \lesssim m_\nu^{\text{heaviest}} \lesssim 0.23 \text{ eV} \)

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1 Planck 2015 results  
2 Particle Data Group
Motivations to the EW-$\nu_R$ Model

$\nu_R$ is a singlet under $SU(2)_L \times U(1)_Y$.

$M_R \sim$ Grand Unified (GUT) mass scale of $10^{16}$ GeV naturally.

$\to M_R$ is too large.

Therefore, one cannot produce and detect $\nu_R$ at the LHC.

Or, the seesaw mechanism is not testable!

Questions

Can we make the Seesaw testable?

Can $M_R$ be of the order of $\Lambda_{EW}$ (246 GeV)?

Keeping the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$?

No new gauge interactions added?

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Non-perturbative phenomena

Such important non-perturbative phenomena

- EW phase transition (from $\langle \phi^0 \rangle = 0$ to $\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}$)
- related Physics ("sphaleron", ...)

can be study by a usual approach: **Lattice regularization**

Can we put the SM gauge theory $(SU(2)_L \times U(1)_Y)$ on the lattice?

- Nielsen-Ninomiya no-go theorem: there appear an equal number of right- and left-handed particles of given quantum numbers in a regularized theory with a chirality invariant action.
- Since the Standard Model is chiral. Left- and right-handed fermions are treated differently by weak interactions, for example, only left-handed doublets coupled to $W$’s. The Nielsen-Ninomiya theorem implies that one cannot put the SM on the lattice.
Anomalies Cancelation

• The SM contains gauge triangle anomalies which breaks gauge invariance. Anomalies cancelation in the SM gives

\[ \sum_i Q_i = 0 \]  

(1)

for each family → cancelation between quarks and leptons.
Anomalies Cancelation

- The SM contains gauge triangle anomalies which breaks gauge invariance. Anomalies cancelation in the SM gives

\[ \sum_i Q_i = 0 \]  \hspace{1cm} (1)

for each family \( \rightarrow \) cancelation between quarks and leptons.

- **Witten anomaly**: the theory is trivial unless the number of doublets is even.
  - SM has 4 doublets per family (1 lepton and 3 color quark doublets) \( \rightarrow \) Witten anomaly free.
  - SM with mirror particles: not a chiral gauge theory \( \rightarrow \) No Witten anomaly.
Can we solve the problems?

Introducing...
Can we solve the problems?

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*The Non-sterile Electroweak-scale Right-handed neutrino (EW $\nu_R$) Model*

[P. Q. Hung, PLB 649 (2007)]
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- Can $M_R$ be of the order of $\Lambda_{EW}$ (246 GeV)?
- Keeping the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$?
- No new gauge interactions added?
- Study non-perturbative phenomena by using lattice regularization?
- Is carefree toward anomalies?
Minimal EW-$\nu_R$ Model $^3$

What is it?

Model in which right-handed neutrinos have Majorana masses of the order of $\Lambda_{\text{EW}}$ naturally.

Gauge group $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$

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$^3$P.Q. Hung, 2007

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Minimal EW-$\nu_R$ Model

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\[^3\text{P.Q. Hung, 2007}\]
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Gauge group

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$^3$P.Q. Hung, 2007
Leptons

\[ \nu_L \rightarrow \nu_R \]

Quarks

\[ u_L \rightarrow u_R, d_L \rightarrow d_R \]
$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$
Leptons

\[ l_L = \left( \begin{array}{c} \nu_L \\ e_L \end{array} \right) \quad \leftrightarrow \quad l_R^M = \left( \begin{array}{c} \nu_R \\ e_R^M \end{array} \right), \]
Leptons

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e_R

Mirror particles are totally different from the SM particles!
Leptons

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\[ e_R \quad \leftrightarrow \quad e_L^M \]

Mirror particles are totally different from the SM particles!
Leptons

\[ l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \leftrightarrow \quad l_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}, \]

\[ e_R \quad \leftrightarrow \quad e^M_L \]

Quarks

\[ q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \leftrightarrow \quad q_R = \begin{pmatrix} u_R \\ d_M \end{pmatrix}, \]

\[ u_R, \quad d_R \quad \leftrightarrow \quad u_L, \quad d_L \]
Leptons

\[ l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \leftrightarrow \quad l_R^M = \begin{pmatrix} \nu_R^M \\ e_R^M \end{pmatrix}, \]

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\[ u_R, d_R \quad \leftrightarrow \quad u_L^M, d_L^M \]
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\[ u_R, d_R \leftrightarrow u_L^M, d_L^M \]

Mirror particles are totally different from the SM particles!
**EW precision**


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**Implications of the 125-GeV SM-like scalar Dr Jekyll (SM-like) & Mr Hyde (very different from SM)**

What are Higgs sectors for Majorana and Dirac masses?
Majorana mass of $\nu_R$

From (2), the Majorana mass $M_R = g M_{\nu M}$ where $\langle \chi_0 \rangle = v_{\nu M} \sim \Lambda_{\text{EW}}$ couples to Z-boson and contributes to $\Gamma_Z$'s constraint (number of light neutrinos = 3) implies $M_R > M_Z/2$. 

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Majorana mass of $\nu_R$

$$L_M = g_M \left( l_R^{M,T} \sigma_2 \right) (i \tau_2 \tilde{\chi}) l_R^M$$  \hspace{1cm} (2)
Majorana mass of $\nu_R$

\[ L_M = g_M \left( l_{R}^{M,T} \sigma_2 \right) (i \tau_2 \tilde{\chi}) l_{R}^{M} \]

\[ = g_M \nu_{R}^{T} \sigma_2 \nu_{R} \chi^0 - \frac{1}{\sqrt{2}} \nu_{R}^{T} \sigma_2 e_{R}^{M} \chi^+ + \ldots \]

\[ \tilde{\chi} = (3, Y/2 = 1) \]

\[ \tilde{\chi} = \frac{1}{\sqrt{2}} \vec{\tau}.\tilde{\chi} = \begin{pmatrix} \frac{1}{\sqrt{2}} \chi^+ \\ \chi^0 \\ -\frac{1}{\sqrt{2}} \chi^+ \end{pmatrix} \]
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\tilde{\chi} = \frac{1}{\sqrt{2}} \tau.\bar{\chi} = \left( \begin{array}{c}
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\chi^0 \\
-\frac{1}{\sqrt{2}} \chi^+
\end{array} \right)
\]

From (2), the Majorana mass $M_R = g_M v_M$ where $\langle \chi^0 \rangle = v_M \sim \Lambda_{EW}$

$\nu_R$ couples to Z-boson and contribute to $\Gamma_Z$

$\Gamma_Z$'s constraint (number of light neutrinos = 3) implies $M_R > M_Z/2$
Dirac mass

The singlet scalar field $\phi$ couples to fermion bilinear. 

$$L_S = g_S \bar{l}_L \phi S l_R + h.c.$$  (3)

From (3), Dirac mass:

$$m_\nu = g_S v$$

where $\langle \phi_S \rangle = v_S$. 

$$m_\nu \approx (m_D_\nu)^2 \lesssim \frac{0.23}{\text{eV}}$$

$v_S \sim 10^{5-6}$ eV with $g_S \sim O(1)$

$v_S \sim \Lambda_{\text{EW}}$ with $g_S \sim O(10^{-6})$.
Dirac mass

The singlet scalar field $\phi_S$ couples to fermion bilinear.

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L_S = g_{S\ell} \bar{\ell}_L \phi_S \ell_R + h.c.
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From (3), Dirac mass: $m^D_\nu = g_{S\ell} \nu_S$ where $\langle \phi_S \rangle = \nu_S$. 

$\phi_S (1, \ Y/2 = 0)$
The singlet scalar field $\phi_S$ couples to fermion bilinear.

\begin{equation}
L_S = g_S \bar{l}_L \phi_S l_R^M + h.c. \\
= g_S \bar{\nu}_L \phi_S \nu_R + ... + h.c.
\end{equation}

$\phi_S (1, \ Y/2 = 0)$

From (3), Dirac mass: $m^D_\nu = g_S \nu_S$ where $\langle \phi_S \rangle = \nu_S$.

$$m_\nu \approx \frac{(m^D_\nu)^2}{M_R} \lesssim 0.23 \text{ eV}$$
Dirac mass

The singlet scalar field $\phi_S$ couples to fermion bilinear.

\[
L_S = g_{Sl} \bar{l}_L \phi_S l^M_R + h.c. \\
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- $v_S \sim 10^{5-6} \text{ eV with } g_{Sl} \sim \mathcal{O}(1)$
- $v_S \sim \Lambda_{EW}$ with $g_{Sl} \sim \mathcal{O}(10^{-6})$
We also need a **Higgs doublet** for charged fermion masses (leptons and quarks)

\[
L_{Y_l} = g_l \bar{l}_L \Phi_2 e_R + h.c. \\
L_{Y_q} = g_q \bar{q}_L \Phi_2 u_R + h.c.
\]

\[
\Phi_2 = \begin{pmatrix}
\phi^+ \\
\phi^0
\end{pmatrix}, \quad \langle \phi^0 \rangle = \frac{\nu_2}{\sqrt{2}}
\]
Experimentally

For the quark sector we use the Cabibbo-Kobayashi-Maskawa (CKM) matrix

\[ V_{CKM} = \begin{pmatrix}
0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347 + 0.00016 - 0.00012 \\
0.2252 \pm 0.0007 & 0.97345 + 0.00015 - 0.00016 & 0.00862 + 0.00026 - 0.00020 \\
0.00862 + 0.00026 - 0.00020 & 0.00347 + 0.00016 - 0.00012 & 0.999152 + 0.000030 - 0.000045
\end{pmatrix} \]

which is really close to a unit matrix.

For the lepton sector, the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix is used to study the mixings

\[ U_{PMNS} = \begin{pmatrix}
0.779 & 0.848 & 0.510 & 0.122 & 0.190 \\
0.183 & 0.568 & 0.385 & 0.728 & 0.589 \\
0.200 & 0.576 & 0.408 & 0.742 & 0.575
\end{pmatrix} \]

\(^4\text{Werner Rodejohann, 2012}\)
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0.200\ldots0.576 & 0.408\ldots0.742 & 0.589\ldots0.775
\end{pmatrix}
\]

\[\text{Werner Rodejohann, 2012}\]
Model of neutrino masses

It was conjectured by Cabibbo\textsuperscript{5} and Wolfenstein\textsuperscript{6} independently that

\[ U_{CW} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \] (6)

\textsuperscript{5}N. Cabibbo, 1978
\textsuperscript{6}L. Wolfenstein, 1978
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(6)

Experimentally, \( U_{PMNS} \simeq U_{CW} \)

Is there a symmetry that can give rise to \( U_{CW} \)?

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Is there a symmetry that can give rise to \( U_{CW} \)?

For instance, \( A_4 \) Symmetry

\textsuperscript{5}N. Cabibbo, 1978
\textsuperscript{6}L. Wolfenstein, 1978
$A_4$ Symmetry

With 3 families, we need a group containing a 3 representation. The simplest one is $A_4$. 
Why $A_4$?

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Why $A_4$?

With 3 families, we need a group containing a $3$ representation. The simplest one is $A_4$. 
What is $A_4$?
What is $A_4$?

- Non-Abelian discrete group

- Four irreducible representations: **Three** 1-dimension representations called $1, 1', 1''$ and **One** 3-dimension representation called $3$
A$_4$ Symmetry

If denoting 3 as (1, 2, 3) then

**Multiplication rule**

$$3 \times 3 = 1(11 + 22 + 33) + 1'(11 + \omega^2 22 + \omega 33) + 1''(11 + \omega 22 + \omega^2 33) + 3(23, 31, 12) + 3(32, 13, 21)$$

where $\omega = e^{i2\pi/3}$

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$^7$Ernest Ma, 2007
In a standard scenario, one usually requires three Higgs doublets to couple to SM charged fermions.

LHC 125-GeV SM-like Higgs boson put a very very tight constraint on the scalar sector. So it’s hard to satisfy those data when 2 or more Higgs doublets are present in the standard scenario.
• In a standard scenario, one usually requires three Higgs doublets to couple to SM charged fermions.

• LHC 125-GeV SM-like Higgs boson put a very very tight constraint on the scalar sector. So it’s hard to satisfy those data when 2 or more Higgs doublets are present in the standard scenario.

⇒ Our model of neutrino masses: minimal EW $\nu_R$ model + 1 Higgs doublet + 2 Higgs triplets $\tilde{\chi}, \xi$ becomes more relevant.

The form of $U_{CW}$ in our work is contained in $\nu$ sector, NOT in charged lepton sector as in some generic models.
Model of neutrino masses

Assignments of the model’s content

<table>
<thead>
<tr>
<th>Field</th>
<th>$(\nu, l)_L$</th>
<th>$(\nu, l^M)_R$</th>
<th>$e_R$</th>
<th>$e^M_L$</th>
<th>$\phi_0S$</th>
<th>$\tilde{\phi}_S$</th>
<th>$\Phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_4$</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Notice: An extension to four Higgs singlet fields $\rightarrow$ No constraints from the LHC!
Neutrino Dirac mass

The Yukawa interactions

\[ L_S = \bar{l}_L \left( g_{0S} \phi_{0S} + g_{1S} \tilde{\phi}_S + g_{2S} \tilde{\phi}_S \right) l_{R}^M + h.c. \]  

(7)
Neutrino Dirac mass

The Yukawa interactions

\[ L_S = \bar{l}_L \left( g_{0S} \phi_{0S} + g_{1S} \tilde{\phi}_S + g_{2S} \tilde{\phi}_S \right) l^M_R + h.c. \]  

\[ 3 \otimes ( 1 \ 3 \ 3 )^T 3 \]

where \( g_{1S} \) and \( g_{2S} \) reflect the two different ways that \( \tilde{\phi}_S \) couples to the product of \( \bar{l}_L \) and \( l^M_R \).
Neutrino Dirac mass

The Yukawa interactions

\[
L_S = \bar{l}_L \left( g_{0S} \phi_{0S} + g_{1S} \phi_S + g_{2S} \phi_S \right) l_R^M + h.c.
\]

where \( g_{1S} \) and \( g_{2S} \) reflect the two different ways that \( \phi_S \) couples to the product of \( \bar{l}_L \) and \( l_R^M \).

Multiplication rule

\[
3 \times 3 = 1(11 + 22 + 33) + 1'(11 + \omega^2 22 + \omega 33) + 1''(11 + \omega 22 + \omega^2 33) + 3(23, 31, 12) + 3(32, 13, 21)
\]

\( ^8 \) Ernest Ma, 2007
Neutrino Dirac mass matrix:

$$M^D = \begin{pmatrix} g_0 v_0 & g_1 v_3 & g_2 v_2 \\ g_2 v_3 & g_0 v_0 & g_1 v_1 \\ g_1 v_2 & g_2 v_1 & g_0 v_0 \end{pmatrix}$$

where $v_0 = \langle \phi_0 \rangle$ and $v_i = \langle \phi_i \rangle$ with $i = 1, 2, 3$. 
Neutrino Dirac mass

If \( v_1 = v_2 = v_3 = v \sim O(10^5 \ eV) \), \( M_D^\nu \) can be diagonalized as follows

\[
U^\dagger_{\nu_L} M_D^\nu U_{\nu_R} = U^\dagger_{\nu_R} M_D^\nu U_{\nu} = \begin{pmatrix}
m_{1D} & 0 & 0 \\
0 & m_{2D} & 0 \\
0 & 0 & m_{3D}
\end{pmatrix}
\]

where \( U_{\nu} = U^{\dagger}_{CW} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \)

Notice that \( U_{\nu_L} = U_{\nu_R} = U_{\nu} \).

---

\(^9\)P.Q. Hung, 2007
The neutrino Dirac masses are

\[
\begin{align*}
    m_{1D} &= g_0 S v_0 + g_1 S v + g_2 S v \\
    m_{2D} &= g_0 S v_0 + g_1 S v \omega^2 + g_2 S v \omega \\
    m_{3D} &= g_0 S v_0 + g_1 S v \omega + g_2 S v \omega^2
\end{align*}
\]

Reality of the masses require that

\[
g_{2S} = g_{1S}^* \]

(13)
Neutrino Majorana Mass

From the Lagrangian

\[ L_M = g_M \left( l_{iR}^M, T \sigma_2 \right) (i \tau_2 \tilde{\chi}) l_{jR}^M + h.c. \quad (14) \]

Because of the constraints from 125-GeV SM-like boson, the Higgs triplet \( \tilde{\chi} \) transforms as \( 1 \). Right-handed Majorana mass matrix

\[
M_R = \begin{pmatrix}
g_M \langle \chi^0 \rangle & 0 & 0 \\
0 & g_M \langle \chi^0 \rangle & 0 \\
0 & 0 & g_M \langle \chi^0 \rangle
\end{pmatrix} = g_M v_M I \quad (15)
\]
The $3 \times 3$ see-saw mass matrix for the light neutrinos ($\nu_e, \nu_\mu, \nu_\tau$) becomes

$$m_\nu \sim -M^D_\nu M^{-1}_R M^D_\nu, T$$

(16)
Charged-lepton mass

• Charged leptons can couple to singlet Higgs field which give rise to a mass mixing between charged SM and mirror leptons. However, the mixing is very small so its contribution to the charged-lepton mass matrix can be negligible\(^\text{10}\).

• The Yukawa couplings (with Higgs doublet)

\[
L_{Yl} = g_l \bar{l} l L \Phi^2 e^R + \text{h.c.} \quad (17)
\]

\(^{10}\)P.Q. Hung, 2007
Charged-lepton mass

- Charged leptons can couple to singlet Higgs field which give rise to a mass mixing between charged SM and mirror leptons. However, the mixing is very small so its contribution to the charged-lepton mass matrix can be negligible \(^{10}\).

\(^{10}\)P.Q. Hung, 2007
Charged-lepton mass

- Charged leptons can couple to singlet Higgs field which give rise to a mass mixing between charged SM and mirror leptons. However, the mixing is very small so its contribution to the charged-lepton mass matrix can be negligible \(^{10}\).

- The Yukawa couplings (with Higgs doublet)

\[
L_{Y_l} = gl \bar{l}_L \Phi_2 e_R + h.c. \tag{17}
\]

\[
= 3 \otimes 1 \otimes 3
\]

\(^{10}\)P.Q. Hung, 2007
The charged-lepton mass matrix is

\[ M_l = g_l \frac{v_2}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]  

(18)

which gives rise to

\[ U_{IL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]  

(19)

This is not satisfactory because it causes degenerate charged leptons. We will modify this later.
Phenomenological implications

Why is $U_{PMNS}$ different from $V_{CKM}$?

Ansätz for $U_{ll}$, Toward $M_{l}M_{l}^*$

Lepton Flavor Violating (LFV) processes:
- $\mu \rightarrow e\gamma$
- $\mu$-e conversion
Why is the $U_{PMNS}$ different from the $V_{CKM}$?

$$U_{\nu L} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} ; \quad U_{IL} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The PMNS Matrix

$$U_{PMNS} = U_{\nu L}^\dagger U_{IL} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

which mainly comes from neutrino mixing matrix.
Why is the $U_{PMNS}$ different from the $V_{CKM}$?

- It has known that $V_{CKM} = U_{U,L}^\dagger U_{D,L}$ comes totally from couplings between quarks and Higgs doublet.

- We are showing that the $U_{PMNS} = U_{\nu L}^\dagger U_{IL}$ comes from
  - $U_{IL} \leftarrow$ couplings between leptons and Higgs doublet
  - $U_{\nu L} \leftarrow$ couplings between leptons and Higgs singlets
Why is the $U_{PMNS}$ different from the $V_{CKM}$?

In a nutshell

There are two different sources of PMNS matrix whereas the CKM matrix comes totally from one source.

One expects a natural difference between $V_{CKM}$ and $U_{PMNS}$. 
Ansätze for $U_{\ell L}$
Ansätz for $U_{IL}$

$A_4$ requires degenerate charged leptons $e, \mu, \tau \Rightarrow U_{IL} = I$.
Ansätz for $U_{IL}$

$A_4$ requires degenerate charged leptons $e, \mu, \tau \Rightarrow U_{IL} = \mathbb{I}$.

Charged leptons are not degenerate $\rightarrow$ Breaking $A_4$ in order to make $U_{IL}$ deviated from $\mathbb{I}$.
Ansätze for $U_{IL}$

$A_4$ requires **degenerate charged leptons** $e, \mu, \tau \Rightarrow U_{IL} = \mathbb{I}$.

Charged leptons are not degenerate $\rightarrow$ Breaking $A_4$ in order to make $U_{IL}$ deviated from $\mathbb{I}$.

We can use **Wolfenstein-like parametrization** to construct $U_{IL}$.

$$U_{IL} \rightarrow U_{IL} = \begin{pmatrix} 1 - \frac{\lambda_l^2}{2} & \lambda_l & A_l \lambda_l^3 (\rho_l - i \eta_l) \\ -\lambda_l & 1 - \frac{\lambda_l^2}{2} & A_l \lambda_l^2 \\ A_l \lambda_l^3 (1 - \rho_l - i \eta_l) & -A_l \lambda_l^2 & 1 \end{pmatrix}$$

(21)

where $A_l, \rho_l, \eta_l$ are real parameters of $O(1)$. 
Ansätz for $U_{IL}$

$$U_{PMNS} = U_{\nu L}^\dagger U_{IL} =$$

$$\frac{1}{\sqrt{3}} \begin{pmatrix}
A_l \lambda^3_l (1 - \rho_l - i\eta_l) - \frac{\lambda^2_l}{2} - \lambda_l + 1 & - (A_l + \frac{1}{2}) \lambda^2_l + \lambda_l + 1 & A_l \lambda^3_l (\rho_l - i\eta_l) + A_l \lambda^2_l + 1 \\
\omega^2 A_l \lambda^3_l (1 - \rho_l - i\eta_l) - \frac{\lambda^2_l}{2} - \omega \lambda_l + 1 & - (\omega^2 A_l + \frac{\beta}{2}) \lambda^2_l + \lambda_l + \omega & A_l \lambda^3_l (\rho_l - i\eta_l) + \omega A_l \lambda^2_l + \omega^2 \\
\omega A_l \lambda^3_l (1 - \rho_l - i\eta_l) - \frac{\lambda^2_l}{2} - \omega^2 \lambda_l + 1 & - (\omega A_l + \frac{\omega^2}{2}) \lambda^2_l + \lambda_l + \omega^2 & A_l \lambda^3_l (\rho_l - i\eta_l) + \omega^2 A_l \lambda^2_l + \omega
\end{pmatrix}$$

Combine with the experimental data, we are able to constrain parameters $A_l, \lambda_l, \rho_l, \eta_l$. 
Toward $\mathcal{M}_I \mathcal{M}_I^\dagger$

Diagonalizing mass matrices $\mathcal{M}_I$ and $\mathcal{M}_I^\dagger$ as follows.

$$U_{IL}^\dagger \mathcal{M}_I U_{IR} \quad ; \quad U_{IR}^\dagger \mathcal{M}_I^\dagger U_{IL}$$

Therefore,

$$U_{IL}^\dagger \mathcal{M}_I \mathcal{M}_I^\dagger U_{IL} = \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix}$$

Thus,

$$\mathcal{M}_I \mathcal{M}_I^\dagger = U_{IL} \cdot \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix} \cdot U_{IL}^\dagger$$
Toward $\mathcal{M}_l\mathcal{M}_l^\dagger$

$\Rightarrow$ Up to $O(\lambda_l^2)$

$$
\begin{pmatrix}
(1 - \lambda_l^2) m_e^2 + \lambda_l m_\mu^2 & \lambda_l (m_\mu^2 - m_e^2) & 0 \\
\lambda_l (m_\mu^2 - m_e^2) & (1 - \lambda_l^2) m_\mu^2 + \lambda_l m_e^2 & A\lambda_l^2 (m_\tau^2 - m_\mu^2) \\
0 & A\lambda_l^2 (m_\tau^2 - m_\mu^2) & m_\tau^2
\end{pmatrix}
$$

(22)

$A_l, \lambda_l$ are extracted from $U_{PMNS}$ and experimental values $m_e, m_\mu, m_\tau$. 
The differences between CKM and PMNS matrices come from the fact that $U_{PMNS}$ is constructed by couplings with Higgs singlets and mainly comes from neutrinos.
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The simplicity of our approach as compared with previous works is due to the source of the neutrino Dirac masses which comes from the Higgs singlets as opposed to Higgs doublets.
The differences between CKM and PMNS matrices come from the fact that $U_{PMNS}$ is constructed by couplings with Higgs singlets and mainly comes from neutrinos.

The simplicity of our approach as compared with previous works is due to the source of the neutrino Dirac masses which comes from the Higgs singlets as opposed to Higgs doublets.

By slightly breaking $A_4$ symmetry, we avoided the case of degenerate charged-lepton mass and were able to extract $\mathcal{M}_l\mathcal{M}_l^\dagger$ for the charged-lepton sector (as well as the quark sector).
$\mu \rightarrow e\gamma$
$B(\mu^+ \rightarrow e^+ \gamma) < 5.7 \times 10^{-13}$

Projected Sensitivity $= 4.0 \times 10^{-14}$
It was argued in model of neutrino masses\textsuperscript{11} that the appropriate set of singlet scalars is composed of an $A_4$-singlet $\phi_0S$ and an $A_4$-triplet $\{\phi_iS\} \ (i = 1, 2, 3)$.

The total Yukawa interactions can be written as

$$\mathcal{L}_S = -\bar{l}_L U_{PMNS}^{\dagger} \tilde{M}_\phi U_{PMNS}^M l_R^M - \bar{l}_R U_{PMNS}^{\dagger} \tilde{M}'_\phi U_{PMNS}^M l_L^M + \text{H.c.} \ (23)$$

where

- $\tilde{M}_\phi = U_{\nu}^{\dagger} M_\phi U_{\nu}$, $\tilde{M}'_\phi = U_{\nu}^{\dagger} M'_\phi U_{\nu}$ and $M'_\phi$ is the same as $M_\phi$.
- $U_{PMNS} = U_{\nu}^{\dagger} U_L^I$, $U_{PMNS}^M = U_{\nu}^{\dagger} U_R^I$
- $U'_{PMNS} = U_{\nu}^{\dagger} U_R^I$, $U'_{PMNS}^M = U_{\nu}^{\dagger} U_L^I$

\textsuperscript{11}P.Q. Hung, T. Le, JHEP 1509, 001 (2015)
$l_i \to l_j \gamma$

**Figure**: One-loop induced Feynman diagram for $l_i \to l_j \gamma$ in EW-scale $\nu_R$ model.
The relevant Yukawa couplings between the leptons, mirror leptons and the $A_4$ singlet and triplet scalars can be deduced by recasting the Lagrangian.

\[
L_S = - \sum_{k=0}^{3} \sum_{i,m=1}^{3} \left( \bar{l}_i U_{im}^L l_R^M + \bar{l}_R U_{im}^R l_L^M \right) \phi_k + H.c. \quad (24)
\]

where

\[
U_{im}^L \equiv \left( U_{PMNS}^\dagger \cdot M^k \cdot U_{PMNS}^L \right)_{im}, \quad (25)
\]

\[
= \sum_{j,n=1}^{3} \left( U_{PMNS}^\dagger \right)_{ij} M_{jn}^k \left( U_{PMNS}^L \right)_{nm}, \quad (26)
\]

and

\[
U_{im}^R \equiv \left( U'_{PMNS}^\dagger \cdot M' \cdot U_{PMNS}^L \right)_{im}, \quad (27)
\]

\[
= \sum_{j,n=1}^{3} \left( U_{PMNS}^\dagger \right)_{ij} M'_{jn}^k \left( U_{PMNS}^L \right)_{nm}. \quad (28)
\]
For the process $l_i^- (p) \to l_j^- (p') + \gamma (q)$

The amplitude

$$
\mathcal{M} \left( l_i^- \to l_j^- \gamma \right) = \epsilon^*_\mu (q) \bar{u}_j (p') \left\{ i \sigma^{\mu \nu} q_\nu \left[ C^{ij}_L P_L + C^{ij}_R P_R \right] \right\} u_i (p),
$$

(29)
For the process $l_i^- (p) \rightarrow l_j^- (p') + \gamma (q)$

- The amplitude

$$\mathcal{M} \left( l_i^- \rightarrow l_j^- \gamma \right) = \epsilon_\mu^* (q) \bar{u}_j (p') \left\{ i \sigma^{\mu \nu} q_\nu \left[ C_L^{ij} P_L + C_R^{ij} P_R \right] \right\} u_i (p) ,$$

(29)

- The partial width

$$\Gamma \left( l_i \rightarrow l_j \gamma \right) = \frac{1}{16 \pi} m_{l_i}^3 \left( 1 - \frac{m_{l_j}^2}{m_{l_i}^2} \right)^3 \left( |C_L^{ij}|^2 + |C_R^{ij}|^2 \right) .$$

(30)
\begin{align}
C^L_{ij} &= + \frac{e}{16\pi^2} \sum_{k=0}^{3} \sum_{m=1}^{3} \left\{ \frac{1}{m^2_{l^M_m}} \left[ m_i U^{R^k}_{jm} (U^{R^k}_{im})^* + m_j U^{L^k}_{jm} (U^{L^k}_{im})^* \right] I \left( \frac{m^2_{\phi_{KS}}}{m^2_{l^M_m}} \right) \\
&\quad + \frac{1}{m^2_{l^M_m}} U^{R^k}_{jm} (U^{L^k}_{im})^* J \left( \frac{m^2_{\phi_{KS}}}{m^2_{l^M_m}} \right) \right\}, \quad (31) \\
C^R_{ij} &= + \frac{e}{16\pi^2} \sum_{k=0}^{3} \sum_{m=1}^{3} \left\{ \frac{1}{m^2_{l^M_m}} \left[ m_i U^{L^k}_{jm} (U^{L^k}_{im})^* + m_j U^{R^k}_{jm} (U^{R^k}_{im})^* \right] I \left( \frac{m^2_{\phi_{KS}}}{m^2_{l^M_m}} \right) \\
&\quad + \frac{1}{m^2_{l^M_m}} U^{L^k}_{jm} (U^{R^k}_{im})^* J \left( \frac{m^2_{\phi_{KS}}}{m^2_{l^M_m}} \right) \right\}. \quad (32)
\end{align}
Considering $m_{\ell i m} \gg m_{i,j}$ and setting $m_{i,j} \to 0$.

\begin{align*}
\mathcal{I}(r) &= \frac{1}{12(1 - r)^4} \left[ -6r^2 \log r + r(2r^2 + 3r - 6) + 1 \right] , \\
\mathcal{J}(r) &= \frac{1}{2(1 - r)^3} \left[ -2r^2 \log r + r(3r - 4) + 1 \right] .
\end{align*}
The branching ratio $B(\mu \to e\gamma)$ is given by

$$B(\mu \to e\gamma) = \tau_\mu \cdot \Gamma (l_i \to l_j \gamma)$$  \hspace{1cm} (35)$$

where $\tau_\mu$ is the lifetime of the muon

$$\tau_\mu = (2.1969811 \pm 0.0000022) \times 10^{-6} \text{ s}$$ \hspace{1cm} (36)$$
Numerical Analysis

- For the masses of the singlet scalars $\phi_k S$
  
  $$m_{\phi_0 S} : m_{\phi_1 S} : m_{\phi_2 S} : m_{\phi_3 S} = M_S : 2M_S : 3M_S : 4M_S$$
  with $M_S = 10$ MeV.

- For the masses of the mirror lepton $l_m^M$
  
  $$m_{l_m^M} = M_{\text{mirror}} + \delta_m$$
  with $\delta_1 = 0$, $\delta_2 = 10$ GeV, $\delta_3 = 20$ GeV and $100$ GeV $\leq M_{\text{mirror}} \leq 800$ GeV

- Scenario 1 $U_{PMNS}^M = U'_{PMNS} = U''_{PMNS} = U_{PMNS}^\dagger$

- Scenario 2 $U_{PMNS}^M = U'_{PMNS} = U''_{PMNS} = U_{PMNS}$
Figure: Contour plots of $\log_{10} B(\mu \rightarrow e \gamma)$ on the $(\log_{10}(g_0 S), M_{\text{mirror}})$ plane for normal (left panel) and inverted (right panel) hierarchy in scenarios 1 (red curves) and 2 (blue curves) with $g_{0S} = g'_{0S}$ and $g_{1S} = g'_{1S} = 0$. 

Numerical Analysis

Some examples
Figure: Same as previous figure with $g_{0S} = g^{'}_{0S} = g_{1S} = g^{'}_{1S}$ instead.
In our analysis, we are showing that constraints from $\mu \rightarrow e\gamma$ imply Yukawa couplings $< 10^{-3}$.

- The small couplings now brings the singlet VEV up to $\mathcal{O}(100 \text{ MeV})$ or even $\mathcal{O}(1 \text{ GeV})$. There does not appear to be much of a hierarchy problem in EW-$\nu_R$ model.

Search for mirror quarks at the LHC
In our analysis, we are showing that constraints from $\mu \rightarrow e\gamma$ imply Yukawa couplings $< 10^{-3}$.

- The small couplings now brings the singlet VEV up to $\mathcal{O}(100 \text{ MeV})$ or even $\mathcal{O}(1 \text{ GeV})$. There does not appear to be much of a hierarchy problem in EW-$\nu_R$ model.

- Due to small couplings, searching for mirror particles of this model at the LHC would be quite interesting since they might decay outside the beam pipe and inside silicon vertex detectors.

**Search for mirror quarks at the LHC**

$\mu$-e conversion
SINDRUM II: $B(\mu^- + Au \rightarrow e^- + Au) < 7 \times 10^{-13}$

SINDRUM II: $B(\mu^- + Ti \rightarrow e^- + Ti) < 6.1 \times 10^{-13}$

Projected Sensitivity for $B(\mu^- + Al \rightarrow e^- + Al)$

Mu2e: $6 \times 10^{-17}$

COMET: $3 \times 10^{-17}$
Effective Lagrangian for $\mu$-e Conversion

\begin{align*}
\mathcal{L}_{\text{eff}} &= - \frac{1}{\Lambda^2} \left[ \left( C_{D\mu} m_\mu \bar{e} \sigma^{\alpha\beta} P_L \mu + C_{DL} m_\mu \bar{e} \sigma^{\alpha\beta} P_R \mu \right) F_{\alpha\beta} \right. \\
&+ \sum_{q=u,d,s} \left( C_{VR}^{(q)} \bar{e} \gamma^\alpha P_R \mu + C_{VL}^{(q)} \bar{e} \gamma^\alpha P_L \mu \right) \bar{q} \gamma^\alpha q \\
&+ \sum_{q=u,d,s} m_\mu m_q G_F \left( C_{SR}^{(q)} \bar{e} P_R \mu + C_{SL}^{(q)} \bar{e} P_L \mu \right) \bar{q} q \\
&+ m_\mu \left( C_{GQR} G_F \bar{e} P_L \mu + C_{GQL} G_F \bar{e} P_R \mu \right) \frac{\beta_L}{2 g_3^2} G^{a\alpha\beta} G_{a\alpha\beta} + \text{H.c.} \right] \\
&= (37)
\end{align*}

where $C_{D(L,R)}$, $C_{V(L,R)}^{(q)}$, $C_{S(L,R)}^{(q)}$ and $C_{GQ(L,R)}$ are dimensionless coupling constants depending on specific LFV model.
\( \mu \)-\( e \) Conversion

Conversion rate (general formula) \(^{12}\)

\[
\Gamma_{\text{conv}} = \frac{m_\mu^5}{4\Lambda^4} \left( |C_{DR}D + 4\tilde{C}_{VR}^{(p)}V^{(p)} + 4\tilde{C}_{VR}^{(n)}V^{(n)} + 
\left. + 4G_F m_\mu \left( m_p \tilde{C}_{SR}^{(p)} S^{(p)} + m_n \tilde{C}_{SR}^{(n)} S^{(n)} \right) \right| \right)^2 \right. 
\]

\[
+ |C_{DL}D + 4\tilde{C}_{VL}^{(p)}V^{(p)} + 4\tilde{C}_{VL}^{(n)}V^{(n)} + 
\left. + 4G_F m_\mu \left( m_p \tilde{C}_{SL}^{(p)} S^{(p)} + m_n \tilde{C}_{SL}^{(n)} S^{(n)} \right) \right| \right)^2 \right) \). \quad (38)
\]

where D, V, S are overlap integrals of the relativistic wave functions of \( \mu \) and e in the electric field of nucleus.

\(^{12}\)R. Kitano, M. Koike, Y. Okada (2007)
Contributions to the conversion rate

- Photonic contributions
  \[ \mu^-(p) \rightarrow e^-(p')\gamma^*(q) \] with an off-shell photon.

- Four-fermion coupling constants from
  - \( \gamma \) exchange
  - \( Z \) exchange
  - box diagrams
  - scalar Higgs exchange
Contributions to the conversion rate

- Photonic contributions
  \[ \mu^-(p) \rightarrow e^-(p') \gamma^*(q) \] with an off-shell photon.

- Four-fermion coupling constants from
  - \( \gamma \) exchange
  - \( Z \) exchange
  - box diagrams
  - scalar Higgs exchange
$$\mu-e \text{ Conversion}$$

The formula for the conversion rate (from $\gamma$ contributions ONLY)

$$\Gamma_{\text{conv}} \simeq \frac{m_\mu^5}{4\Lambda^4} \left( \left| C_{DR} D + 4 \tilde{C}^{(p)}_{VR} V^{(p)} + 4 \tilde{C}^{(n)}_{VR} V^{(n)} \right| \right)^2$$

$$+ \left| C_{DL} D + 4 \tilde{C}^{(p)}_{VL} V^{(p)} + 4 \tilde{C}^{(n)}_{VL} V^{(n)} \right|^2 \right) \ . \quad (39)$$

Question

Is there any relation between $\mu$-e conversion and $\mu \rightarrow e\gamma$?
μ-e Conversion

The formula for the conversion rate (from γ contributions ONLY)

\[ \Gamma_{\text{conv}} \simeq \frac{m^5_\mu}{4\Lambda^4} \left( \left| C_{DR} D + 4 \tilde{C}^{(p)}_V V^{(p)} + 4 \tilde{C}^{(n)}_V V^{(n)} \right|^2 
+ \left| C_{DL} D + 4 \tilde{C}^{(p)}_V V^{(p)} + 4 \tilde{C}^{(n)}_V V^{(n)} \right|^2 \right) . \] (39)

Question

Is there any relation between μ-e conversion and μ → eγ?

THE ANSWER IS YES
\( \mu \rightarrow e \gamma \) Conversion and \( \mu \rightarrow e \gamma \)

The relation is

\[
\Gamma_{\gamma^*}^{\text{conv}}(q^2 \rightarrow 0) \approx \pi D^2 \Gamma_{\gamma} \tag{40}
\]

So in terms of the branching ratio, we have

\[
B_{\mu N \rightarrow eN} = \frac{\Gamma_{\gamma^*}^{\text{conv}}}{\Gamma_{\text{capt}}} = \pi D^2 \frac{\Gamma_{\mu}}{\Gamma_{\text{capt}}} B_{\mu \rightarrow e\gamma} \tag{41}
\]
$\mu$-e Conversion

\begin{center}
\includegraphics[width=\textwidth]{contour_plot.png}
\end{center}

**Figure**: Contour plots of $\log_{10} B(\mu \rightarrow e \text{ conversion})$ on the $(\log_{10}(g_0S), M_{\text{mirror}})$ plane for normal hierarchy in scenario 1 (left panel) and scenario 2 (right panel) with $g_0S = g'_0S$ and $g_1S = g'_1S = 10^{-2} g_0S$. 

Trinh Le (UVA)  
HEP Seminar  
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Let's zoom in
μ-e Conversion

Let's zoom in

\[ \text{Mirror Lepton Mass } M_{\text{mirror}} (\text{GeV}) \]

\[ \text{Log}_{10} B(\mu \rightarrow e_{\text{conv Scen1}}) \text{ (Normal)} \]

- **Al_COMET**: BR = $3 \times 10^{-17}$
- **Al_Mu2e**: BR = $6 \times 10^{-17}$
- **Ti_Sindrum2**: BR = $6.1 \times 10^{-13}$
- **Au_Sindrum2**: BR = $7 \times 10^{-13}$
- **MEG**: BR = $5.7 \times 10^{-13}$
Figure: Contour plots of $\log_{10} B(\mu \rightarrow e \text{ conversion})$ on the $(\log_{10}(g_{0S}), M_{\text{mirror}})$ plane for normal hierarchy in scenario 1 (left panel) and scenario 2 (right panel) with $g_{0S} = g'_{0S} = g_{1S} = g'_{1S}$.
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$\mu$-$e$ Conversion in Mirror Fermion Model with Electroweak Scale Right-handed Neutrinos,
P.Q. Hung, T. Le, V.Q. Tran and T.C. Yuan (paper in preparation).
On-going project

We are working on a Quarks Project using the similar ansätz that we have made for leptons.

Stay tuned. We have more things coming up.
Conclusion

- We present a model of neutrino masses in the framework of the Electroweak scale Right-handed neutrinos (EW-$\nu_R$) model, which is constructed with a horizontal $A_4$ symmetry. Such a model has several interesting phenomenological implications.

- We not only obtain the experimentally desired form of the PMNS matrix but also provide an explanation of why $U_{PMNS}$ is very different from $V_{CKM}$. By making a simple ansatz we extract $M_l M_l^{\dagger}$ for the charged lepton sector. A similar ansatz is proposed for the quark sector.

- The one-loop induced lepton flavor violating radiative decays $l_i \rightarrow l_j \gamma$ and $\mu$-e conversion in an extended mirror model might be related to each other under a good approximation that we have established.

- Implications concerning the possible detection of mirror leptons at the LHC and the ILC as well as future searches for $\mu$-e conversion at Fermilab and J-PARC COMET are also discussed.
1. Characters of $A_4$ representations

where $\omega = e^{i\frac{2\pi}{3}}$ which is the cube root of unity.
Appendix

(1) \[0.779 < \frac{1}{\sqrt{3}} |A\lambda^3(1 - \rho - i\eta) - \frac{\lambda^2}{2} - \lambda + 1| < 0.848\]

(2) \[0.510 < \frac{1}{\sqrt{3}} |-\left(A + \frac{1}{2}\right)\frac{\lambda^2}{2} + \lambda + 1| < 0.604\]

(3) \[0.122 < \frac{1}{\sqrt{3}} |A\lambda^3(\rho - i\eta) + A\lambda^2 + 1| < 0.190\]

(4) \[0.183 < \frac{1}{\sqrt{3}} |\omega^2A\lambda^3(1 - \rho - i\eta) - \frac{\lambda^2}{2} - \omega\lambda + 1| < 0.568\]

(5) \[0.385 < \frac{1}{\sqrt{3}} |-\left(\omega A + \frac{\omega}{2}\right)\frac{\lambda^2}{2} + \lambda + \omega| < 0.728\]

(6) \[0.613 < \frac{1}{\sqrt{3}} |A\lambda^3(\rho - i\eta) + \omega A\lambda^2 + \omega^2| < 0.794\]

(7) \[0.200 < \frac{1}{\sqrt{3}} |\omega A\lambda^3(1 - \rho - i\eta) - \frac{\lambda^2}{2} - \omega^2\lambda + 1| < 0.576\]

(8) \[0.408 < \frac{1}{\sqrt{3}} |-\left(\omega A + \frac{\omega^2}{2}\right)\frac{\lambda^2}{2} + \lambda + \omega^2| < 0.742\]

(9) \[0.589 < \frac{1}{\sqrt{3}} |A\lambda^3(\rho - i\eta) + \omega^2 A\lambda^2 + \omega| < 0.775\]

\[-4.8517 < A < -4.4580, \quad -0.2404 < \lambda < -0.1882,\]

\[-5.6339 < \rho < -5.5712, \quad -4.7160 < \eta < 4.8912\]
3. Sample numerical results
Taking upper limit values of $A = -4.4580$, $\lambda = -0.1882$, $\rho = -5.5712$ and $\eta = 4.8912$

$$U_l = \begin{pmatrix} 0.9823 & -0.1882 & -0.1656 - 0.1454i \\ 0.1882 & 0.9823 & -0.1579 \\ 0.1953 - 0.1454i & 0.1579 & 1 \end{pmatrix}$$

$$U_l U_l^\dagger = \begin{pmatrix} 1.0489 & 0.0261 + 0.0230i & -0.0035 - 0.0026i \\ 0.0261 - 0.0230i & 1.0253 & 0.0340 + 0.0274i \\ -0.0035 + 0.0026i & 0.0340 - 0.0274i & 1.0842 \end{pmatrix}$$

$$\simeq I$$
Using the above numerical $U_l$ and putting in the values of $m_e = 0.51 \times 10^{-3}$ GeV, $m_\mu = 0.1057$ GeV and $m_\tau = 1.7768$ GeV we get

$$M_l M_l^\dagger \simeq \begin{pmatrix}
0.1537 & 0.0805 + 0.0725i & -0.5231 - 0.4590i \\
0.0805 - 0.0725i & 0.0895 & -0.4968 \\
-0.5231 + 0.4590i & -0.4968 & 3.1573
\end{pmatrix}$$
Appendix

4. Possible signature of EW $\nu_R$ model

The fact

1. $\nu_R$ interacts with the W and Z (part of a doublet)
2. Both $\nu_R$ and $e_R^M$ interact with $\nu_L$ and $e_L$ through the singlet scalar field $\phi_S$

Since $m_{\phi_S} \sim O(10^5 \text{ eV})$, it’s possible

\[
\begin{align*}
\nu_R & \rightarrow \nu_L + \phi_S \\
e_R^M & \rightarrow e_L + \phi_S
\end{align*}
\]

If $m_{\nu_R} \lesssim m_{e_R^M}$:

\[
\begin{align*}
e_M^R & \rightarrow \nu_R + e_L + \bar{\nu}_L \\
\nu_R & \rightarrow \nu_L + \phi_S
\end{align*}
\]
Possible signature of EW $\nu_R$ model

The heaviest $\nu_R$ could be pair produced

$$q + \bar{q} \rightarrow Z \rightarrow \nu_R + \nu_R$$

$$\nu_R \rightarrow e_R^M + W^*(W)$$

$$e_R^M \rightarrow e_L + \phi_S$$

at a 'displaced' vertex.

If $\nu_R$ is Majorana

$$e_R^{M, -} + W^+ + e_R^{M, -} + W^+ \rightarrow e_L + e_L + W^+ + W^+ + 2\phi_S$$

same-sign dilepton event which is distinctively different from the Dirac case!
5. How to stop neutrinos?

**Q:** If one uses a wall of lead, how thick should it be to stop a beam of neutrinos?

**A:** Typical low energy (MeV) cross section $\sigma \approx 10^{-47} \ m^2$. Mean free path for neutrinos going through e.g. lead:

- Number density of nucleons in Pb: $n = \frac{11400 \ \text{kg}/m^3}{1.76 \times 10^{-27} \ \text{kg}}$

- Number of interaction per meter:
  $$\sigma \times n = 10^{-47} \ m^2 \times \frac{11400 \ \text{kg}/m^3}{1.76 \times 10^{-27} \ \text{kg}}$$

- Mean free path: $\lambda = \frac{1}{\sigma \times n} \approx 1.5 \times 10^{17} \ m \approx 1.6 \ light \ years$