Atom Interferometry
Measurements of Atomic Polarizabilities and Tune-out Wavelengths

Alex Cronin, University of Arizona

UVA Physics Colloquium March 18, 2016
Our Atom Beam Interferometer

- Space Domain
- Mach-Zehnder
- Nanogratings
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Applications for Atom Interferometry:

- Inertial sensing:
- Atomic properties:
- Quantum phenomena:
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Applications for Atom Interferometry:

- Inertial sensing: $g$, $\Omega$, $\nabla g$, $\Delta \Omega$, $G$, WEP tests, Grav. waves?, $DE$?, $DDM$?
- Atomic properties: $\alpha(0)$, $\tau$, $\langle |D| i \rangle$, $f_{ik}$, $C_6$, $\alpha(\omega)$, $\lambda_{zero}$, $C_3$, $C_6$, hyperpol. $\gamma$?
- Quantum phenomena: (de)coherence, topological phases, $h/m$, $\alpha_{fs}$
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Applications for Atom Interferometry:

- Inertial sensing: $g$, $\Omega$, $\nabla g$, $\Delta \Omega$, $G$, WEP tests, Grav. waves?, $DE$?, $DDM$?
- Atomic properties: $\alpha(0)$, $\tau$, $\langle |D|D| \rangle$, $f_{ik}$, $C_6$, $\alpha(\omega)$, $\lambda_{zero}$, $C_3$, $C_6$, hyperpol. $\gamma$?
- Quantum phenomena: (de)coherence, topological phases, $\hbar/m$, $\alpha_{fs}$

“Atom interferometry’s precision could make it the Swiss Army knife of physics”
-- Science News
Our Atom Beam Interferometer

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Gaussian Schell Model (GSM) Atom Beam Simulations

Parameters:
- atom $\lambda$ dB
- grating period
- Transverse coherence
- Longitudinal coherence
- Beam width
- dB wavefront curvature

References:
- PRA 78, 013601 (2008)
- PRA 89, 033612 (2014)
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Parameters:
- atom $\lambda dB$
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- Transverse coherence
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*PRA* 78, 013601 (2008)
*PRA* 89, 033612 (2014)
Nanograting w/ gap for C₃ expt
Atom Beam Interferometer Machine

Phase chopper

1m

Position [nm]

Intensity [kcopnts/sec]

C = 24.7%
University of Arizona Atom Interferometry Laboratory
Front Row (from left): Alex Cronin, Ivan Hromada, Raisa Trubko, Maxwell Gregoire
Back Row (from left): Will Holmgren, James Greenberg
Atomic Polarizability ($\alpha$) Experiment

\[ U = -\frac{1}{2} \alpha E^2 \]
Tune Out Wavelength ($\lambda_{\text{zero}}$) Experiment

Interaction Laser for $\lambda_{\text{zero}}$ measurement

\[ U = -\frac{\alpha(\omega)I}{2\varepsilon_0 c} \]

phase shift (rad)

wavelength (nm)
Van der Waals ($C_3$) Experiment

Interaction Grating for vdW $C_3$ studies

$$U = -\frac{C_3}{r^3}$$
Physics Motivations:

- Meas. of $\alpha(0)$ let us report lifetimes $\tau$, dipole matrix elements $\langle k | r | i \rangle$, and $f_{ik}$
- Combinations of C6 and $\alpha(0)$ measurements let us report $\alpha_{\text{core}}$
- Meas. of $\lambda_{\text{zero}}$ let us report ratios of dipole matrix elements $\langle P_{3/2} | r | S_{1/2} \rangle / \langle P_{1/2} | r | S_{1/2} \rangle$
- Meas. of $\alpha(0)$, $C_3$, and $\lambda_{\text{zero}}$ probe different functions of oscillator strengths $f_{ik}$
Measurements of $\alpha(0)$, $C_3$, and $\lambda_{\text{zero}}$ probe different functions of atomic oscillator strengths $f$.

$$\alpha_i(\omega) = \frac{e^2}{m} \sum \frac{f_{ik}}{\omega_{ik}^2 - \omega^2}$$

$$\alpha_i(0) = \frac{e^2}{m} \sum \frac{f_{ik}}{\omega_{ik}^2}$$

$$C_3 = \frac{\hbar}{4\pi} \int \alpha(i\omega)g(i\omega)$$

$$C_3 = \frac{\hbar e^2}{8m} \sum \frac{f_{ik}}{\omega_{ik}}$$

$$\alpha(\omega_{\text{zero}}) = 0$$

$$\frac{\omega_{\text{zero}}^2 - \omega_1^2}{\omega_2^2 - \omega_{\text{zero}}^2} \approx \frac{f_1}{f_2}$$
Measurements of $\alpha(0)$, $C_3$, and $\lambda_{\text{zero}}$ probe different functions of atomic oscillator strengths $f$. 

\[
\alpha_i(\omega) = \frac{e^2}{m} \sum \frac{f_{ik}}{\omega_{ik}^2 - \omega^2}
\]

$C_3 = \frac{\hbar}{4\pi} \int \alpha(i\omega)g(i\omega)$

$\lambda_{\text{zero}}$ in more detail.

$\alpha(\omega)=0$ at $\omega = 2\pi c / \lambda_{\text{zero}}$
Measurements of $\alpha(0)$, $C_3$, and $\lambda_{\text{zero}}$ probe different functions of atomic oscillator strengths $f$.

\[ \alpha_i(\omega) = \frac{e^2}{m} \sum f_{ik} \frac{\omega_i^2}{\omega_{ik}^2 - \omega^2} \]

\[ C_3 = \frac{\hbar}{4\pi} \int \alpha(i\omega)g(i\omega) \]

$\lambda_{\text{zero}}$ in more detail.

$\alpha(\omega) = 0$ at $\omega = 2\pi c/\lambda_{\text{zero}}$
Historical measurements of $\alpha_{\text{Cs}}$

<table>
<thead>
<tr>
<th>$\alpha$ [Å$^3$]</th>
<th>Year</th>
<th>Author</th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>1934</td>
<td>Stark</td>
</tr>
<tr>
<td>51</td>
<td>1954</td>
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<td>53.7</td>
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<tr>
<td>48</td>
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<tr>
<td>52.5</td>
<td>1961</td>
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</tr>
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<td>61.3</td>
<td>1993</td>
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</tr>
<tr>
<td>59.6</td>
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<td>Molof</td>
</tr>
<tr>
<td>63.3</td>
<td>1974.5</td>
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</tr>
<tr>
<td>59.26</td>
<td>1999</td>
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<tr>
<td>58.68</td>
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<tr>
<td>59.13</td>
<td>2013</td>
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<td>59.42</td>
<td>2003</td>
<td>Amini+Gould</td>
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<tr>
<td>63.72</td>
<td>1999.5</td>
<td>Lim</td>
</tr>
<tr>
<td>59.42</td>
<td>1993</td>
<td>Fuentealba</td>
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<tr>
<td>59.8</td>
<td>2011</td>
<td>Holmgren NJ</td>
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<tr>
<td>59.72</td>
<td>2014</td>
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<td>59.04</td>
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<td>58.38</td>
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<td>60.61</td>
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<td>Patil+Tanq</td>
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<td>66.5</td>
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<td>Reitz</td>
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<td>61</td>
<td>1965</td>
<td>Crown</td>
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<td>64.31</td>
<td>2002</td>
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<tr>
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<tr>
<td>59.57</td>
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<td>Knize SBT11</td>
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<td>59.36</td>
<td>2010</td>
<td>Porsev PBD1</td>
</tr>
</tbody>
</table>

\[ U = -\frac{1}{2}\alpha E^2 \]

\[
\alpha_i(0) = \frac{e^2}{m} \sum_k \frac{f_{ik}}{\omega_{ik}^2} = \frac{e^2}{3} \sum_k \frac{|\langle i | r | k \rangle|^2}{E_i - E_k}
\]
Historical polarizability measurement techniques

• Beam deflection
  • Miller and Bederson (1974)
  • Hall and Zorn (1974)

• Interferometry
  • Pritchard (1995)
  • Vigue (2006)
Our 2010 E-field apparatus

Two paths through interferometer

E-fields cause a phase $\phi$ for atom waves that depends on atomic polarizability $\alpha$.

$$\phi_\Gamma = \frac{\alpha}{2\hbar v} \int E^2 d\ell$$

Differential phase shift $\phi_{\Gamma 2} - \phi_{\Gamma 1}$ depends on $\alpha$ and $v^2$. 
Phase and contrast measurements with cylinder-plane electrodes

\[ \alpha_K = 43.73 \pm 0.04 \]

\[ \phi = \alpha \frac{V^2}{\nu^2} f(x) \]

To report polarizability \( \alpha \)

Get \( f(x) \) from apparatus geometry.

Measure \( \phi \) and velocity distribution \( P(\nu) \)
Our 2015 $\alpha(0)$ measurement apparatus: 2-cylinders & 2 phase choppers
Contrast vs phase due to E-gradient enables measurement of $\Omega_e$ to 5%, or $g\sin\theta$ to 2%.

BS Thesis 2014 by James Greenberg

- Broad $P(v)$ sharpens C peak
- Can measure both $\Omega_e$ and $g$ by using multiple $v$
- Contrast loss can also affect Polarizability measurements
- E-gradient can improve dynamic range of aIFM gyroscopes (Dispersion Compensation)

Figure 3.1: Two sets of raw contrast vs. phase data with different beam conditions and Gaussian fits. $\Phi_{C_{\text{max}}}$ is denoted by the dotted vertical lines. The larger $v_0$ leads to a smaller $\Phi_{C_{\text{max}}}$. Larger $v_r$ leads to slower loss of contrast.
20 hours of velocity and $\alpha_K$ measurements

TABLE III. Absolute measurements of Cs, Rb, and K static, ground-state polarizabilities.

<table>
<thead>
<tr>
<th>Atom</th>
<th>$v_0$ (m/s)</th>
<th>$v_r$</th>
<th>$\alpha_{\text{stat.}}$(sys.) (Å$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cs</td>
<td>1585</td>
<td>19.8</td>
<td>59.45(3)(11)</td>
</tr>
<tr>
<td>Rb</td>
<td>1890</td>
<td>22.9</td>
<td>47.44(3)(9)</td>
</tr>
<tr>
<td>K</td>
<td>2113</td>
<td>13.2</td>
<td>42.97(2)(8)</td>
</tr>
</tbody>
</table>

TABLE IV. Ratio measurements of Cs, Rb, and K static, ground-state polarizabilities. The systematic errors in each ratio, which arise from the fact that the systematic errors in different measurements are not perfectly correlated, are negligible compared to the statistical errors.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Value(stat.)</th>
<th>Sys. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{\text{Cs}}/\alpha_K$</td>
<td>1.3835(9)</td>
<td>$4 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$\alpha_{\text{Cs}}/\alpha_{\text{Rb}}$</td>
<td>1.2532(10)</td>
<td>$5 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>$\alpha_{\text{Rb}}/\alpha_K$</td>
<td>1.1040(9)</td>
<td>$3 \cdot 10^{-5}$</td>
</tr>
</tbody>
</table>
3 methods to measure atom beam velocity

• Atom diffraction
• Phase Choppers
• Pulsed beam TOF
Pulsed atom beam velocity measurement with TOF

Figure 2.8: TOF measurement for a metastable argon beam. Two fits are made to the data: A Gaussian fit in black and a convolution of Equation 2.25 with a 0.35 ms impulse and metastable lifetime decay function in blue. The corresponding velocity distribution results are shown in the boxes.

Figure 2.1: A comparison of the Maxwell-Boltzmann, effusive, and supersonic distribution functions for a sodium source with reservoir temperature of 800 K. Mach numbers of 5, 10, and 20 are used for the supersonic distributions.

TOF data $\Rightarrow$ Velocity distributions $P(v)$ $\frac{\delta v}{v} \leq 0.8\%$
Atom beam velocity measurement with diffraction

Measure de Broglie wavelength using atom beam diffraction

\[ \lambda_{dB} = \frac{\hbar}{mv} \]

\[ \frac{\delta v}{v} \leq 0.25\% \]
Phase choppers for velocity measurement

- Pulsed electric fields periodically apply $\pi$ phase shifts.
- Interferometer contrast oscillates as we change the chopping frequency $f$.
- Max contrast when $f = n \frac{v}{L} = n f_0$.

Published in: Holmgren et al. NJP 2011
Velocity measurement with phase choppers

\[ \frac{\delta v}{v} \leq 0.06\% \]

Cesium (3/17/2011)

\[ v_0 = 1654.5(9) \text{ m/s} \]
\[ r = 26.9(9) \]
Error budget for Our 2015 polarizability Measurements.

- $w_{\text{det}} = 100 \pm 3 \, \mu m$
- $d_g = 99.90 \pm 0.5 \, \text{nm}$
- $V_{\text{pillars}}: \delta V/V = 0.0005$
- $a_{\text{pillars}} = 1999.9 \pm 0.5 \, \mu m$
- $R_{\text{pillars}} = 6350 \pm 0.5 \, \mu m$
- $v_0: \delta v_0/v_0$ given in Fig. 5
- $v_r: \delta v_r/v_r$ given in Fig. 5
- $Z_{g1,\text{pillars}} = 833.50 \pm 0.25 \, \text{mm}$
- dimer fraction $< 0.01$
- total sys. error $\times 1000$

$\delta \alpha/\alpha \times 1000$
2015 Results: $\alpha$ measurements for Cs, Rb, and K with 0.2% unc.
2015 Results: $\alpha$ measurements for Cs, Rb, and K with 0.2% unc.
Our measured ratios of polarizabilities serve as a benchmark test for theoretical calculations [c]-[o]:

- Holmgren et al. PRA 81, 053607 (2010)

- [p] theory from DJSB99 without core electrons
Polarizability $\alpha$ in terms of oscillator strengths ($f_{ik}$), lifetimes ($\tau$), dipole matrix elements $<k| r |i>$, and line strengths $S$

\[ \alpha_i(\omega) = \frac{e^2}{m} \sum_{k \neq i} \frac{f_{ik}}{\omega_{ik}^2 - \omega^2} \]

\[ \alpha_i(\omega) = 2 \pi e_0 c^3 \sum_{k \neq i} \frac{A_{ik} \omega_{ik}^{-2}}{\omega_{ik}^2 - \omega^2} \frac{g_k}{g_i} \]

\[ \alpha_i(\omega) = \frac{2}{3 \hbar} \sum_{k \neq i} \frac{|\langle k| e \vec{r} |i \rangle|^2 \omega_{ik}}{\omega_{ik}^2 - \omega^2} \]

\[ \alpha_i(\omega) = \frac{1}{3 \hbar} \sum_{k \neq i} \frac{S_{ik} \omega_{ik}}{\omega_{ik}^2 - \omega^2} \]

\[ \alpha(0) = \frac{e^2}{m} \left[ \frac{f_{1/2}}{\omega_{D1}^2} + \frac{f_{3/2}}{\omega_{D2}^2} \right] + \alpha_r \]

\[ \alpha(0) = 2 \pi e_0 c^3 \left[ \frac{\tau_{1/2}^{-1}}{\omega_{D1}^4} + \frac{2 \tau_{3/2}^{-1}}{\omega_{D2}^4} \right] + \alpha_r \]

\[ \alpha(0) = \frac{1}{3} \left( \frac{|D_{D1}|^2}{\hbar \omega_{D1}} + \frac{|D_{D2}|^2}{\hbar \omega_{D2}} \right) + \alpha_r \]

\[ \alpha(0) = \frac{1}{3 \hbar} \left[ \frac{S_{D1}}{\omega_{D1}} + \frac{S_{D2}}{\omega_{D2}} \right] + \alpha_r \]

$\alpha_r = \alpha_{\nu'} + \alpha_{\text{core}} + \alpha_{\text{cv}}$
Results based on our 2015 Polarizability measurements

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Cs</th>
<th>Rb</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{1/2}$</td>
<td>4.510(4)</td>
<td>4.255(4)</td>
<td>4.116(4)</td>
</tr>
<tr>
<td>$D_{3/2}$</td>
<td>6.347(6)</td>
<td>5.989(6)</td>
<td>5.793(6)</td>
</tr>
<tr>
<td>$\tau_{1/2}$ (ns)</td>
<td>34.75(6)</td>
<td>27.39(5)</td>
<td>26.61(5)</td>
</tr>
<tr>
<td>$\tau_{3/2}$ (ns)</td>
<td>30.34(6)</td>
<td>26.15(5)</td>
<td>26.51(5)</td>
</tr>
<tr>
<td>$f_{1/2}$</td>
<td>0.3453(6)</td>
<td>0.3459(7)</td>
<td>0.3342(6)</td>
</tr>
<tr>
<td>$f_{3/2}$</td>
<td>0.7179(14)</td>
<td>0.6981(14)</td>
<td>0.6649(13)</td>
</tr>
<tr>
<td>$S_{1/2}$</td>
<td>20.34(4)</td>
<td>18.11(5)</td>
<td>16.94(3)</td>
</tr>
<tr>
<td>$S_{3/2}$</td>
<td>40.29(8)</td>
<td>35.87(7)</td>
<td>33.56(6)</td>
</tr>
<tr>
<td>$C_6$</td>
<td>6877(23)</td>
<td>4734(31)</td>
<td>3891(21)</td>
</tr>
<tr>
<td>$\alpha_{NP_{1/2}}$ ($\text{Å}^3$)</td>
<td>196.87(11)</td>
<td>120.38(9)</td>
<td>89.96(8)</td>
</tr>
</tbody>
</table>

\[
f_{1/2} = \frac{(\alpha - \alpha_r)}{\left(e^2/m\omega_{D1}^2\right)} \left(\frac{1}{1 + R\omega_{D1}/\omega_{D2}}\right)
\]

\[
f_{3/2} = \frac{(\alpha - \alpha_r)}{\left(e^2/m\omega_{D2}^2\right)} \left(\frac{1}{R^{-1}\omega_{D2}/\omega_{D1} + 1}\right)
\]

\[
\tau_{1/2} = \frac{2\pi\varepsilon_0 c^3}{[\alpha(0) - \alpha_r]} \left[\frac{1}{\omega_{D1}^4} + \frac{R}{\omega_{D2}^3\omega_{D1}}\right]
\]

\[
\tau_{3/2} = \frac{2\pi\varepsilon_0 c^3}{[\alpha(0) - \alpha_r]} \left[\frac{2}{R\omega_{D1}^3\omega_{D2}^2} + \frac{2}{\omega_{D2}^4}\right]
\]

\[
|D_{D1}|^2 = \frac{3[\alpha - \alpha_r]}{\frac{1}{\hbar\omega_{D1}} + \frac{R}{\hbar\omega_{D2}}}
\]

\[
|D_{D2}|^2 = \frac{3[\alpha - \alpha_r]}{\frac{1}{R\hbar\omega_{D1}} + \frac{1}{\hbar\omega_{D2}}}
\]

\[
R \equiv \frac{S_{D2}}{S_{D1}} = \frac{|D_{D2}|^2}{|D_{D1}|^2} = \frac{f_{D2} \omega_{D1}}{f_{D1} \omega_{D2}} = 2\frac{\tau_{1/2}}{\tau_{3/2}} \left(\frac{\omega_{D1}}{\omega_{D2}}\right)^3
\]
Comparison to $\alpha(0)$ values inferred from other measurements
Combine our $\alpha(0)$ meas. w/ other lifetime meas. to report $\alpha_r(0)$.

$$\alpha(0) = 2\pi\varepsilon_0 c^3 \left[ \frac{\tau_{1/2}^{-1}}{\omega_{D1}^4} + 2 \frac{\tau_{3/2}^{-1}}{\omega_{D2}^4} \right] + \alpha_r$$
Combine C6 meas. w/ our $\alpha(0)$ meas. to report $\alpha_r(0)$.

$$C_6 = \frac{3\hbar}{\pi} \int_0^\infty [\alpha(i\omega)]^2 d\omega$$

$$C_6 = \frac{3\hbar}{\pi} \int_0^\infty [\alpha_p(i\omega) + \alpha_r(i\omega)]^2 d\omega$$
Combine C6 meas. w/ our $\alpha(0)$ meas. to report $\alpha_r(0)$.

\[ C_6 = \frac{3\hbar}{\pi} \int_{0}^{\infty} [\alpha(i\omega)]^2 d\omega \]

\[ C_6 = \frac{3\hbar}{\pi} \int_{0}^{\infty} [\alpha_p(i\omega) + \alpha_r(i\omega)]^2 d\omega \]
We thus measured $\alpha_r$ with <10% uncertainty.
Static Dipole Polarizability

SUMMARY:
• Measured $\alpha(0)$ for Na, K, Rb, Cs with 0.2% unc. Ratios with 0.1%.
• Listed in CRC handbook, sum-check MBPT calculations of $\langle k|r|i \rangle$

NEXT STEPS:
• Use this method for Sr and Yb to improve atomic clocks
• Use Li and He* to anchor ratios
• Measure $\alpha(0)$ for molecules to test DFT
• Use $\nabla E$ for dispersion compensation in gyroscopes / accelerometers

SURPRISES:
• Few ab initio theories are this accurate
• Several other experiments can be used to deduce $\alpha(0)$
• Velocity measurements were challenging because $\nabla E$ is a $\lambda_{dB}$ Lens
• We can report $\alpha_{core}$ by combining measurements.
Precision Measurements of Tune-Out Wavelengths with an Atom Interferometer

λzero

atom beam

laser

nanogratings

Raisa Trubko, Maxwell D. Gregoire, and Alexander D. Cronin

College of Optical Sciences and Department of Physics
University of Arizona
DAMOP 2015
2015 Tune-Out Wavelength Measurement for K atoms

\[ \lambda_{\text{zero}} = 768.9701(5) \text{ nm} \]

<table>
<thead>
<tr>
<th>Source of error</th>
<th>( \lambda_{\text{zero}} ) error (pm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>laser wavelength</td>
<td>0.1</td>
</tr>
<tr>
<td>broadband light</td>
<td>0.3</td>
</tr>
<tr>
<td>laser polarization &amp; lab rotation rate</td>
<td>0.2</td>
</tr>
<tr>
<td>Doppler shift</td>
<td>0.1</td>
</tr>
<tr>
<td>Total systematic error</td>
<td>0.4</td>
</tr>
<tr>
<td>Total statistical error</td>
<td>0.3</td>
</tr>
<tr>
<td>Total error</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Origin of tune-out wavelengths and motivation for measurements

\[ \alpha (\lambda_{\text{zero}}) = 0 \]

\[ R = \frac{|\langle 4s | d | 2p_{3/2} \rangle|^2}{|\langle 4s | d | 2p_{1/2} \rangle|^2} \]

\[ \alpha(\omega) = \frac{1}{3\hbar} \left( \frac{|\langle 4s | d | 4p_{1/2} \rangle|^2}{\omega_{D1}^2 - \omega^2} \omega_{D1} + \frac{|\langle 4s | d | 4p_{3/2} \rangle|^2}{\omega_{D2}^2 - \omega^2} \omega_{D2} \right) + \alpha_{\text{other}} \]
Origin of tune-out wavelengths and motivation for measurements

\[ \alpha(\omega) = \frac{1}{3h} \sum_k \left| \langle k || d || g \rangle \right|^2 \frac{\omega_k}{\omega_k^2 - \omega^2} + \alpha_{\text{core}} + \alpha_{\text{other}} \]

\[ \alpha(\omega) = \sum_k \frac{S_k \omega_k}{\omega_k^2 - \omega^2} + \alpha_{\text{core}} + \alpha_{\text{other}} \]

\[ \alpha(\omega) = 6\pi \varepsilon_0 c^3 \sum_k \frac{\Gamma_k}{(\omega_k^2 - \omega^2) \omega_k^2} + \alpha_{\text{core}} + \alpha_{\text{other}} \]

\[ \alpha(\omega) = \frac{e^2}{m} \sum_k \frac{f_k}{\omega_k^2 - \omega^2} + \alpha_{\text{core}} + \alpha_{\text{other}} \]
Origin of tune-out wavelengths and motivation for measurements

\[ \alpha(\omega) = \frac{1}{3\hbar} \sum_k \frac{|\langle k | d | g \rangle|^2 \omega_k}{\omega_k^2 - \omega^2} + \alpha_{\text{core}} + \alpha_{\text{other}} \]

\[ \alpha(\omega) = \sum_k \frac{S_k \omega_k}{\omega_k^2 - \omega^2} + \alpha_{\text{core}} + \alpha_{\text{other}} \]

\[ \alpha(\omega) = 6\pi \epsilon_0 c^3 \sum_k \frac{\Gamma_k}{(\omega_k^2 - \omega^2) \omega_k^2} + \alpha_{\text{core}} + \alpha_{\text{other}} \]

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Origin of tune-out wavelengths and motivation for measurements

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\]
### Science Impact and Experimental Sensitivities Comparison

<table>
<thead>
<tr>
<th>( \lambda_{\text{zero}} ) theory (nm)</th>
<th>relative exper. uncertainty (pm)</th>
<th>slope ( d\alpha/d\lambda ) (a.u.)</th>
<th>( \sigma_{\text{theory}}/\sigma_{\text{experiment}} )</th>
<th>( \Delta \lambda_{\text{zero}} ) due to ( \alpha_{\text{core}} ) (pm)</th>
<th>what we learn</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^7\text{Li}) 670.971626(1)</td>
<td>0.0004</td>
<td>9,500,000</td>
<td>2.5</td>
<td>&lt;0.01</td>
<td>Hyper-polarizability</td>
</tr>
<tr>
<td>(^{39}\text{K}) 768.791 (3)</td>
<td>0.10</td>
<td>42,000</td>
<td>30</td>
<td>0.15(1)</td>
<td>R1</td>
</tr>
<tr>
<td>(^{39}\text{K}) 405.980 (40)</td>
<td>51</td>
<td>83</td>
<td>0.8</td>
<td>66(4)</td>
<td>R1, (core)</td>
</tr>
<tr>
<td>(^{39}\text{K}) 404.720 (40)</td>
<td>0.68</td>
<td>5,700</td>
<td>57</td>
<td>0.96(5)</td>
<td>R2 &amp; R3</td>
</tr>
<tr>
<td>(^{87}\text{Rb}) 790.034 (7)</td>
<td>1.7</td>
<td>2,500</td>
<td>4</td>
<td>2.3(1)</td>
<td>R1, (core)</td>
</tr>
<tr>
<td>(^{87}\text{Rb}) 423.050 (80)</td>
<td>56</td>
<td>75</td>
<td>1.4</td>
<td>132(7)</td>
<td>R2, (R3 &amp; core)</td>
</tr>
<tr>
<td>(^{87}\text{Rb}) 421.080 (50)</td>
<td>8.4</td>
<td>500</td>
<td>6</td>
<td>18(1)</td>
<td>Core &amp; R3, (R2)</td>
</tr>
<tr>
<td>(^{133}\text{Cs}) 880.250 (40)</td>
<td>11</td>
<td>400</td>
<td>3.8</td>
<td>38(2)</td>
<td>R1, (core)</td>
</tr>
<tr>
<td>(^{133}\text{Cs}) 460.220 (20)</td>
<td>38</td>
<td>110</td>
<td>0.5</td>
<td>124(6)</td>
<td>R2, (R3, core)</td>
</tr>
<tr>
<td>(^{133}\text{Cs}) 457.310 (30)</td>
<td>38</td>
<td>110</td>
<td>0.8</td>
<td>138(7)</td>
<td>Core, (R2 &amp; R3)</td>
</tr>
<tr>
<td>(^{133}\text{Sr}) 689.230 (30)</td>
<td>2.6</td>
<td>1,600</td>
<td>11.5</td>
<td>4.0(2)</td>
<td>Intercom. ( f_{ik} )</td>
</tr>
</tbody>
</table>

*Adapted from Cronin 2012 NSF proposal*
Precision Measurements with Atom Interferometry

atom beam

nanogratings

atom flux

position
Precision Measurements with Atom Interferometry

atom beam

laser

nanogratings

$\phi_{light}$

atom flux

position
Tune-out wavelength measurements with atom interferometry

\[ \phi(\omega) = \frac{\alpha(\omega)}{\epsilon_0 c \hbar \nu} \int_{-\infty}^{\infty} I(\omega, z) dz \]

\[ v = \text{atom beam velocity} \]

\[ I(\omega, z) = \text{laser beam irradiance} \]

\[ z = \text{coordinate along atom beam propagation} \]

2015 Experimental Set-up

- Laser system
- Wavemeter
- Fiber
- Atom beams
- Optical cavity
- Laser beam
- Atom detector
- Vacuum
- Polarizer
- Nanogratings
- Wavevector $B \perp k$
2015 Experimental Set-up

- Laser system
- Fiber
- Wavemeter
- Atom beam
- Nanogratings
- Optical cavity
- Atom detector
- B ⊥ k
Doppler Shift & Wavemeter Calibration

Decoherence Spectroscopy

0.22(5) pm shift
Doppler Shift & Wavemeter Calibration

Decoherence Spectroscopy

0.22(5) pm shift
Measurement of broadband light from TA

\[
\text{Power} = P_{\text{mono}} \delta(\omega_L) + P_{\text{BB}} e^{\frac{-(\lambda - \lambda_0)^2}{2\sigma^2}}
\]
Error in $\lambda_{\text{zero}}$ due to broadband light from TA

$$\text{Power} = P_{\text{mono}} \delta(\omega_L) + P_{\text{BB}} e^{-\frac{(\lambda - \lambda_0)^2}{2\sigma^2}}$$

$$\phi = \int [P_{\text{mono}}(\omega) + P_{\text{BB}}(\omega)] \alpha(\omega) \, d\omega$$

$$\Delta \lambda_{\text{zero}} = \frac{\Delta \lambda}{\Delta \phi} \int P_{\text{BB}}(\omega) \alpha(\omega) \, d\omega$$

The figure shows a graph with $\Delta \lambda_{\text{zero}}$ (in pm) on the y-axis against $\lambda_0$ (in nm) on the x-axis. The graph indicates a 0.1(3) pm shift.
Error in $\lambda_{\text{zero}}$ due to elliptical polarization and Earth's rotation

Rotation Rate Systematic Shifts in $\lambda_{\text{zero}}$ measurements due to laser polarization and magnetic field orientation

<table>
<thead>
<tr>
<th></th>
<th>circular polarization</th>
<th>linear polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B \parallel k$</td>
<td>$B \perp k$</td>
</tr>
<tr>
<td>$\lambda_{\text{zero}}$ right - $\lambda_{\text{zero}}$ left</td>
<td>416 pm</td>
<td>64 pm</td>
</tr>
<tr>
<td></td>
<td>$B \parallel k$</td>
<td>$B \perp k$</td>
</tr>
<tr>
<td>$\lambda_{\text{zero}}$ right - $\lambda_{\text{zero}}$ left</td>
<td>17 pm</td>
<td>2.6 pm</td>
</tr>
</tbody>
</table>
How rotation affects $\lambda_{\text{zero}}$ measurements

- Sagnac phase depends on atom velocity

$$\Phi_S = \frac{4\pi L^2 \Omega}{vd_g}$$

Physics: spin-dependent dispersion compensation

Ingredients
- Atom beam with a spread in velocity
How rotation affects $\lambda_{\text{zero}}$ measurements

- Sagnac phase depends on atom velocity

$$\Phi_S = \frac{4\pi L^2 \Omega}{v d_g}$$

- Light-induced phase depends on atom velocity and spin

$$\Phi_L = \frac{\alpha(\omega)}{2\epsilon_0 c h v} \int s \cdot \left[ \frac{d}{dx} I(x, \omega) \right] dz$$

**Physics:** spin-dependent dispersion compensation

**Ingredients**
- Atom beam with a spread in velocity
- Atom beam with multiple spin states
How rotation affects $\lambda_{\text{zero}}$ measurements

- Sagnac phase depends on atom velocity
  $$\Phi_S = \frac{4\pi L^2 \Omega}{v d_g}$$

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**Physics**: spin-dependent dispersion compensation

**Ingredients**
- Atom beam with spread in velocity
- Atom beam with multiple spin states
- Circularly polarized light
- Magnetic field parallel to optical k-vector
How rotation affects $\lambda_{\text{zero}}$ measurements

- Sagnac phase depends on atom velocity
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**Physics:** spin-dependent dispersion compensation

**Ingredients**
- Atom beam with a spread in velocity
- Atom beam with multiple spin states
- Circularly polarized light
- Magnetic field parallel to optical k-vector
2015 Tune-Out Wavelength Measurement for K atoms

\[ \lambda_{\text{zero}} = 768.9701(5) \text{ nm} \]

### Source of error

<table>
<thead>
<tr>
<th>Source of error</th>
<th>( \lambda_{\text{zero}} ) error (pm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>laser wavelength</td>
<td>0.1</td>
</tr>
<tr>
<td>broadband light</td>
<td>0.3</td>
</tr>
<tr>
<td>laser polarization &amp; lab rotation rate</td>
<td>0.2</td>
</tr>
<tr>
<td>Doppler shift</td>
<td>0.1</td>
</tr>
<tr>
<td>Total systematic error</td>
<td>0.4</td>
</tr>
<tr>
<td>Total statistical error</td>
<td>0.3</td>
</tr>
<tr>
<td>Total error</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Rb light-induced phase shift data
Tune-Out wavelengths $\lambda_{\text{zero}}$

Measurement of $\lambda_{\text{zero}}$ lets us report $R = (S_{D2}/S_{D1})$.
\[ R_1 = \left| \frac{\langle 4s || d || 2p_{3/2} \rangle}{\langle 4s || d || 2p_{1/2} \rangle} \right|^2 \]

\[ = S_{D2}/S_{D1} \]

\[ = 1.9976(13) \]

In preparation (2016)
Tune-Out Wavelength Experiments:

SUMMARY:

- Measured $\lambda_{\text{zero}} = 768.9695(5)$ for K with 0.5 picometer uncertainty
- Benchmark test for ratios of $\langle 4p_j | r | 4s \rangle$ calculated with MBPT
- Novel gyroscope demonstrated using $\lambda_{\text{zero,lab}}$

NEXT STEPS:

- Other $\lambda_{\text{zero}}$ will test other $\langle k | r | i \rangle$
- Hyperpolarizability measurement idea
- Intercombination line strength measurement for Sr clocks
Talk Summary:

- Polarizability ratios (e.g. \( \frac{\alpha^K}{\alpha^Na} \)) measured with 0.1% uncertainty.

- Tune-out wavelength \( \lambda_{\text{zero}} \) measured with 0.5 pm uncertainty.

- C3 ratios (e.g. \( \frac{C_3^K}{C_3^Na} \)) measured with 2% uncertainty.

- Each type of measurement tests different functions of atomic \( f_{ik} \).

- Nanogratings work for several atomic species → steps towards a “universal atom interferometer”
Atom interferometry
studies of atomic structure

Alex Cronin, University of Arizona

John Perreault, PhD 2005
Ben McMorrn, PhD 2009
Vincent Lonij, PhD 2011
Will Holmgren, PhD 2013
Ivan Hromada, PhD 2014
Raisa Trubko, current PhD student
Maxwell Gregoire, current PhD student

Robert Wild, BS 2004
Melissa Revelle, BS 2009
Cathy Klauss, BS 2011
James Greenberg, BS 2014

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“New applications for atom interferometry with material gratings”
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Atom Interferometry: “Swiss Army Knife of Physics”
Thanks!