Astromaterial Science

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Astromaterials

- Stars freeze. But not all stars. Only parts of some stars freeze.
Astromaterial Science and Nuclear Pasta

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(Submitted on 12 Jun 2016)

The heavens contain a variety of materials that range from conventional to extraordinary and extreme. For this colloquium, we define Astromaterial Science as the study of materials, in astronomical objects, that are qualitatively denser than materials on earth. Astromaterials can have unique properties, related to their density, such as extraordinary mechanical strength, or alternatively be organized in ways similar to more conventional materials. The study of astromaterials may suggest ways to improve terrestrial materials. Likewise, advances in the science of conventional materials may allow new insights into astromaterials. We discuss Coulomb crystals in the interior of cold white dwarfs and in the crust of neutron stars and review the limited observations of how stars freeze. We apply astromaterial science to the generation of gravitational waves. According to Einstein's Theory of General Relativity, accelerating masses radiate gravitational waves. However, very strong materials may be needed to vigorously accelerate large masses in order to produce continuous gravitational waves that are observable in present detectors. We review large-scale molecular dynamics simulations of the breaking stress of neutron star crust that suggest it is the strongest material known, some ten billion times stronger than steel. Nuclear pasta is an example of a soft astromaterial. It is expected near the base of the neutron star crust at densities of ten to the fourteen grams per cubic centimeter. Competition between nuclear attraction and Coulomb repulsion rearrange neutrons and protons into complex non-spherical shapes such as flat plates (lasagna) or thin rods (spaghetti). We review semi-classical molecular dynamics simulations of nuclear pasta. We illustrate some of the shapes that are possible and discuss transport properties including shear viscosity and thermal and electrical conductivities.
Neutron Stars
Supernova

- The star implodes
- Outer layers rebound off of the core (bounce)
- Neutrinos heat and push the outer shell off
- Kablowy!
Supernova
Without the pressure from fusion to support the core, it will collapse.
Collapse

- Without the pressure from fusion to support the core, it will collapse
• Without the pressure from fusion to support the core, it will collapse

\[ {^{56}\text{Fe}} \rightarrow e^- \]

\[ e^- + p \rightarrow \nu_e + n \]
Without the pressure from fusion to support the core, it will collapse.

\[ ^{56}\text{Fe} - \nu_e \]

\[ e^- + p \rightarrow \nu_e + n \]
• Without the pressure from fusion to support the core, it will collapse

\[ e^- + p \rightarrow \nu_e + n \]
Collapse

• Without the pressure from fusion to support the core, it will collapse

\[ e^{-} + p \rightarrow \nu_{e} + n \]
Phase Diagram
Neutron Stars

• How much does the volume of the star change?
• Nucleus: $R \sim 10^{-15}$ m
• Atom: $R \sim 10^{-10}$ m

Image Credit: Google maps
Neutron Stars

- Neutron stars are so dense that Mt. Everest would fit in a cup of coffee.
- If you dropped a solar mass neutron star on the Rotunda…

Image Credit: Google maps
Neutron Stars

- Neutron stars are so dense that Mt. Everest would fit in a cup of coffee.
- If you dropped a solar mass neutron star on the Rotunda... it wouldn't even reach Shenandoah Natl. Park.

Image Credit: Google maps
Neutron stars

- Neutron stars form in supernovae, when a massive star collapses.
- The electrons get squeezed into the protons to make a big ball of neutrons, about 12 miles across.

So what physics changes after collapse?

\[ R \rightarrow 10^{-5} R \]
Neutron stars

- Neutron stars form in supernovae, when a massive star collapses.
  - The electrons get squeezed into the protons to make a big ball of neutrons, about 12 miles across.

So what physics changes after collapse?

1. Rotation: Cons of Ang Mom:

\[ R \rightarrow 10^{-5} R \]

\[ I = MR^2 \]

\[ L_1 = L_2 \]

\[ MR^2 w_1^2 = M (10^{-5} R)^2 w_2^2 \]
Neutron stars

- So what physics changes after collapse?

1. Rotation: Cons of Ang Mom: \[ L = Iw \]
   \[ I = MR^2 \]
   \[ L_1 = L_2 \]
   \[ MR^2 w_1^2 = M (10^{-5} R)^2 w_2^2 \]
   \[ 10^{10} w_1 = w_2 \]
   \[ T_2 = 10^{-10} T_1 \]
   \[ T_1 = \text{A few days?} \]
   \[ T_2 = \text{A few milliseconds} \]
Neutron stars

• So what physics changes after collapse?

(1) Rotation: Millisecond pulsars!
Neutron stars

- Neutron stars form in supernovae, when a massive star collapses.
  - The electrons get squeezed into the protons to make a big ball of neutrons, about 12 miles across.

So what physics changes after collapse?

1. Rotation: Millisecond pulsars!
2. Escape Velocity:

\[ v_{esc} = \sqrt{\frac{2GM}{R}} \]

\[ R \rightarrow 10^{-5} R \]

\[ M_{\odot} = 2 \times 10^{30} \text{ kg} \]

\[ R = 12 \text{ km} \]

\[ v_{esc} = 0.5c \]
Neutron stars

- Neutron stars form in supernovae, when a massive star collapses.
- The electrons get squeezed into the protons to make a big ball of neutrons, about 12 miles across.

So what physics changes after collapse?

1. Rotation: Millisecond pulsars!
2. Escape Velocity: Half the speed of light!
Neutron stars

- So what physics changes after collapse?
  1. Rotation: Millisecond pulsars!
  2. Escape Velocity: Half the speed of light!
  3. Magnetic Field: Conserve flux:

\[ R \rightarrow 10^{-5} R \]

\[ \Phi_B = BA \]
\[ B_1 R^2 = B_2 (10^{-5} R)^2 \]
\[ 10^{10} B_1 = B_2 \]

\[ B_1 = 10^3 \text{ G} \]
\[ B_2 = 10^{13} \text{ G} \]
Neutron stars

- So what physics changes after collapse?
  (1) Rotation: Millisecond pulsars!
  (2) Escape Velocity: Half the speed of light!
  (3) Magnetic Field: Literally nothing for comparison…

\[
B_1 = 10^3 \text{ G} \quad B_2 = 10^{13} \text{ G}
\]
The Four Forces

- Neutron stars are the only objects in the universe where all four forces play notable roles!

<table>
<thead>
<tr>
<th>Weak Force: Neutrinos</th>
<th>Electromagnetism: Strong B field</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong Force: Nuclear interactions</td>
<td>Gravity: So much gravity.</td>
</tr>
</tbody>
</table>
What’s inside a neutron star?
Neutron Star Structure

• What’s inside a neutron star?

Not just a “giant nucleus in space!”
Neutron Star Structure

• What’s inside a neutron star?

- outer crust 0.3–0.5 km: ions, electrons
- inner crust 1–2 km: electrons, neutrons, nuclei
- outer core ~ 9 km: neutron-proton Fermi liquid, few % electron Fermi gas
- inner core 0–3 km: quark gluon plasma?
Low Mass X-Ray Binaries

COMPANION (HOT)

MASS FLOW

NEUTRON STAR (COLD)

HOT CRUST

ACCRETION
X-ray bursts

- As matter accretes, it is compressed, buried, and heated
- Explosive nuclear burning produces a mix of heavy nuclei (rp-process)
- Ash is buried, and crystallizes
X-ray bursts

- Accretion Rate
- Nuclear Burning
- Crustal Heating
- Crust Composition
- Thermal Conductivity
- Crustal Cooling

Accretion flow

Liquid ocean, low-Z
Solid crust, high-Z
Liquid core

rp-process burning in an energetic novae (Champagne & Wiescher 1992)
Hard Astromaterials
Crystallization
Crystallization
Crystallization

• Physics is set by an ‘impurity parameter’

\[ Q_{\text{imp}} \equiv n_{\text{ion}}^{-1} \sum_i n_i (Z_i - \langle Z \rangle)^2 \]

• Low impurity parameter implies thermally conductive crust
• High impurity parameter implies thermally resistive crust

• rp-ash has a large impurity parameter (30-50)
• What does observation favor?
Low Mass X-Ray Binaries

- Companion (HOT)
- Neutron Star (Cold)
- Mass Flow
- Accretion
- Hot Crust
Low Mass X-Ray Binaries

COMPANION (HOT)

NEUTRON STAR (COLD)

COOLING CRUST

Crust Temperature

Core Temp

Time
Observables – Thermal Properties

• Find an effective impurity parameter and try to fit neutron star cooling curves
• Cooling curves: low mass X-ray binary MXB 1659-29

  • Blue: Conductive crust
    $Q_{imp} = 3.5$
    $T_c = 3.05 \times 10^7$ K
A problem?

• rp-ash has a large impurity parameter (30-50) while observation favors a low impurity parameter (<10)
• How do we reconcile this? Purify the crust with phase separation! (Mckinven et al 2016)
A problem?

Accretion

Core

Crust

Ocean

H/He

X-ray burst

rp-ash

Phase Separation

Low Z

High Z

Liquid

Solid
A problem?

Accretion

H/He

X-ray burst

rp-ash

Phase Separation

Low Z

High Z

Core

Solid

Liquid
• Lattice site hops in simulations of the crystal allow for diffusion near the melting temperature
• Broken power law in D(Z)
• If $D = 10^{-6} w_p a^2$, then 3 cm layers with 100 yrs accretion (Mckinven et al 2016)
Radial Distribution Functions
Radial Distribution Functions

First peak in $g(r)$ shifts depending on Coulomb force
Second peaks are not seen to shift
Screening of impurities? Long range correlations?
Soft Astromaterials
Neutron stars

- The crust is a crystalline lattice, while the core is uniform nuclear matter, like a solid nucleus. What’s in between these two phases?
Non-Spherical Nuclei

- First theoretical models of the shapes of nuclei near $n_0$
  1983: Ravenhall, Pethick, & Wilson
  1984: Hashimoto, H. Seki, and M. Yamada

- **Frustration**: Competition between proton-proton Coulomb repulsion and strong nuclear attraction

- Nucleons adopt non-spherical geometries near the saturation density to minimize surface energy
Nuclear Pasta

FIG. 1. Candidates for nuclear shapes. Protons are confined in the hatched regions, which we call nuclei. Then the shapes are: (a) sphere, (b) cylinder, (c) board or plank, (d) cylindrical hole and (e) spherical hole. Note that many cells of the same shape and orientation are piled up to form the whole space, and thereby the nuclei are joined to each other except for the spherical nuclei (a).
\[ n = 0.1200 \text{fm}^{-3} \]
Phases

- i-Antignocchi
- i-Antispaghetti
- i-Lasagna
- i-Spaghetti
- i-Gnocchi

- Uniform
- Defects
- Waffles

- r-Antignocchi
- r-Antispaghetti
- r-Lasagna
- r-Spaghetti
- r-Gnocchi
Classical Pasta Formalism

• Classical Molecular Dynamics with IUMD on Big Red II

\[ V_{np}(r_{ij}) = ae^{-r_{ij}^2/\Lambda} + [b - c]e^{-r_{ij}^2/2\Lambda} \]
\[ V_{nn}(r_{ij}) = ae^{-r_{ij}^2/\Lambda} + [b + c]e^{-r_{ij}^2/2\Lambda} \]
\[ V_{pp}(r_{ij}) = ae^{-r_{ij}^2/\Lambda} + [b + c]e^{-r_{ij}^2/2\Lambda} + \alpha e^{-r_{ij}/\lambda} \]

<table>
<thead>
<tr>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( \Lambda )</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>110 MeV</td>
<td>-26 MeV</td>
<td>24 MeV</td>
<td>1.25 fm(^2)</td>
<td>10 fm</td>
</tr>
</tbody>
</table>

• Short range nuclear force
• Long range Coulomb force

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Monte-Carlo ( \langle V_{tot} \rangle ) (MeV)</th>
<th>Experiment (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{16}\text{O})</td>
<td>-7.56±0.01</td>
<td>-7.98</td>
</tr>
<tr>
<td>(^{40}\text{Ca})</td>
<td>-8.75±0.03</td>
<td>-8.45</td>
</tr>
<tr>
<td>(^{90}\text{Zr})</td>
<td>-9.13±0.03</td>
<td>-8.66</td>
</tr>
<tr>
<td>(^{208}\text{Pb})</td>
<td>-8.2 ±0.1</td>
<td>-7.86</td>
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Classical Pasta Formalism

• Classical Molecular Dynamics with IUMD on Big Red II

\[ V_{np}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b - c] e^{-r_{ij}^2/2\Lambda} \]
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Short range nuclear force
Long range Coulomb force
Classical and Quantum MD

- We can use the classical pasta to initiate the quantum codes
- Classical structures remain stable when evolved via Hartree-Fock

Classical Points → Folded with Gaussian → Equilibrated Wavefunctions
Classical and Quantum MD

800 nucleons
24 fm
Classical and Quantum MD

800 nucleons
24 fm

100 fm

51,200 nucleons

\( n = 0.312n_0 \)
Molecular Dynamics

- We have evolved simulations of 409,600 nucleons, 819,200 nucleons, 1,638,400 nucleons, and 3,276,800 nucleons.
Nuclear Pasta

• Important to many processes:
  • Thermodynamics: Late time crust cooling
  • Magnetic field decay: Electron scattering in pasta
  • Gravitational wave amplitude: Pasta elasticity and breaking strain
  • Neutrino scattering: Neutrino wavelength comparable to pasta spacing
  • R-process: Pasta is ejected in mergers
Defects

• In the same way that crystals have defects, pasta does too!
• Electrons don’t scatter from order, they scatter from disorder

• Horowitz et al, PRL.114.031102 (2015)
Self Assembly

• Left: Electron microscopy of helicoids in mice endoplasmic reticulum

(a)  (b)


• Right: Defects in nuclear pasta MD simulations

Pasta Defects

- Defects act as a site for *scattering*
Pasta Defects

- The magnetic field decays within about 1 million years, indicating that there is an electrically resistive layer in neutrons stars (Pons et al 2013)
Lepton Scattering

• Lepton scattering from pasta influences a variety of transport coefficients:

• Shear viscosity:

\[ \eta = \frac{\pi v_F^2 n_e}{20 \alpha^2 \Lambda_{\eta ep}'}, \]

• Electrical conductivity:

\[ \sigma = \frac{v_F^2 k_F}{4 \pi \alpha \Lambda_{\sigma ep}'} \]
\[ \Lambda_{\eta ep}' = \int_0^{2k_F} dq \left( 1 - \frac{q^2}{4 k_F^2} \right) \left( 1 - \frac{v_F^2 q^2}{4 k_F^2} \right) S_p(q) \]

• Thermal conductivity:

\[ \kappa = \frac{\pi v_F^2 k_F k_B T}{12 \alpha^2 \Lambda_{\kappa ep}'}. \]
Lepton Scattering

\[ S_i(q) = \langle \rho_i^*(q, t) \rho_i(q, t) \rangle_t - \langle \rho_i^*(q, t) \rangle_t \langle \rho_i(q, t) \rangle_t \]

\[ \rho_i(q, t) = N_i^{-1/2} \sum_{j=1}^{N_i} e^{i \mathbf{q} \cdot \mathbf{r}_j(t)} \]

\[ \Lambda_{ep} \approx \frac{\Delta q^* Z^*}{q^*} \]

\begin{table}[h]
\begin{tabular}{|c|c|c|c|}
\hline
Simulation & $\bar{\eta}$ (fm$^{-3}$) & $\bar{\kappa}$ (10$^3 k_B$ MeV/fm) & $\bar{Z}$* \\
\hline
perfect & 87.7 & 6.66 & 5.5 \\
defects & 55.5 & 4.15 & 50.2 \\
\hline
\end{tabular}
\end{table}
Crust Cooling
An Example

SUN

Cold Reservoir (AC)

Insulation Walls

Cold Reservoir (AC)

Walls
An Example

Cold Reservoir (AC)

Insulation
Walls

Cold Reservoir (AC)

Walls

MOON
An Example

Cold Reservoir (AC)

Walls

MOON

Wall Temperature

AC

Time

Cold Reservoir (AC)

Walls
Low Mass X-Ray Binaries

COMPANION (HOT)

MASS FLOW

HOT CRUST

NEUTRON STAR (COLD)

ACCRETION
Low Mass X-Ray Binaries

COMPANION (HOT)

NEUTRON STAR (COLD)

COOLING CRUST

Crust Temperature

Core Temp

Time
Guess an effective impurity parameter for defects and try to fit neutron star cooling curves

Cooling curves: low mass X-ray binary MXB 1659-29

- Blue: normal isotropic matter
  \[ Q_{imp} = 3.5 \]
  \[ T_c = 3.05 \times 10^7 \text{ K} \]

- Red: impure pasta layer
  \[ Q_{imp} = 1.5 \text{ (outer crust)} \]
  \[ Q_{imp} = 30 \text{ (inner crust)} \]
  \[ T_c = 2 \times 10^7 \text{ K} \]
To interpret observations of neutron stars, we must first develop microscopic models of their interiors. By simulating the kinds of matter we expect to find in the crust we can calculate properties of the star, and potentially constrain fundamental physics.
Backup Slides!
Self Assembly
Self Assembly

- Well studied in phospholipids: hydrophilic heads and hydrophobic tails self assemble in an aqueous solution
Self Assembly

- Well studied in phospholipids: hydrophilic heads and hydrophobic tails self assemble in an aqueous solution.
Self Assembly

Seddon, BBA 1031, 1–69 (1990)
Self Assembly

• Left: Electron microscopy of helicoids in mice endoplasmic reticulum

  Terasaki et al, Cell 154.2 (2013)

• Right: Defects in nuclear pasta MD simulations

  Horowitz et al, PRL.114.031102 (2015)

  Parking Garage Structures in astrophysics and biophysics (arXiv:1509.00410)
Pulsars slowly *spin down*, meaning their period gets longer

Occasionally, they ‘glitch’ and start to spin faster

Is this crust breaking? Is this a *starquake*?

The breaking strain of the crust determines the frequency and ‘intensity’ of glitches
$l_z = 100.80 \text{ fm}$

$l_x = l_y = 100.80 \text{ fm}$
Gravitational Waves
Gravitational Waves

- LIGO has confirmed that direct detection is viable!
- First detected via a binary pulsar:
Neutron star mergers
## Event Rate

### TABLE II: Compact binary coalescence rates per Milky Way Equivalent Galaxy per Myr.

<table>
<thead>
<tr>
<th>Source</th>
<th>$R_{low}$</th>
<th>$R_{re}$</th>
<th>$R_{high}$</th>
<th>$R_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS-NS (MWE$^{-1}$ Myr$^{-1}$)</td>
<td>$1_{[1]^a}$</td>
<td>$100_{[1]^b}$</td>
<td>$1000_{[1]^c}$</td>
<td>$4000_{[16]^d}$</td>
</tr>
<tr>
<td>NS-BH (MWE$^{-1}$ Myr$^{-1}$)</td>
<td>$0.05_{[18]^e}$</td>
<td>$3_{[18]^f}$</td>
<td>$100_{[18]^g}$</td>
<td></td>
</tr>
<tr>
<td>BH-BH (MWE$^{-1}$ Myr$^{-1}$)</td>
<td>$0.01_{[14]^h}$</td>
<td>$0.4_{[14]^i}$</td>
<td>$30_{[14]^j}$</td>
<td></td>
</tr>
<tr>
<td>IMRI into IMBH (GC$^{-1}$ Gyr$^{-1}$)</td>
<td>$3_{[19]^k}$</td>
<td>$20_{[19]^l}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IMBH-IMBH (GC$^{-1}$ Gyr$^{-1}$)</td>
<td>$0.007_{[20]^m}$</td>
<td>$0.07_{[20]^n}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE V: Detection rates for compact binary coalescence sources.

<table>
<thead>
<tr>
<th>IFO</th>
<th>Source$^a$</th>
<th>$\hat{N}_{low}$ (\text{yr}^{-1})</th>
<th>$\hat{N}_{re}$ (\text{yr}^{-1})</th>
<th>$\hat{N}_{high}$ (\text{yr}^{-1})</th>
<th>$\hat{N}_{max}$ (\text{yr}^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>NS-NS</td>
<td>$2 \times 10^{-4}$</td>
<td>0.02</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>NS-BH</td>
<td>$7 \times 10^{-5}$</td>
<td>0.004</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BH-BH</td>
<td>$2 \times 10^{-4}$</td>
<td>0.007</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IMRI into IMBH</td>
<td>&lt; 0.001$^b$</td>
<td></td>
<td>0.01$^c$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IMBH-IMBH</td>
<td>$10^{-4}d$</td>
<td></td>
<td>$10^{-3}e$</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>NS-NS</td>
<td>0.4</td>
<td>40</td>
<td>400</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>NS-BH</td>
<td>0.2</td>
<td>10</td>
<td>300</td>
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<td>0.4</td>
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<td>$10^b$</td>
<td></td>
<td>$300^c$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IMBH-IMBH</td>
<td>$0.1^d$</td>
<td></td>
<td>$1^e$</td>
<td></td>
</tr>
</tbody>
</table>

(Abadie, 2010)
Neutron star mergers

- When the binary separation is similar to the neutron star radius, gravitational waves get strong

Inspiral

Merger
$t = O(10 \text{ ms})$

Ringdown
Mountains

• What if the surface is lumpy? Are there mountains?
• Dense, fast lump produces ripples in spacetime

• How big can they be? A few centimeters?
• How long do they last?

• The pasta is the densest stuff, therefore, it’s the stiffest. Could pasta support mountains?
R-mode instability

- *Rotational*-mode – toroidal oscillation of neutron star that is unstably driven by gravitational wave emission

\[ \vec{u} = (\omega_l \hat{r} \times \nabla Y_{ll} + v_{l+1} \nabla Y_{l+1} l + u_{l+1} Y_{l+1} l \hat{r}) e^{i\omega t} \]

- Primarily the \(l=m=2\) mode
- Solution: Is the damping from the crust enough to stabilize the star?

Youtube: LSU_Astrophysics
Nucleosynthesis
Recipe: Neutron star mergers

(1) Start with a binary of massive stars
(2) Make them supernova
(3) Merge the neutron stars’ by radiating gravitational waves
Neutron star mergers

- Makes a LOT of observables:
  - Short Gamma Ray Burst
  - Gravitational Waves
  - Black Hole
  - Neutron Rich Ejecta?
  - Kilonova?
Ejecta Evolution

- Nuclear matter is ejected from the crust and decompresses.
Ejecta Evolution

• The protons form small clusters which ‘seed’ the neutron gas
Ejecta Evolution

- Neutrons capture onto the seeds, forming neutron rich isotopes
• Neutrons capture onto the seeds, forming neutron rich isotopes
Ejecta Evolution

• These seeds beta decay: 
  \[ n \rightarrow p + e^- + \bar{\nu}_e \]
  and continue to capture neutrons
Ejecta Evolution

- These seeds beta decay: $n \rightarrow p + e^- + \bar{\nu}_e$
  and continue to capture neutrons
Ejecta Evolution

• These seeds beta decay: $n \rightarrow p + e^- + \bar{\nu}_e$
  and continue to capture neutrons...
  ntil they fission and repeat the process

Neutron Gas
Ejecta Evolution

- These seeds beta decay:
  \[ n \rightarrow p + e^- + \bar{\nu}_e \]
  and continue to capture neutrons...
  Until they fission and repeat the process
• r-process: the rapid neutron capture process
• Occurs in supernova and neutron star mergers

• Source of neutrons?
  • Neutron star mergers – obvious
  • Supernova – Neutrino driven wind

\[ e^- + p \leftrightarrow n + \nu_e \]
\[ e^+ + n \leftrightarrow p + \bar{\nu}_e \]

• Key parameter: Neutron to seed ratio (i.e. neutron to proton ratio)
  • Supernova: 4:1?
  • Neutron Stars: 100:1
Ejecta Evolution
Ejecta Evolution

- Decompress pasta to simulate ejecta evolution
- Count the number of protons and neutrons in each cluster after fission
• Pasta gnocchi produce realistic distributions of nuclei

What fuses the elements heavier than iron?
Phase Diagrams
Linear Elasticity

- Simulate pasta with constant temperature and proton fraction
- Observe phase transitions as a function of density

\( Y_p = 0.2 \)

\( Y_p = 0.4 \)

Decreasing Density
“Thermodynamic” Curvature

• Use curvature as a thermodynamic quantity
• Discontinuities in curvature indicate phase changes

\[ \int_M K \, dA + \int_{\partial M} k_g \, ds = 2\pi \chi(M) \]
\[ \chi(M) = 2 - 2g \]

• Pieces + Cavities - Holes

<table>
<thead>
<tr>
<th>Volume</th>
<th>Surface Area</th>
<th>Mean Breadth</th>
<th>Euler Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V )</td>
<td>( A = \int_{\partial K} dA )</td>
<td>( B = \int_{\partial K} (\kappa_1 + \kappa_2) / 4\pi dA )</td>
<td>( \chi = \int_{\partial K} (\kappa_1 \cdot \kappa_2) / 4\pi dA )</td>
</tr>
</tbody>
</table>

| \( V \) | \( A = \int_{\partial K} dA \) | \( B = \int_{\partial K} (\kappa_1 + \kappa_2) / 4\pi dA \) | \( \chi = \int_{\partial K} (\kappa_1 \cdot \kappa_2) / 4\pi dA \) |

| Sphere | 2 |
| Torus (Product of two circles) | 0 |
| Double torus | -2 |
| Triple torus | -4 |
“Thermodynamic” Curvature

• Use curvature as a thermodynamic quantity
• Discontinuities in curvature indicate phase changes

\[
\begin{align*}
V & = \int_{\partial K} dA \\
A & = \int_{\partial K} (\kappa_1 + \kappa_2) / 4\pi \\nB & = \int_{\partial K} (\kappa_1 \cdot \kappa_2) / 4\pi \\
\chi & = \int_{\partial K} (\kappa_1 \cdot \kappa_2) / 4\pi \\
\end{align*}
\]
Phases

- i-Antignocchi
- i-Antispaghetti
- i-Lasagna
- i-Spaghetti
- i-Gnocchi

- Uniform
- Defects
- Waffles

- r-Antignocchi
- r-Antispaghetti
- r-Lasagna
- r-Spaghetti
- r-Gnocchi
Phases (T=1 MeV)
Lepton Scattering

• Why does it matter?
• Lepton scattering from pasta influences a variety of transport coefficients:

• Shear viscosity:
\[ \eta = \frac{\pi v_F^2 n_e}{20 \alpha^2 \Lambda_{\eta}} , \]

\[ \Lambda_\eta = \int_0^{2k_F} dq \frac{d q}{q \varepsilon(q)} \left( 1 - \frac{q^2}{4k_F^2} \right) \left( 1 - \frac{v_F^2 q^2}{4k_F^2} \right) S_p(q) \]

• Electrical conductivity:
\[ \sigma = \frac{v_F^2 k_F}{4\pi\alpha \Lambda_{\sigma}} \]

\[ \Lambda_{\sigma} = \int_0^{2k_F} dq \frac{d q}{q \varepsilon(q)} \left( 1 - \frac{v_F^2 q^2}{4k_F^2} \right) S_p(q). \]

• Thermal conductivity:
\[ \kappa = \frac{\pi v_F^2 k_F k_B^2 T}{12 \alpha^2 \Lambda_{\kappa}} . \]
Lepton Scattering

\[ S_i(q) = \langle \rho_i^*(q, t) \rho_i(q, t) \rangle_t - \langle \rho_i^*(q, t) \rangle_t \langle \rho_i(q, t) \rangle_t \]

\[ \rho_i(q, t) = N_i^{-1/2} \sum_{j=1}^{N_i} e^{i \mathbf{q} \cdot \mathbf{r}_j(t)} \]

\[ \Lambda_{ep} \approx \frac{\Delta q^* Z^*}{q^*} \]
Lepton Scattering

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$\bar{n}$ (fm$^{-3}$)</th>
<th>$\bar{\kappa}$ ($10^3 k_B$ MeV/fm)</th>
<th>$Z^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>perfect</td>
<td>87.7</td>
<td>6.66</td>
<td>5.5</td>
</tr>
<tr>
<td>defects</td>
<td>55.5</td>
<td>4.15</td>
<td>50.2</td>
</tr>
</tbody>
</table>
Self Assembly

- The Helfrich Hamiltonian describes the bending energy, can be found with the principal curvatures, $k_1$ and $k_2$, and curvature energies $B$ and $\bar{B}$

$$H_0 = \frac{1}{2} B \int dA (k_1 + k_2)^2 + \bar{B} \int dA (k_1 k_2)^2$$

- Relate the curvature energy to a curvature term in the SEMF

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(A - 2Z)^2}{A} + a_K A^{1/3} + \delta(A, Z)$$

- Bottom line: minimal surfaces minimize surface energy and the curvature energy settles the tie

Helfrich, Z. Naturforsch. 28 (1973)
Self Assembly

• What minimal surfaces do we see in pasta?

1) \( k_1 = k_2 = 0 \), Flat plates

2) \( k_1 = -k_2 \), Hyperbola

3) Other minimal surfaces:
   - Helicoids:
   - Gyroids

\[
H_0 = \frac{1}{2} B \int dA (k_1 + k_2)^2 + B \int dA (k_1 k_2)^2
\]