Gravitational waves from binary black holes across the spectrum

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1915: GR
1916: GWs; Schwarzschild metric
1919: Eddington’s expedition

1939: gravitational collapse

1957: Chapel Hill conference
1960: Weber bars
1967: “black hole,” no-hair theorem
1971: Cygnus X-1
1972: GW interferometer design
1974: PSR B1913+16

1990, 1999: LIGO approved, inaugurated

2002: Sgr A* as black hole
2002–2010: initial LIGO runs

2015: aLIGO; GW150914
What are gravitational waves?

GW150914: detection and companion papers at papers.ligo.org

[LVC 2016]
GW151216: see PRL and O1 BBH paper
[LVC 2016]

55 cycles over 1 s
35–450 Hz
Gravitational waves and their detection

To first approximation... indeed, in the linearized approximation...

\[ \Box \bar{h}^{\alpha \beta} = -16\pi G^{\alpha \beta} \]

Gravitational waves and their detection

[Nature 2016]
GWs are transverse and traceless tidal fields

[ESA 2016]
• **Modified Michelson interferometer** with ~300x resonant arm cavities (in 1µPa vacuum), power and signal recycling
• 40-kg fused silica mirrors on quadruple-pendulum suspensions with active seismic isolation
• 20 W input power, 100 kW circulating in O1
• **Servos** in 300 control loops to maintain resonance and alignment; **calibration** achieved by measuring response to induced test-mass motion
• **Environmental monitors**: seismometers, accelerometers, microphones, magnetometers, radio receivers, weather sensors, ac-power line monitors, cosmic-ray detector
High-vacuum tubes and chambers
### Advanced LIGO & Advanced Virgo

#### The History of LIGO

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>National Science Foundation (NSF) funds Caltech and MIT for laser interferometer research and development.</td>
</tr>
<tr>
<td>1980</td>
<td>Site construction begins in Hanford, WA and Livingston, LA.</td>
</tr>
<tr>
<td>1990</td>
<td>Early work on gravitational-wave detection by laser interferometers begins with a 1972 MIT study describing a kilometer-scale interferometer and estimates of its noise sources.</td>
</tr>
<tr>
<td>2000</td>
<td>During an engineering test a few days before the first official search begins, Advanced LIGO detects strong gravitational waves from collision of two black holes.</td>
</tr>
<tr>
<td>2010</td>
<td>Construction of Advanced LIGO components begins.</td>
</tr>
</tbody>
</table>
GW150914: inspiral, merger, and ringdown.

8 cycles increasing in frequency. Recognizable as inspiraling binary. Evolution characterized by chirp mass.

Estimating f and $\dot{f}$ yields chirp mass 30 Msun, so total mass > 70 Msun.

Sum of Schwarzschild radii at least 210 km; at 75 Hz (orbital frequency), radial separation would be 350 km. Thus these objects must be very compact.

Only BHs and NSs known to exist. NS impossible, since total mass would be much larger and merge at lower frequencies.

Hints of BH decay seen.
GW150914: numerical relativity simulation
[SXS collaboration 2016]
GW150914: matched-filter inspiral search

- Binaries with masses 1–99 M⊙, total mass < 100 M⊙, dimensionless spin < 0.99
- 250,000 PN and EOB signal templates. Matched-filter SNR + χ² statistic
- Measured on 608,000-yr background, false-alarm rate < 1 in 203,000 yr (2x10⁻⁷ false alarm = 5.1σ)

GW150914: matched-filter inspiral search
[LVC 2016]
LIGO O1 BBH: parameter estimation

[SNR] [masses] [effective spin] [D/Mpc] [z]

<table>
<thead>
<tr>
<th>Event</th>
<th>SNR</th>
<th>masses</th>
<th>effective spin</th>
<th>D/Mpc</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>GW150914</td>
<td>23.7</td>
<td>36 + 29</td>
<td></td>
<td>420</td>
<td>0.1</td>
</tr>
<tr>
<td>LVT151012</td>
<td>9.7</td>
<td>23 + 13</td>
<td></td>
<td>1000</td>
<td>0.2</td>
</tr>
<tr>
<td>GW151226</td>
<td>13</td>
<td>14 + 7.5</td>
<td>0.2</td>
<td>440</td>
<td>0.1</td>
</tr>
</tbody>
</table>

LIGO O1 BBH: parameter estimation

[LVC 2016]
• Primordial: density fluctuations after Big Bang
• Pop III: first massive stars (1% of stars in Universe)
• Pop II/I: classic field binary evolution (90%)
• Pop II/I: rapid rotation (homogeneous evol.) (10%)
• Pop II/I: dynamical formation in globular clusters (0.1%)
• Exotic: e.g., single-star core splitting

Origin of massive GW150914-like BHs
[LVC 2016, Belczynski 2016]
equal-mass BBH
Inspiral: PN equations

waveform models

GW searches

statistical inference

[Babak, MV et al. 2013]

[Cutler & MV 2007]

[MV 2008, 2012]
GW150914 and GW151226: merger rate estimates

<table>
<thead>
<tr>
<th>Mass distribution</th>
<th>( \frac{R}{(\text{Gpc}^{-3} \text{yr}^{-1})} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PyCBC</td>
</tr>
<tr>
<td>Event based</td>
<td></td>
</tr>
<tr>
<td>GW150914</td>
<td>(3.2^{+8.3}_{-2.7})</td>
</tr>
<tr>
<td>LVT151012</td>
<td>(9.2^{+30.3}_{-8.5})</td>
</tr>
<tr>
<td>GW151226</td>
<td>(35^{+92}_{-29})</td>
</tr>
<tr>
<td>All</td>
<td>(53^{+100}_{-40})</td>
</tr>
<tr>
<td>Astrophysical</td>
<td></td>
</tr>
<tr>
<td>Flat</td>
<td>(31^{+43}_{-21})</td>
</tr>
<tr>
<td>Power Law</td>
<td>(100^{+136}_{-69})</td>
</tr>
</tbody>
</table>

2016, 6 months
2017, 9 months

\(N > 10\)
\(N > 35\)
\(N > 70\)
• **Consistency**: useful sanity checks, hard to interpret statistically. P values are possible with much work. But would we ever believe an inconsistent result?

• **Parametric tests**: constraints on GR “constants” (PN coefficients, graviton mass)—useful proxies for increasing resolving power, but again hard to interpret. Apparent violations may focus our search for new physics.

• **Alternative theories**: new physics will be established by model comparison of GR with fully predictive alternative theories. (However, it is a problem to establish Bayesian priors for alternative gravity, and for alternative-gravity parameters.)

A hierarchy of tests of GR with GW observations

[MV in preparation]
“Consistency” test: residual

[B. Allen 2016]
an actual null-hypothesis test (with $P$-value 0.3), which implies that GR prediction is verified to 4%; i.e., no GR violations above 4% of waveform

\[
\text{SNR}_{\text{res}}^2 = \frac{1 - \text{FF}^2}{\text{FF}^2} \text{SNR}_{\text{det}}^2
\]

Fitting Factor: parameter-maximized waveform overlap

\[
\text{SNR}_{\text{res}} \leq 7.3 \Rightarrow \text{FF} \geq 0.96
\]

(for violations not absorbed by physical parameters)

“Consistency” test: residual [LVC 2016]
answers question: if we estimate QNM parameter directly and compare them with values deduced from the preferred binary parameters, are the resulting estimates “consistent”?

“Consistency” test: quasinormal modes

[LVC 2016]
answers question: what are the preferred values of individual waveform coefficients in a set of hypothetical theories in which each in turn is free?

\[ h(f) = \frac{1}{D} \frac{A}{\sqrt{F}} f^{2/3} e^{i\Psi(f)} \]

\[ \Psi(f) = \sum_l [\psi_i + \psi_{il} \log f] f^{(i-5)/3} + \Phi_{MR}[\beta_i, \alpha_i] \]

Parametric test: PN coefficients

[Yunes, Yagi, Pretorius 2016]

[Yunes, Yagi, Pretorius 2016]

[Parametric test: PN coefficients]

[Parametric test: PN coefficients]
answers question: what is the preferred value of the “dispersion” $m_g$ in a hypothetical theory of gravity where it is a free parameter?

\begin{align*}
  h(f) &= \frac{1}{D} \frac{A}{\sqrt{F}} f^{2/3} e^{i \Psi(f)} \\
  \Psi(f) &= \sum_i \left[ \psi_i + \psi_i \log f \right] f^{(i-5)/3} + \Phi_{\text{MR}}[\beta_i, \alpha_i]
\end{align*}

Parametric test: graviton mass

[LVC 2016]
**new physics** follows from establishing an **anomaly**: we need to obtain convincing evidence that the data prefers an alternative theory of gravity over GR.
for a fixed false-alarm rate, we ask what SNR is needed to detect AG with 50% probability as a function of fitting factor FF, using the Bayesian odds ratio as “detection” statistic.

Detection SNR limits GR test sensitivity  
[MV 2012]
Modified theories of gravity

[Berti et al. 2016]
Axions with Compton wavelength large compared to the size of the BH have an approximately hydrogenic spectrum of bound states around the BH.

When a spinning BH is born, the number of axions in superradiant levels will grow exponentially, seeded by spontaneous emission. The fastest-growing level, generally one with the minimum $l$ and $m$ such that Eq. 2 is satisfied, will extract energy and angular momentum from the BH until Eq. 2 is saturated. This process repeats for the next-fastest-growing level, until the time it takes for the next level to grow is longer than the accretion timescale of the BH or the age of the universe.

The absence of rapidly rotating old BHs is a signal that SR has taken place. The spin vs. mass distribution of BHs should be empty in the region affected by SR, with a large number of BHs populating the curve $\omega = m\Omega_H$.

Direct emission: two axions can annihilate into a single graviton of energy $2\mu_a$, creating a quasi monochromatic emission.

\[ \mu_a \left(1 - \frac{a^2}{2m^2}\right) \]

\[ \Omega_H = \frac{1}{2r_g} \frac{a_*}{\sqrt{1-a_*^2}} \]

\[ h_{ann} \approx 6 \times 10^{-23} \left(\frac{\alpha}{0.3}\right)^7 \left(\frac{a_*}{0.9}\right) \left(\frac{M_{BH}}{60M_\odot}\right) \left(\frac{1\text{ Mpc}}{d}\right) \]

\[ \tau_{ann} \approx 0.1 \text{ yr} \left(\frac{0.3}{\alpha}\right)^{15} \left(\frac{0.9}{a_*}\right) \left(\frac{M_{BH}}{60M_\odot}\right) \]

**GWs from superradiant axions in gravitational “atoms”**

[Arvanitaki et al. 2016]
Advanced LIGO roadmap

[O1] 65-80 Mpc

[O2] 60-100 Mpc

[O3] 120-170 Mpc (target)

[O3] 200 Mpc (target)

Binary Neutron Star range

[GW150914] 30M☉ Black Hole Binaries

S6 run
O1 run
Adv. LIGO design
Future upgrades

SNR

Strain Noise (1/√Hz)

Frequency (Hz)

Redshift

Advanced LIGO roadmap

[LVC 2016, 2017]
Adv. LIGO Plus (A+): x1.7 range increase over aLIGO
leverage existing technology and infrastructure

LIGO Voyager: x2 sensitivity broadband improvement
larger Si masses, cryogenic operation, shorter laser wavelength

Future LIGO enhancements
[LVC 2016]
Gravitational-wave detectors

- Galactic binaries
- massive black-hole binaries
- captures into MBHs
- merging NS, BH
- rotating NS
- early-Universe quantum fluctuations

Frequency range:
- Hz
- $10^{-16}$
- $10^{-14}$
- $10^{-12}$
- $10^{-10}$
- $10^{-8}$
- $10^{-6}$
- $10^{-4}$
- $10^{-2}$
- 1
- $10^2$

- CMB
- pulsar timing
- LISA-like
- future space
- LIGO
The last century has seen enormous progress in our understanding of the Universe. We know the life cycles of stars, the structure of galaxies, the remnants of the big bang, and have a general understanding of how the Universe evolved. We have come remarkably far using electromagnetic radiation as our tool for observing the Universe. However, gravity is the engine behind many of the processes in the Universe, and much of its action is dark. Opening a gravitational window on the Universe will let us go further than any alternative. Gravity has its own messenger: Gravitational waves, ripples in the fabric of spacetime. They travel essentially undisturbed and let us peer deep into the formation of the first seed black holes, exploring redshifts as large as $z \sim 26$, prior to the epoch of cosmic re-ionisation. Exquisite and unprecedented measurements of black hole masses and spins will make it possible to trace the histories of black holes across all eras.
Massive BH binaries

Galactic white-dwarf binaries

Extreme mass-ratio inspirals

Cosmological backgrounds

LISA GW sources
LISA sensitivity and sources

[LISA proposal 2017]
LISA payload and LPF performance

[LISA proposal 2017]
The LISA science analysis

[MV 2011]
Gravitational-wave detectors

- LIGO
- LISA-like
- Pulsar timing
- CMB
- Future space

Hz

- $10^{-16}$
- $10^{-14}$
- $10^{-12}$
- $10^{-10}$
- $10^{-8}$
- $10^{-6}$
- $10^{-4}$
- $10^{-2}$
- 1
- $10^2$

- Galactic binaries
- massive black-hole binaries
- early-Universe quantum fluctuations
- rotating NS
- merging NS, BH

- Early-Univers quantum fluctuations
- Massive black-hole binaries
- Galactic binaries

- Rotating NS
- Merging NS, BH

- Early-Univers quantum fluctuations
- Massive black-hole binaries
- Galactic binaries
pulsars: Nature’s precision clocks

[Manchester 2015]
Pulsar-timing multiphysics

\[ L_{12} = L_{12}^{\text{no gw}} \]

J1909-3744

B1937+21

[NANOGrav soon]
Pulsar-timing arrays [Foster and Backer 1990]

NANOGrav 11-Year Data Set
MJD 53187.5-53370.1
Year 2004.500-2005.000

[Nice 2016, NANOGrav soon]
Pulsar science: individual SMBH binaries

[Graham et al. 2015]  [Babak et al. 2016]
Pulsar science: relic radiation

Planck + BICEP2 + SPTpol
... + PPTA
... + LIGO/Virgo
... + indirect
... + aLIGO
+ PPTA(2020)

Lasky et al. 2016
Pulsar science: cosmic strings

recombination vs string tension

string tension vs loop size

[NANOGrav 2016]
\[ \Omega_{gw}(f) = \frac{1}{\rho_c} \frac{d\rho_{gw}(f)}{d\ln f} \]

\[ \frac{d\rho_{gw}(f)}{d\ln f} = \frac{\pi}{4} f^2 h_c^2(f) = \int_0^\infty dz \frac{dn}{dz} \frac{1}{1+z} \frac{dE_{gw}}{d\ln f_r} \bigg|_{f_r = f(1+z)} \]

\[ h_c^2(f) = \frac{4}{\pi f^2} \int_0^\infty dz \int_0^\infty dM \frac{d^2 n}{dz dM} \frac{1}{1+z} \frac{dE_{gw}(M)}{d\ln f_r} \]

\[ h_c(f) = h_{1yr} \left( \frac{f}{\text{yr}^{-1}} \right)^{-2/3} \]

\[ \frac{dE_{gw}}{d\ln f_r} = \frac{\pi^{2/3}}{3} M^{5/3} f_r^{2/3} \]

\[ h = \frac{8\pi^{2/3} M^{5/3}}{10^{1/2} d_L(z)} f_r^{2/3} \]

\[ \frac{df_r}{dt_r} = \frac{96}{5} \pi^{8/3} M^{5/3} f_r^{11/3} \]

Stochastic background from SMBH mergers

[Phinney 2001, Sesana et al. 2008]
Stochastic background from SMBH mergers

Isotropic SMBH background: NANOGrav 9-year analysis

[McWilliams et al. (2014)]
[Ravi et al. (2014)]
[Sesana et al. (2013)]
Isotropic SMBH background: NANOGrav 9-year analysis

\[ h_c(f) = A \frac{(f/f_{yr})^\alpha}{(1+(f_{\text{bend}}/f)^\kappa)^{1/2}} \]

McWilliams et al. (2014) Model

\[ B = 22 \]

Sesana et al. (2013) Model

\[ B = 2.2 \]

[NANOGrav 2016, Sampson et al. 2015]
Gravitational waves from binary supermassive black holes missing in pulsar observations

Detection probability given the PPTA limit

[Taylor, Vallisneri, et al. 2015]
A PTA noise model: everything is a Gaussian process

$y_{gp} = Fa$

$p(a) \propto e^{-a^T \Phi(\theta)^{-1} a/2}$

$K(\theta) = F \Phi(\theta) F^T$

Basis picture
Search over basis coefficients and hyperparameters

timing residuals = radiometer noise (white) + timing-model errors + jitter noise (white, epoch) + DM + timing noise (red) + GWs

Kernel picture
Marginalize over basis coefficients, search over hyperparameters

$y_{gp} \sim e^{-y_{gp}^T K(\theta)^{-1} y_{gp}/2}$

van Haasteren & MV
PRD 90, 104012 (2014)
Stochastic GWs as correlated Gaussian process

\[ \text{timming residuals} = \text{radiometer noise (white)} \]
\[ + \text{timming-model errors} \]
\[ + \text{jitter noise (white, epoch)} \]
\[ + \text{DM} \]
\[ + \text{timming noise (red)} \]
\[ + \text{GWs} \]

pulsar #1

\[ \text{timming residuals} = \text{radiometer noise (white)} \]
\[ + \text{timming-model errors} \]
\[ + \text{jitter noise (white, epoch)} \]
\[ + \text{DM} \]
\[ + \text{timming noise (red)} \]
\[ + \text{GWs} \]

pulsar #2

\[ \text{timming residuals} = \text{radiometer noise (white)} \]
\[ + \text{timming-model errors} \]
\[ + \text{jitter noise (white, epoch)} \]
\[ + \text{DM} \]
\[ + \text{timming noise (red)} \]
\[ + \text{GWs} \]

pulsar #3

\[ \text{timming residuals} = \text{radiometer noise (white)} \]
\[ + \text{timming-model errors} \]
\[ + \text{jitter noise (white, epoch)} \]
\[ + \text{DM} \]
\[ + \text{timming noise (red)} \]
\[ + \text{GWs} \]

pulsar #4

Environmental Coupling
- Stellar hardening
- Gas-driven inspiral
- Eccentricity

Galaxy Population Uncertainties
- Merger timescale
- SMBH - host relations
- Pair fraction
- Redshift evolution

Characteristics strain, $h_0$

Gravitational Wave Frequency, $f$ (Hz)

[Burke-Spolaor 2015]

Expected correlation

Angle between pulsars (degrees)

[Jenet et al. 2015]
Ephemeris systematics
[NANOGrav 2017, PRELIMINARY]

DE421 (2008): targets Mars
DE430 (2014): ICRF 2.0, Moon++
DE435 (2016): targets Cassini
DE436 (2016): targets Juno
J1713+0747 noise model

[NANOGrav 2017, PRELIMINARY]
GWB amplitude posteriors

[NANOGrav 2017, PRELIMINARY]
PTA outlook

- **GW detection with PTAs** offers a very beautiful, yet extremely difficult challenge: building a detector the size of our galaxy, exploiting nature’s most precise clocks, millisecond pulsars.

- Barring surprises (cosmic strings, nonstandard relic radiation, GW memory from early-Universe events), PTAs will observe first the **stochastic background** from the cosmological population of **supermassive black-hole binaries** in Galactic nuclei.

- **Improvements in sensitivity** are limited by the increasing span of datasets and by the continued discovery of new pulsars.

- The most recent **upper limits** on the background are in tension with **theoretical expectations**, suggesting “last-parsec” physics, or faulty assumptions. Nevertheless, if theoretical models are correct, detection is expected within 10 years.

- Establishing confident detection requires sophisticated statistical techniques and superior control of systematics. Unfortunately, recent hints of a signal seem to be subsiding.
I've been talking to schoolkids about gravitational waves, so I'm providing a translation of my title. I bet Einstein did not see this one coming, either.