Strongly Coupled Plasmas and Gauge/String Duality

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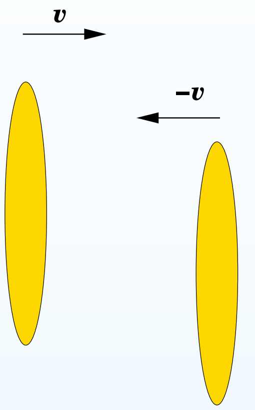
Outline

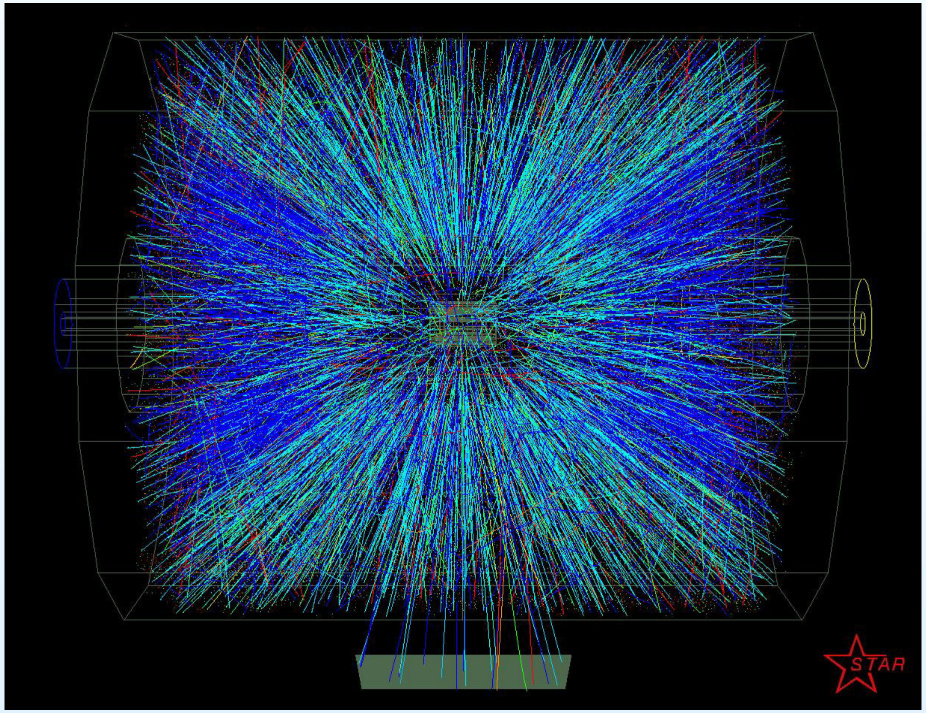
1. Heavy ion collisions and quark gluon plasma (QGP)
2. Strongly coupled versus weakly coupled systems
3. Theoretical techniques
4. Gauge/string duality
5. Applications to strongly coupled non-Abelian plasmas
6. Outlook

Relativistic heavy ion collisions

RHIC: Au+Au, 200 GeV/nucleon
running since 2000

LHC: Pb+Pb, 5.5 TeV/nucleon
starting 2008





Relativistic heavy ion collisions: timeline

0 — “splat” — parton liberation (small x gluons)

very high density gluon “soup”

1 fm/c — “thermalization” — isotropization of phase space distribution

quark-gluon plasma

time

expansion, cooling

hadronization

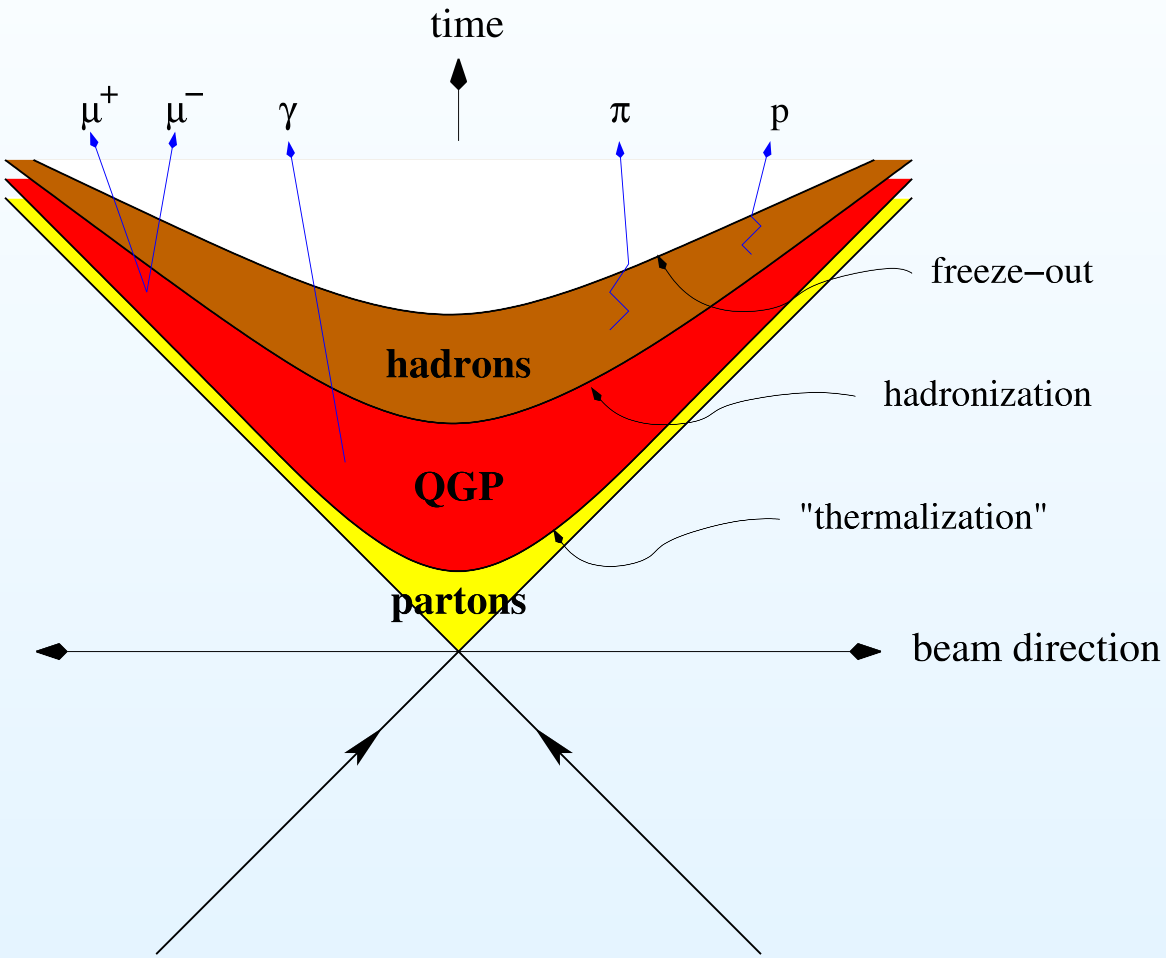
10 fm/c

expanding hadron gas

$\approx$

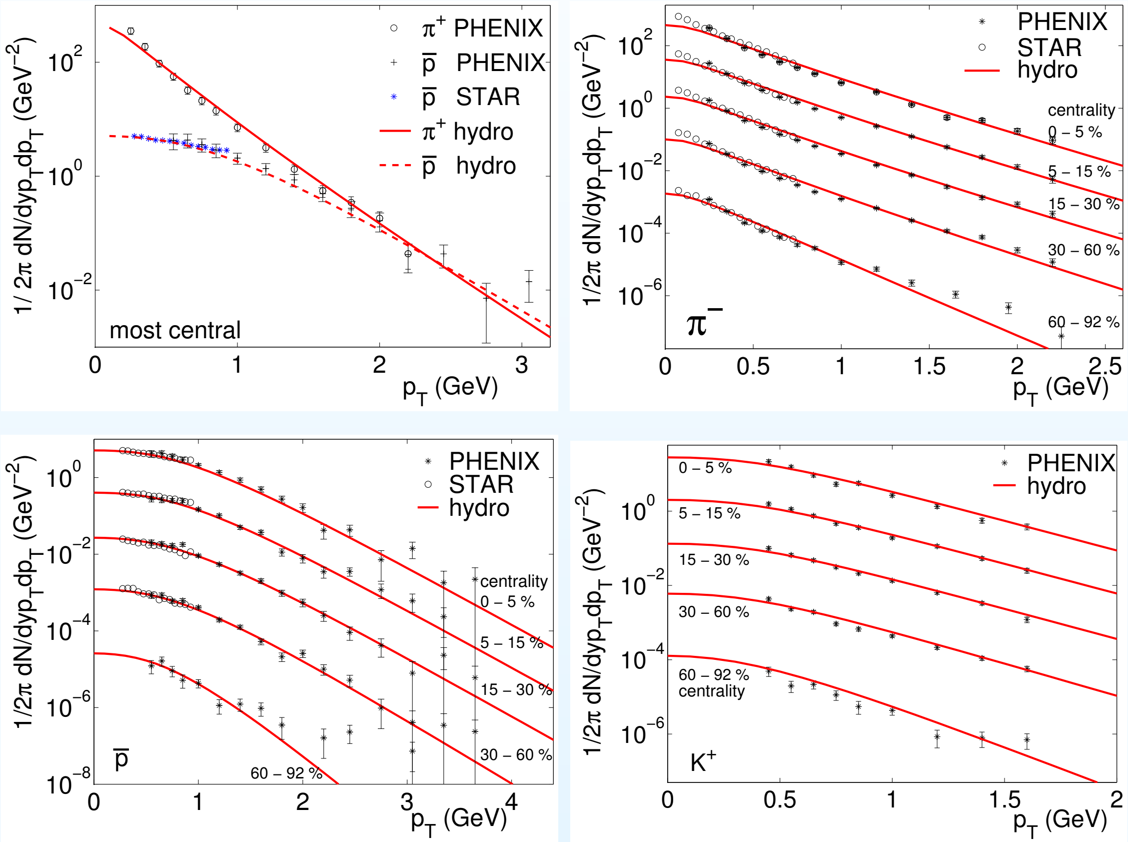
detection

Relativistic heavy ion collisions: spacetime view



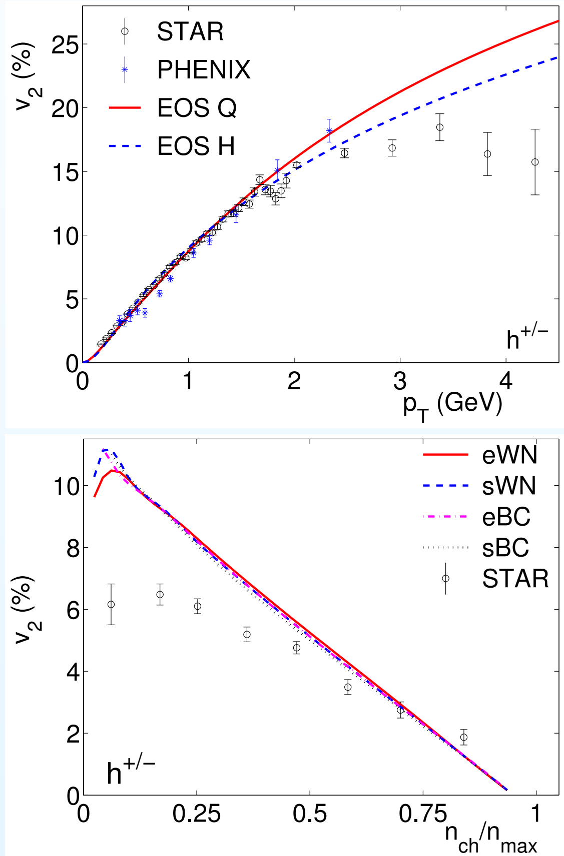
Relativistic heavy ion collisions: ideal hydrodynamics?

Single particle spectra
Au+Au, 130 GeV/nucleon



U. Heinz & P. Kolb, hep-ph/0204061

Elliptic flow: momentum &
centrality dependence



U. Heinz, nucl-th/0512051

QGP at RHIC: nearly ideal fluid

QGP at RHIC:

Nearly ideal fluid $\Longrightarrow$ low viscosity, short mean free path

$\Longrightarrow$ strongly coupled plasma

Theoretical challenge: predict dynamic properties of strongly coupled QGP

viscosity

energy loss (dE/dx)

emission spectra

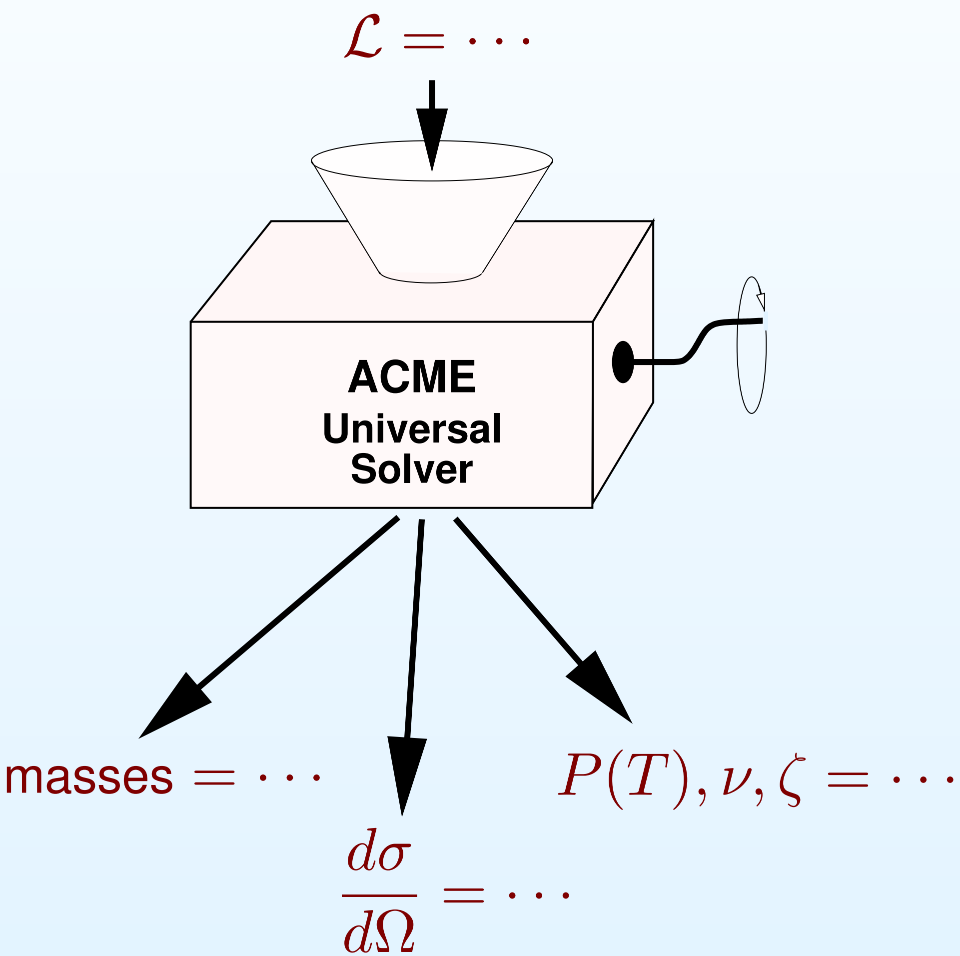
$\vdots$

Weak coupling versus strong coupling

Weak coupling	Strong coupling
interactions "small"	interactions dominant
mean free path, screening length $\gg$ particle separation, deBroglie wavelength	mean free path, screening length $\lesssim$ particle separation, deBroglie wavelength
high mobility & viscosity	low mobility & viscosity
Ex: dilute gases	Ex: liquids
dilute, hot astrophysical plasmas	cool, dense plasmas
QGP at $\underbrace{T \ggg 1 \text{ GeV}}_{\text{experimentally inaccessible}}$	QGP at $T \approx \underbrace{\text{few} \times 100 \text{ MeV}}_{\text{RHIC/LHC}}$

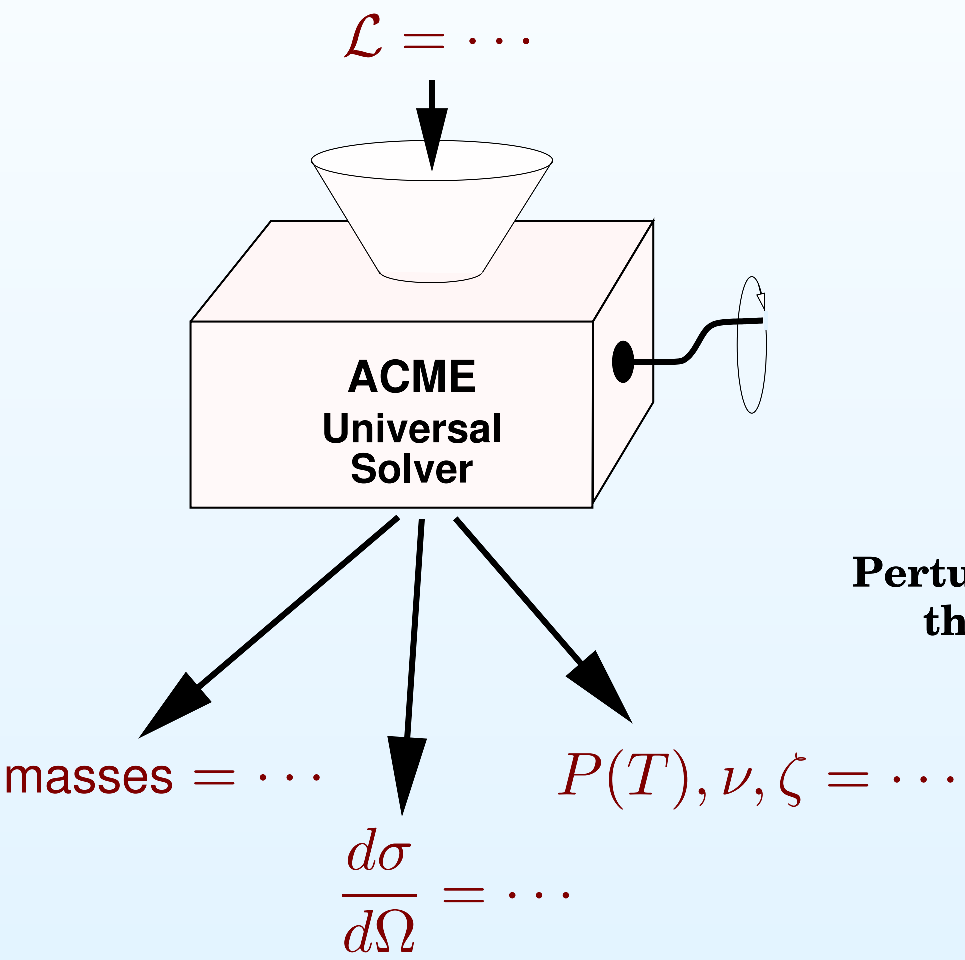
Theoretical physics

Fantasy

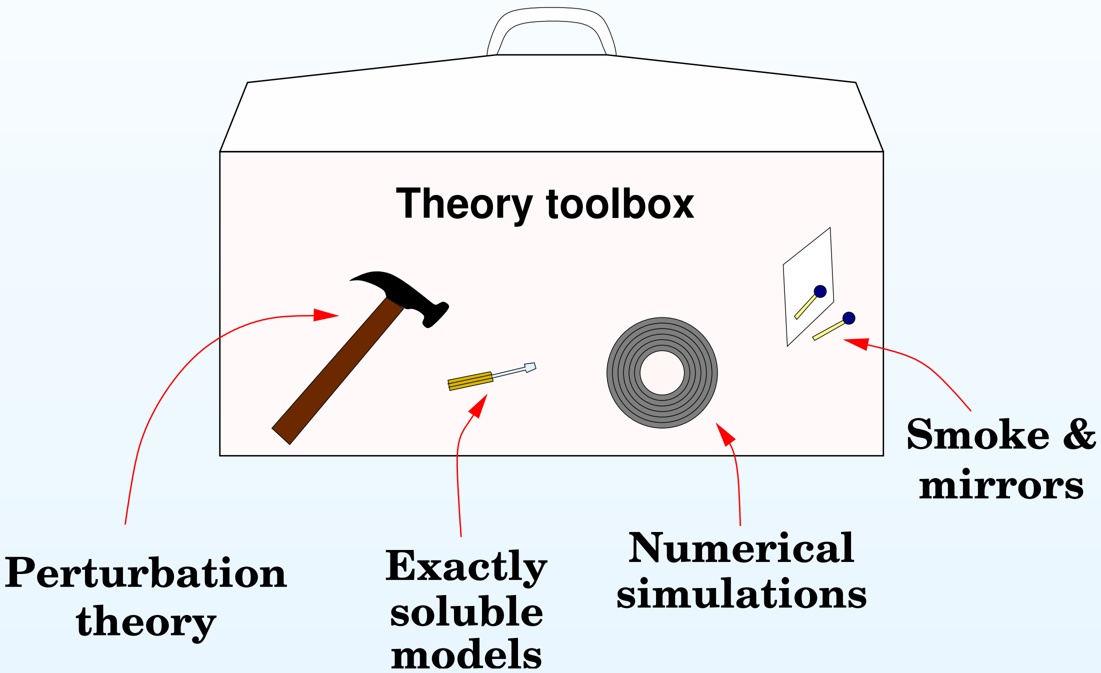


Theoretical physics

Fantasy



Reality



Perturbation theory

QED

$$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137.\cdots}, \qquad \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} - 0.328\left(\frac{\alpha}{\pi}\right)^2 + 1.181\left(\frac{\alpha}{\pi}\right)^3 - 1.510\left(\frac{\alpha}{\pi}\right)^4 + \cdots$$

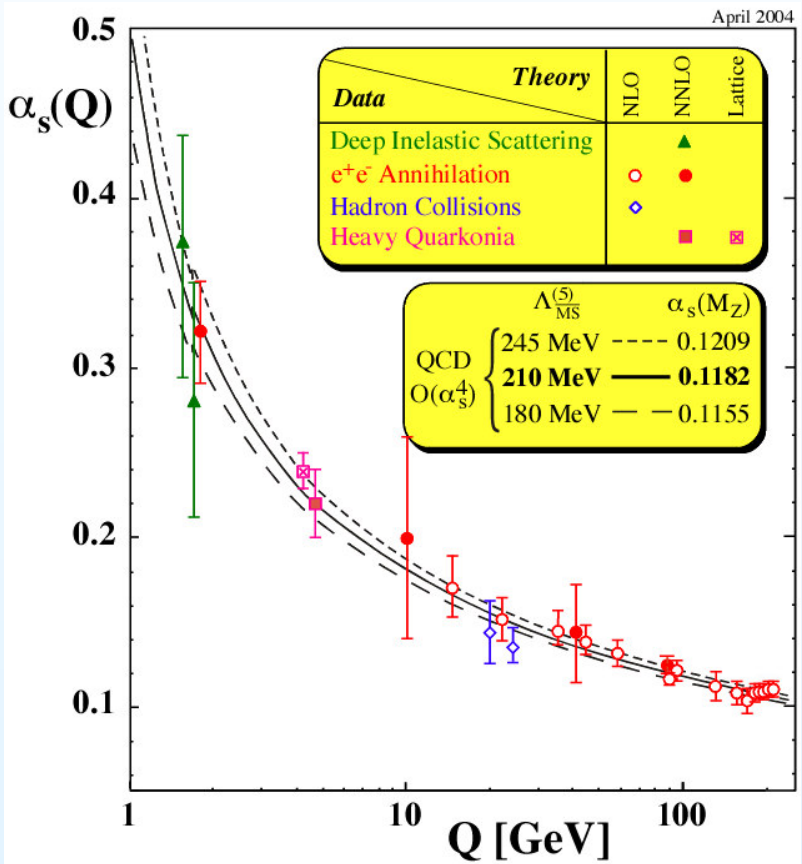
high energy QCD

$$\alpha_s(E) = \frac{g^2(E)}{4\pi} \propto \frac{1}{\ln(E/\Lambda_{QCD})}$$

very high temperature QGP

$$\frac{\lambda_{\mathrm{deBroglie}}}{\lambda_{\mathrm{mfp}}} \sim g^4(T)$$

sometimes works great...



Numerical simulations

Very useful in: Classical dynamical systems

deterministic dynamics, no quantum fluctuations

Equilibrium classical statistical systems

stochastic dynamics $\Rightarrow$ averaging over probability distribution

Equilibrium quantum theories

ground state properties, particle energies, ...

(QCD lattice gauge theory)

Not so good for: Real-time dynamics of quantum field theories

nasty interference in Feynman path integral,

$$\langle \hat{\phi}(x_1)\hat{\phi}(x_2)\cdots\rangle = \frac{\int \mathcal{D}\phi(x)\, e^{iS[\phi]/\hbar}\, \phi(x_1)\,\phi(x_2)\cdots}{\int \mathcal{D}\phi(x)\, e^{iS[\phi]/\hbar}}$$

many degrees of freedom: no good algorithm

String theory

1968 Introduced as phenomenological model of strong interactions — Veneziano, ...

But: unphysical tachyon, massless spin-2 particle, ...

1974 Reinterpreted as possible theory of quantum gravity — Scherk, Schwarz

But: needs 26 dimensions, no apparent connection to real world

1984 Realization that 10-d superstrings are anomaly free — Green, Schwarz, ...

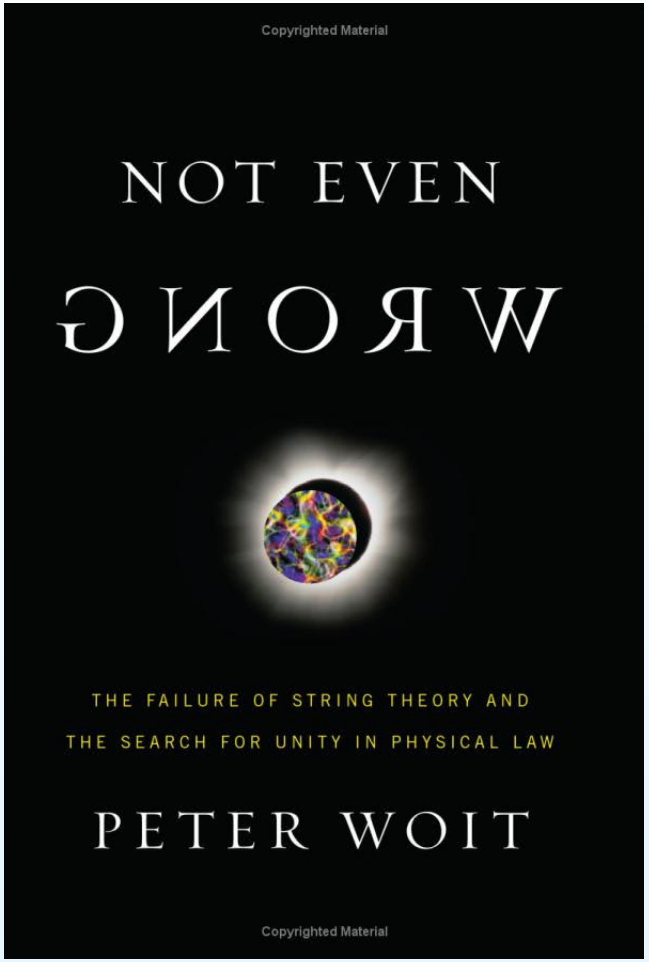
'Anomaly free' $\Rightarrow$ quantum-mechanically consistent

1995 Realization that theory describes "D-branes" too — Polchinski, ...

Dynamical surfaces of dimension > 1 on which strings can end

1997 Realization that string theory on $AdS_5 \times S^5$ spacetime is equivalent to a 4-dimensional quantum field theory — Maldacena, ...

String theory controversy:

The “Theory of Everything,” or  ?

Which view is right?

Progress in physics:

Often comes from asking the right question.

Also comes from learning what are bad questions:

- What is simultaneous position and momentum of an electron?
- What is the bare mass of an electron?

Progress in physics:

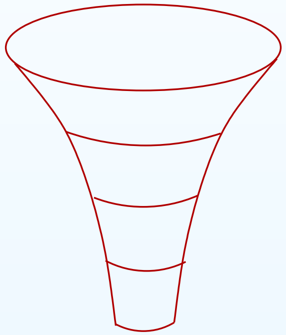
Often comes from asking the right question.

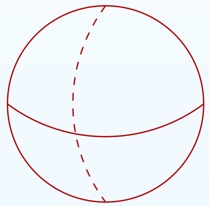
Also comes from learning what are bad questions:

- What is simultaneous position and momentum of an electron?
- What is the bare mass of an electron?
- What is the fundamental gauge group?
- Are there extra spacetime dimensions?
- Is string theory “right”?

Strings on $AdS_5 \times S^5$

$\underbrace{AdS_5}_{\text{const. negative curvature}} \times \underbrace{S^5}_{\text{const. positive curvature}}$: 10-dimensional solution of General Relativity





$$ds^2 = R^2 \left\{ \frac{du^2}{u^2} + u^2(-dt^2 + d\vec{x}^{\,2}) + d\vec{\Sigma}_5^2 \right\}$$

curvature radius

AdS radial coordinate

S^5 metric

$u \to \infty$ = AdS boundary = 4-d Minkowski space

IIB string theory on $AdS_5 \times S^5$: two adjustable (dimensionless) parameters

$R =$ curvature radius of $AdS_5 =$ radius of S^5

$\ell_s =$ fundamental string length $= 1/\sqrt{\text{string tension}}$

$R/\ell_s =$ curvature radius in string units

$g_s =$ string coupling = amplitude for string splitting/joining

"Easy" limit: $g_s \ll 1$ and $R/\ell_s \gg 1 \Longrightarrow$ classical (super)-gravity

Maldacena conjecture:

IIB string theory on $AdS_5 \times S^5$ is *exactly* equivalent to maximally supersymmetric 4-d Yang-Mills theory ("$\mathcal{N}=4$ SYM")

$\mathcal{N}=4$ SYM : Non-Abelian gauge theory w. massless fermions & scalars

Two adjustable parameters:

λ = interaction strength ('t Hooft coupling)

N_c = # "colors" = rank of $SU(N_c)$ gauge group

Scale invariant (conformal) field theory ("CFT")

AdS/CFT duality : Not (yet) rigorously proven, but supported by much evidence

Matching symmetries, operator dimensions, $\cdots$

AdS radius $\sim$ energy scale

Dictionary: $(R/\ell_s)^4 = \lambda, \quad g_s = \lambda/(4\pi N_c)$

Weakly coupled string theory $\Longleftrightarrow$

strongly coupled, large N_c gauge theory

Many generalizations

$\mathcal{N} = 4$ SYM vs. QCD: zero temperature

	QCD	$\mathcal{N} = 4$ SYM
particles:	mesons $(\pi, K, \rho, \cdots)$ baryons $(n, p, \Delta, \cdots)$	**none** no S-matrix
RG flow:	asymptotic freedom $g^2(E) \searrow$ as $E \nearrow$	**none** fixed coupling λ
fundamental scale:	Λ_{QCD}	**none** scale invariant CFT

Completely different properties!

$\mathcal{N}=4$ SYM vs. QCD: high temperature

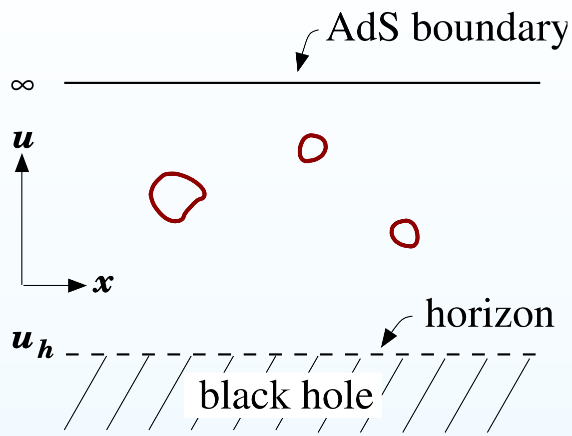
	QCD	$\mathcal{N}=4$ SYM
deconfined non-Abelian plasma	✓	✓
Debye screening	✓	✓
finite spatial correlation length	✓	✓
neutral fluid hydrodynamics	✓	✓

Complete qualitative agreement

Try using $\mathcal{N}=4$ SYM as model for QGP

Works well at weak coupling

Gauge/string duality for hot plasma



$\mathcal{N}=4$ SYM at non-zero temperature dual to strings in AdS black hole geometry

$$ds^2_{\mathrm{AdS-BH}} = R^2 \left\{ \frac{du^2}{f(u)} + f(u)(-dt^2 + d\vec{x}^{\,2}) \right\}, \qquad f(u) = u^2 \left[1 - \left(\frac{u_h}{u} \right)^4 \right]$$

Hawking temperature = plasma temperature, $T = \pi\, u_h$

Classical dynamics in 5-d AdS black hole geometry $\Rightarrow$ exact results in strongly coupled ($\lambda \to \infty$, $N_c \to \infty$) SYM plasma

Strongly coupled SYM plasma: selected results

Shear viscosity

$$\eta = \frac{\pi}{8} N_c^2 T^3$$

From black hole absorption cross-section.

Policastro, Son, Starinets

Viscosity/entropy density

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Universal lower bound?

Kovtun, Son, Starinets

Heavy quark energy loss

$$\frac{dE}{dx} = \frac{dp}{dt} = -\frac{\pi\sqrt{\lambda}T^2}{2m}\,p$$

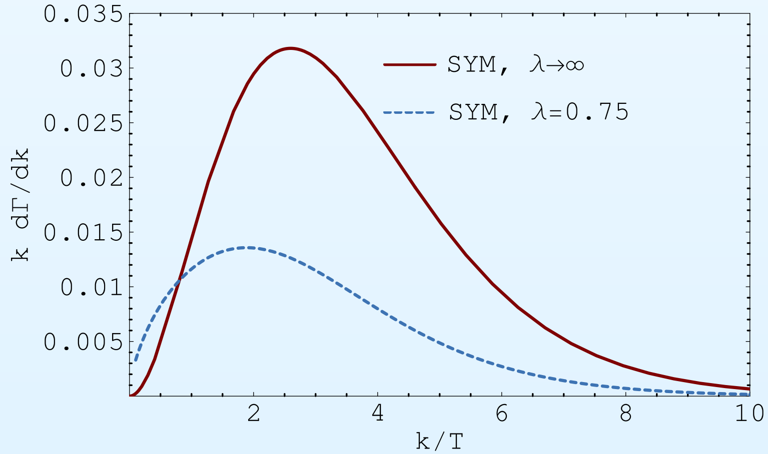
From classical string dynamics

Herzog, Karch, Kovtun, Kozcaz, L.Y.

Photo-emission spectrum

From Maxwell's equations

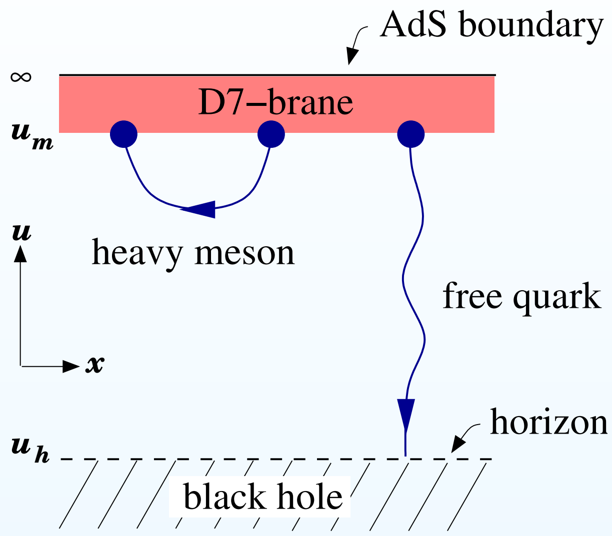
Huot, Kovtun, Moore, Starinets, L.Y.



Heavy quark energy loss

Adding heavy quarks to $\mathcal{N}=4$ SYM $\Leftrightarrow$
adding D7 brane to gravity dual

Quark moving through $\mathcal{N}=4$ plasma $\Leftrightarrow$
open string moving in AdS-BH geometry



String dynamics:

world sheet coordinates: $X^\mu(\sigma,\tau) \Rightarrow x(u,t)$ for planar world sheet, $\sigma{=}u,\ \tau{=}t$

action: $-S = T_0\,(\text{world sheet area}) = T_0 \int d\sigma d\tau\, \sqrt{-g}$

induced metric: $-g = \left(\frac{\partial X}{\partial \tau}\cdot\frac{\partial X}{\partial \sigma}\right)^2 - \left(\frac{\partial X}{\partial \tau}\right)^2\left(\frac{\partial X}{\partial \sigma}\right)^2$

$$= L^4\left[1 - \frac{u^2}{f(u)}\,\dot{x}^2 + u^2 f(u)\, x'^2\right]$$

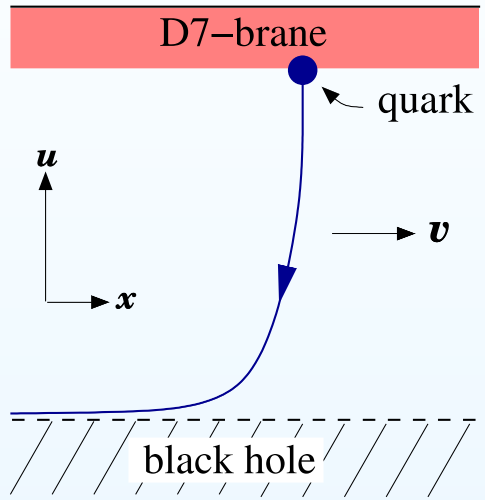
stationary solution: $\dot{x}(u) = v \quad \Longrightarrow$ easy variational problem

first integral: $x'(u) = \pm v\, \frac{u_h^2}{u^2 f(u)}$

Heavy quark energy loss: trailing string solution

$$x(u,t) = x_0 + v(t + F(u))$$

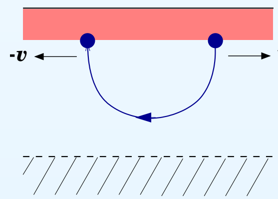
$$F(u) = \frac{1}{2u_h}\left[\frac{\pi}{2} - \arctan\frac{u}{u_h} - \operatorname{arccoth}\frac{u}{u_h}\right]$$



Constant momentum flow down string

= external force needed to maintain velocity

= -(plasma drag force acting on moving quark)



A bit more work:

Separating quark-antiquark pair ($\sim c\bar{c}$ jets)
HKKKY

World sheet fluctuations $\Longrightarrow \frac{d}{dt}\langle p_\perp^2\rangle,\ \frac{d}{dt}\langle p_\parallel^2\rangle$
Gubser; Casalderry-Solana, Teaney

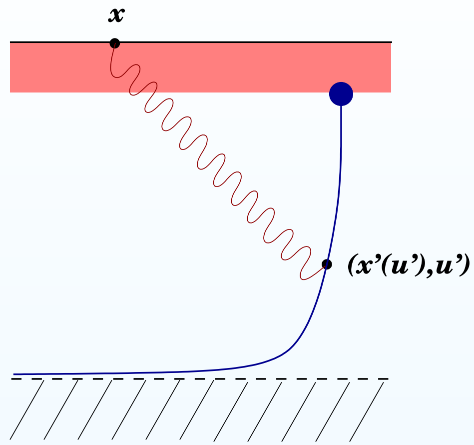
Linearized gravitational corrections $\Longrightarrow \langle T_{\mu\nu}(x)\rangle$
P. Chesler, L.Y.

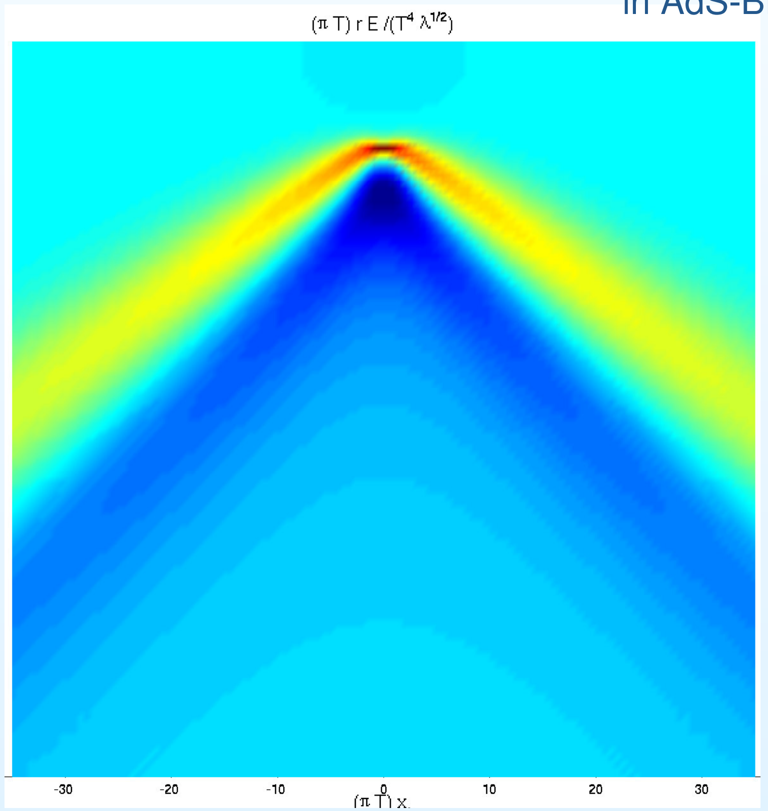
Heavy quark energy loss: wakes

$\langle T_{\mu\nu}(x)\rangle \Leftarrow$ asymptotic near-boundary behavior of $g_{\mu\nu}$

$\delta\langle T_{\mu\nu}(x)\rangle \Leftarrow \delta g_{\mu\nu}(x,u)\Big|^{u=\infty}_{\text{moving string}}$

$$= \int du' \; \underbrace{G_{\mu\nu,\alpha\beta}}_{\text{"bulk-to-boundary" graviton Green's function in AdS-BH background}}(x,\infty;x'(u'),u') \; \underbrace{\pi^{\alpha\beta}(u')}_{\text{stress-energy density of moving string}}$$





Energy density, $v = 0.75\,c$

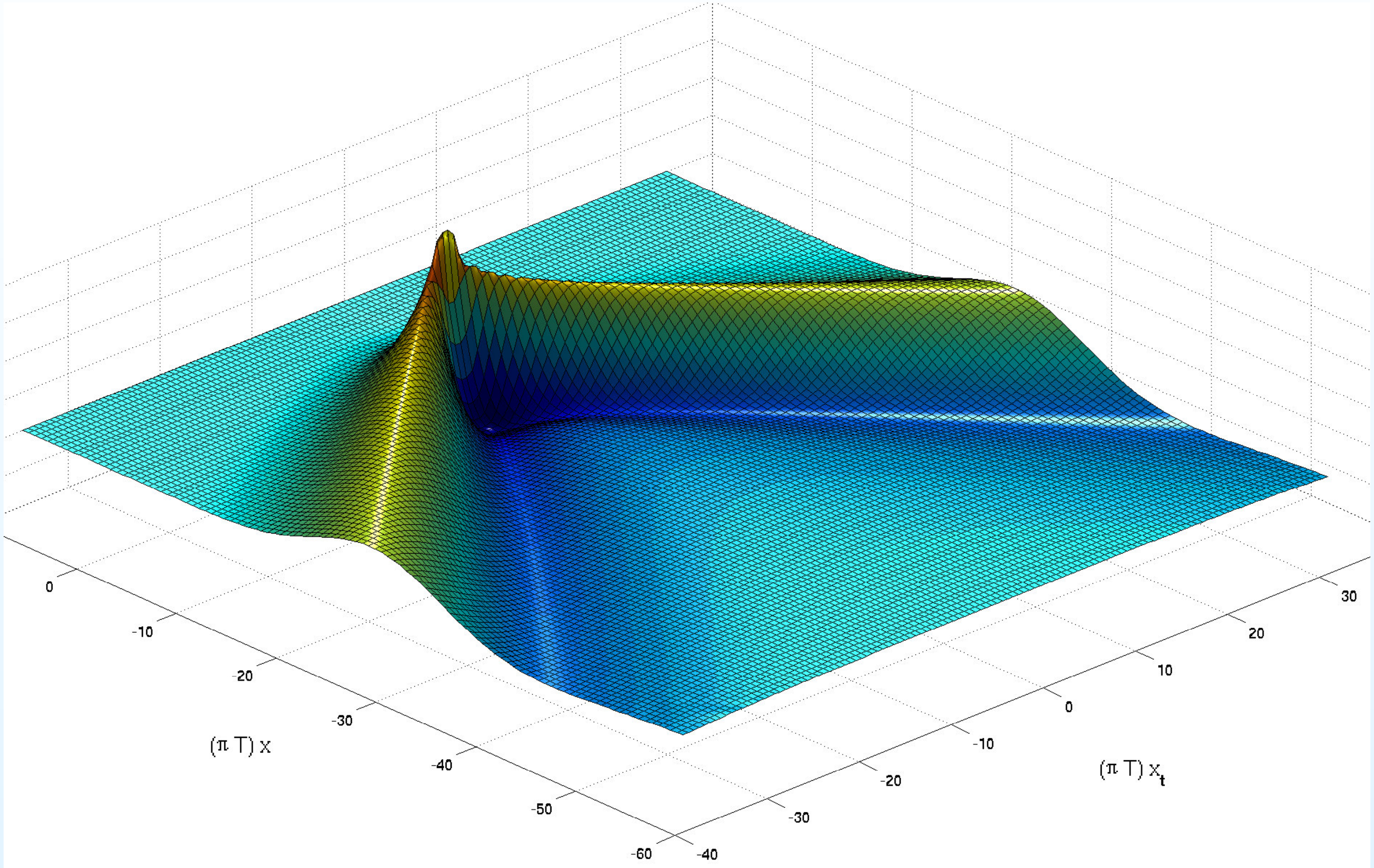
(velocity of sound $v_s = c/\sqrt{3} = 0.577\,c$)

Supersonic shock in viscous plasma

Results valid on *all* length scales

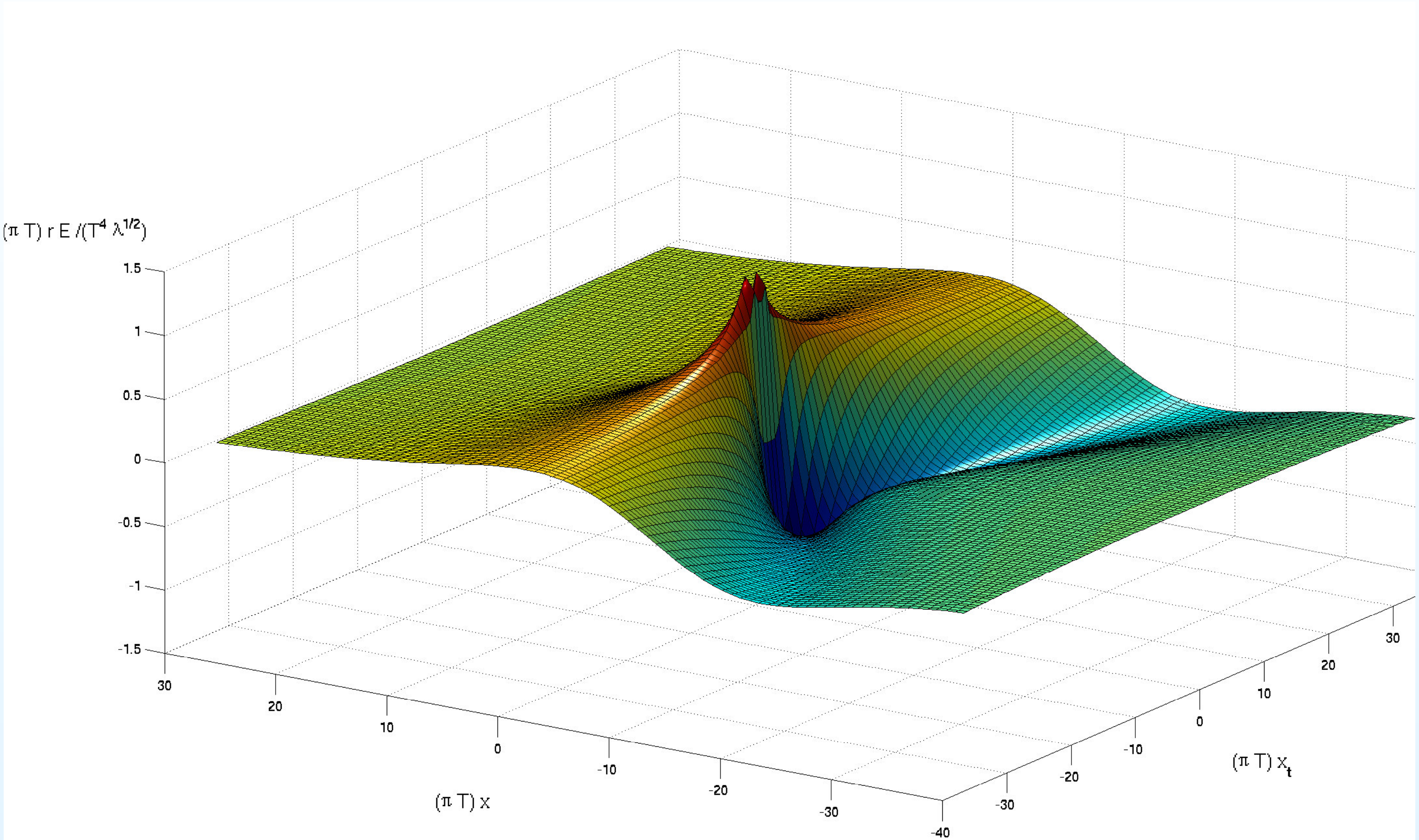
P. Chesler

Heavy quark wake: $v = 0.75\,c$



P. Chesler

Heavy quark wake: $v = v_s = 0.577\,c$



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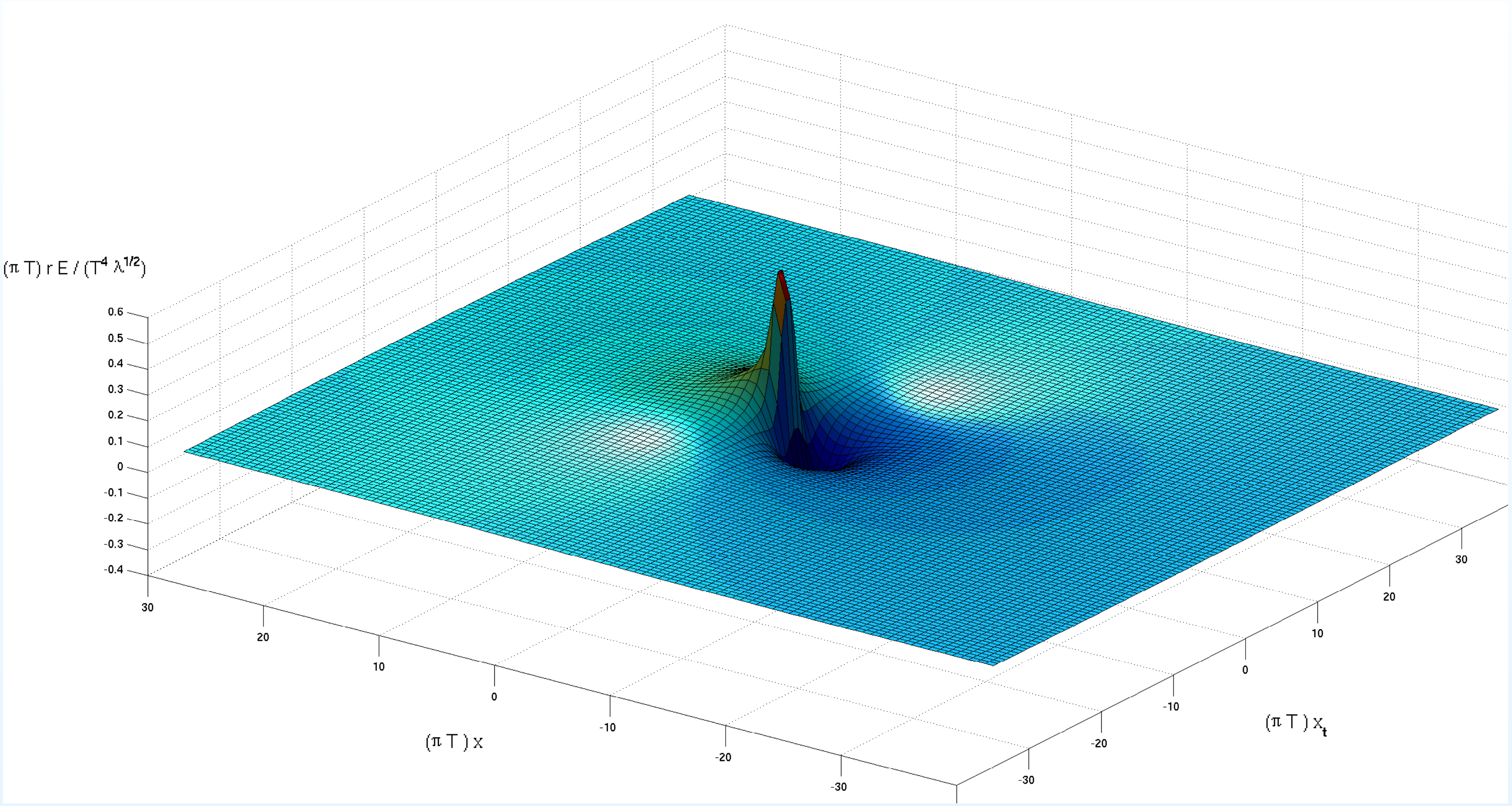
Heavy quark wake: $v = 0.25\,c$



P. Chesler

Outlook

Remarkable recent progress using gauge/string duality to map non-equilibrium, strongly coupled plasma dynamics into “easy” classical dynamics (in curved space).

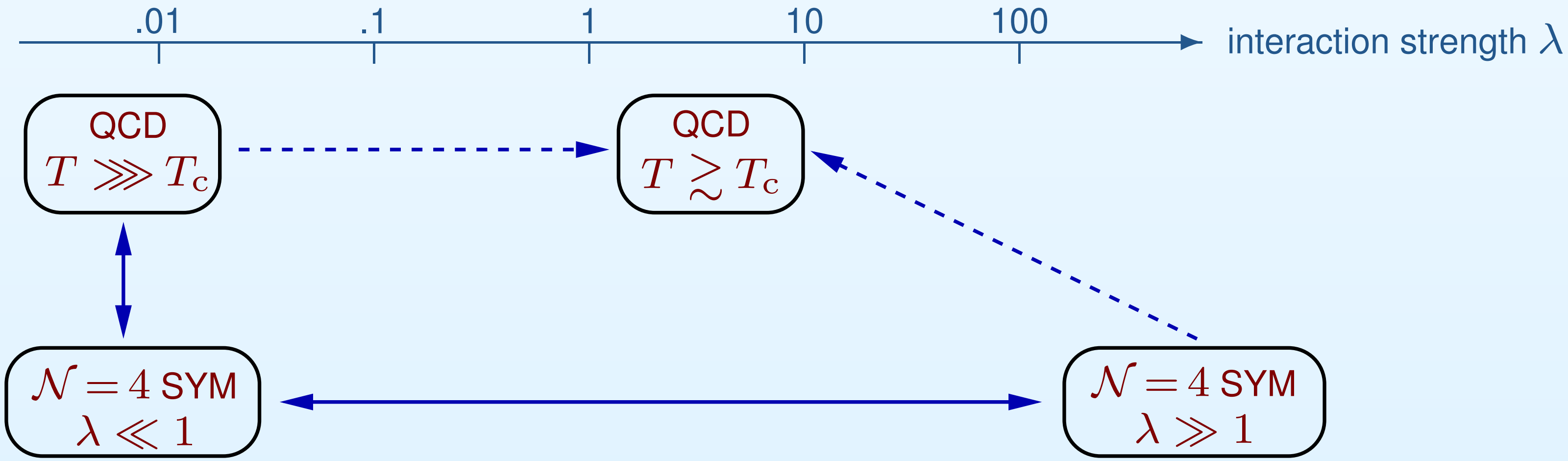
Comparisons of RHIC data and $\mathcal{N}=4$ SYM predictions currently underway.

Much more to do:

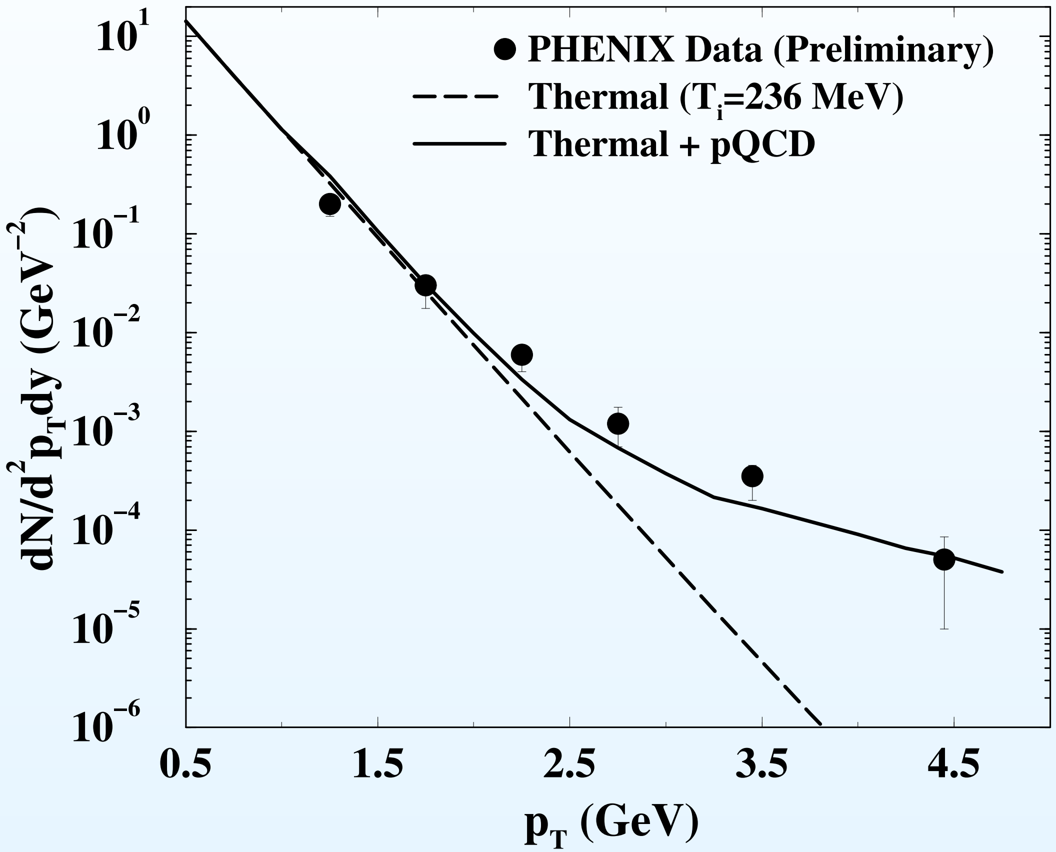
- light quark dynamics,
- $1/\lambda$ corrections,
- $1/N_{\rm c}$ corrections,
- results for more QCD-like theories (with known gravity duals), ...

important!

Goal: controlled extrapolation from $\lambda{=}\infty$, $\mathcal{N}=4$ SYM to real QGP near $T=T_{\rm c}$.



Photon yield: comparison to RHIC data



Preliminary results, Jan-e Alam

$\mathcal{N} = 4$ SYM spectrum plus perturbative QCD for high $p_\perp$ tail

$\Longrightarrow$ good fit with reasonable value of temperature T.