Quantum-limited measurements: One physicist's crooked path from quantum optics to quantum information

I. Introduction
II. Squeezed states and optical interferometry
III. Ramsey interferometry and cat states
IV. Quantum information perspective
V. Beyond the Heisenberg limit

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Quantum circuits in this presentation were set using the LaTeX package Qcircuit, developed at the University of New Mexico by Bryan Eastin and Steve Flammia. Qcircuit is available at http://info.phys.unm.edu/Qcircuit/ .

I. Introduction

In the Sawtooth Range
Central New Mexico

Quantum information science

A new way of thinking

Computer science

Computational complexity depends on physical law.

New physics

Quantum mechanics as liberator.

What can be accomplished with quantum systems that can't be done in a classical world?

Explore what can be done with quantum systems, instead of being satisfied with what Nature hands us.

Quantum engineering

Old physics

Quantum mechanics as nag.

The uncertainty principle restricts what can be done.

Metrology

Taking the measure of things
The heart of physics

New physics

Quantum mechanics as liberator.

Explore what can be done with quantum systems, instead of being satisfied with what Nature hands us.

Quantum engineering

Old physics

Quantum mechanics as nag.

The uncertainty principle restricts what can be done.

Old conflict in new guise

II. Squeezed states and optical interferometry

Oljeto Wash
Southern Utah

(Really) high-precision interferometry

Hanford, Washington

Initial LIGO

$$\begin{pmatrix}\text{differential}\\ \text{strain}\\ \text{sensitivity}\end{pmatrix} \simeq 10^{-21}$$

$$\begin{pmatrix}\text{differential}\\ \text{displacement}\\ \text{sensitivity}\end{pmatrix} \simeq 4\times 10^{-18}\,\text{m}$$

from $10\,\text{Hz}$ to $10^3\,\text{Hz}$.

Laser Interferometer Gravitational Observatory (LIGO)

Livingston, Louisiana

High-power, Fabry-Perot cavity (multipass) interferometers

(Really) high-precision interferometry

Hanford, Washington

Advanced LIGO

$$\begin{pmatrix}\text{differential}\\ \text{strain}\\ \text{sensitivity}\end{pmatrix} \simeq 3 \times 10^{-23}$$

$$\begin{pmatrix}\text{differential}\\ \text{displacement}\\ \text{sensitivity}\end{pmatrix} \simeq 10^{-20}\,\text{m}$$

from $10\,\text{Hz}$ to $10^3\,\text{Hz}$.

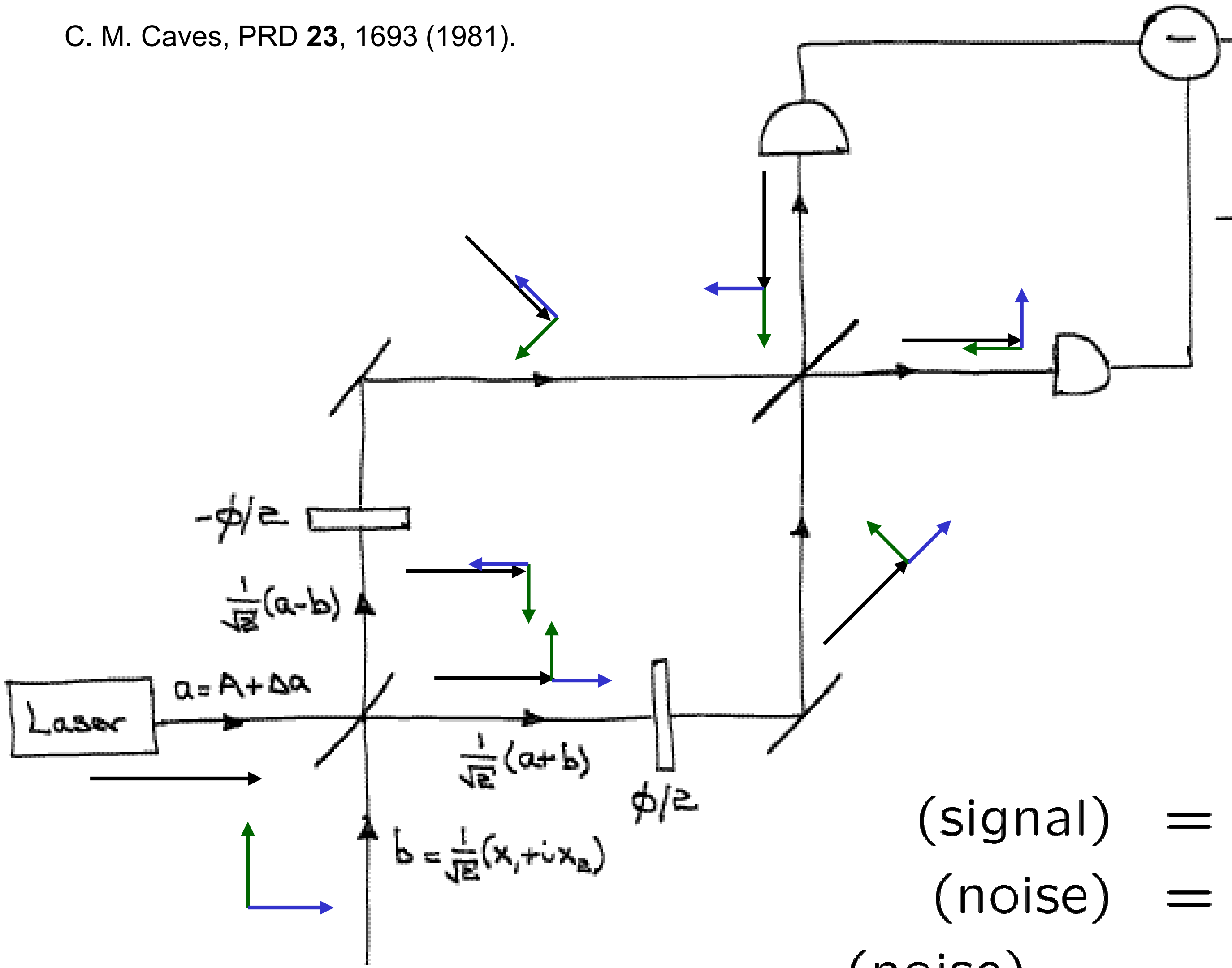
Laser Interferometer Gravitational Observatory (LIGO)

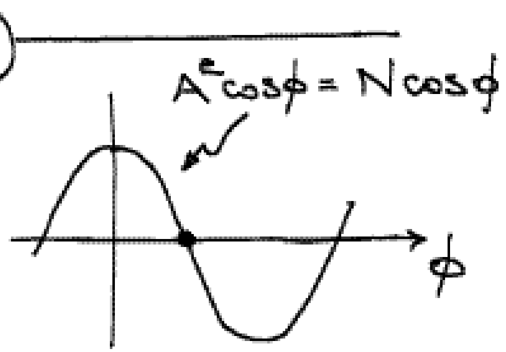
Livingston, Louisiana

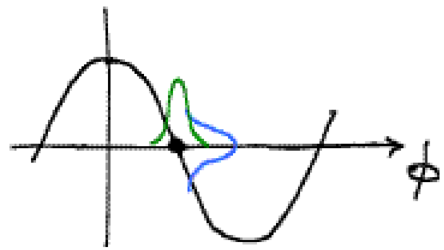
High-power, Fabry-Perot cavity (multipass), recycling, squeezed-state (?) interferometers

Mach-Zender interferometer

C. M. Caves, PRD **23**, 1693 (1981).





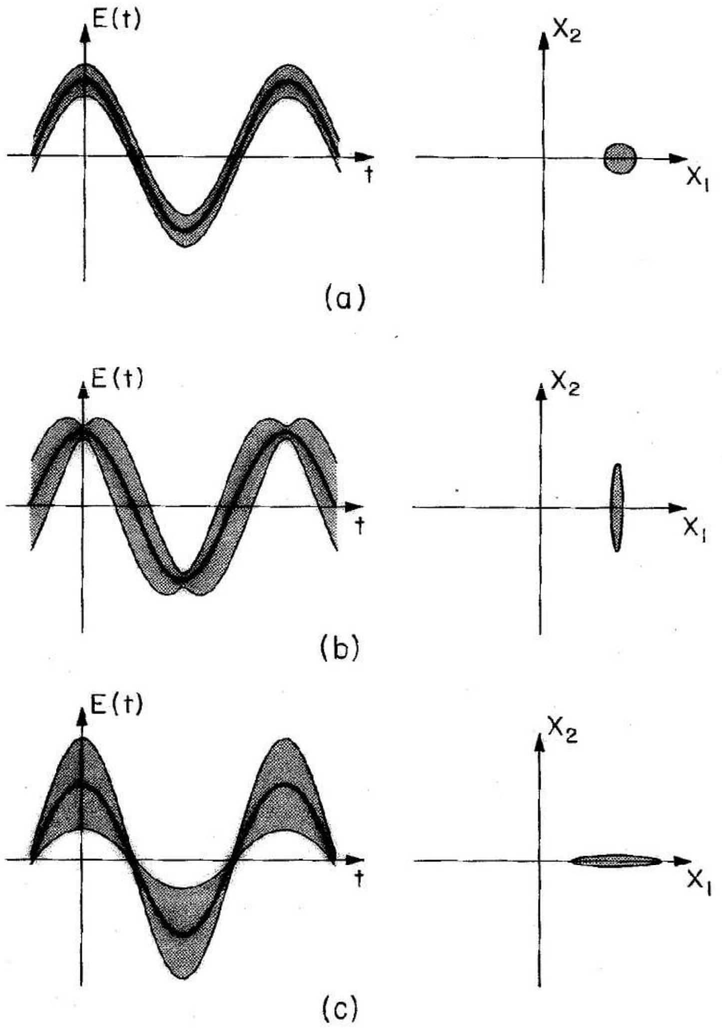


$$
\begin{aligned}
\text{(signal)} &= N\cos\phi \\
\text{(noise)} &= \sqrt{2N}\,\Delta x_2 |\sin\phi|
\end{aligned}
$$

$$
\Delta\phi = \frac{\text{(noise)}}{|d(\text{signal})/d\phi|} = \frac{\sqrt{2}\,\Delta x_2}{\sqrt{N}}
$$

Squeezed states of light

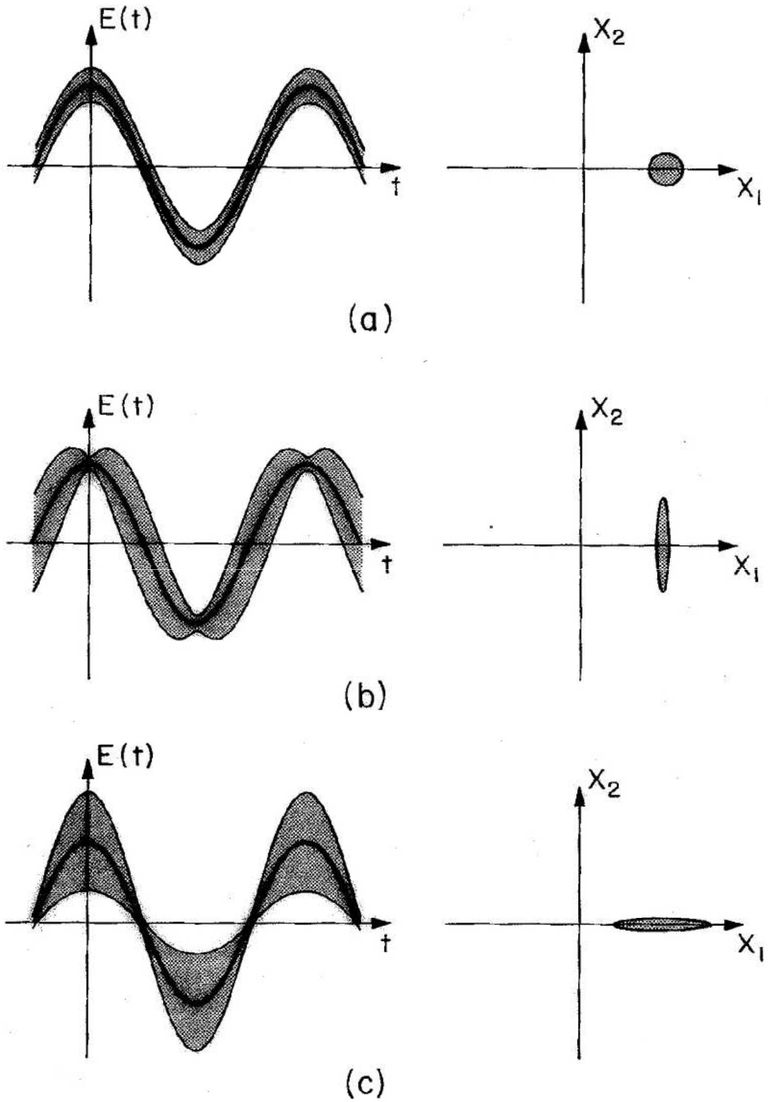
$$\Delta\phi \sim \frac{\Delta x_2}{A} = \frac{\Delta x_2}{\sqrt{N}}$$



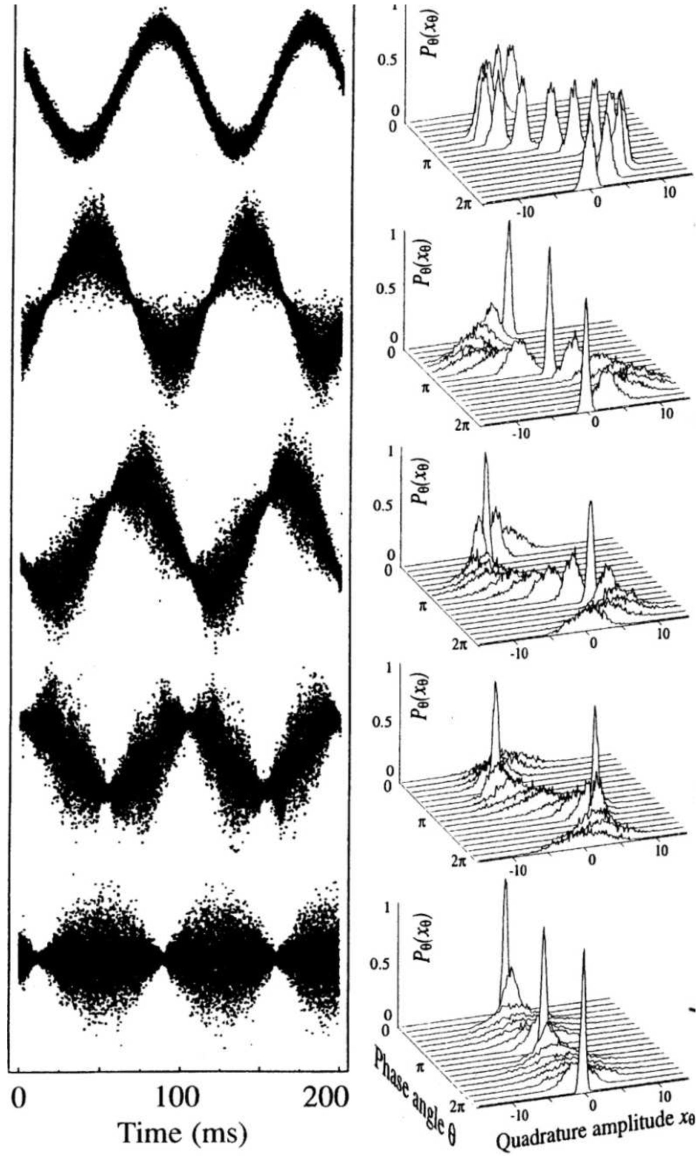
$$\Delta x_1 = \Delta x_2 = \frac{1}{\sqrt{2}}, \quad \Delta\phi \sim \frac{1}{\sqrt{2N}}$$

$$\begin{aligned} \Delta x_1 &= e^{r}/\sqrt{2} \\ \Delta x_2 &= e^{-r}/\sqrt{2} \end{aligned}, \quad \Delta\phi \sim \frac{e^{-r}}{\sqrt{2N}}$$

Squeezed states of light



Groups at ANU, Hannover, and Tokyo continue to push for greater squeezing at audio frequencies for use in LIGO II or III.



Squeezing by a factor of about 3.5

G. Breitenbach, S. Schiller, and J. Mlynek, Nature **387**, 471 (1997).

Quantum limits on interferometric phase measurements

$$\Delta\phi = \frac{\sqrt{2}\Delta x_2}{\sqrt{N}}$$

Standard Quantum Limit (Shot-Noise Limit)

$$\Delta x_1 = \Delta x_2 = \frac{1}{\sqrt{2}}, \quad \boxed{\Delta\phi = \frac{1}{\sqrt{N}}}$$

Heisenberg Limit

As much power in the squeezed light as in the main beam

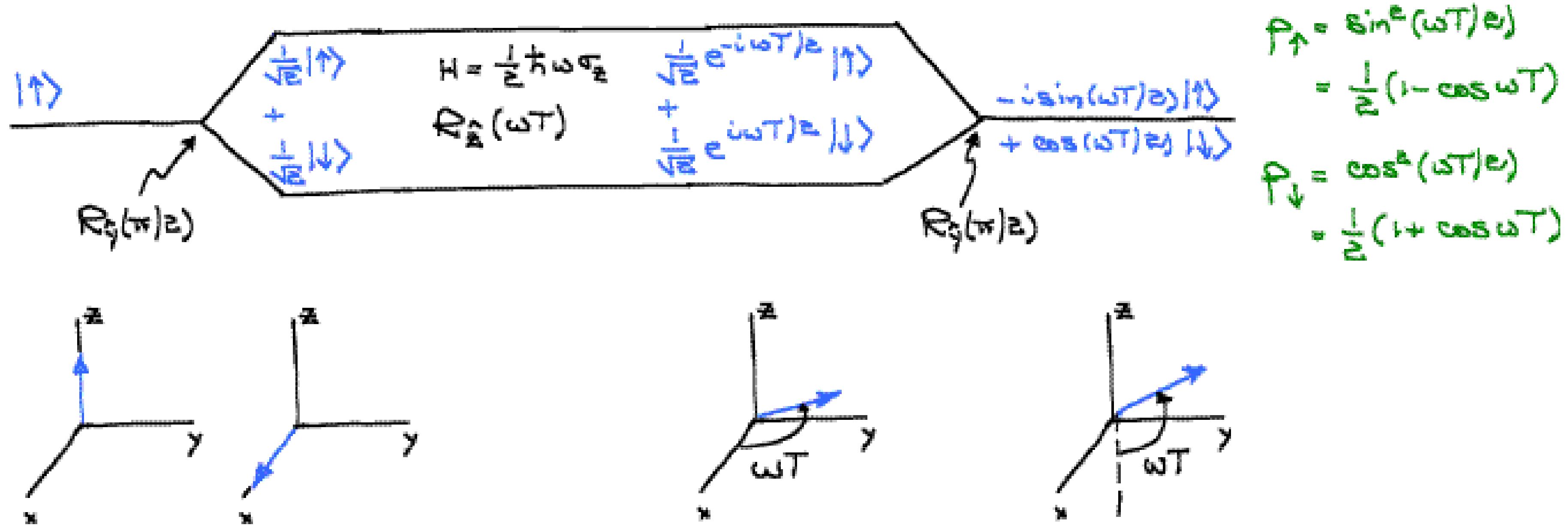
$$\frac{1}{2}(\Delta x_1)^2 \sim N$$

$$\Delta x_2 = \frac{1}{2\Delta x_1} = \frac{1}{2\sqrt{2N}}, \quad \boxed{\Delta\phi = \frac{1}{2N}}$$

III. Ramsey interferometry and cat states

Truchas from East Pecos Baldy
Sangre de Cristo Range
Northern New Mexico

Ramsey interferometry



***N* independent "atoms"**

$$(\text{signal}) = \langle\sigma_z\rangle = -\cos\omega T$$

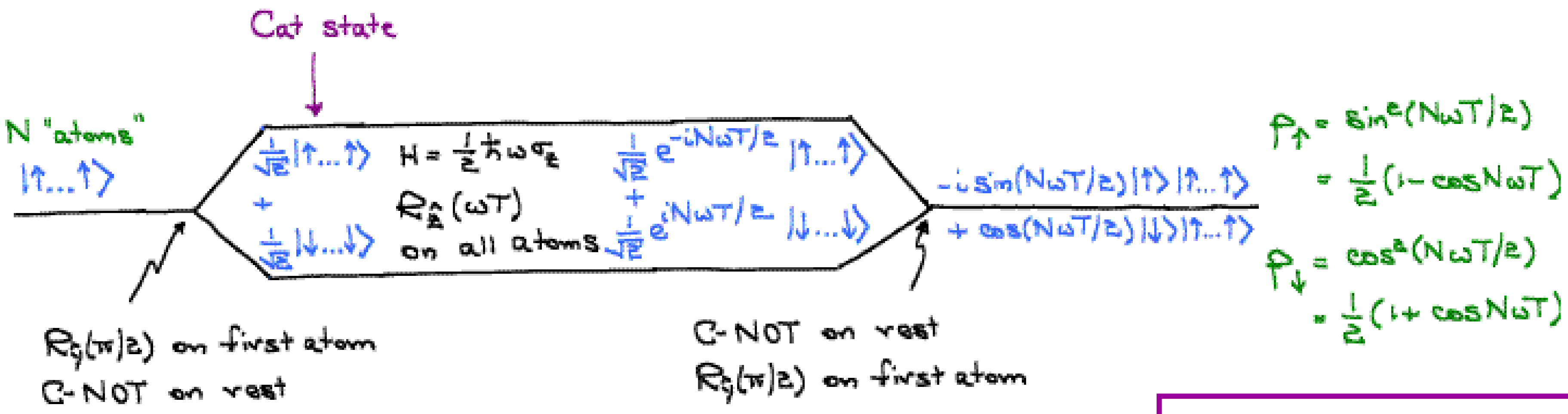
$$(\text{noise}) = \Delta\sigma_z = \sqrt{1-\cos^2\omega T} = |\sin\omega T|$$

$$\Delta(\omega T) = \boxed{\frac{1}{\sqrt{N}}}\,\frac{(\text{noise})}{|d(\text{signal})/d(\omega T)|} = \frac{1}{\sqrt{N}}$$

Frequency measurement
Time measurement
Clock synchronization

Cat-state Ramsey interferometry

J. J. Bollinger, W. M. Itano, D. J. Wineland, and D. J. Heinzen, Phys. Rev. A **54**, R4649 (1996).



Fringe pattern with period $2\pi/N$

$$\text{(signal)} = \langle\sigma_z\rangle = -\cos N\omega T$$

$$\text{(noise)} = \Delta\sigma_z = \sqrt{1-\cos^2 N\omega T} = |\sin N\omega T|$$

$$\Delta(\omega T) = \boxed{\frac{1}{\sqrt{\nu}}}\frac{\text{(noise)}}{|d(\text{signal})/d(\omega T)|} = \frac{1}{\sqrt{\nu}}\boxed{\frac{1}{N}}$$

$\nu =$ (number of trials)

***N* cat-state atoms**

Optical interferometry — Ramsey interferometry

$$\Delta\phi \sim \frac{1}{\sqrt{N}}$$

Standard Quantum Limit (Shot-Noise Limit)

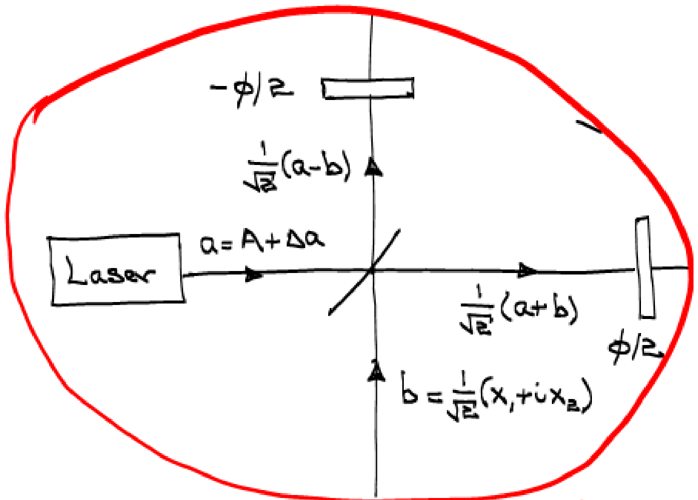
$$\Delta\phi \sim \frac{1}{\sqrt{N}}$$

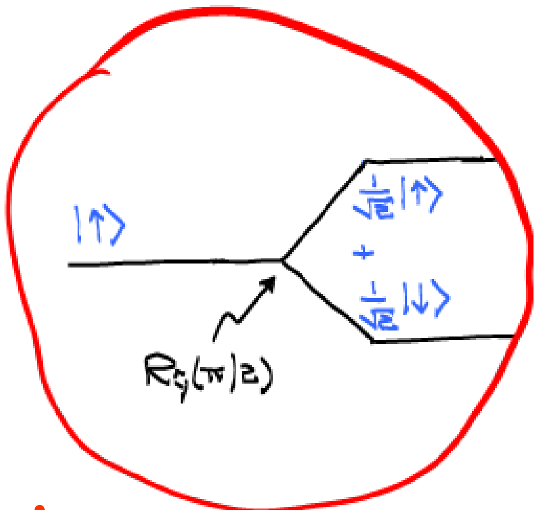
$$\phi = \omega T$$

$$\Delta\phi \sim \frac{1}{N}$$

Heisenberg Limit

$$\Delta\phi \sim \frac{1}{N}$$





Something's going on here.

Optical interferometry Ramsey interferometry

Entanglement?

Optical interferometry

Between arms
(wave entanglement)

$\sim r/\ln 2$ e-bits $\rightarrow \frac{1}{2}\log N$ e-bits

Between photons
(particle entanglement)

?

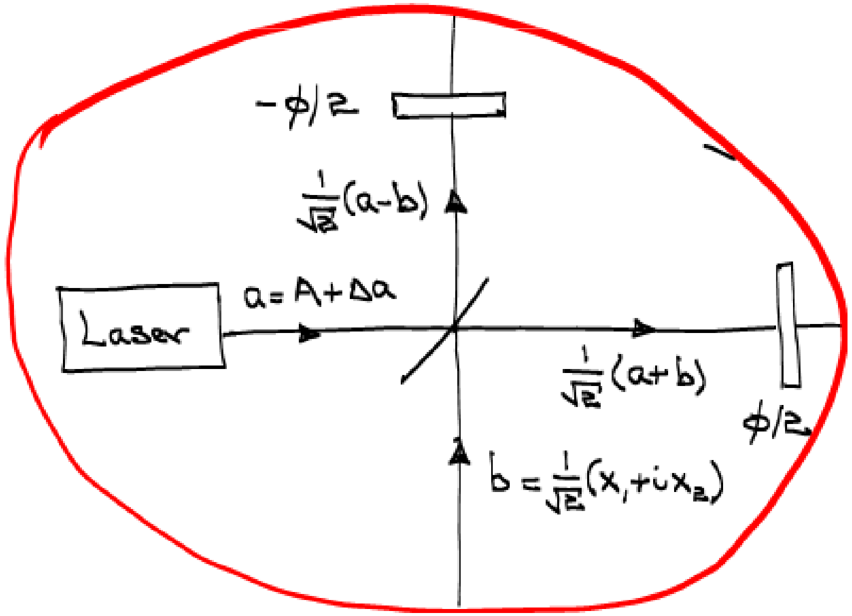
Ramsey interferometry

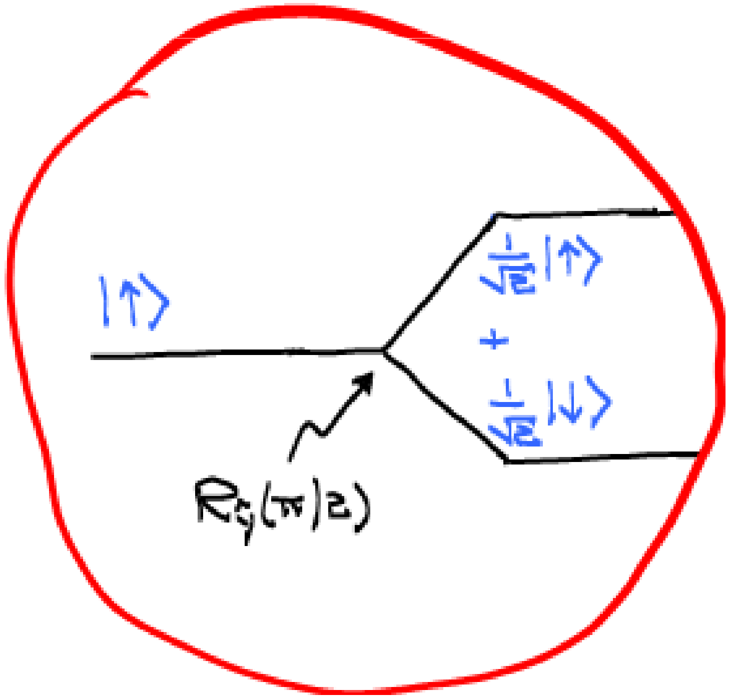
Between atoms
(particle entanglement)

1 e-bit

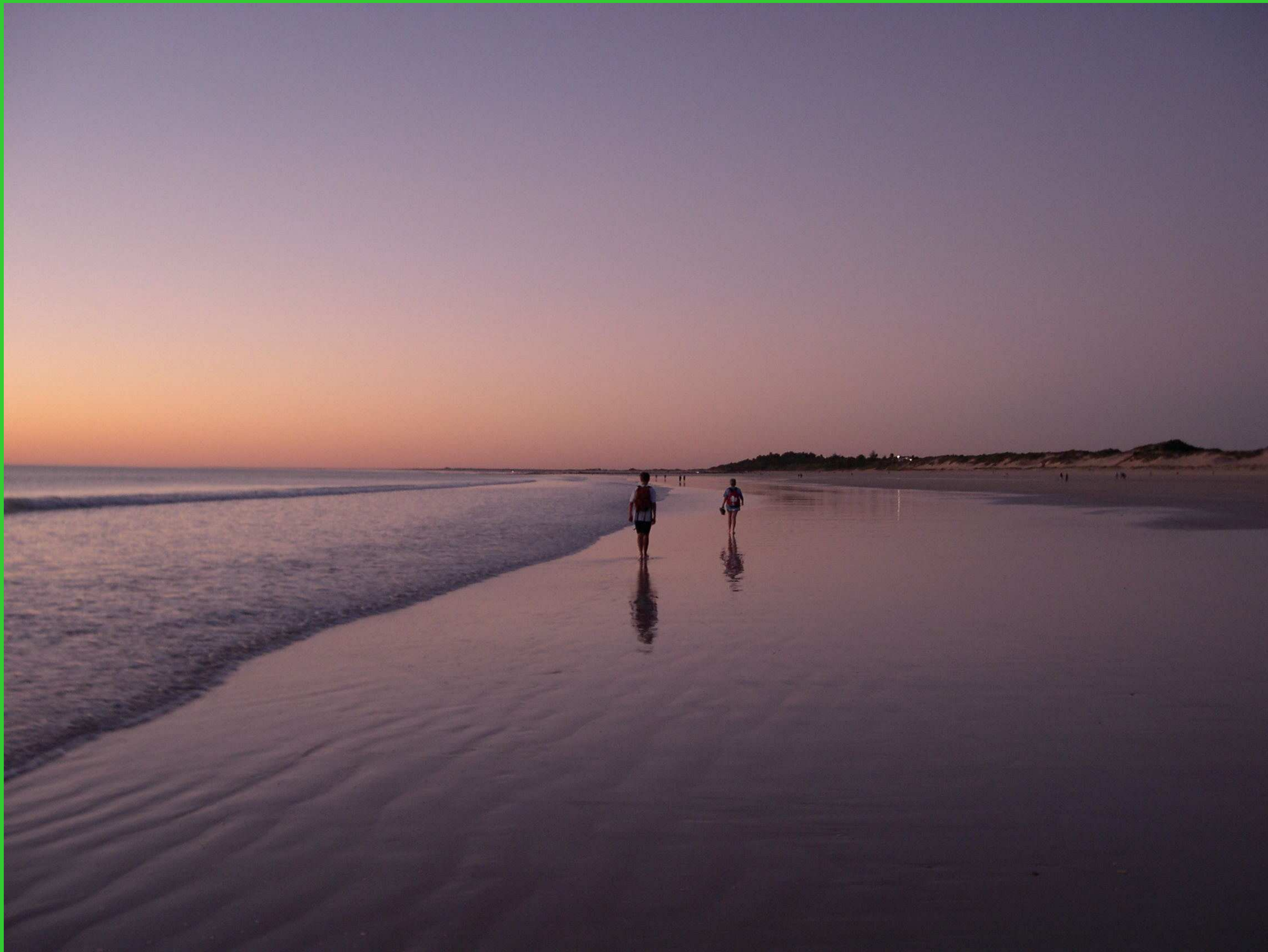
Between arms
(wave entanglement)

1 e-bit



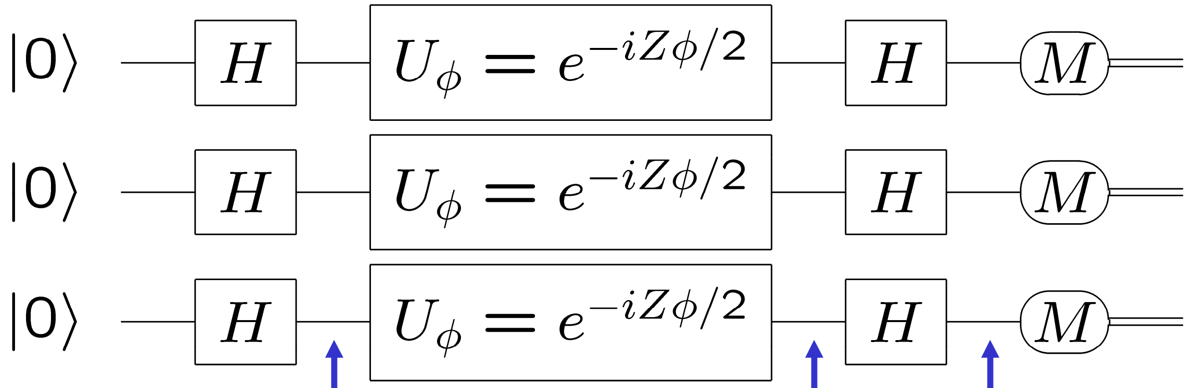


IV. Quantum information perspective



Cable Beach
Western Australia

Quantum information version of interferometry



Standard quantum limit

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$\frac{1}{\sqrt{2}}(e^{-i\phi/2}|0\rangle + e^{i\phi/2}|1\rangle)$

$\cos(\phi/2)|0\rangle - i\sin(\phi/2)|1\rangle)$

Quantum circuits

Heisenberg limit

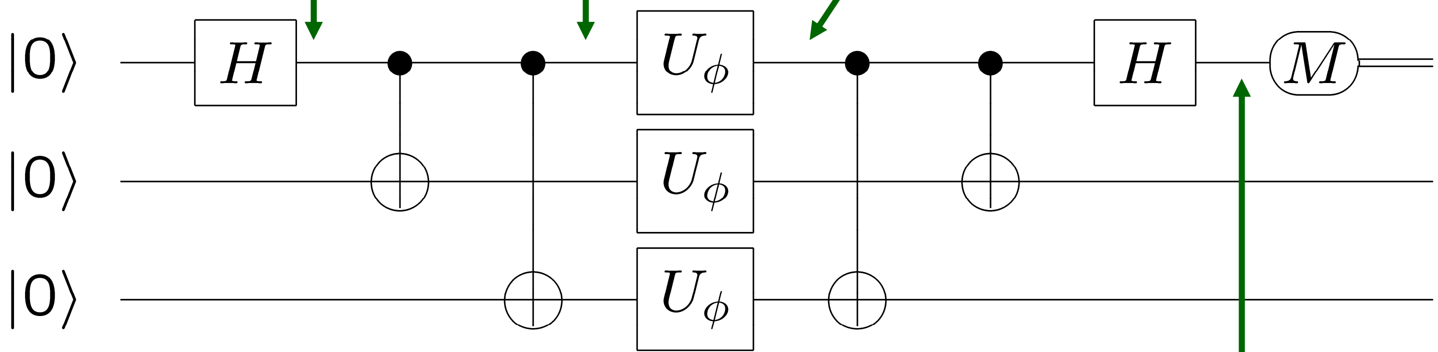
$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|00\rangle$

cat state

$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$

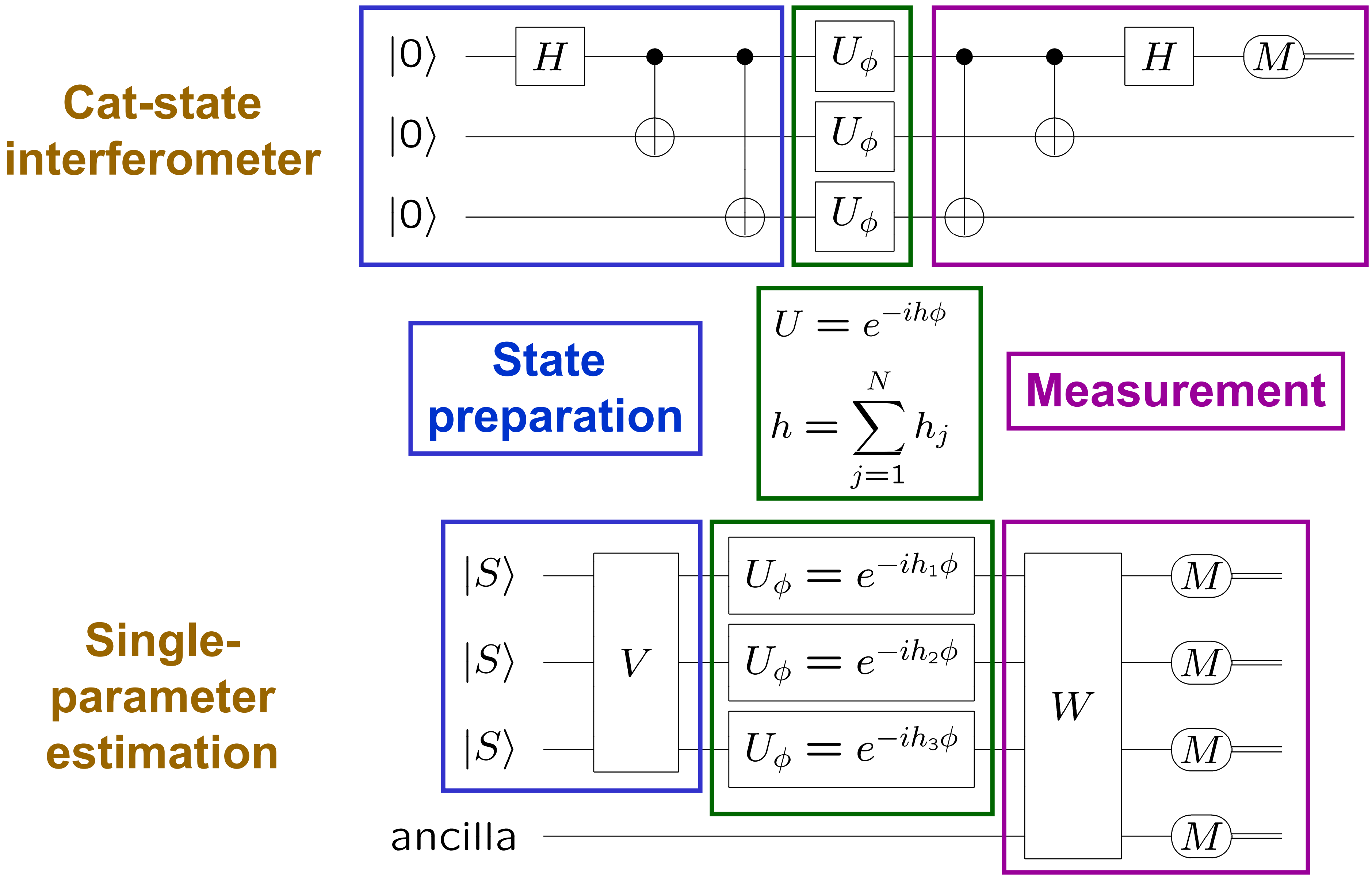
$\frac{1}{\sqrt{2}}(e^{-iN\phi/2}|000\rangle + e^{iN\phi/2}|111\rangle)$

$N = 3$



Fringe pattern with period 2π/*N*

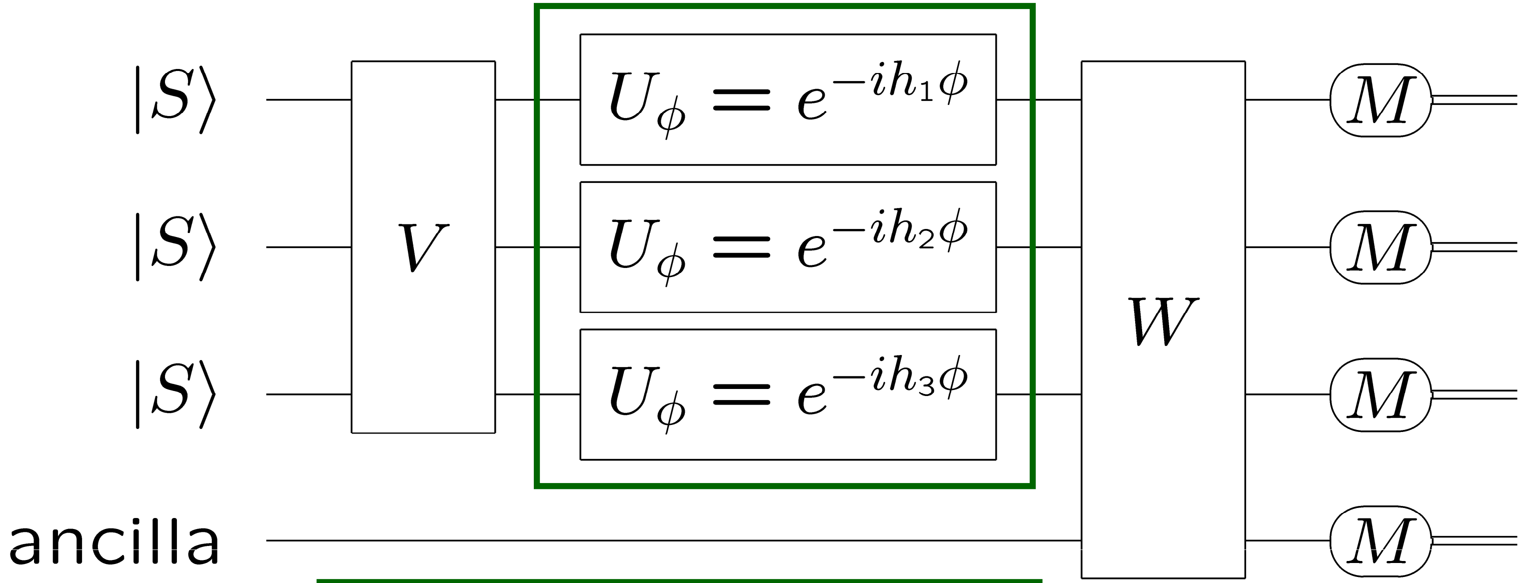
$[\cos(N\phi/2)|0\rangle - i\sin(N\phi/2)|1\rangle]|00\rangle$



Cat-state interferometer
|0⟩
|0⟩
|0⟩
H
U_ϕ
U_ϕ
U_ϕ
H
M
State preparation
$U = e^{-ih\phi}$
$h = \sum_{j=1}^{N} h_j$
Measurement
Single-parameter estimation
|S⟩
|S⟩
|S⟩
V
$U_\phi = e^{-ih_1\phi}$
$U_\phi = e^{-ih_2\phi}$
$U_\phi = e^{-ih_3\phi}$
W
M
M
M
M
ancilla

Heisenberg limit

S. L. Braunstein, C. M. Caves, and G. J. Milburn, Ann. Phys. **247**, 135 (1996).
V. Giovannetti, S. Lloyd, and L. Maccone, PRL **96**, 041401 (2006).



W, U_ϕ, and ancilla measurements can be interleaved.

$$U = e^{-ih\phi}\,, \qquad h = \sum_{j=1}^{N} h_j$$

$$\Delta\phi \geq \frac{1}{2\Delta h} \geq \frac{1}{N(\Lambda - \lambda)}$$

Generalized uncertainty principle
Cramér-Rao bound

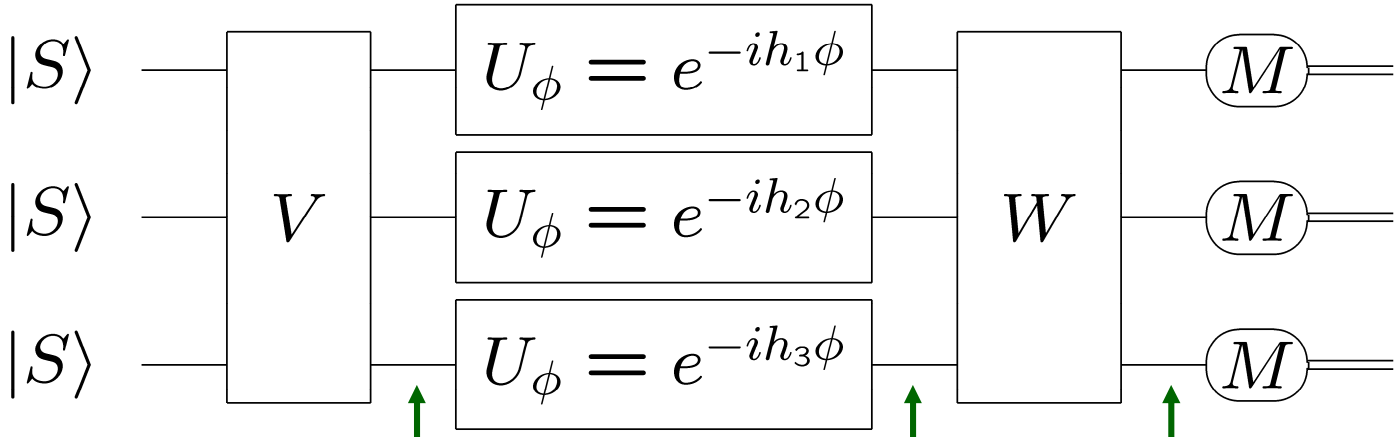
$$\Delta h \leq \frac{1}{2}||h|| = \frac{1}{2}N(\Lambda - \lambda)$$

Separable inputs

$$\Delta h \leq \frac{1}{2}\sqrt{N}(\Lambda - \lambda)$$

$$\Delta\phi \geq \frac{1}{\sqrt{N}(\Lambda - \lambda)}$$

Achieving the Heisenberg limit



cat state

$$\frac{1}{\sqrt{2}}(|\Lambda,\ldots,\Lambda\rangle + |\lambda,\ldots,\lambda\rangle)$$

$$\frac{1}{\sqrt{2}}\left(e^{-iN\Lambda\phi}|\Lambda,\ldots,\Lambda\rangle + e^{-iN\lambda\phi}|\lambda,\ldots,\lambda\rangle\right)$$

$$e^{-iN(\Lambda+\lambda)\phi/2}\Big(\cos[N(\Lambda-\lambda)\phi/2]|\Lambda,\ldots,\Lambda\rangle - i\sin[N(\Lambda-\lambda)\phi/2]|\lambda,\ldots,\lambda\rangle\Big)$$

Fringe pattern with period $2\pi/N(\Lambda-\lambda)$

$$\Delta\phi = \frac{1}{N(\Lambda-\lambda)}$$

Is it entanglement?

It's the entanglement, stupid.

But what about?

- The optimal state for optical interferometry, the optical cat state $(|N,0\rangle+|0,N\rangle)/\sqrt{2}$, called a "N00N" state, does a tiny bit better than inputting the optimal squeezed (Gaussian) state, but has just 1 e-bit of entanglement (either wave or particle), compared with the much larger $\sim \frac{1}{2}\log N$ e-bits of wave entanglement when inputting the optimal squeezed state.

- Flip half the spins in a cat state, and you get a state with the same amount of entanglement, but one that is worthless for metrology.

- Measurement sensitivity and optimal initial state depend on local Hamiltonians h_j, but entanglement measures are usually constructed to be independent of such mundane details.

We need a generalized notion of entanglement that includes information about the physical situation, particularly the relevant Hamiltonian.

V. Beyond the Heisenberg limit

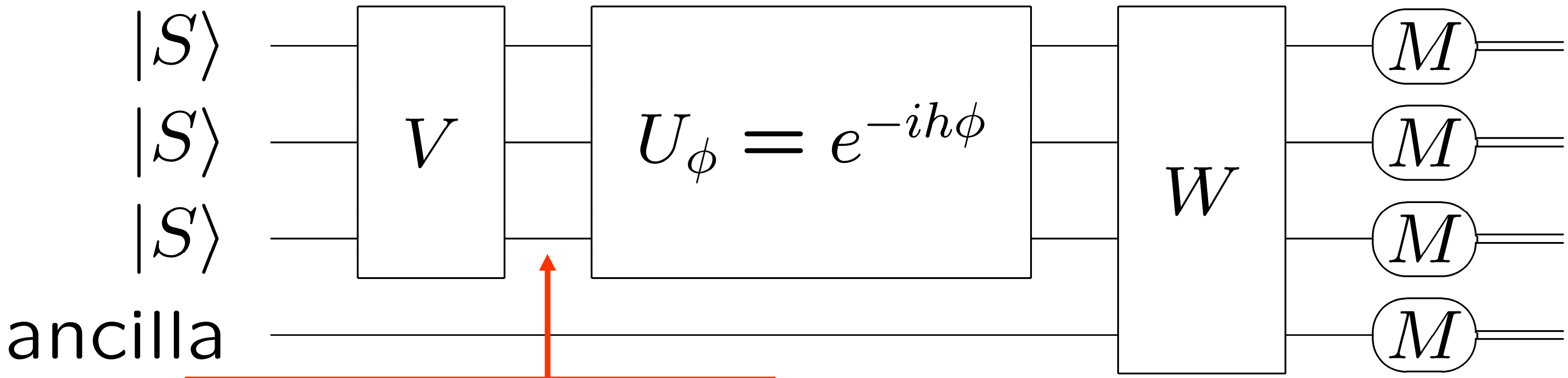
Echidna Gorge
Bungle Bungle Range
Western Australia

Beyond the Heisenberg limit

The purpose of theorems in physics is to lay out the assumptions clearly so one can discover which assumptions have to be violated.

Improving the scaling with N

S. Boixo, S. T. Flammia, C. M. Caves, and JM Geremia, PRL **98**, 090401 (2007).



Cat state does the job.

Nonlinear Ramsey interferometry

$$\Delta\phi \geq \frac{1}{2\Delta h} \geq \frac{1}{||h||} = \frac{1}{N^k(\Lambda^k - \lambda^k)}$$

Metrologically relevant *k*-body coupling

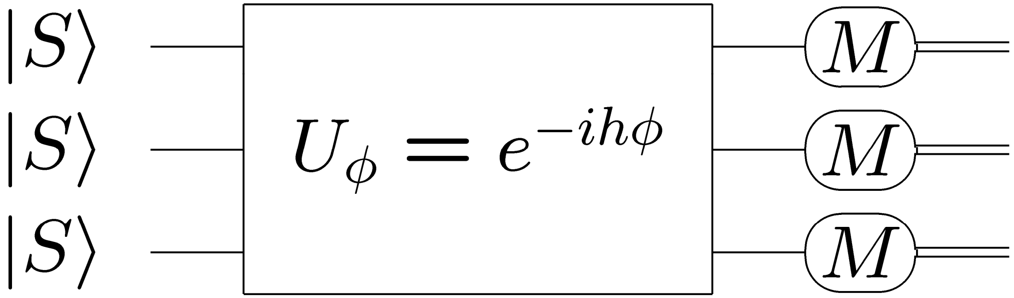
$$h = \left(\sum_{j=1}^{N} h_j\right)^k = \underbrace{\sum_{j_1,\ldots,j_k} h_{j_1} h_{j_2} \cdots h_{j_k}}_{N^k \text{ terms in sum}}$$

$$||h|| = N^k(\Lambda^k - \lambda^k)$$

Improving the scaling with N without entanglement

S. Boixo, A. Datta, S. T. Flammia, A. Shaji, E. Bagan, and C. M. Caves, PRA **77**, 012317 (2008).

Product input



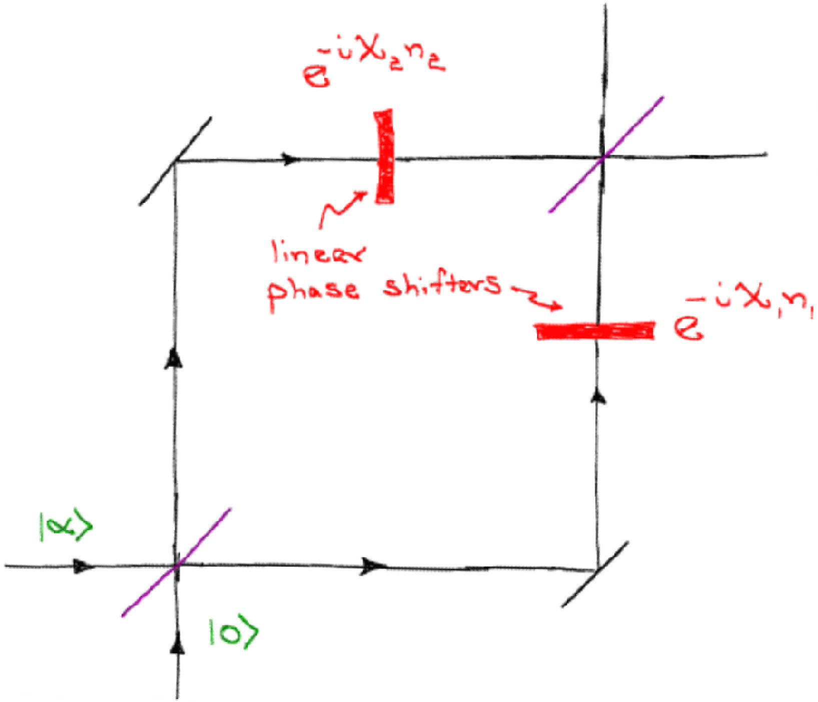
Product measurement

$$h = \left(\sum_{j=1}^{N} Z_j/2 \right)^k = J_z^k$$

$$\Delta\phi \sim \frac{1}{N^{k-1/2}}$$

Improving the scaling with N without entanglement.
Two-body couplings

Linear interferometer



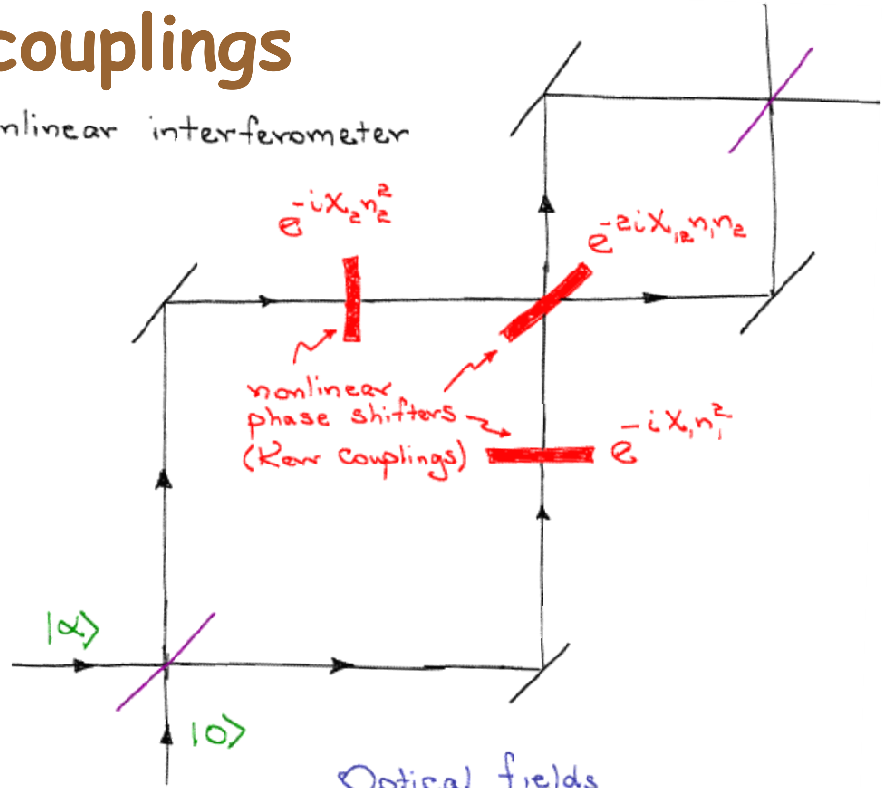
$$\chi_1 n_1 + \chi_2 n_2 = \tfrac{1}{2}(\chi_1+\chi_2)N + \boxed{\underbrace{(\chi_1-\chi_2)\delta n}_{\equiv\,\phi}}$$

$$N = n_1 + n_2\,, \qquad \delta n = \tfrac{1}{2}(n_1 - n_2)$$

$$\Delta\phi = 1/\sqrt{N}$$

M. J. Woolley, G. J. Milburn, and C. M. Caves, New J. Phys., to be published, arXiv:0804.4540 [quant-ph].

Nonlinear interferometer



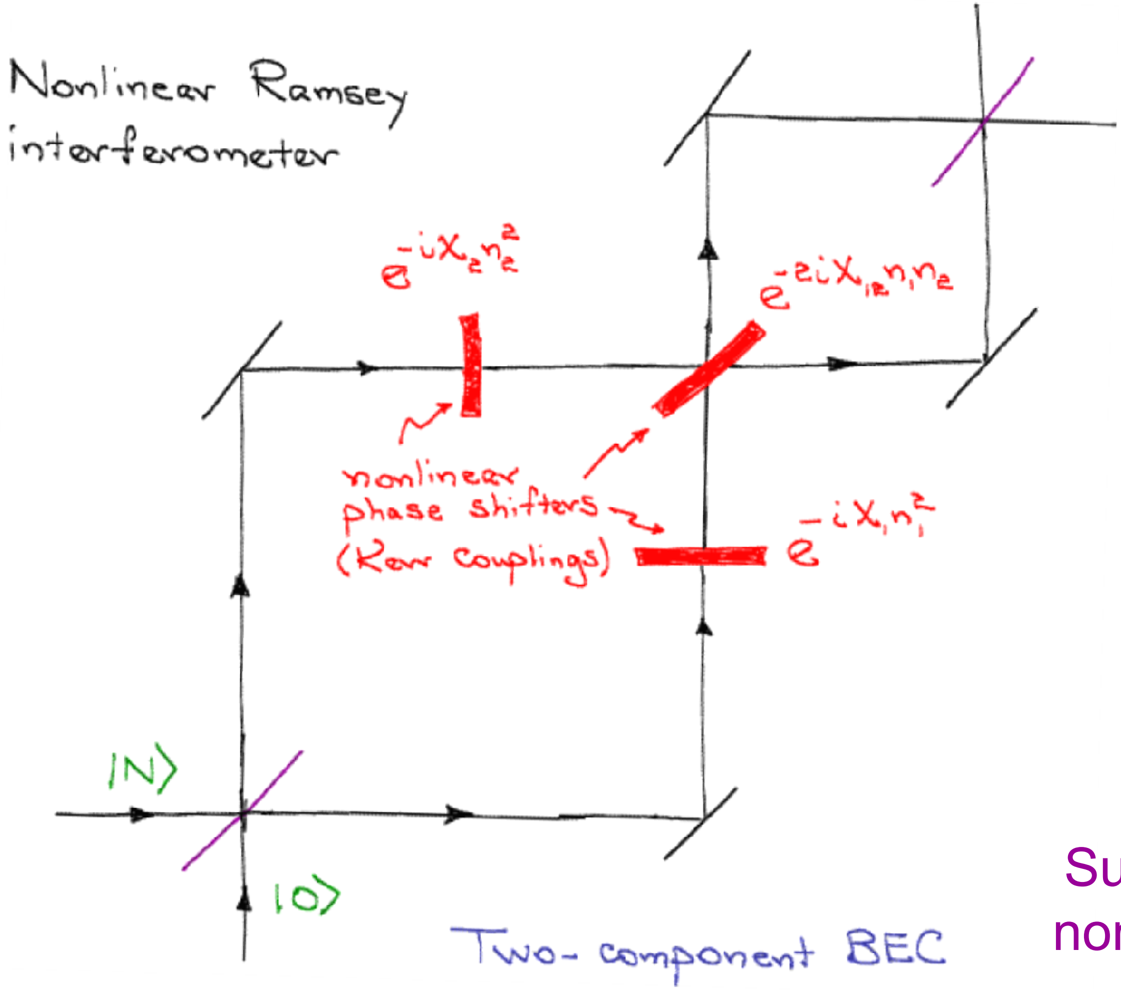
Optical fields

Nanomechanical resonators

$$\begin{aligned}\chi_1 n_1^2 + \chi_2 n_2^2 + 2\chi_{12} n_1 n_2 &= \tfrac{1}{4}(\chi_1+\chi_2+2\chi_{12})N^2 \\ &\quad + \boxed{(\chi_1-\chi_2)N\delta n} \\ &\quad + (\chi_1+\chi_2-2\chi_{12})\delta n^2\end{aligned}$$

$$\Delta\phi = 1/N^{3/2}$$

Improving the scaling with N without entanglement. Two-body couplings



$$\chi_1 n_1^2 + \chi_2 n_2^2 + 2\chi_{12} n_1 n_2$$
$$= \tfrac{1}{4}(\chi_1 + \chi_2 + 2\chi_{12})N^2$$
$$+(\underbrace{\chi_1 - \chi_2}_{\equiv \phi})N\delta n$$
$$+(\underbrace{\chi_1 + \chi_2 - 2\chi_{12}}_{\simeq 0})\delta n^2$$

$$\Delta\phi = 1/N^{3/2}$$

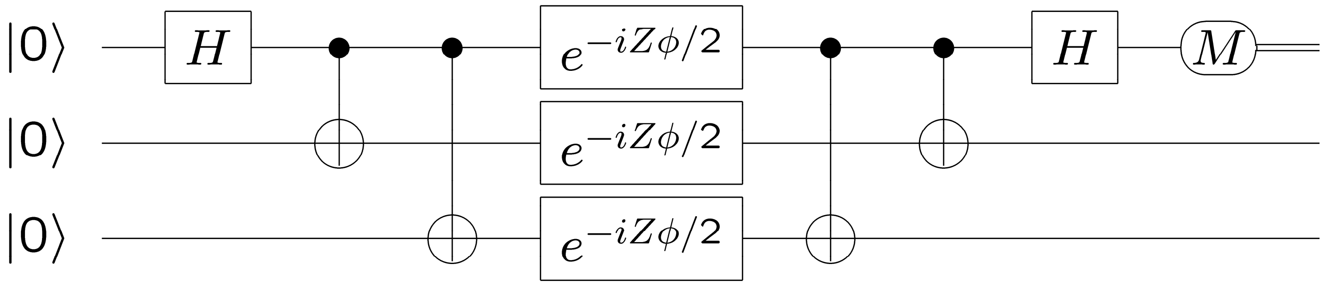
Super-Heisenberg scaling from nonlinear dynamics, without any particle entanglement

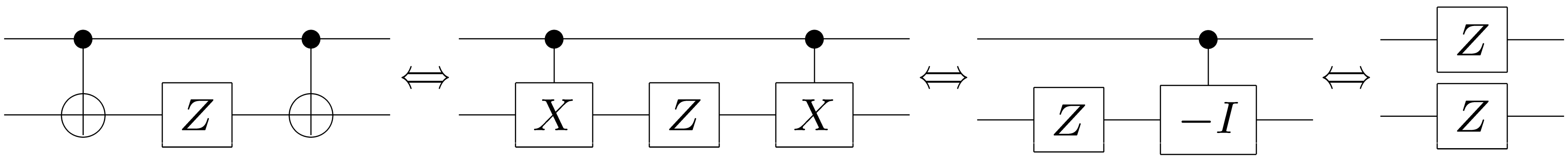
S. Boixo, A. Datta, M. J. Davis, S. T. Flammia, A. Shaji, and C. M. Caves, PRL **101**, 040403 (2008).

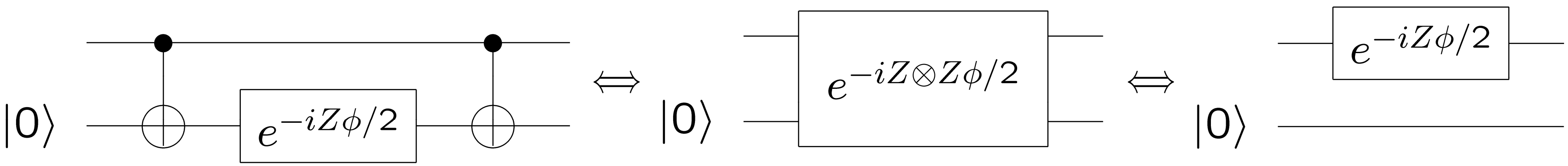
Bungle Bungle Range
Western Australia

Using quantum circuit diagrams

Cat-state interferometer







Cat-state interferometer $\Longleftrightarrow$