

# The Quantum Mechanics of Global Warming

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Brown University

University of Virginia

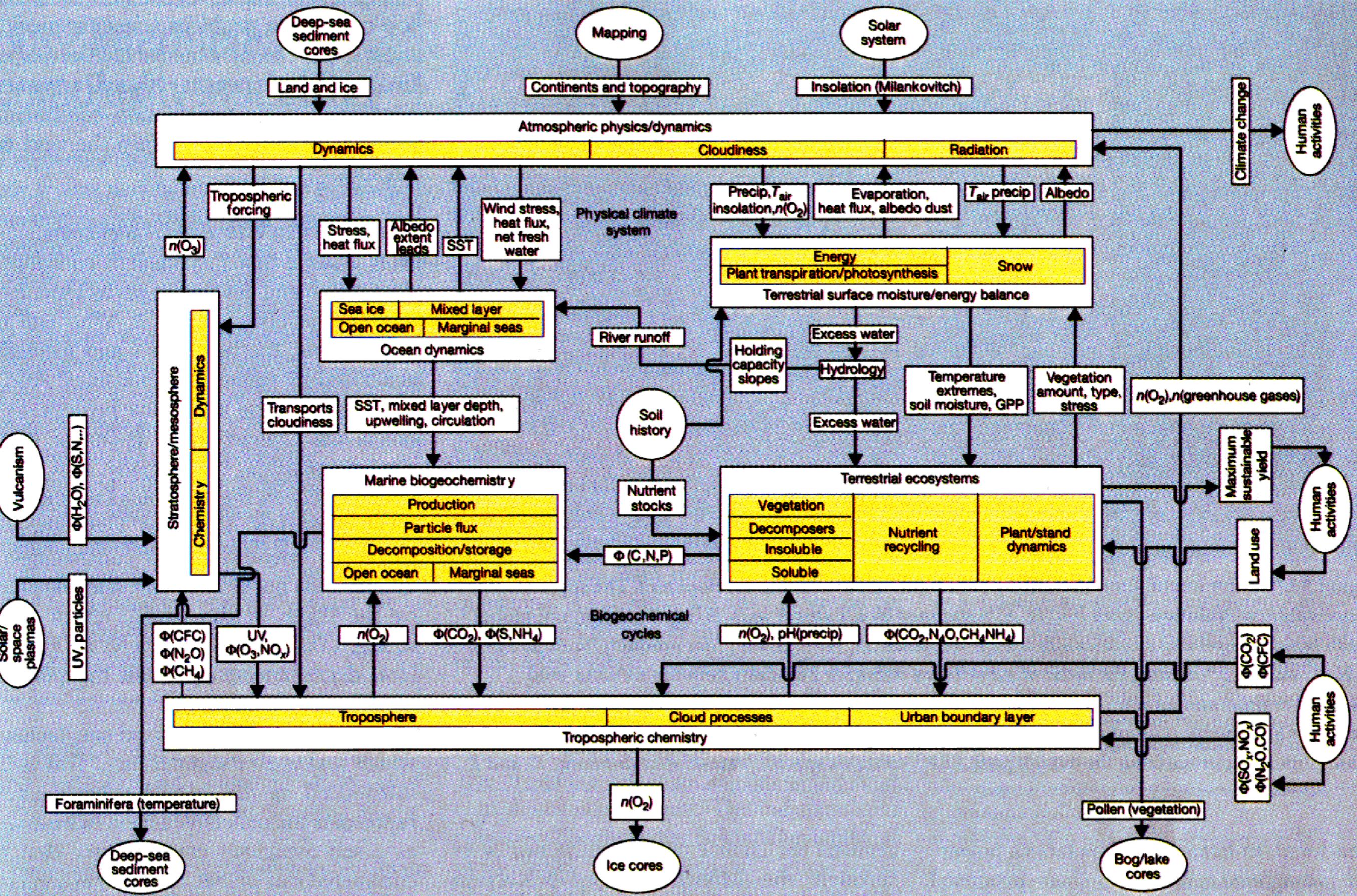
April 17, 2009

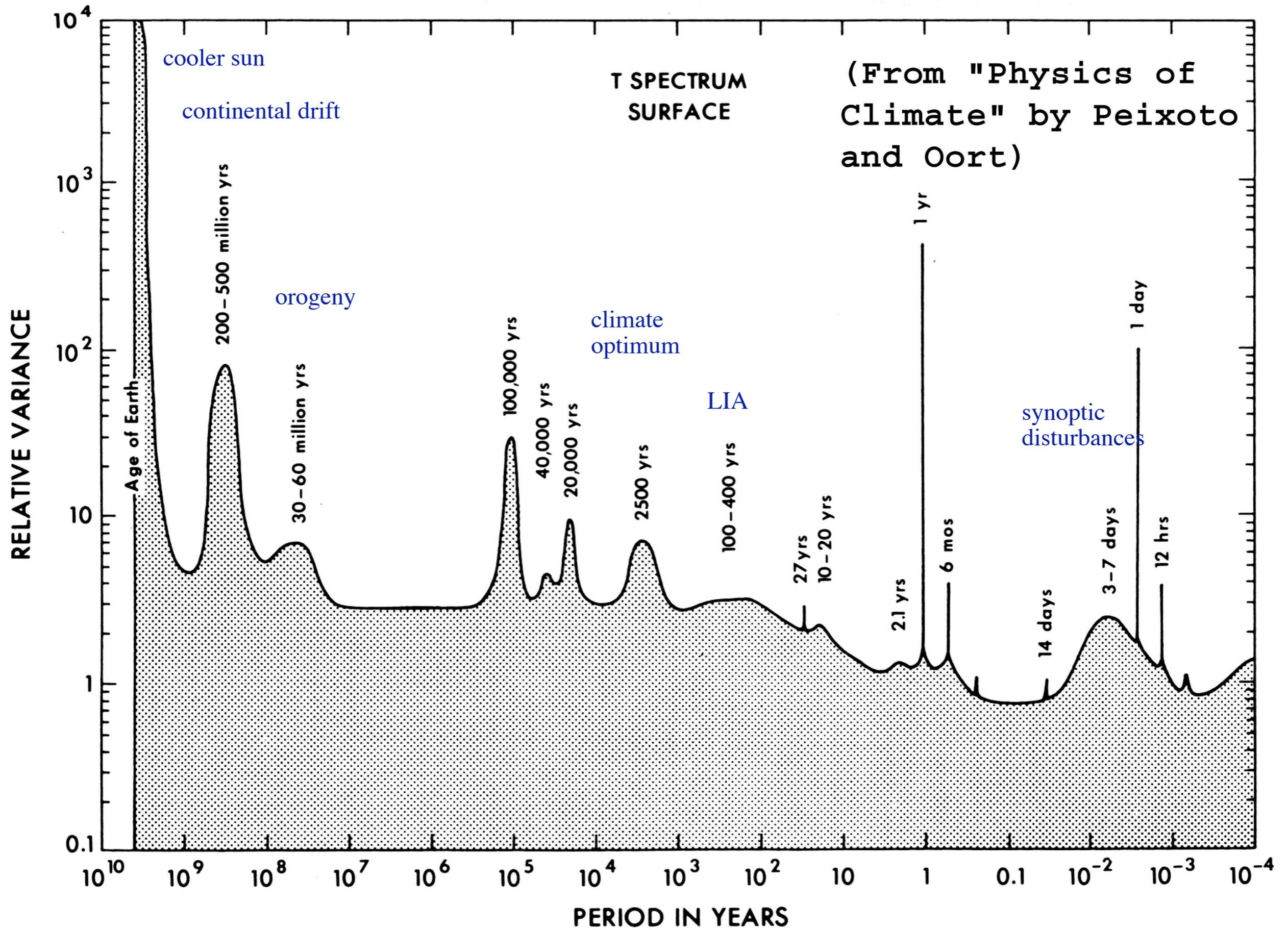


# Richardson's Human Weather Computer (1917 --1922)



“Lewis Fry Richardson’s imaginary ‘forecast factor’ would have employed some 64,000 human computers to keep up with the pace of the weather, The workers sit in tiers inside a great spherical theater; the director, atop a pedestal in the middle, shines a beam of light on those places where the calculation is getting ahead or falling behind.” [Brian Hayes, *American Scientist* **80**, 10 -- 14 (2001).]





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- Ecosystems and feedbacks

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The diagram shows the functional form  $Intensity = function(Temperature, frequency)$  at the top. Two red arrows point downwards from the words "Watts" and "Joules" in the equation below to the corresponding terms in the functional form above. The equation is:  $[I] = \frac{Watts}{meter^2} = \frac{Joules}{m^2 s}$  and  $[k_B T] = J$ .

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*Intensity = function(Temperature, frequency)*

The diagram shows three red arrows pointing downwards from the words 'Intensity', 'Temperature', and 'frequency' in the equation above to their respective units in the equations below.

$$[I] = \frac{\text{Watts}}{\text{meter}^2} = \frac{\text{Joules}}{\text{m}^2 \text{ s}}$$
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**UV Catastrophe!**

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$$\begin{aligned}\Delta I(T, \nu) &= \frac{2\pi h\nu^3}{c^2} \frac{1}{e^{h\nu/k_B T} - 1} \Delta\nu \\ &\rightarrow \frac{2\pi k_B T \nu^2}{c^2} \Delta\nu \text{ for } k_B T \gg h\nu \\ &\rightarrow 0 \text{ for } \nu \rightarrow \infty\end{aligned}$$

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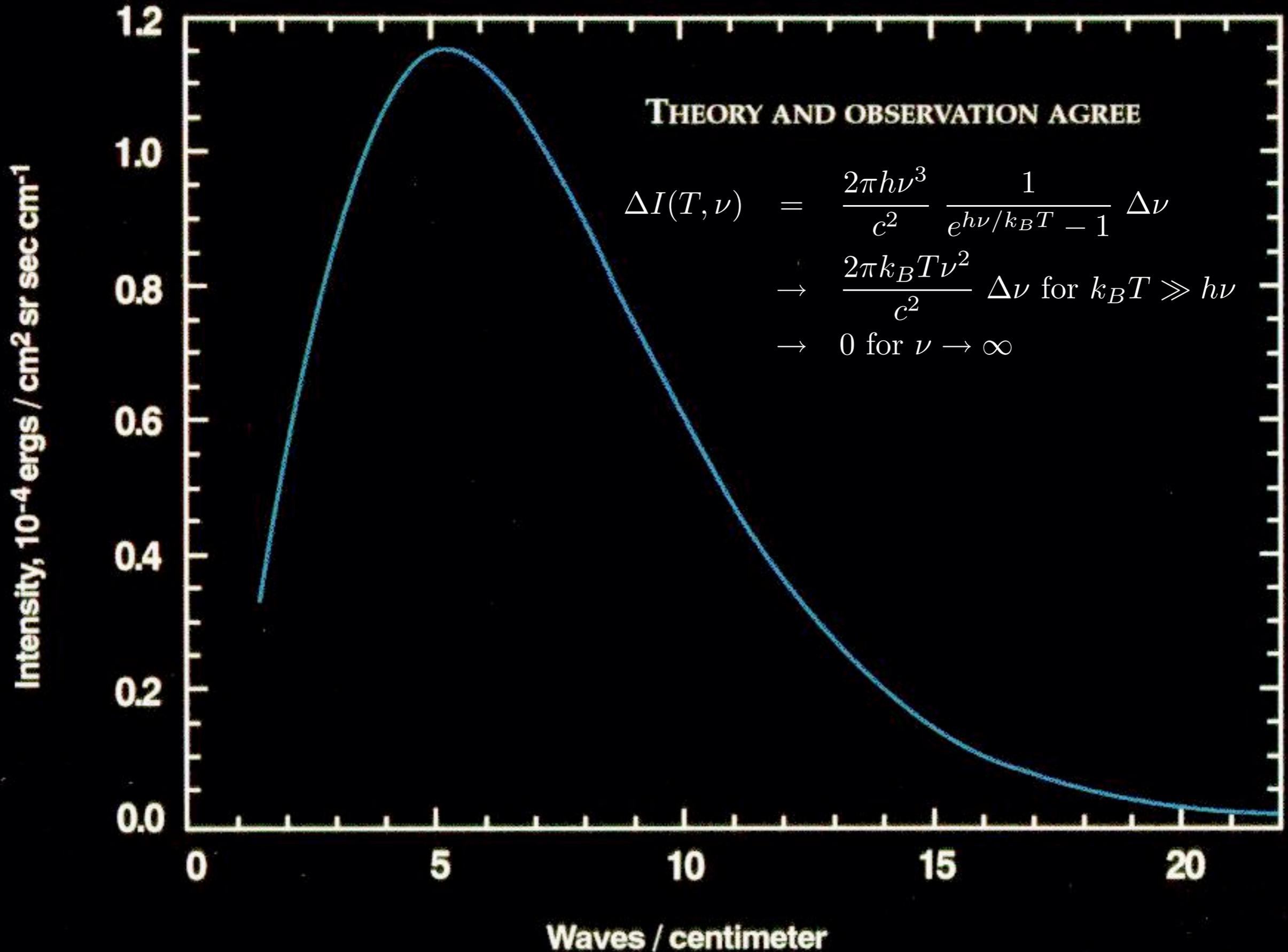
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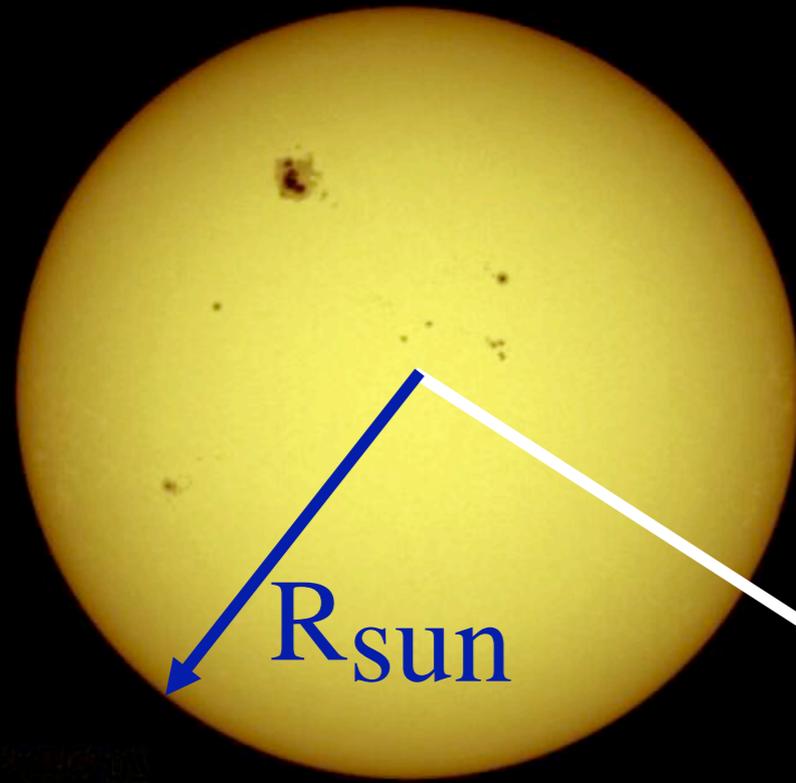
Now we can do that sum over frequency!

$$\begin{aligned} I &= \sigma T^4 \\ \sigma &\equiv \frac{2\pi^5 k_B^4}{15h^3 c^2} = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4} \end{aligned}$$

# COSMIC MICROWAVE BACKGROUND SPECTRUM FROM COBE

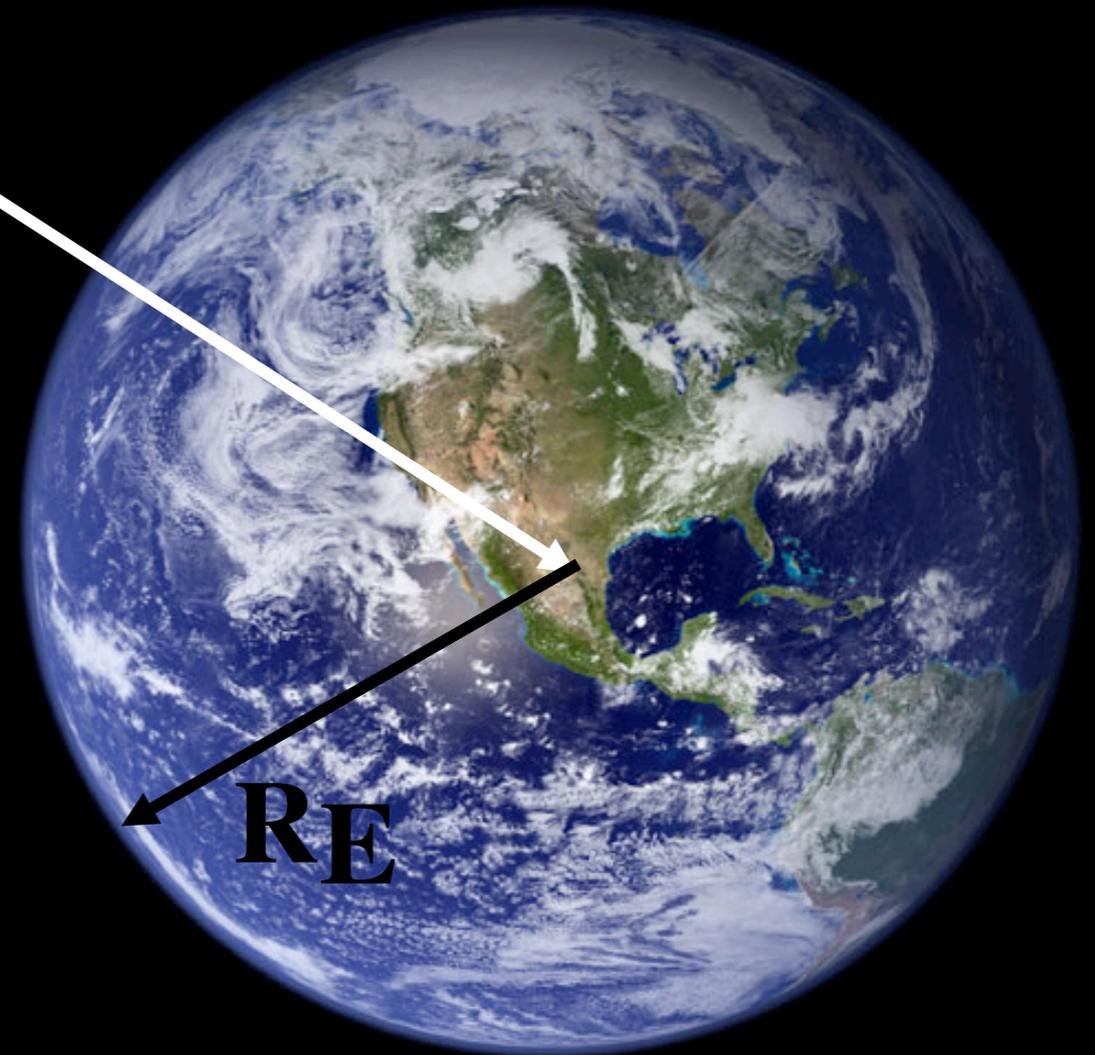


# Temperature of the Earth

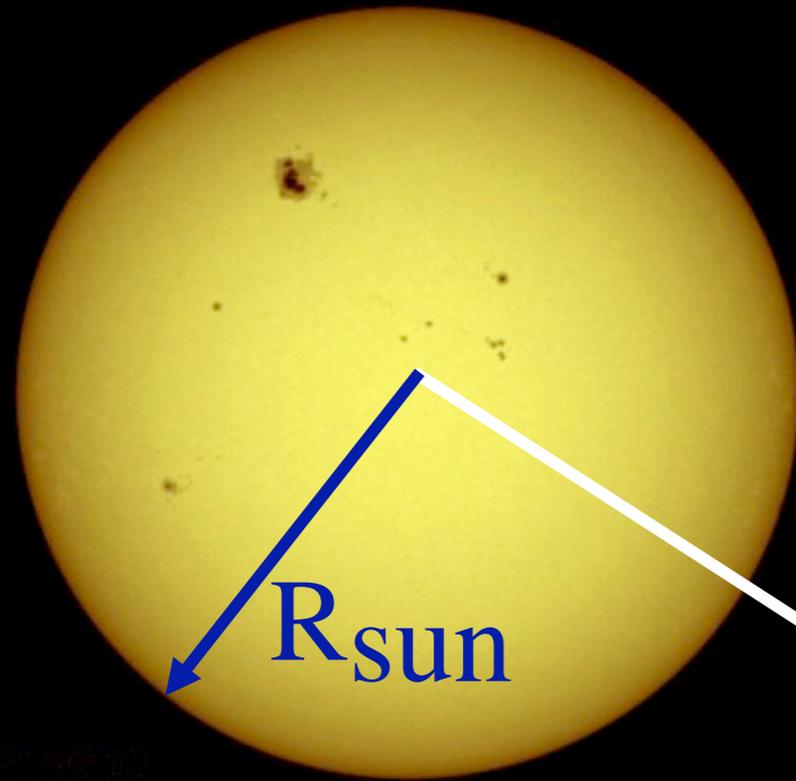


The earth is (almost) in a thermal steady state: it emits as much radiation as it receives from the sun.

$r_E$



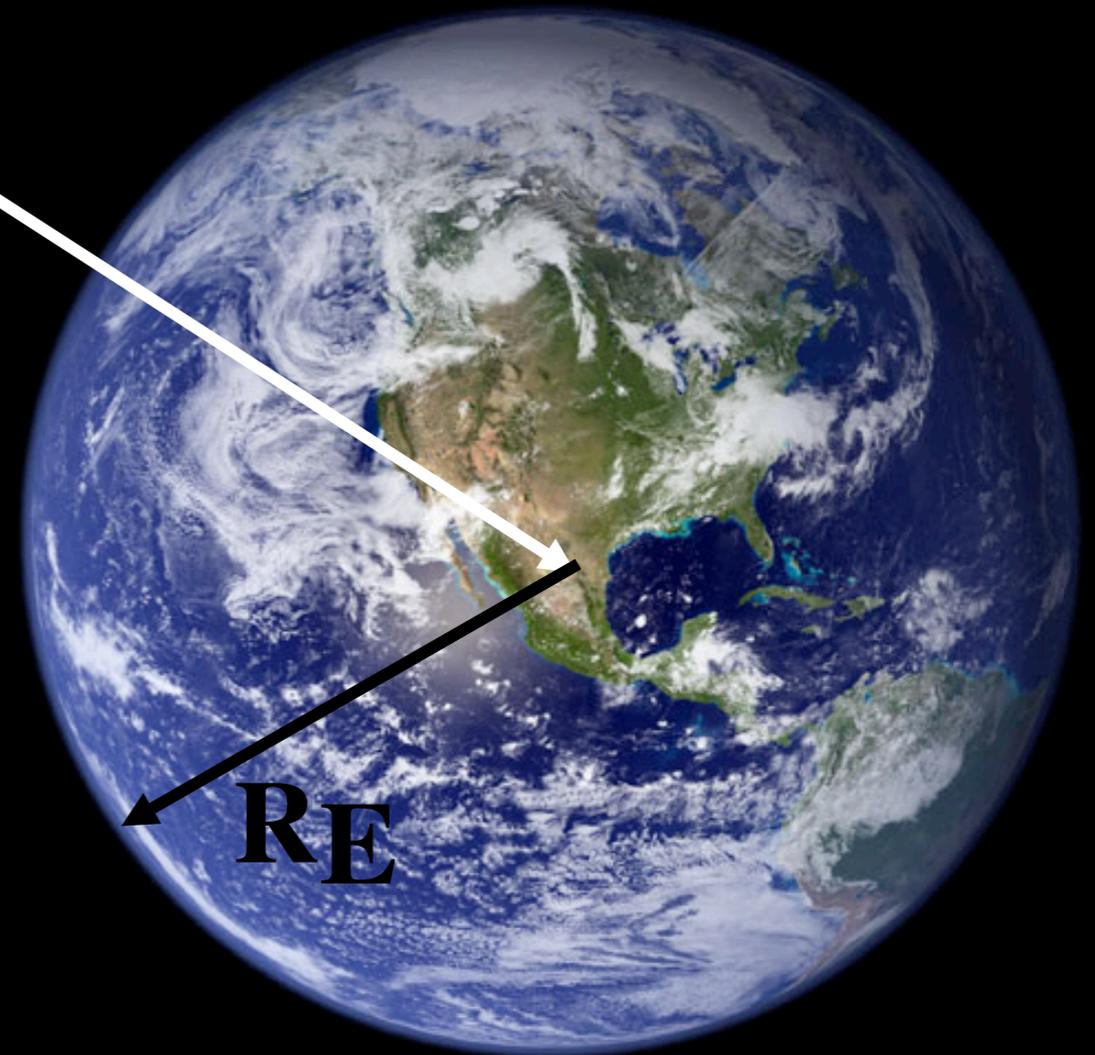
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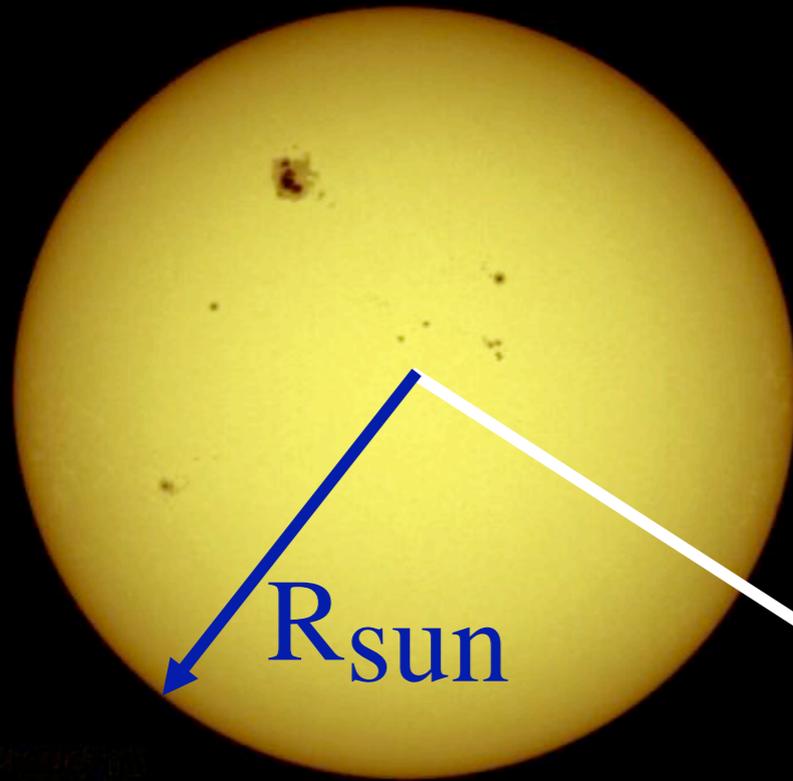
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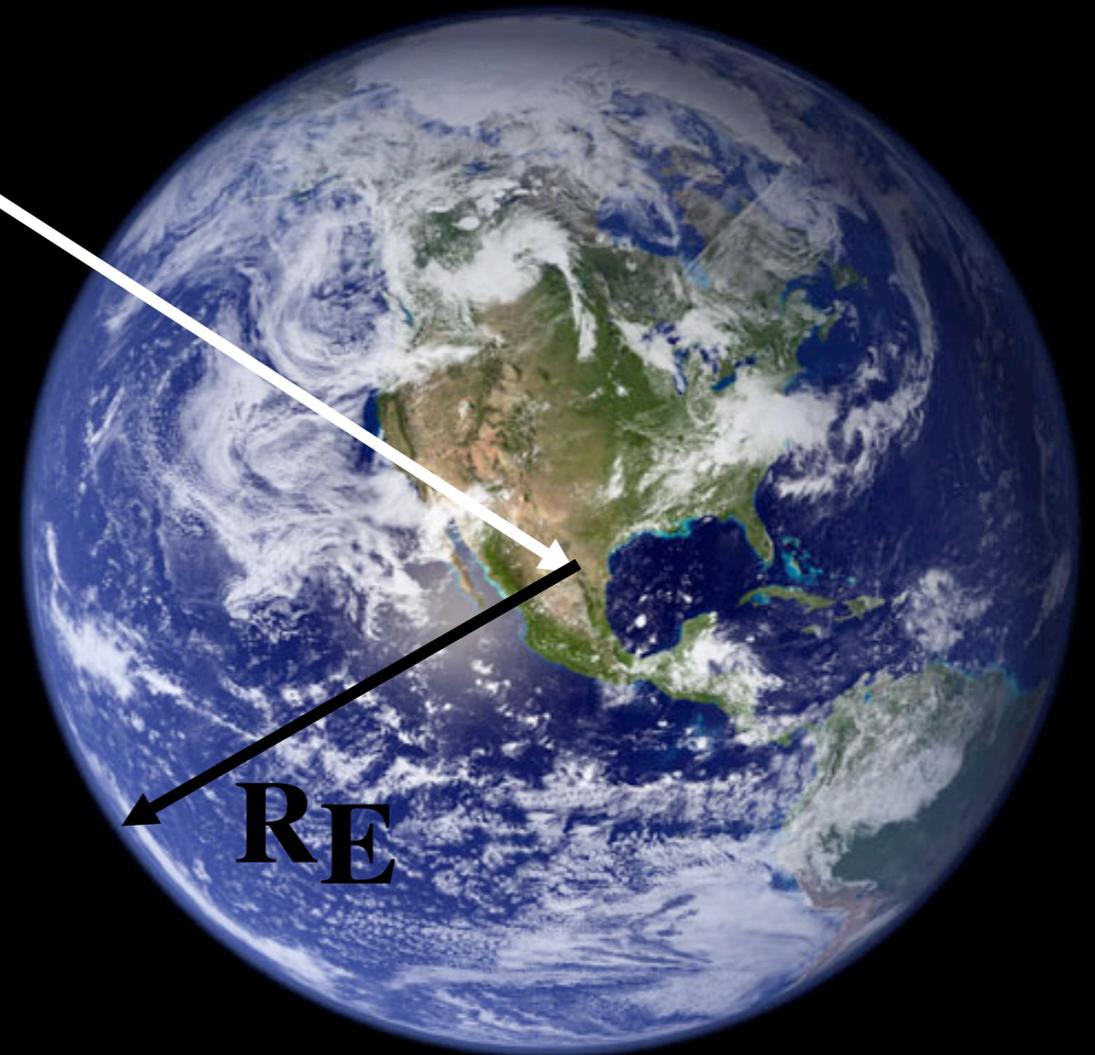


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$$fraction = \frac{\pi R_E^2}{4\pi r_E^2} \times (1 - a)$$



# Energy Balance

$$\begin{aligned} \textit{Luminosity} &= \textit{Area} \times \textit{Intensity} \\ &= 4\pi R_{sun}^2 \times \sigma T_{sun}^4 \end{aligned}$$

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$$r_E = 150 \times 10^9 m$$

$$R_{sun} = 6.96 \times 10^8 m$$

$$T_{sun} = 5,800 K$$

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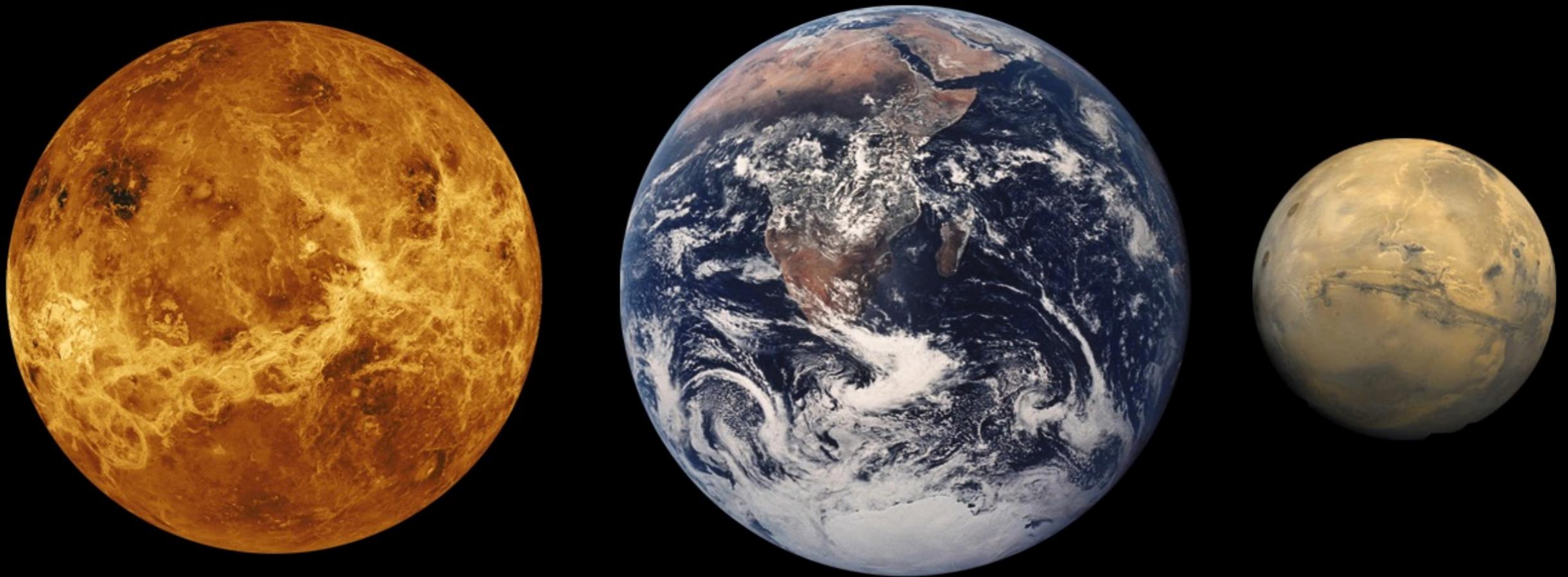


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**FREEZING**

# Terrestrial Planets



Planet	Earth
calculated temperature	-18 °C
actual temperature	15 °C
greenhouse warming	33 °C

Planet	Earth	Mars
calculated temperature	-18 °C	-56 °C
actual temperature	15 °C	-53 °C
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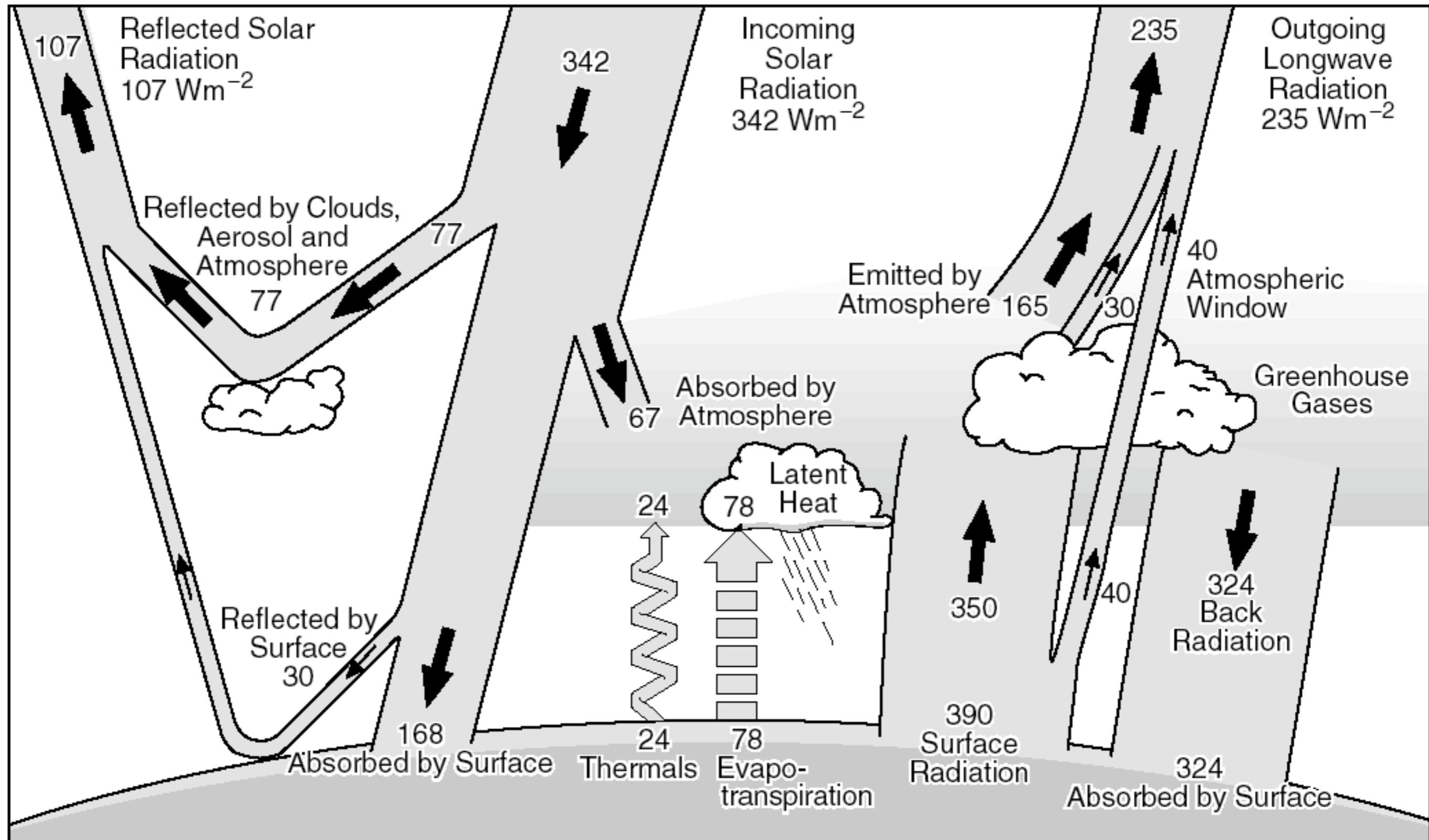
Planet	Earth	Mars	Venus
calculated temperature	-18 °C	-56 °C	-39 °C
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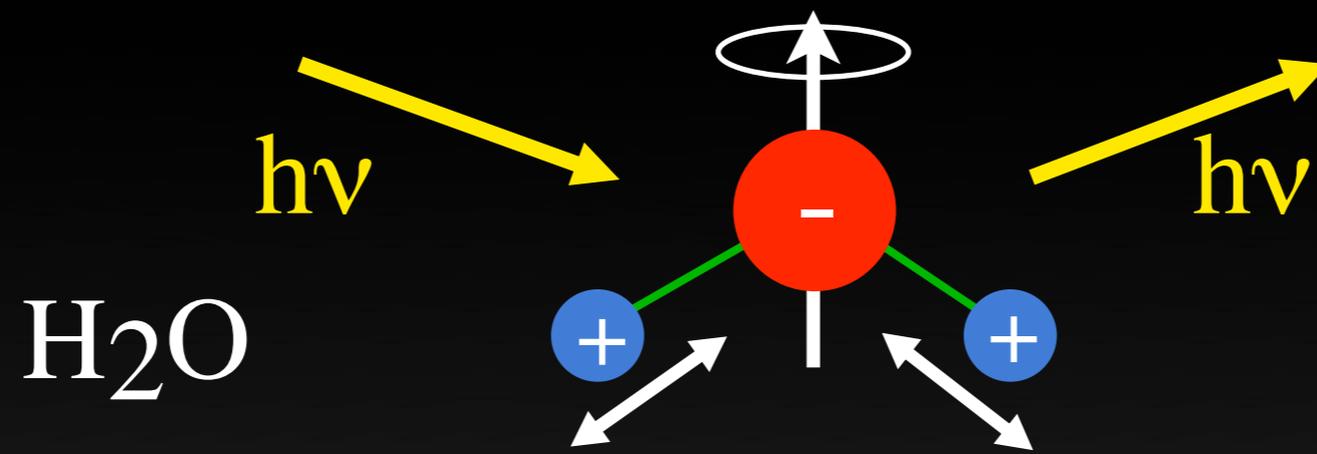
Water Vapour: 65%    Carbon Dioxide: 21%



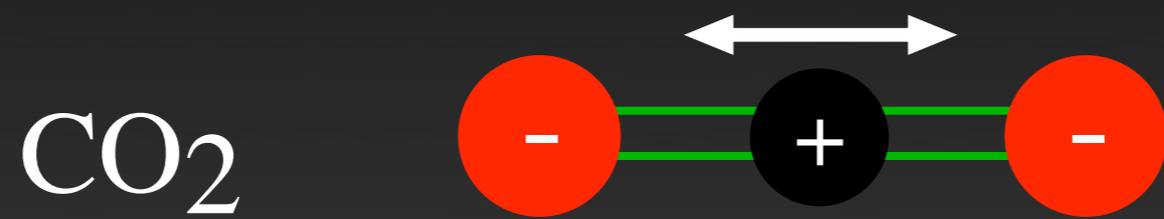
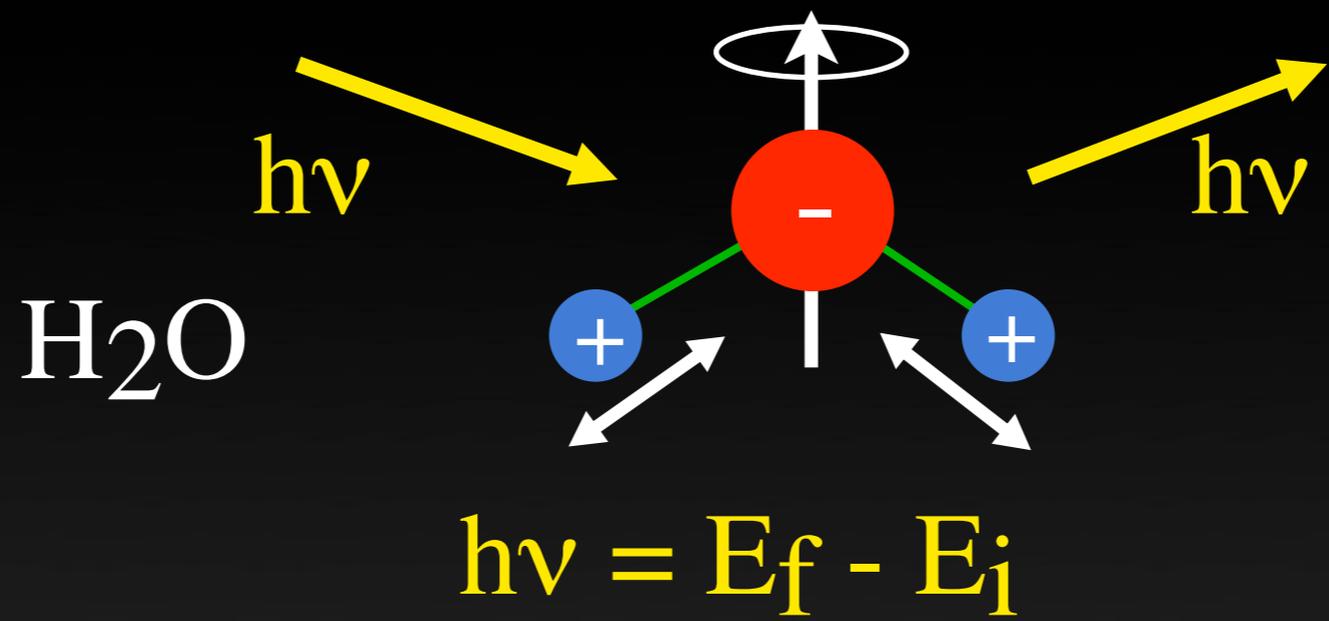
# Why We Aren't Freezing

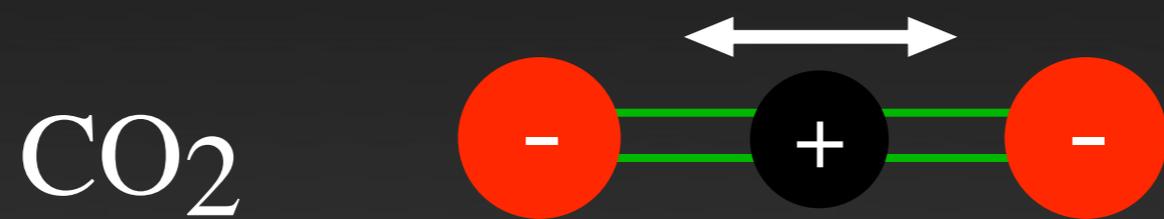
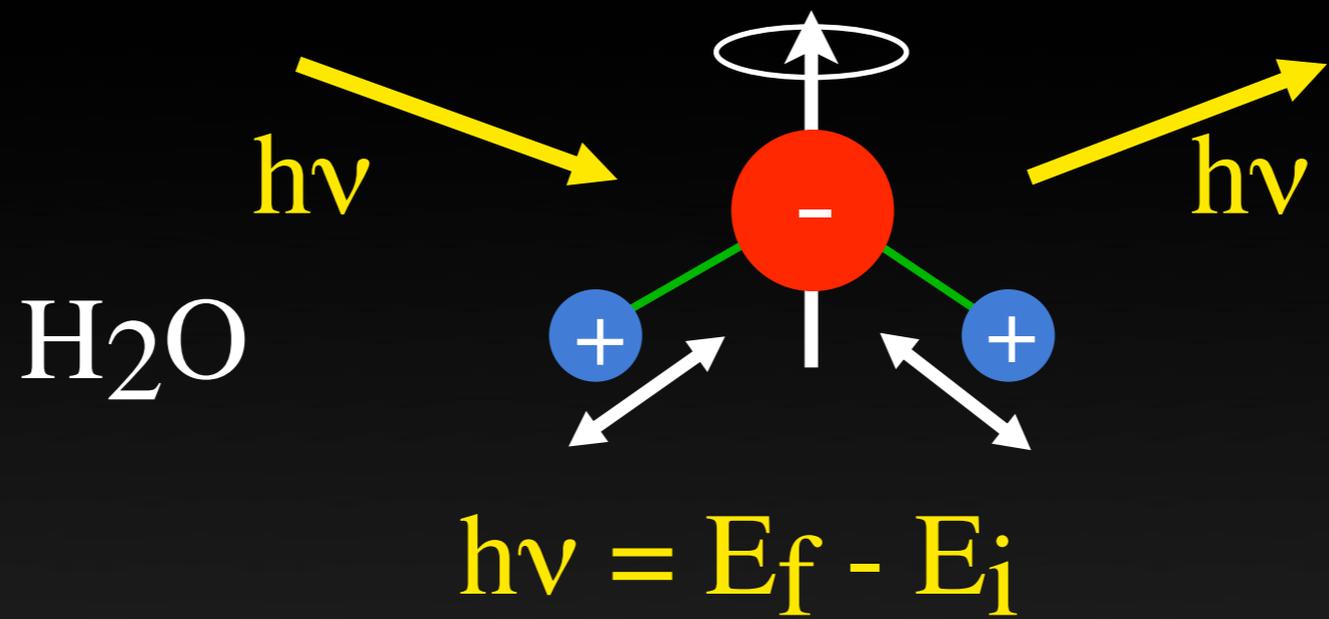


from *Climate Change 1995: The Science of Climate Change*



$$h\nu = E_f - E_i$$

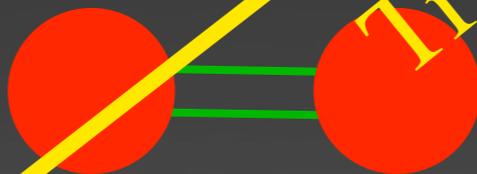




N<sub>2</sub>



O<sub>2</sub>

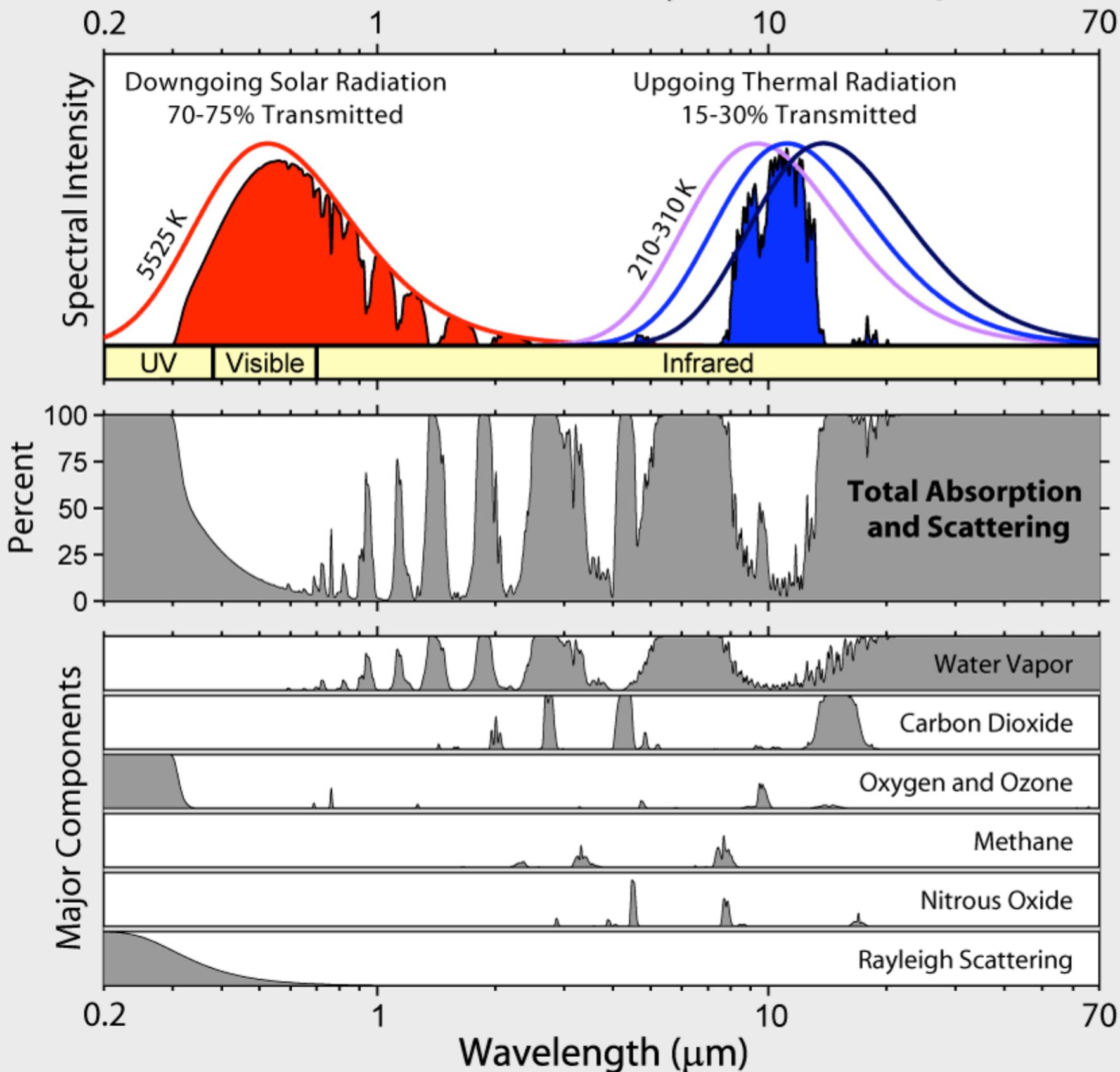


Ar



Transparent

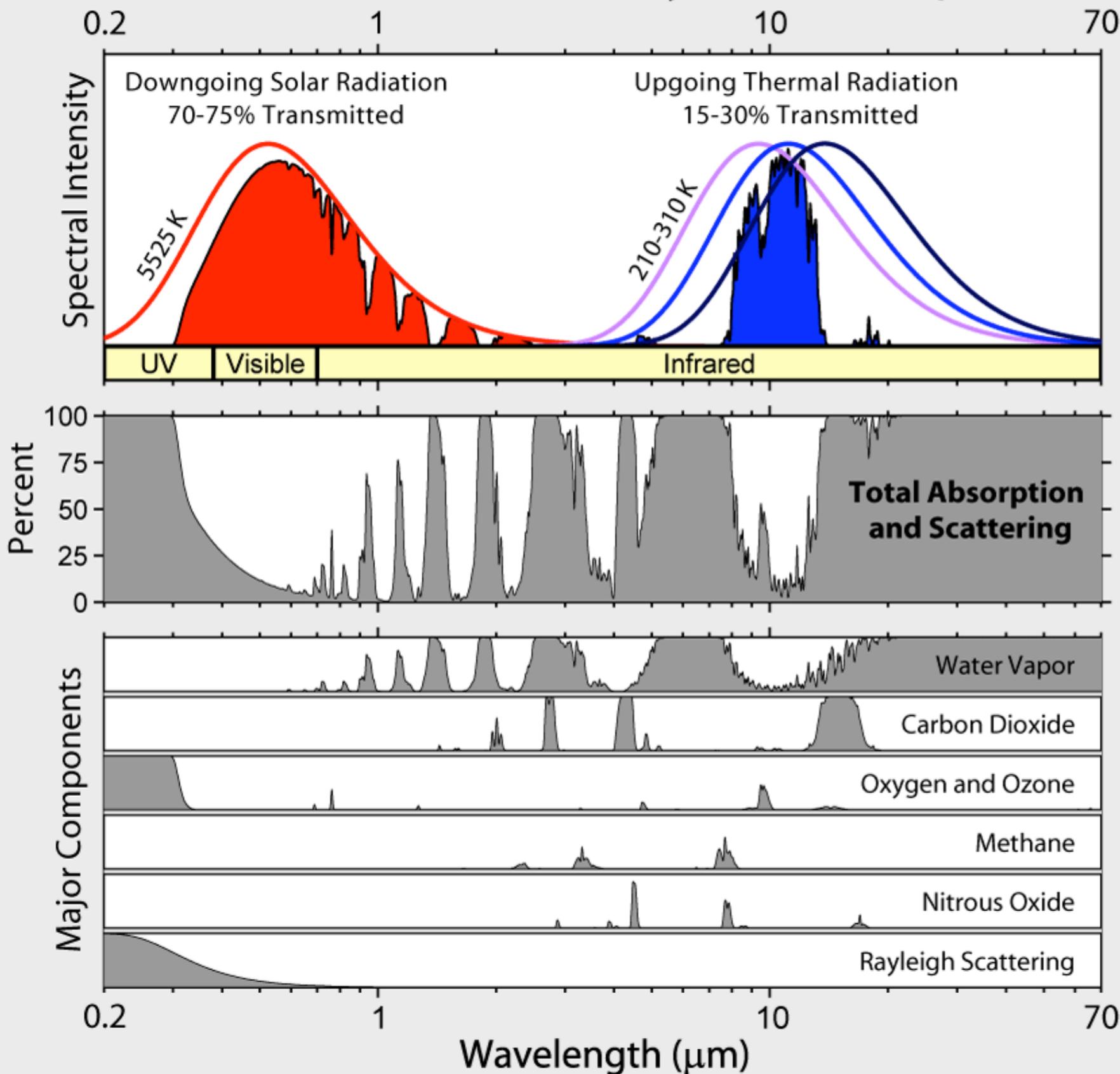
# Radiation Transmitted by the Atmosphere



Principal greenhouse gas: water vapor

Secondary: carbon dioxide, methane, CFC's, ...

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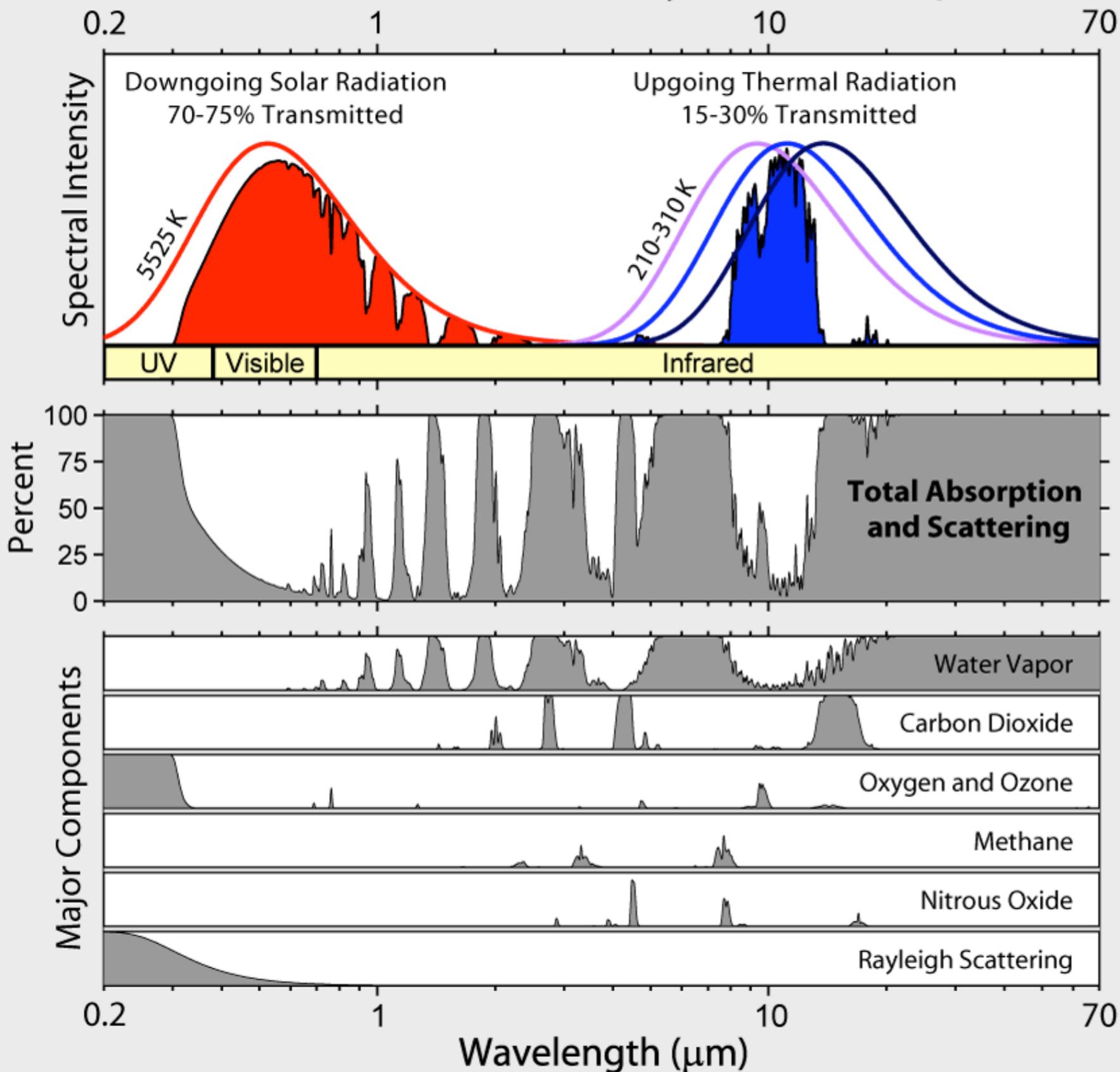


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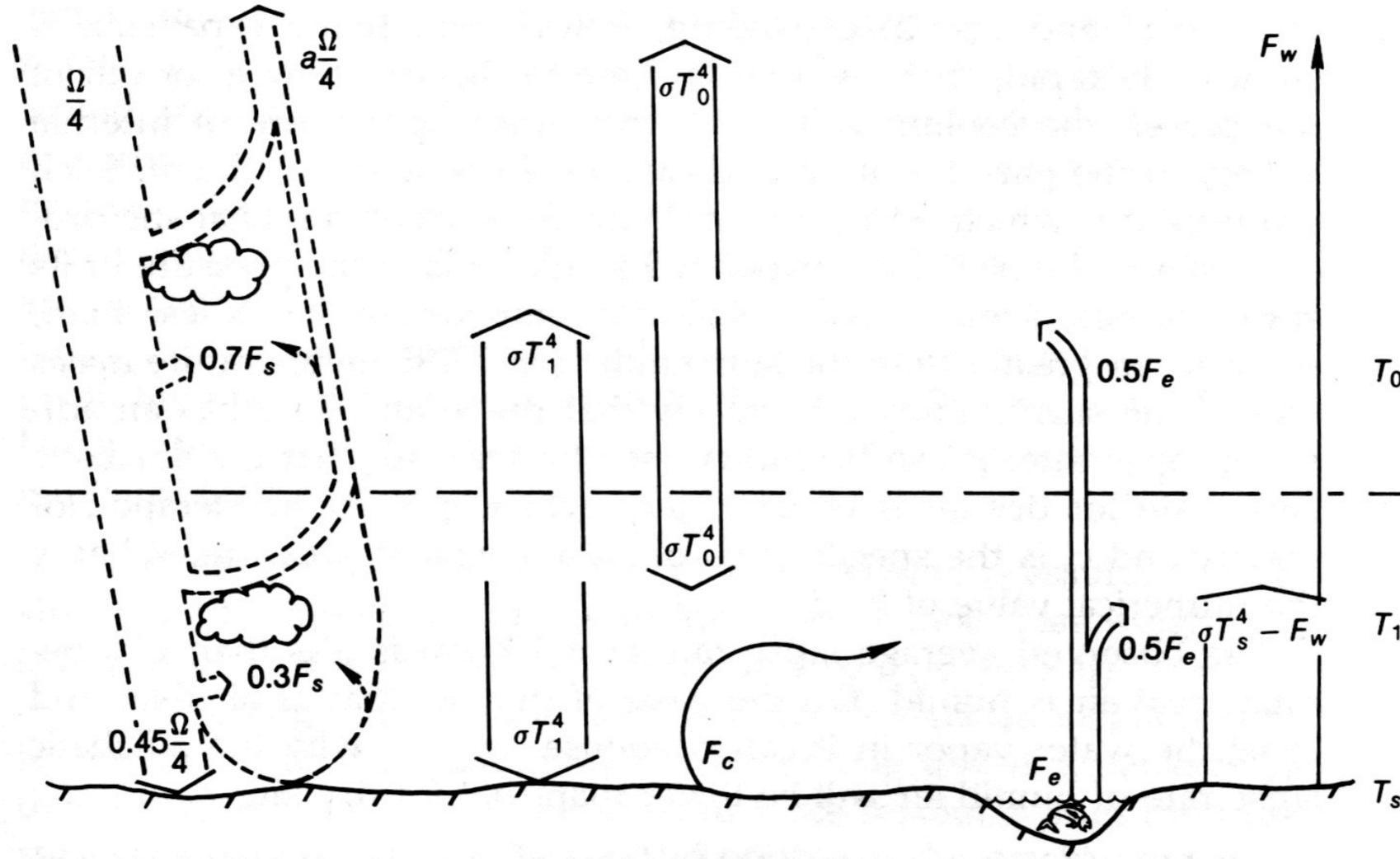
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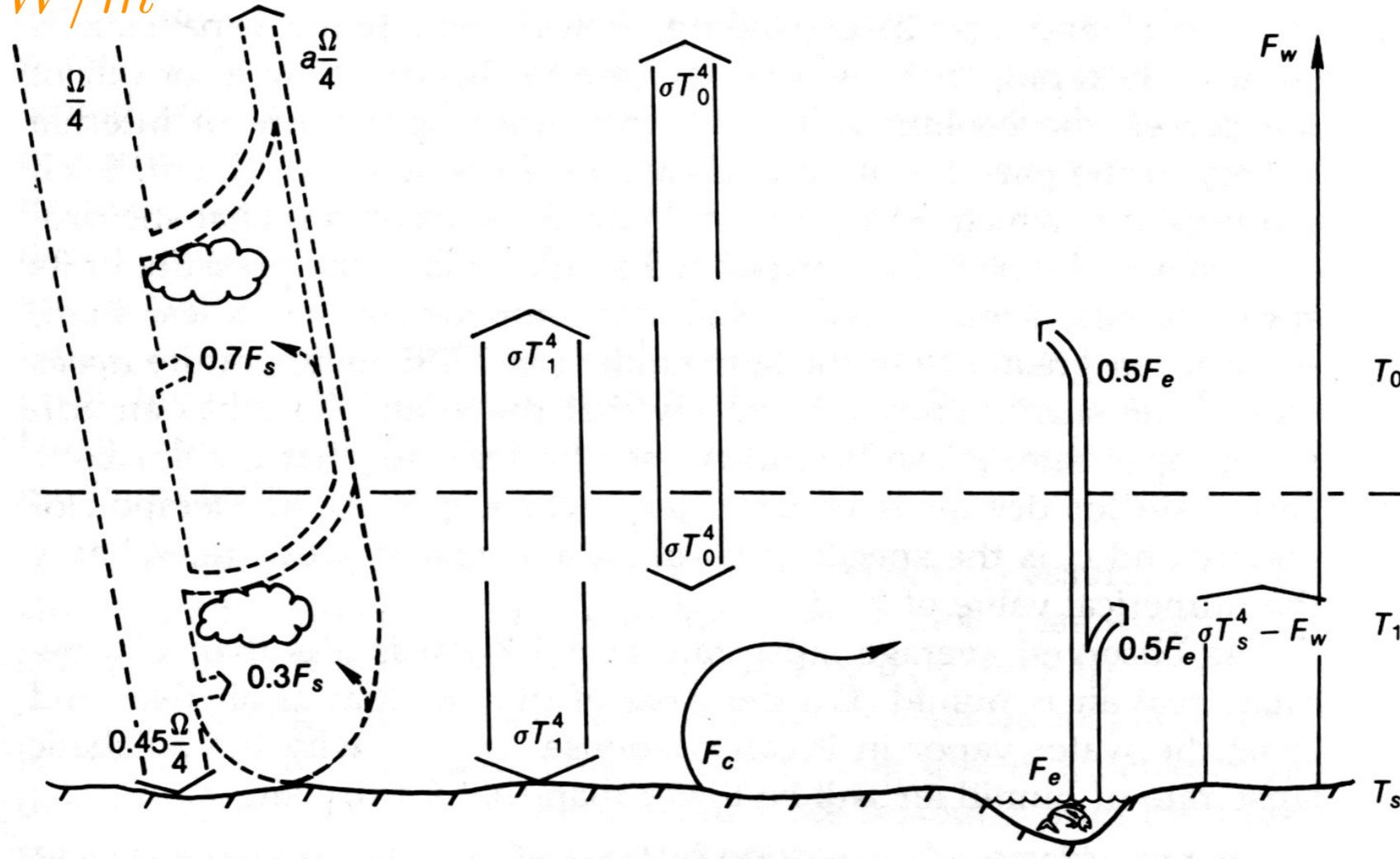
IR photons are on average absorbed and emitted about 2x on way out to space

# John Harte, *Consider a Spherical Cow*



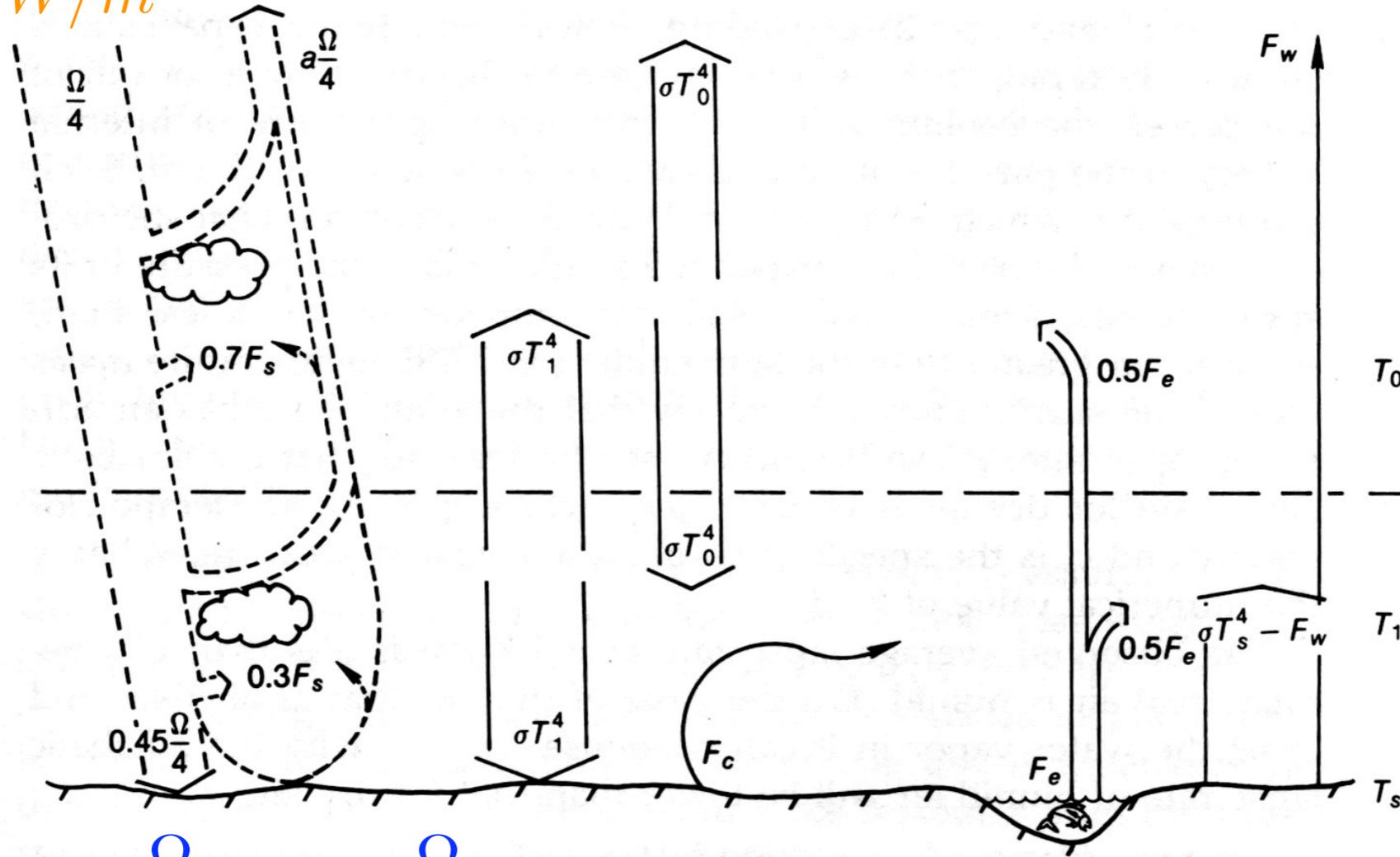
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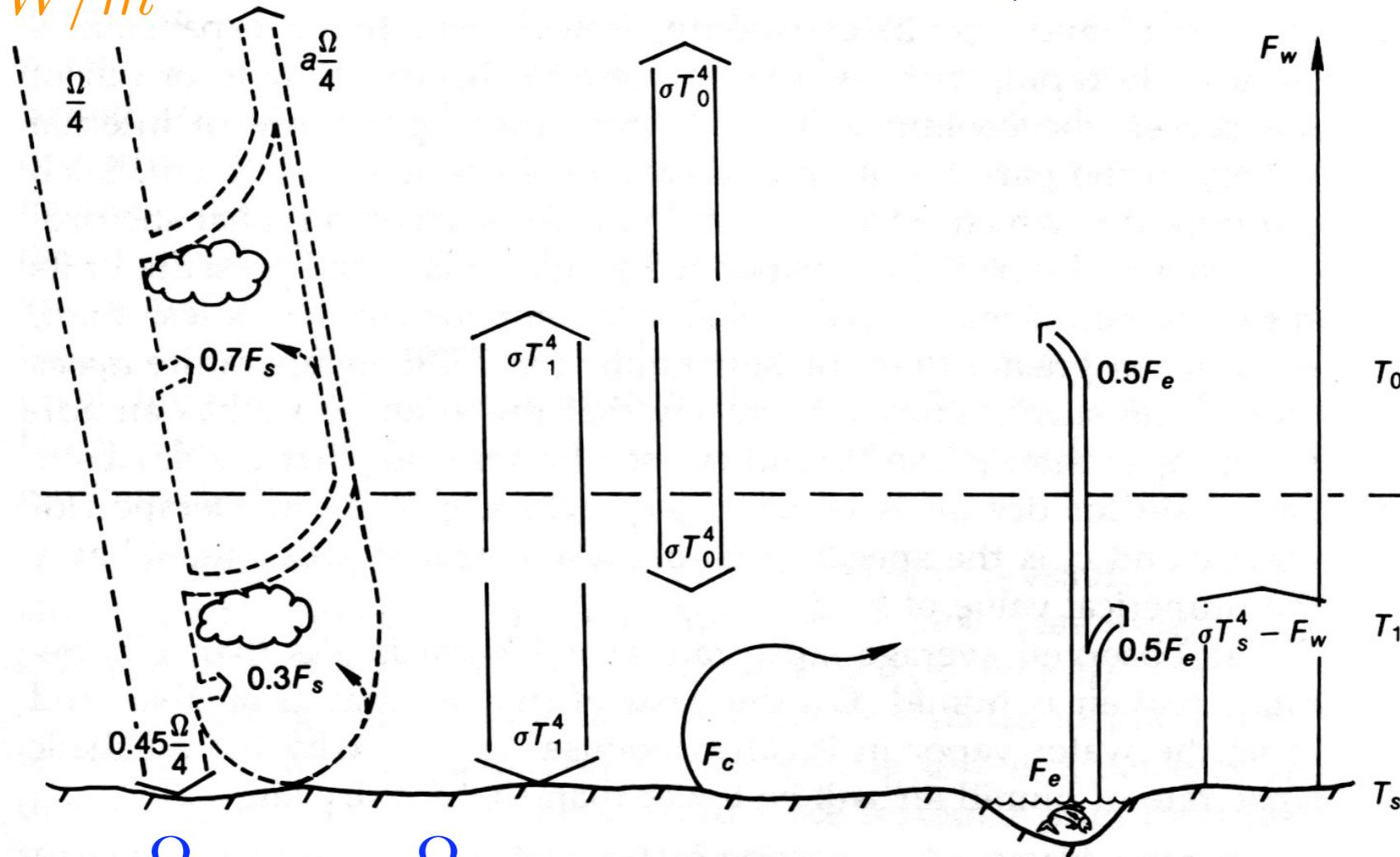
$$\frac{\Omega}{4} = a\frac{\Omega}{4} + \sigma T_0^4 + F_w$$

$$2\sigma T_0^4 = \sigma T_1^4 + 0.5F_e + 0.7F_s$$

$$2\sigma T_1^4 = \sigma T_0^4 + \sigma T_s^4 - F_w + F_c + 0.5F_e + 0.3F_s$$

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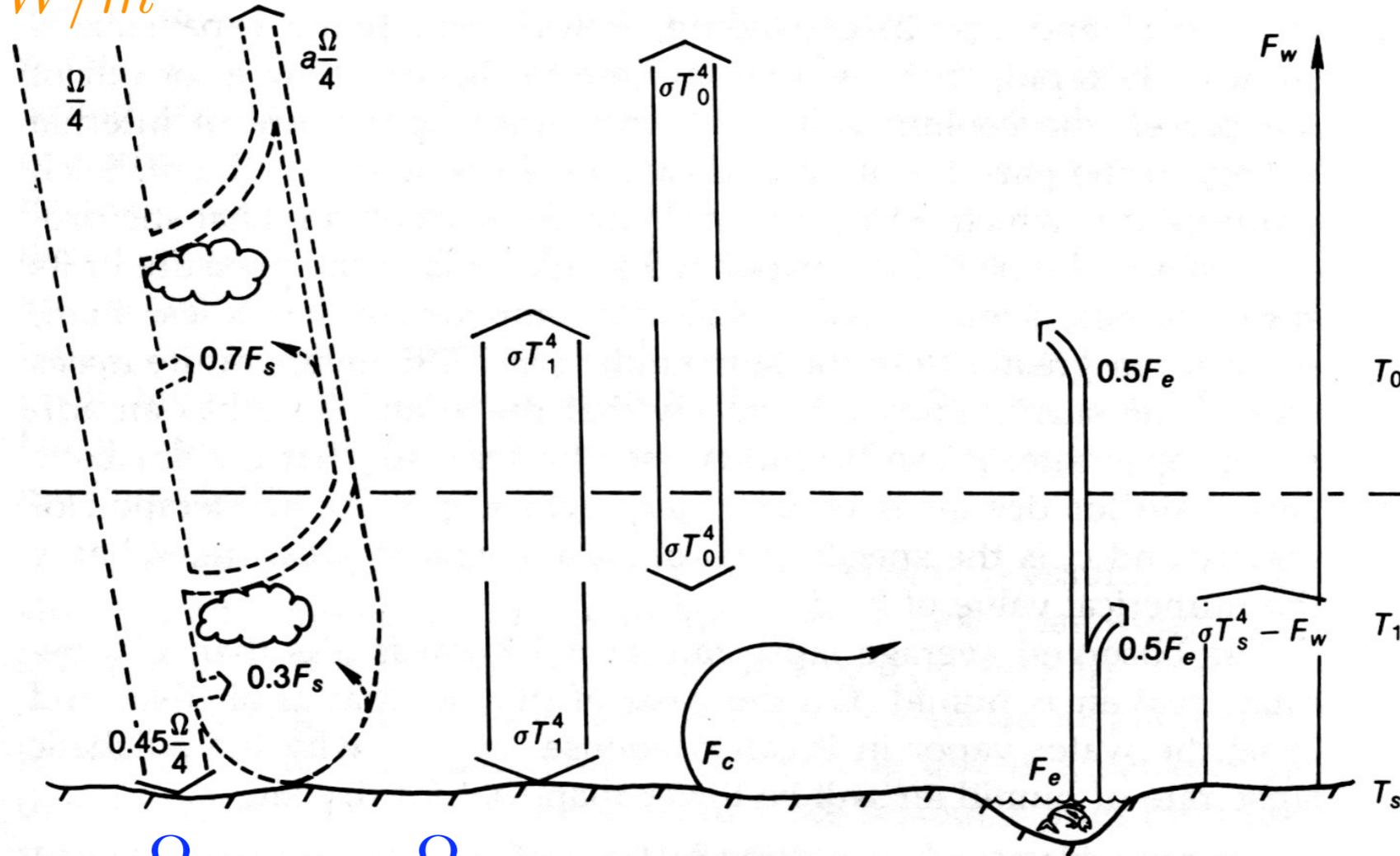
$F_e \approx 80 \text{ W/m}^2$  Latent heat from evaporating water

$F_w \approx 20 \text{ W/m}^2$  IR flux directly to space

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$$T_0 = 250K$$

$$T_1 = 278K$$

$$T_s = 289K = 16C$$

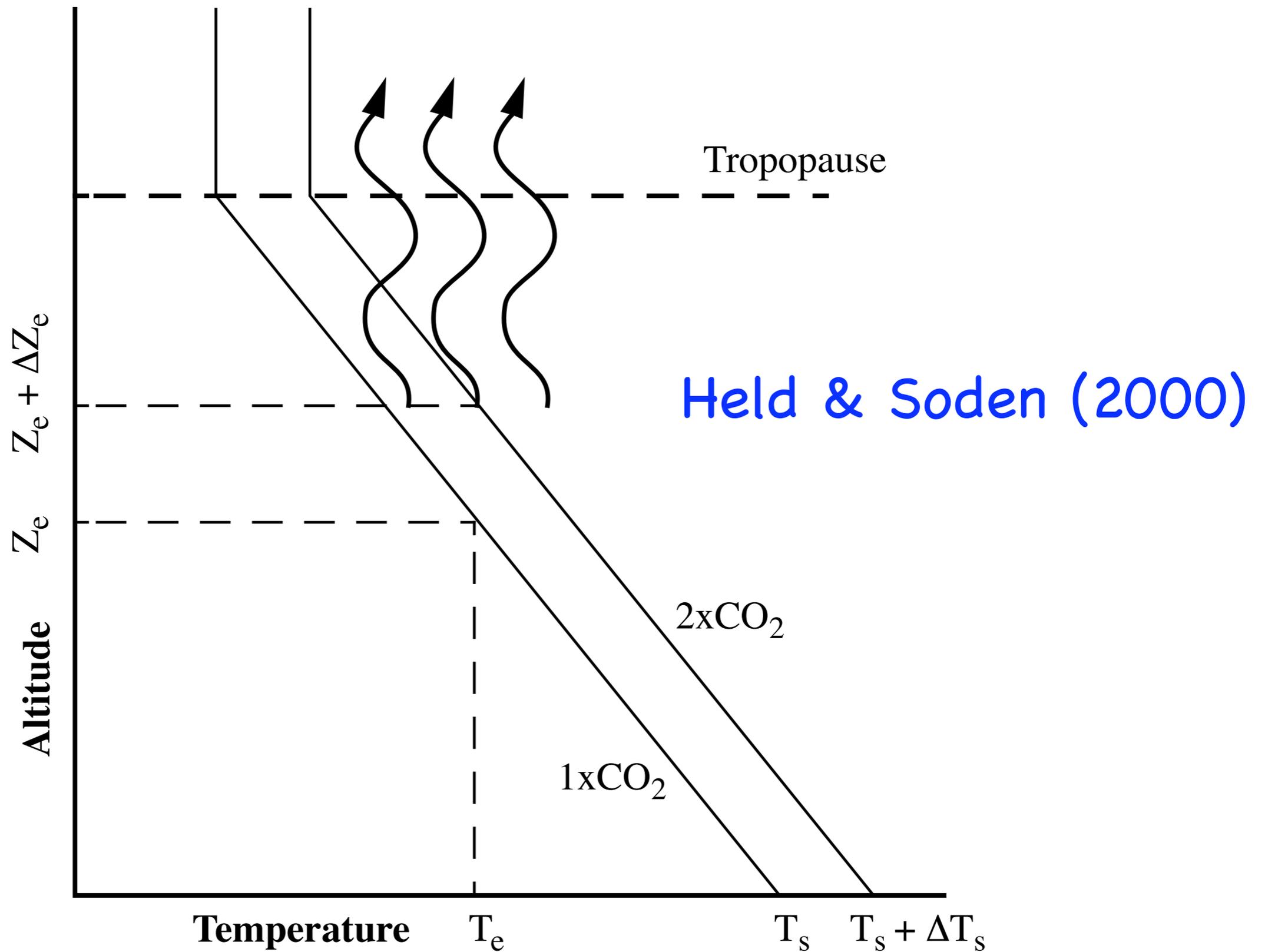
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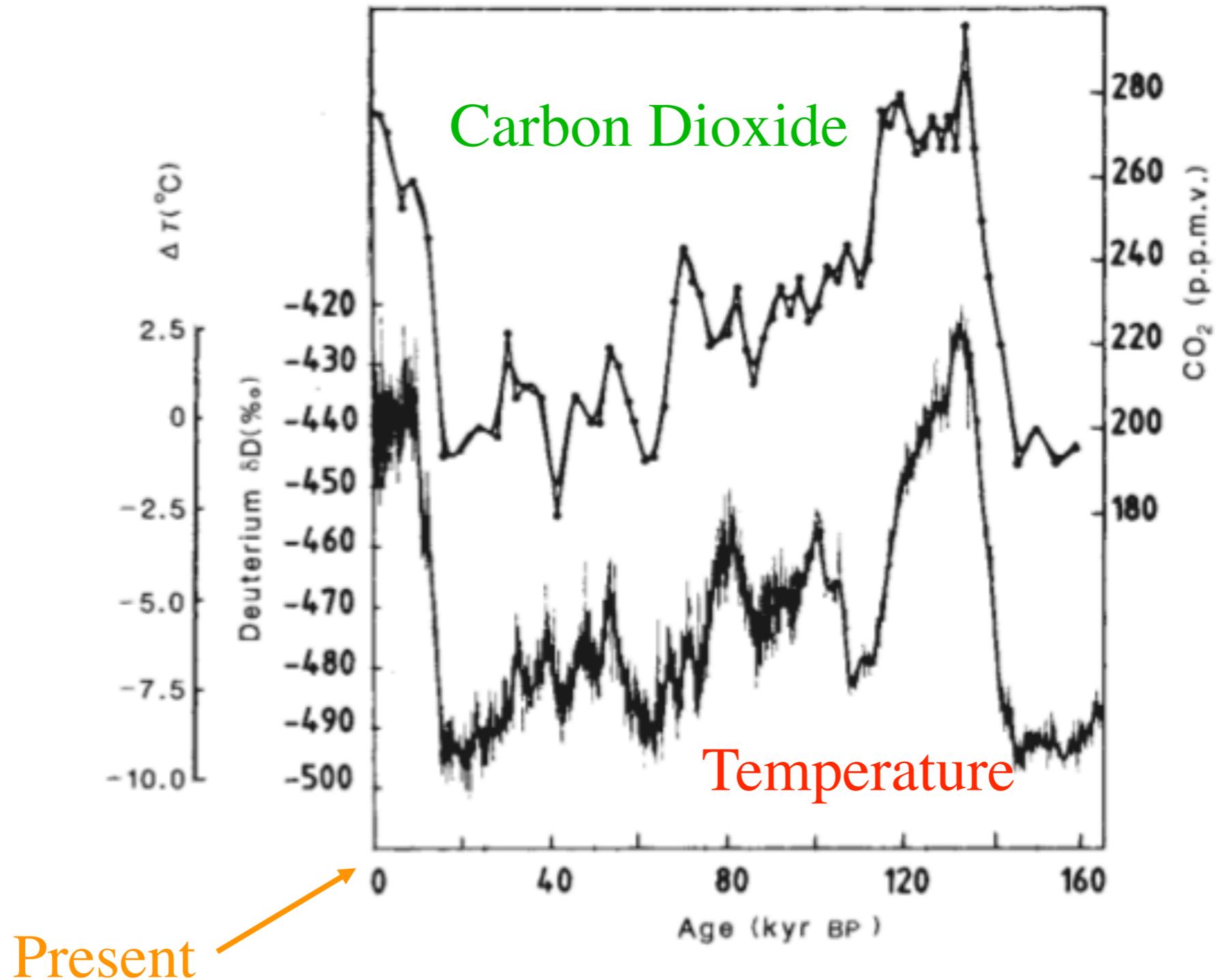
**Excellent agreement  
for such a simple model**



**Figure 1** Schematic illustration of the change in emission level ( $Z_e$ ) associated with an increase in surface temperature ( $T_s$ ) due to a doubling of CO<sub>2</sub> assuming a fixed atmospheric lapse rate. Note that the effective emission temperature ( $T_e$ ) remains unchanged.

# The Past 160,000 Years

J. M. Barnola *et al.*, Nature **329**, 408 (1987)



# Quantum Zero-Point Motion

(Harold Urey, "The thermodynamic properties of isotopic substances," 1946)

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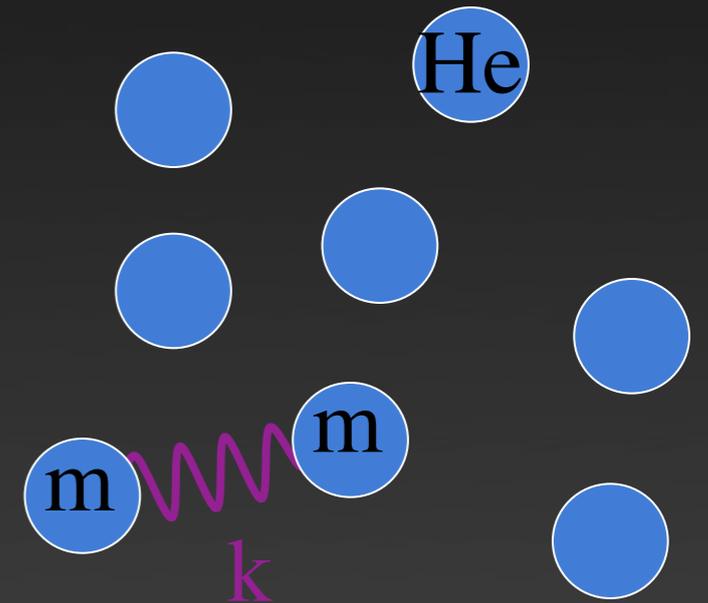
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motion and energy.

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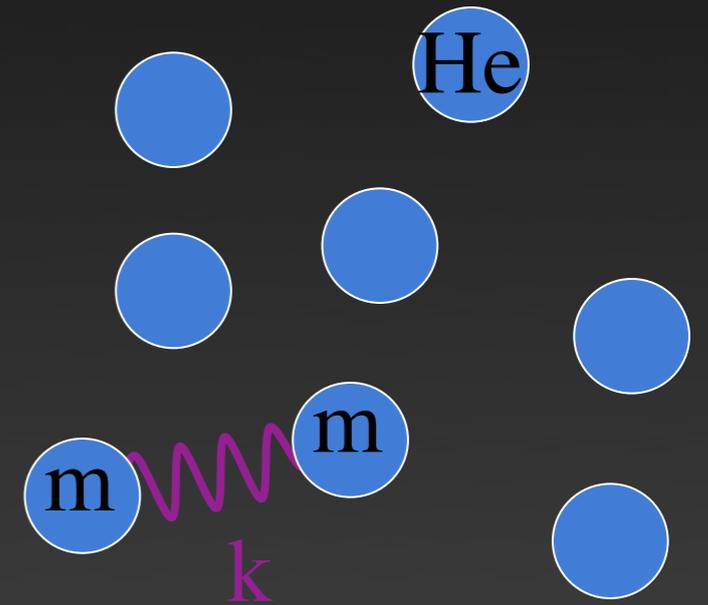
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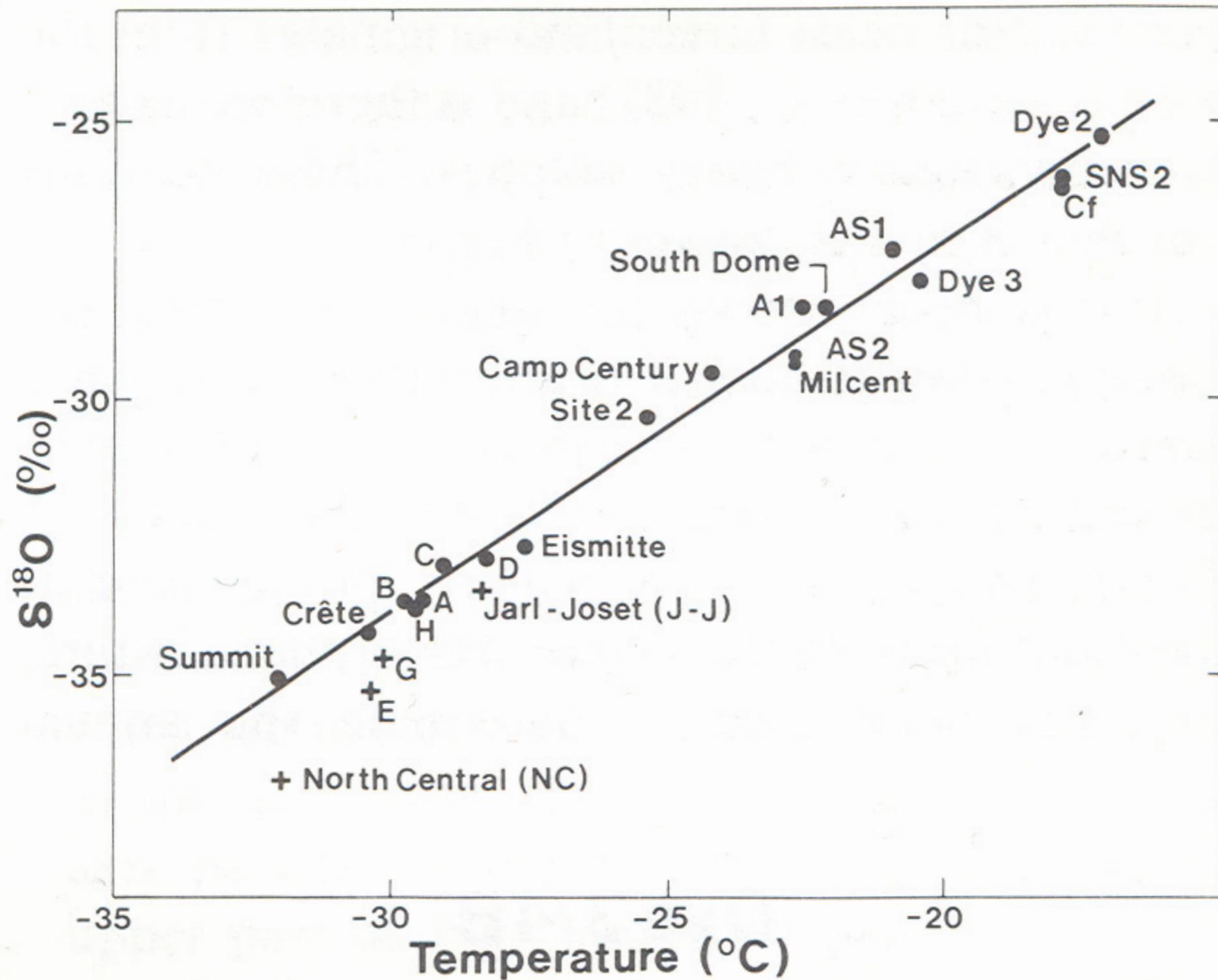
$$\nu = \frac{1}{2\pi} \sqrt{\frac{2k}{m}}$$
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$^{18}\text{O}$  versus  $^{16}\text{O}$  in  $\text{H}_2\text{O}$ : Classically both molecules have same energy.

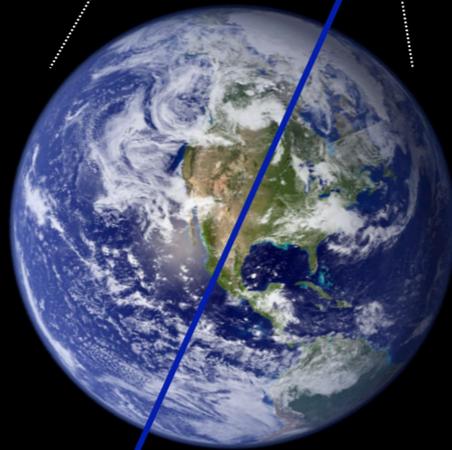
Quantum zero-point energy means that  $^{18}\text{O}$  water is slightly less likely  
to evaporate during cold spells.

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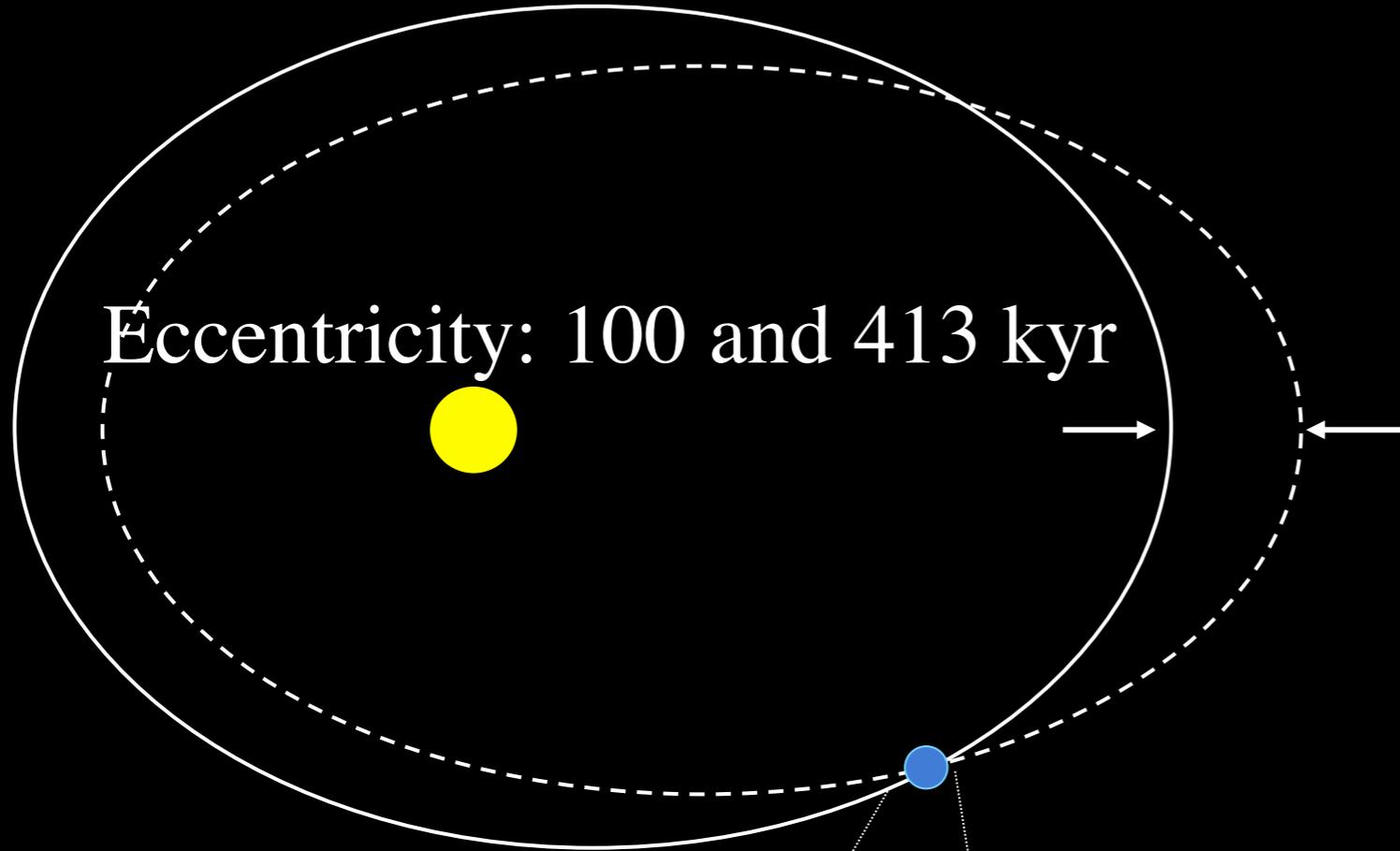
*Fig. 3.* Mean  $\delta^{18}O$  of snow deposited on the Greenland ice sheet plotted against the annual mean surface temperature as represented by the temperature at 10 or 20 m depths.

Eccentricity: 100 and 413 kyr



N

S



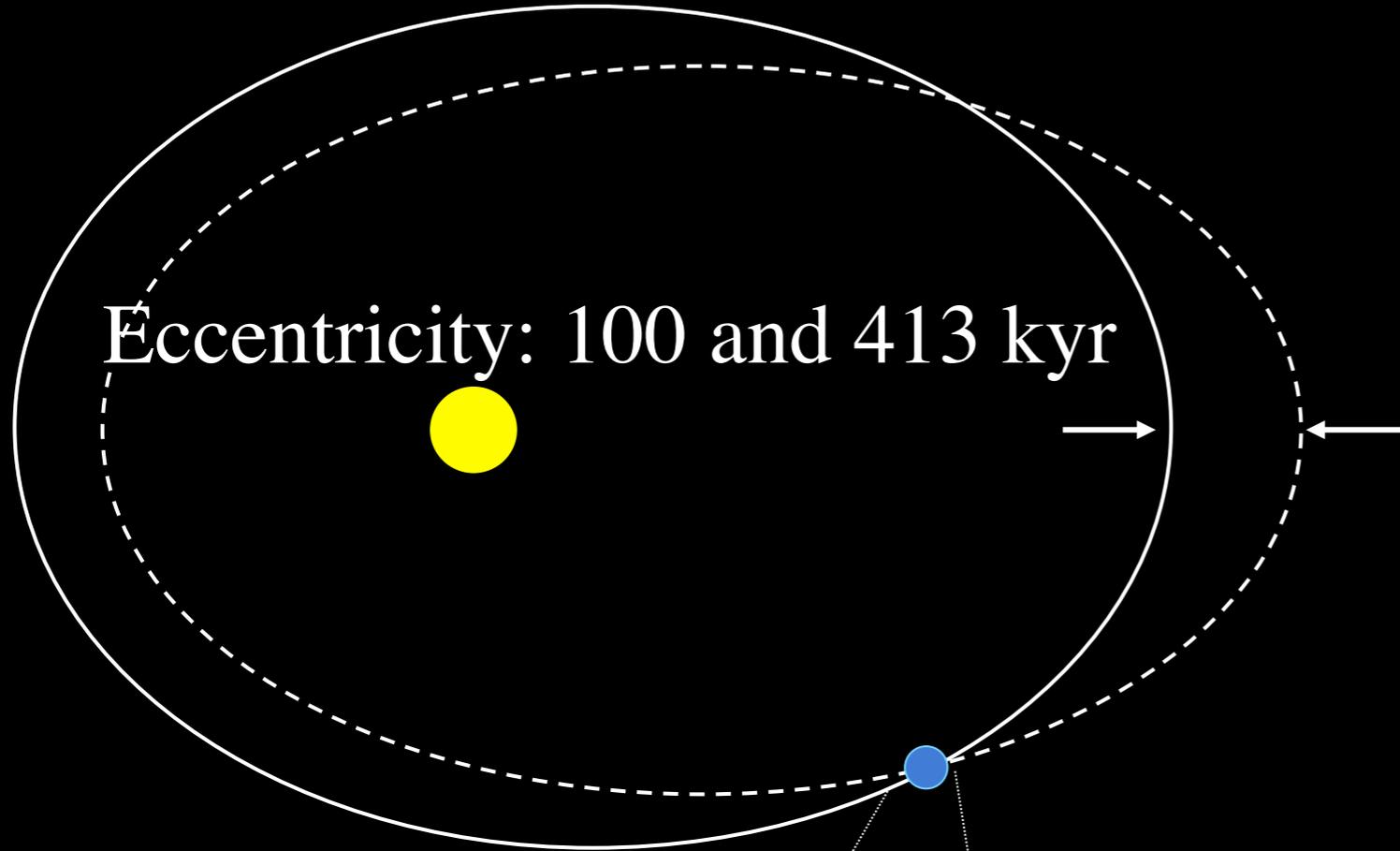
$\alpha$  Draconis  
(2000 BC)



Polaris  
(now)

Precession: 19 to 23 kyr

Eccentricity: 100 and 413 kyr



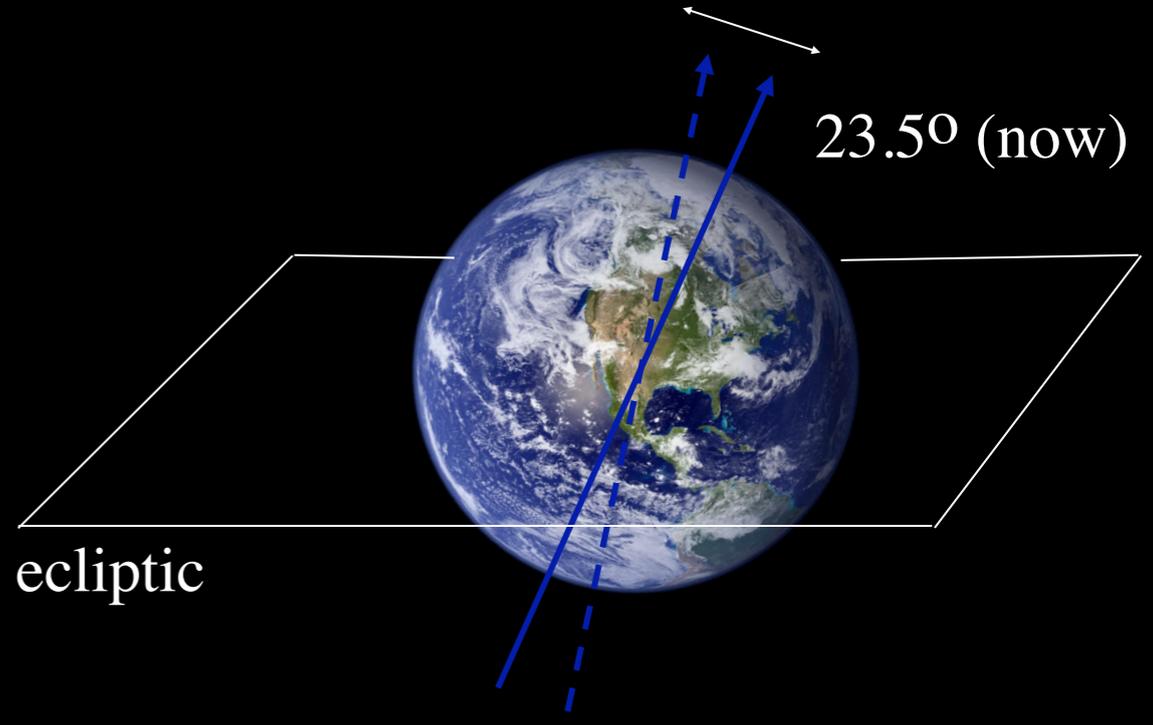
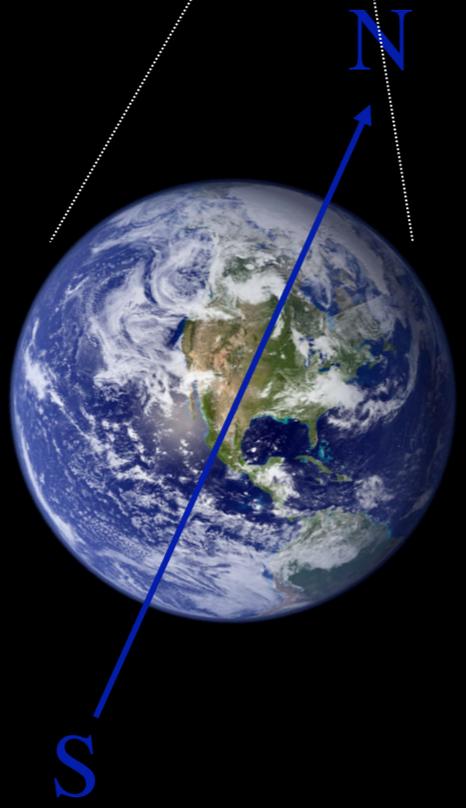
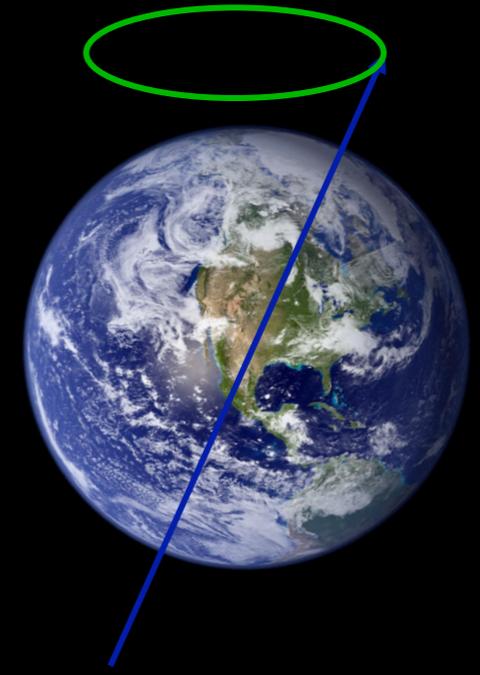
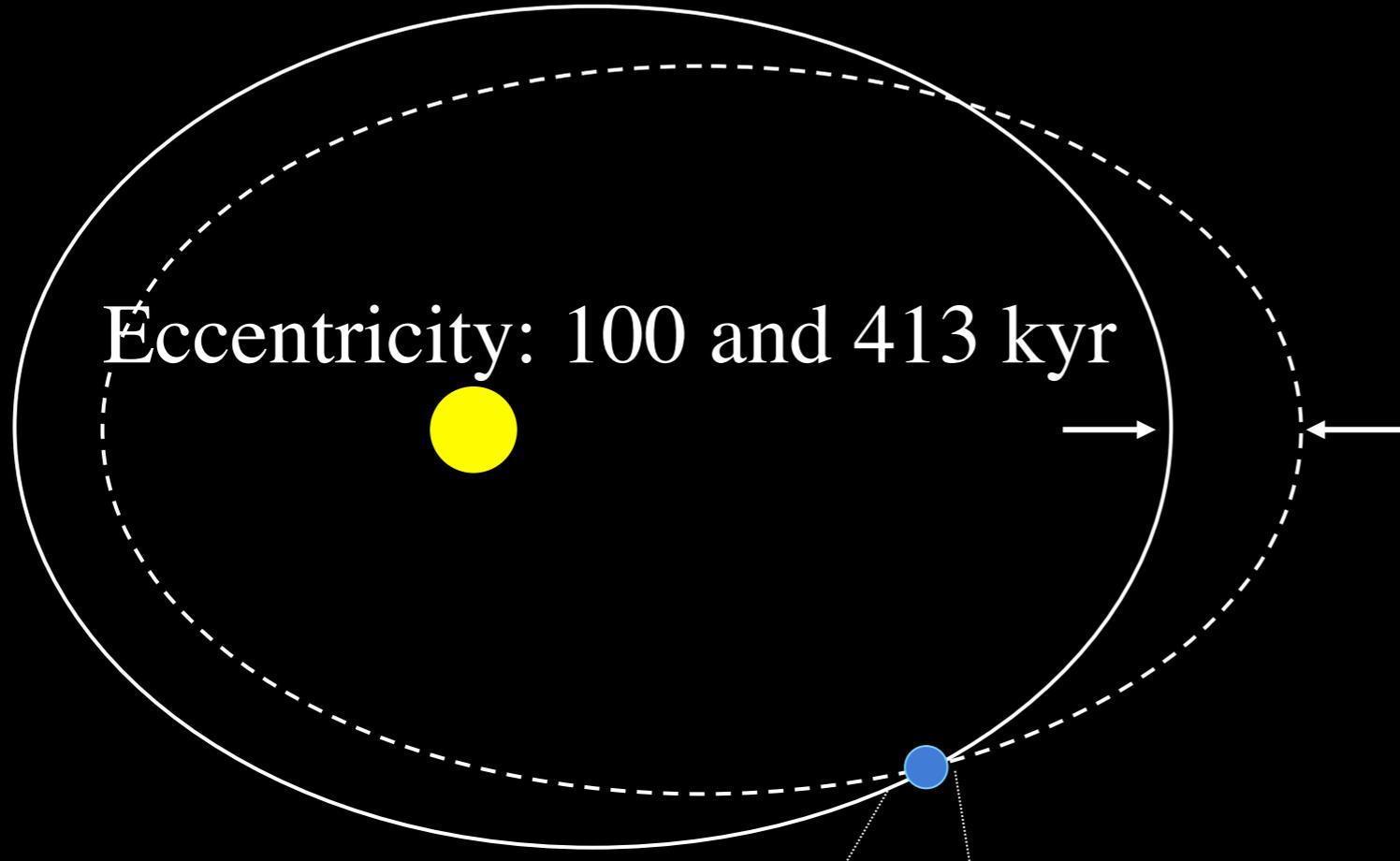
$\alpha$  Draconis  
(2000 BC)



Polaris  
(now)

Precession: 19 to 23 kyr

Eccentricity: 100 and 413 kyr



Change in tilt of axis  
"obliquity": 41 kyr

# Spectral Analysis of Isotope Records

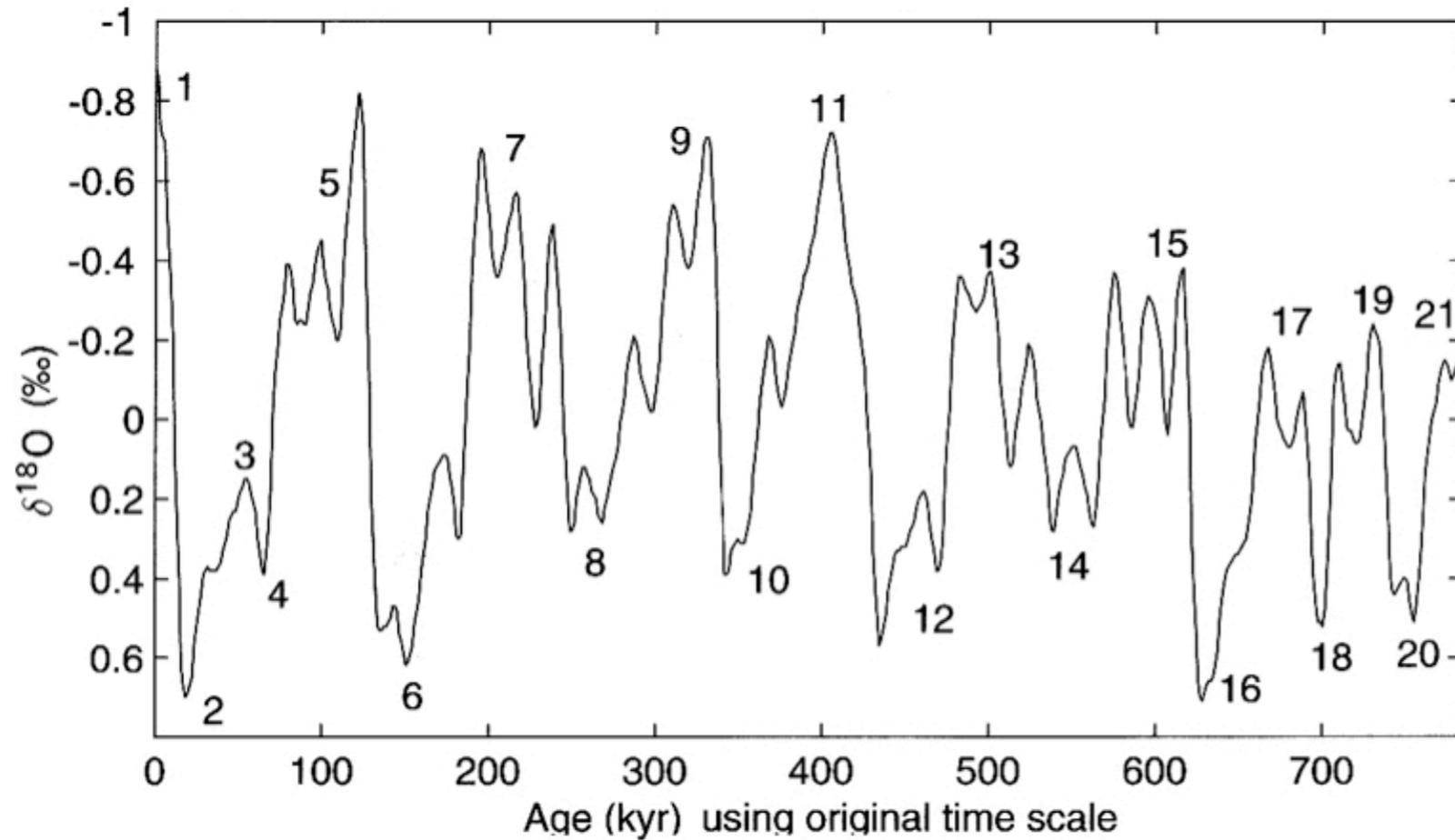


Fig. 4.4. SPECMAP  $\delta^{18}\text{O}$  stack with marine isotope stage numbers.

Figures from *Ice Ages and Astronomical Causes: Data, Spectral Analysis, and Mechanisms* by R. Muller and G. MacDonald

# Spectral Analysis of Isotope Records

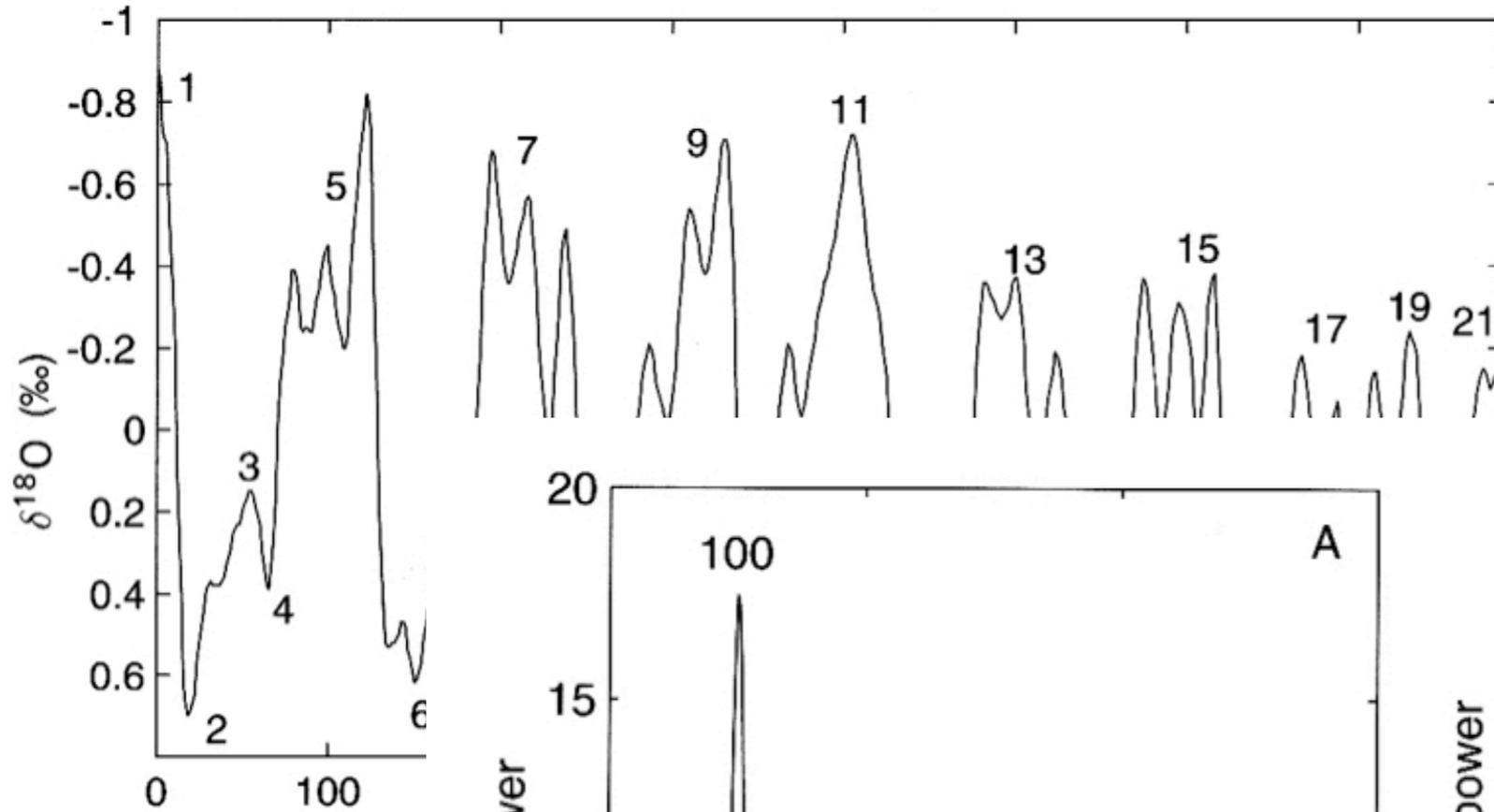


Fig. 4.4. SPECM

Figures from *Ice Ages and Astronomical Causes: Data, Spectral Analysis, and Mechanisms* by R. Muller and G. MacDonald

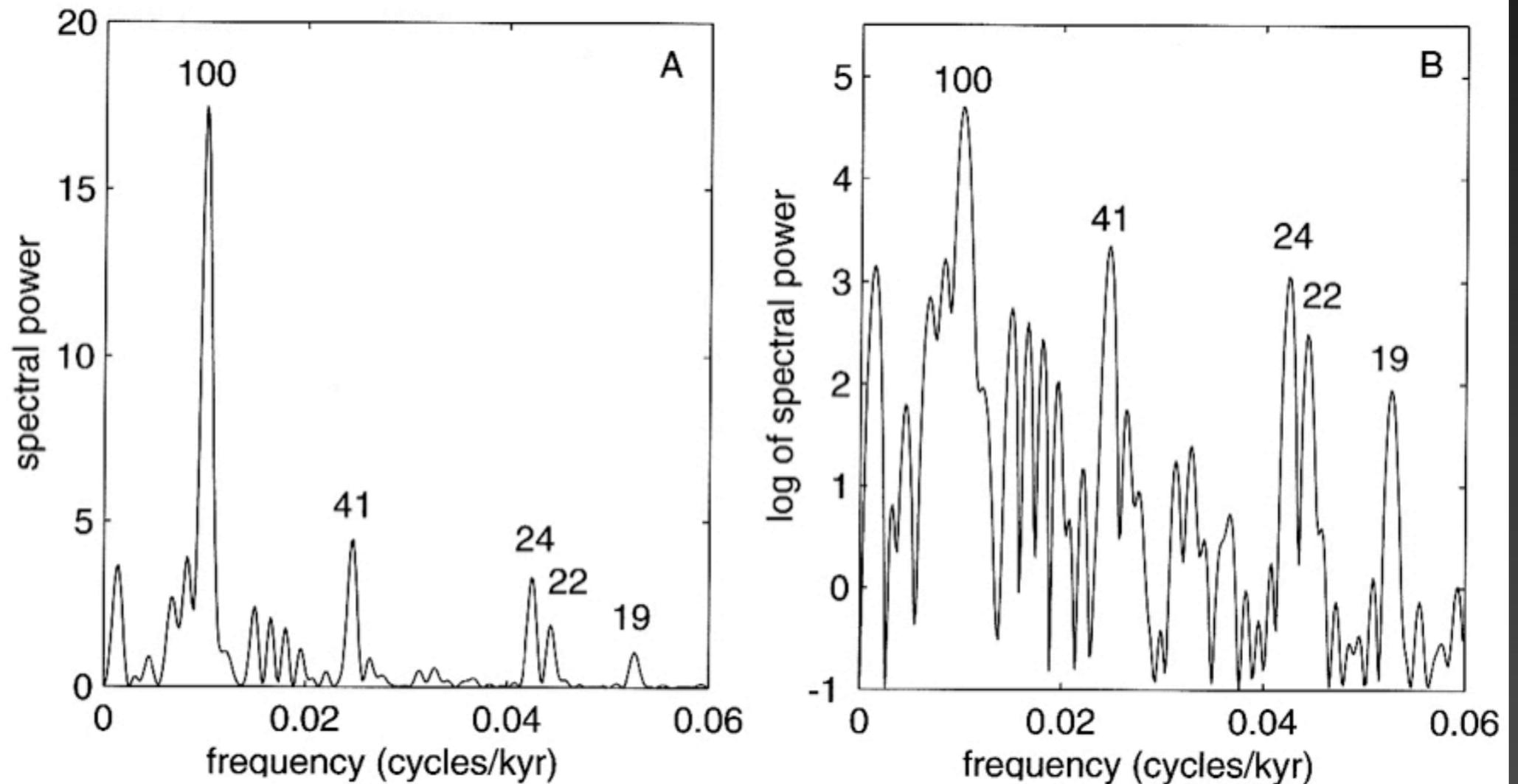


Fig. 4.5. Spectrum of original SPECMAP stack.

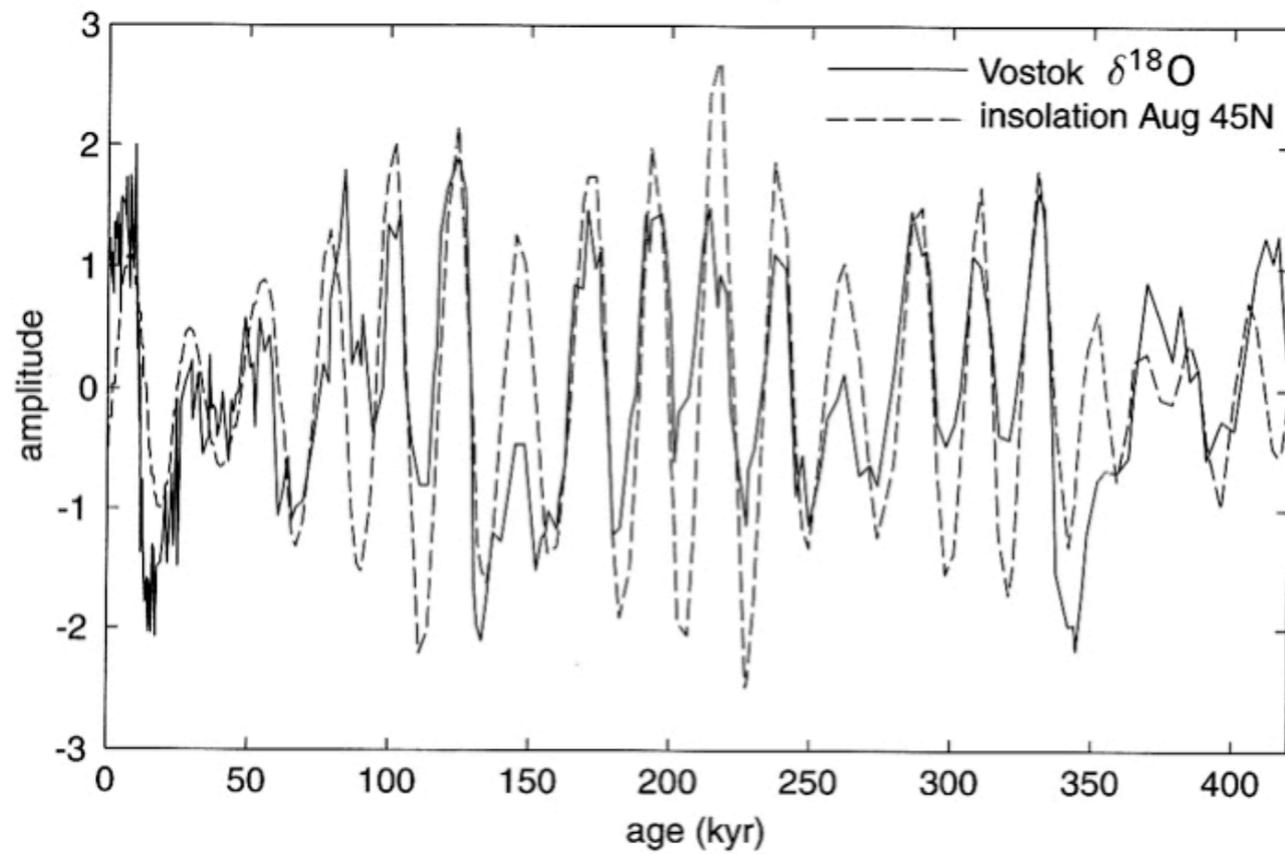


Fig. 4.40. Vostok oxygen and August 45N insolation. No phase lag used.

What amplifies orbital forcing to produce ice ages?

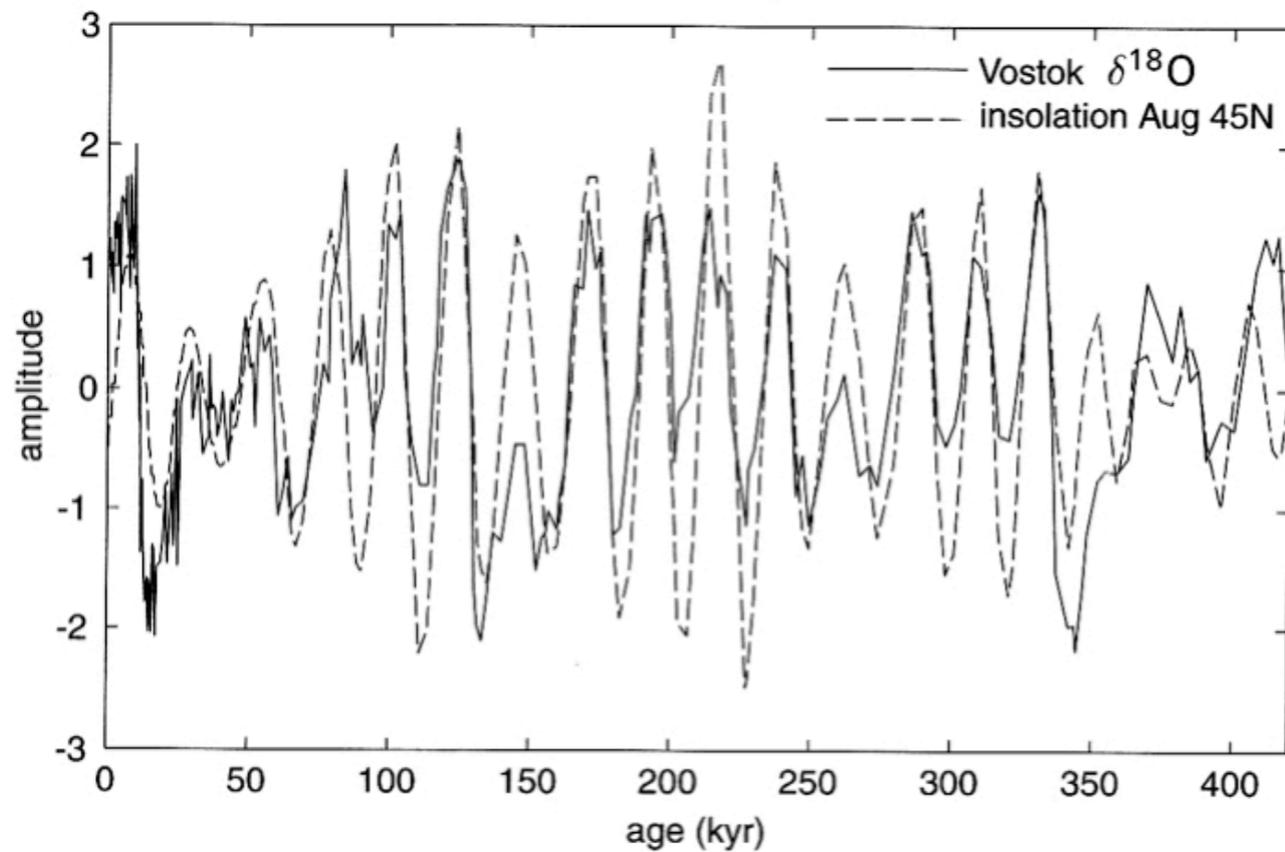


Fig. 4.40. Vostok oxygen and August 45N insolation. No phase lag used.

What amplifies orbital forcing to produce ice ages?

Why does 100 kyr eccentricity period dominate climate signal?

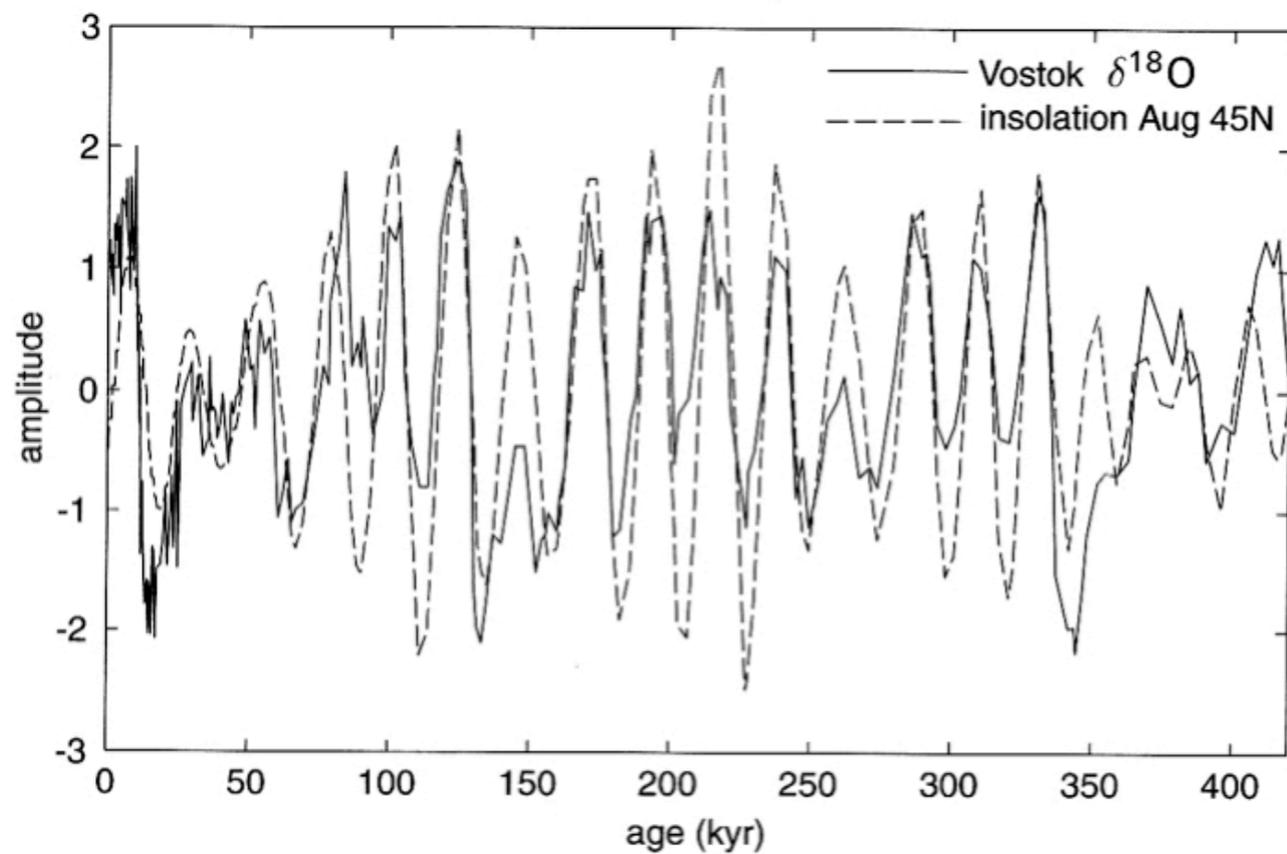
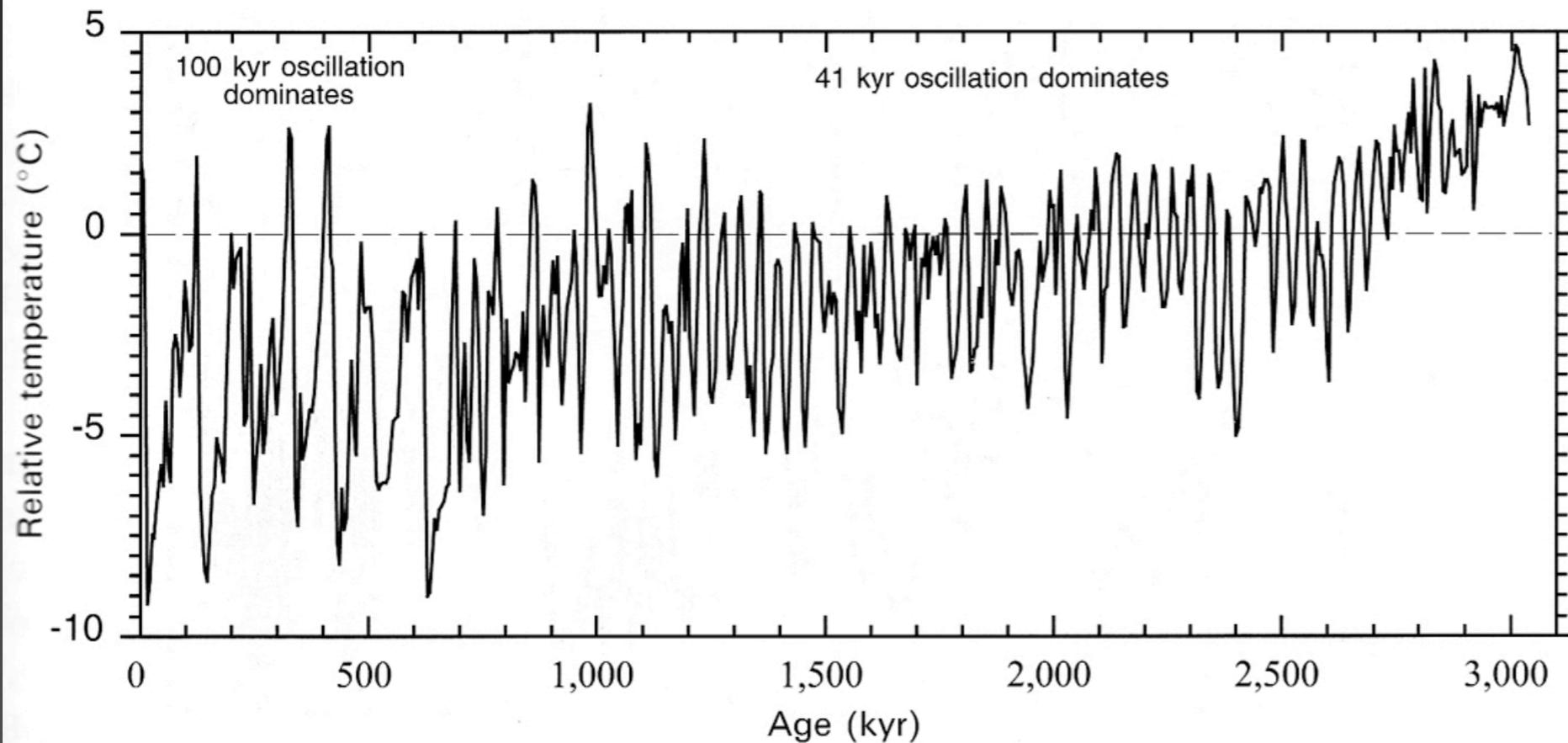


Fig. 4.40. Vostok oxygen and August 45N insolation. No phase lag used.

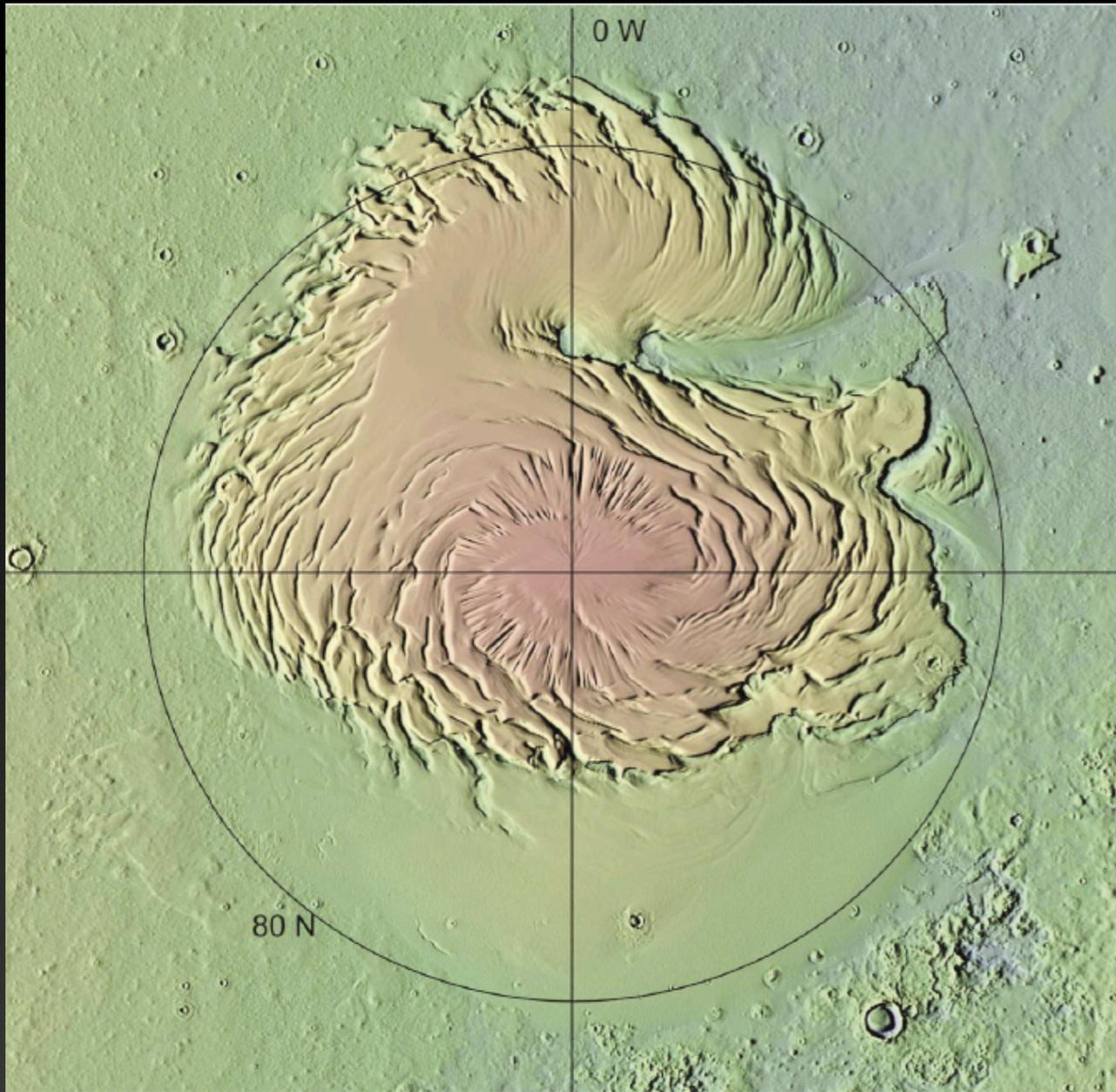
What amplifies orbital forcing to produce ice ages?

Why does 100 kyr eccentricity period dominate climate signal?



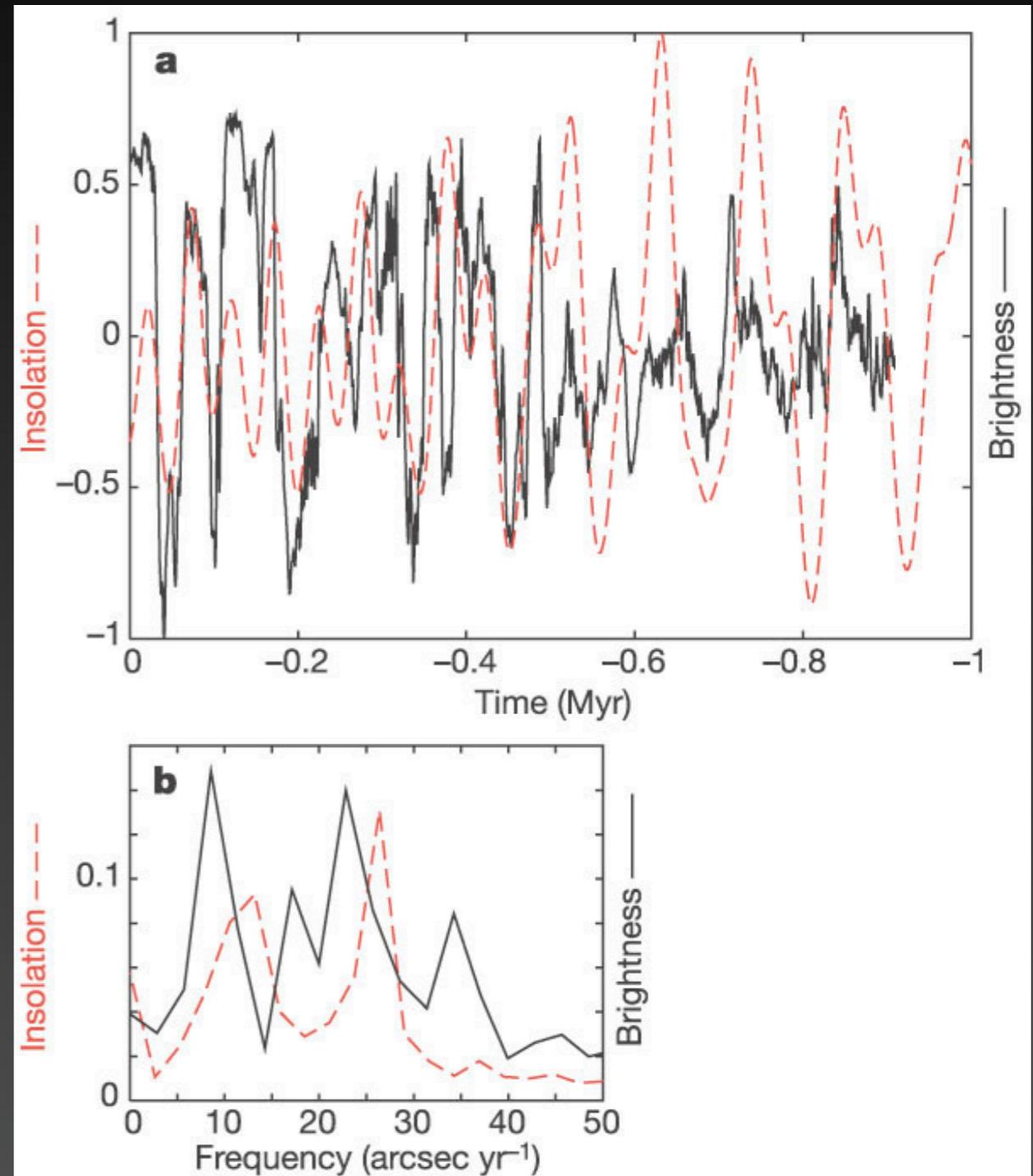
Why did the 41 kyr period dominate 1.5 million years ago?

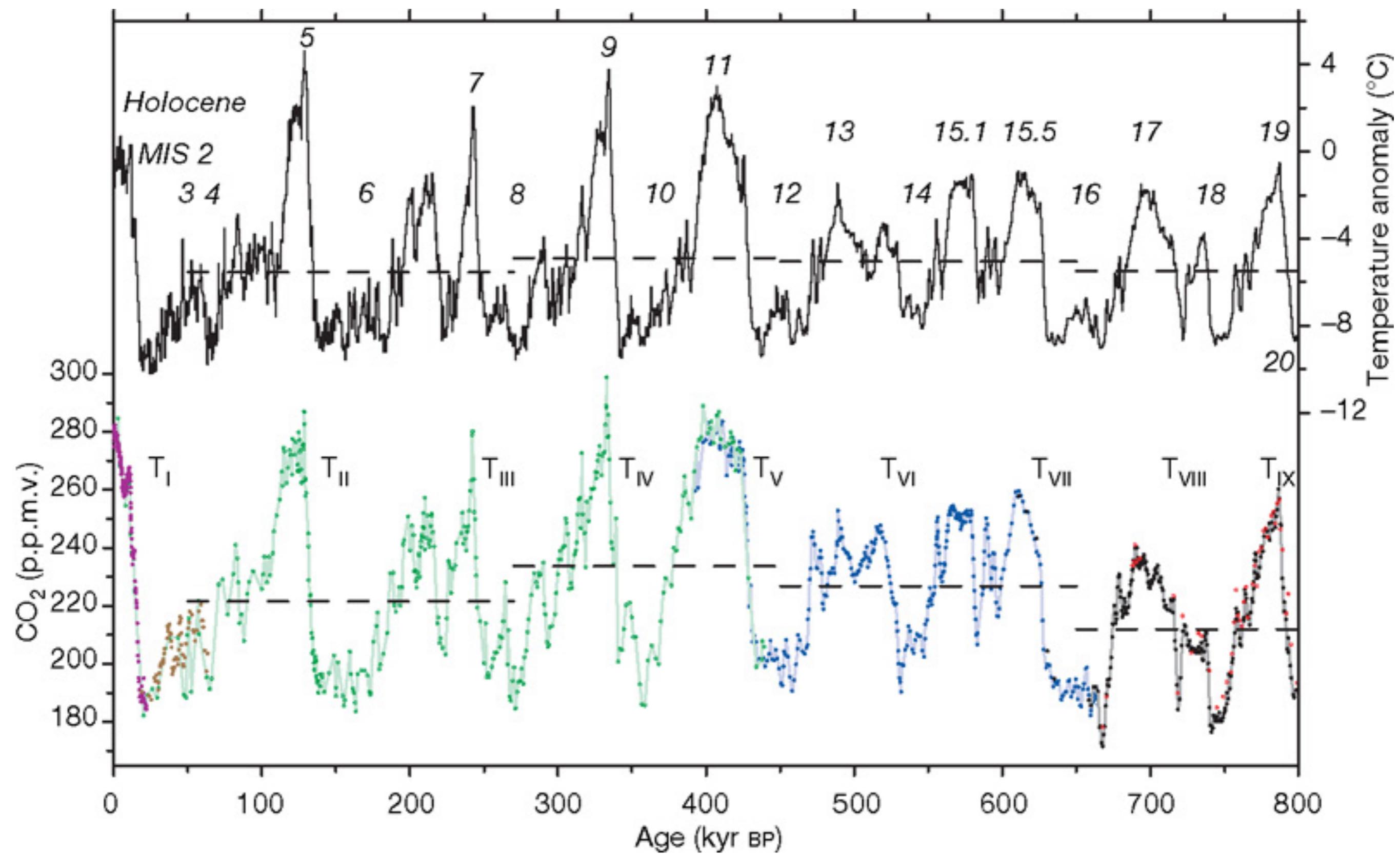
# Martian Climate May Also Show Orbital Forcing



## North Polar Cap of Mars

Laskar, Levrard, and Mustard,  
*Nature* **419**, 375 (2002); Head  
*et al.* *Nature* **426**, 797 (2003).





## Ice Age Climate Forcings ( $\text{W}/\text{m}^2$ )

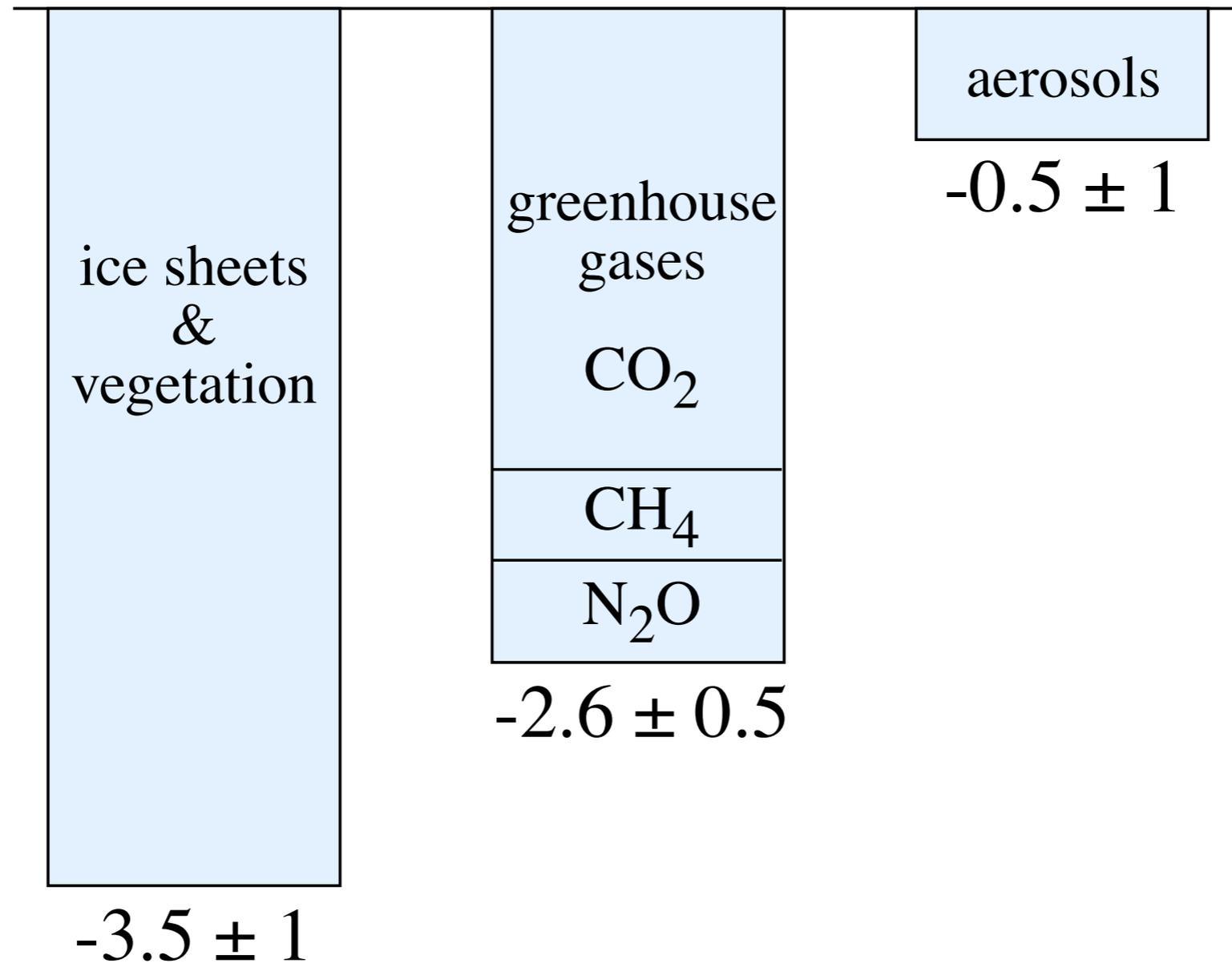
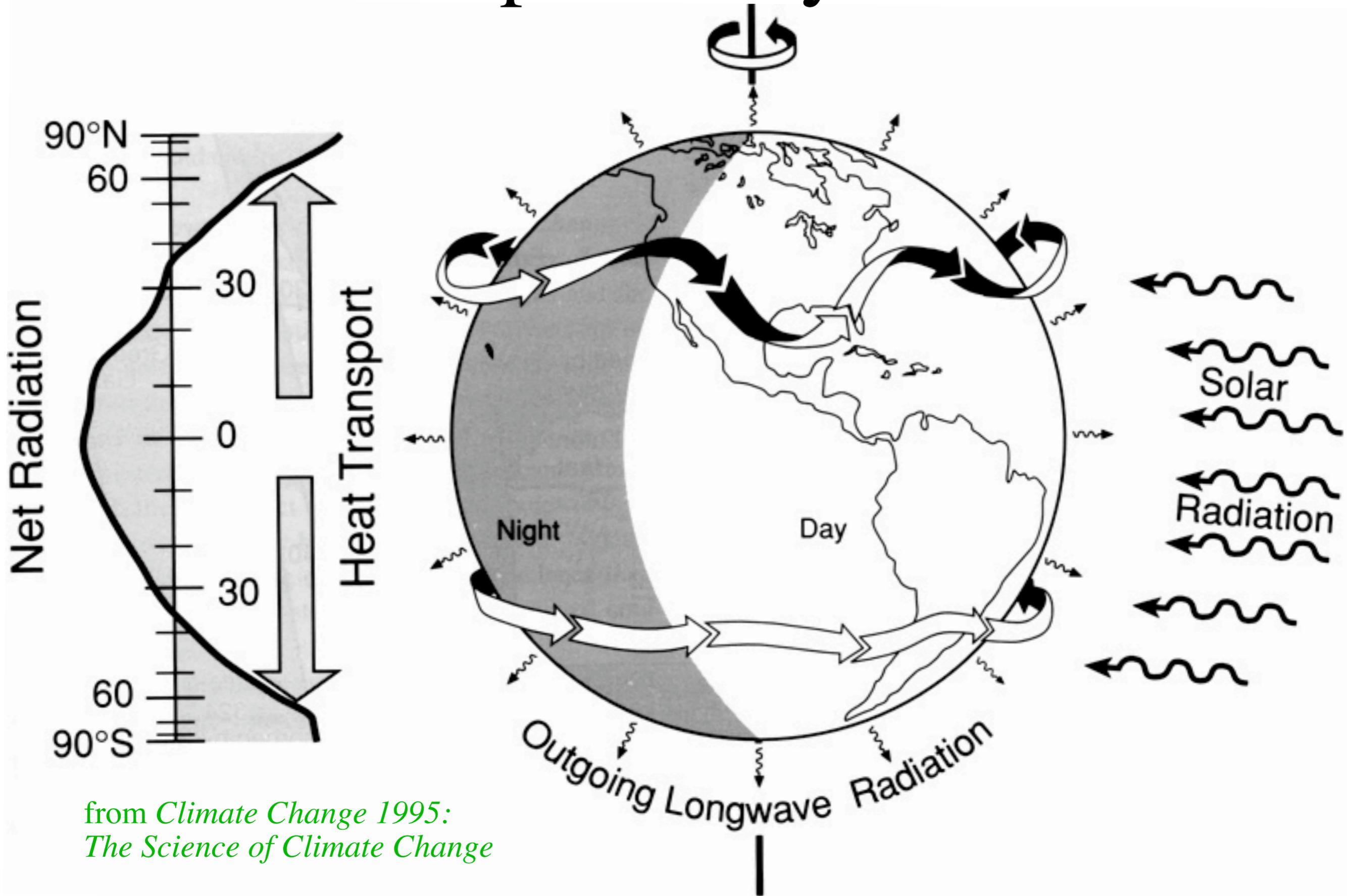


Fig 2. Global radiative forcings during the last ice age relative to the current interglacial period. The total forcing is  $-6.6 \pm 1.5 \text{ W}/\text{m}^2$ . Thus, the  $5^\circ\text{C}$  cooling of the ice age implies a climate sensitivity of  $0.75^\circ\text{C}$  per  $1 \text{ W}/\text{m}^2$  forcing.

Hansen, J. et al., The missing climate forcing, Phil. Trans. R. Soc. London. B, 352, 231-240, 1997.

# Atmospheric Dynamics



from *Climate Change 1995:  
The Science of Climate Change*

# Single Layer Models

# Single Layer Models

$$\frac{D\omega}{Dt} = 0$$

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Vorticity   $\omega = \hat{r} \cdot (\vec{\nabla} \times \vec{v})$

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**Vorticity**   $\omega = \hat{r} \cdot (\vec{\nabla} \times \vec{v})$

$$\frac{\partial \omega}{\partial t} + \vec{v} \cdot \vec{\nabla} \omega = 0$$

$$\vec{v} = \hat{r} \times \vec{\nabla} \psi$$

$$\vec{\nabla} \cdot \vec{v} = 0$$

$$\omega = \nabla^2 \psi$$

# Single Layer Models

$$\frac{D\omega}{Dt} = 0$$

**Vorticity**   $\omega = \hat{r} \cdot (\vec{\nabla} \times \vec{v})$

$$\frac{\partial \omega}{\partial t} + \vec{v} \cdot \vec{\nabla} \omega = 0$$

$$\frac{\partial \omega}{\partial t} + J(\psi, \omega) = 0$$

$$\vec{v} = \hat{r} \times \vec{\nabla} \psi$$

$$\vec{\nabla} \cdot \vec{v} = 0$$

$$\omega = \nabla^2 \psi$$

$$J(\psi, \omega) \equiv \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x}$$

# Freely Decaying Turbulence on Sphere

# Coriolis Force

# Coriolis Force

$$\frac{\partial q}{\partial t} + J(\psi, q) = 0$$

# Coriolis Force

$$\frac{\partial q}{\partial t} + J(\psi, q) = 0$$

Absolute vorticity  $q$  =  $\omega + f$  =  $\nabla^2 \psi + f$

Relative vorticity  $\omega$

Coriolis term  $f$

The diagram illustrates the relationship between absolute vorticity  $q$ , relative vorticity  $\omega$ , and the Coriolis term  $f$ . It shows the equation  $q = \omega + f$  and its equivalent form  $q = \nabla^2 \psi + f$ . Yellow arrows point from the labels 'Absolute vorticity', 'Relative vorticity', and 'Coriolis term' to their respective terms in the equations.

# Coriolis Force

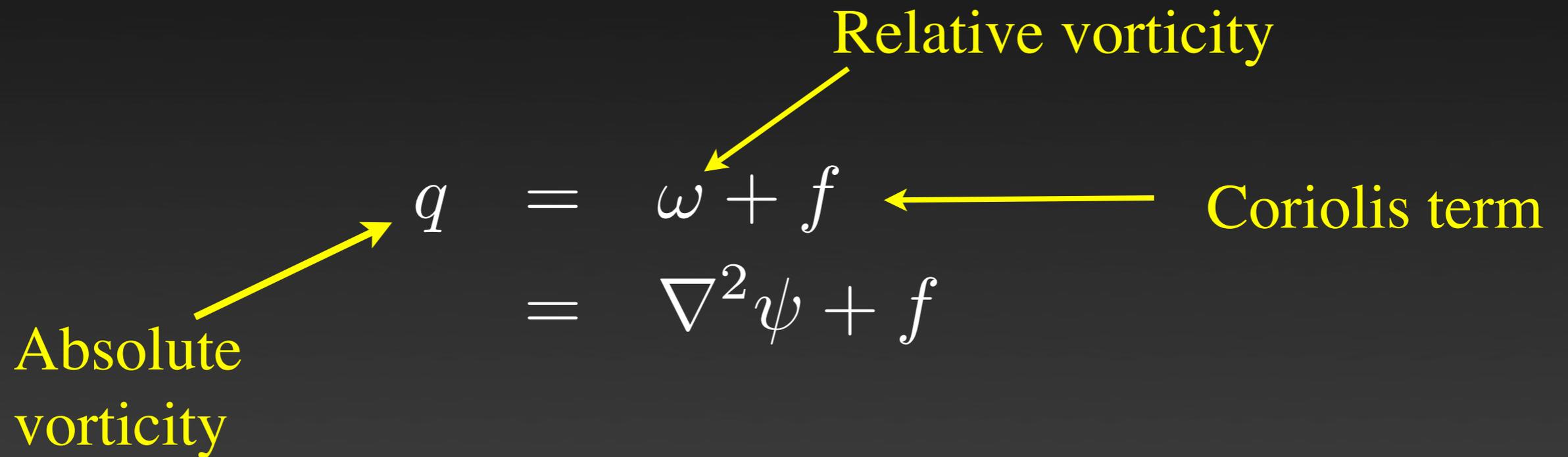
$$\frac{\partial q}{\partial t} + J(\psi, q) = 0$$

Relative vorticity

Absolute vorticity

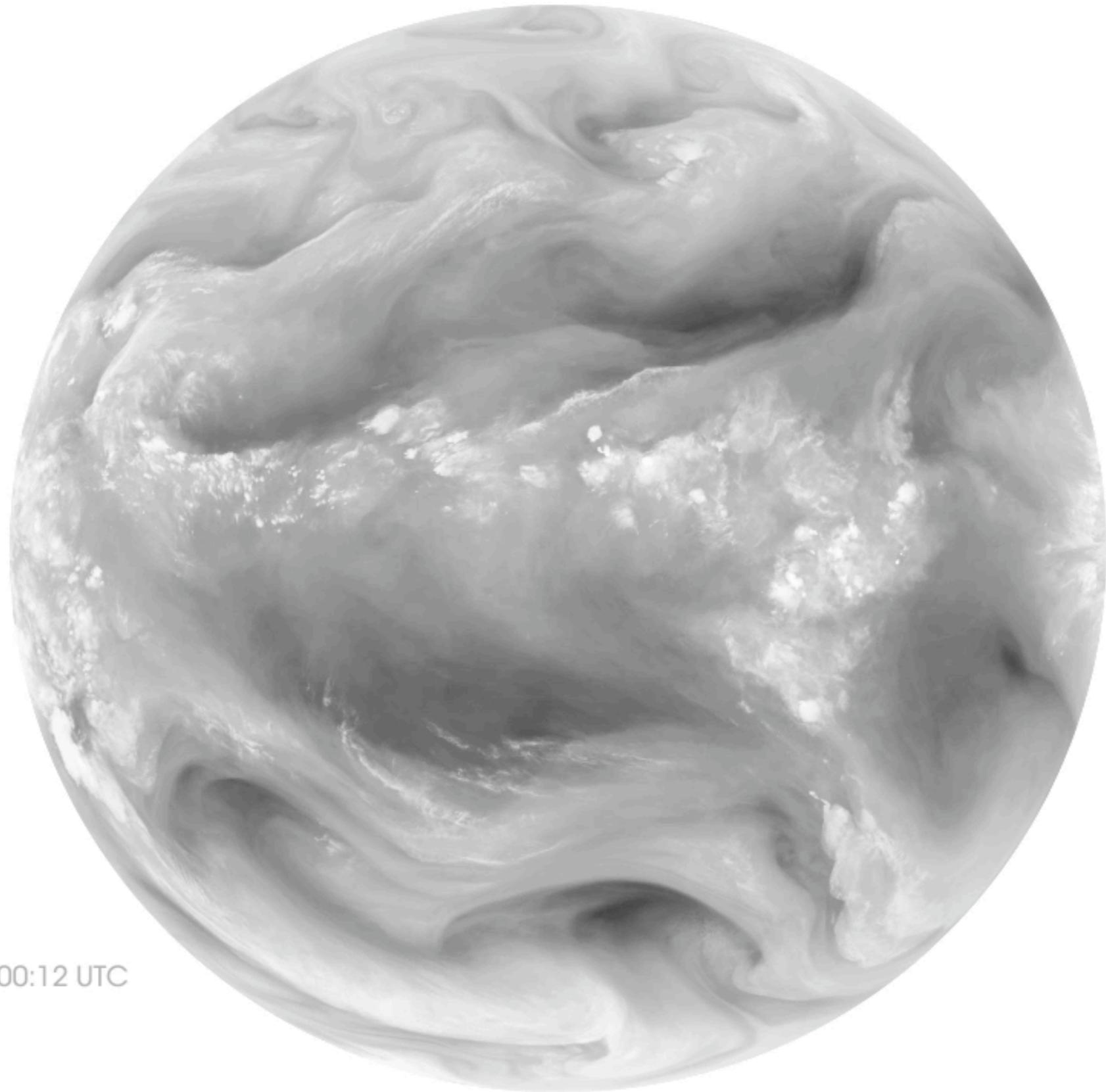
$$q = \omega + f$$
$$q = \nabla^2 \psi + f$$

Coriolis term



$$f = 2\Omega \sin(\phi)$$

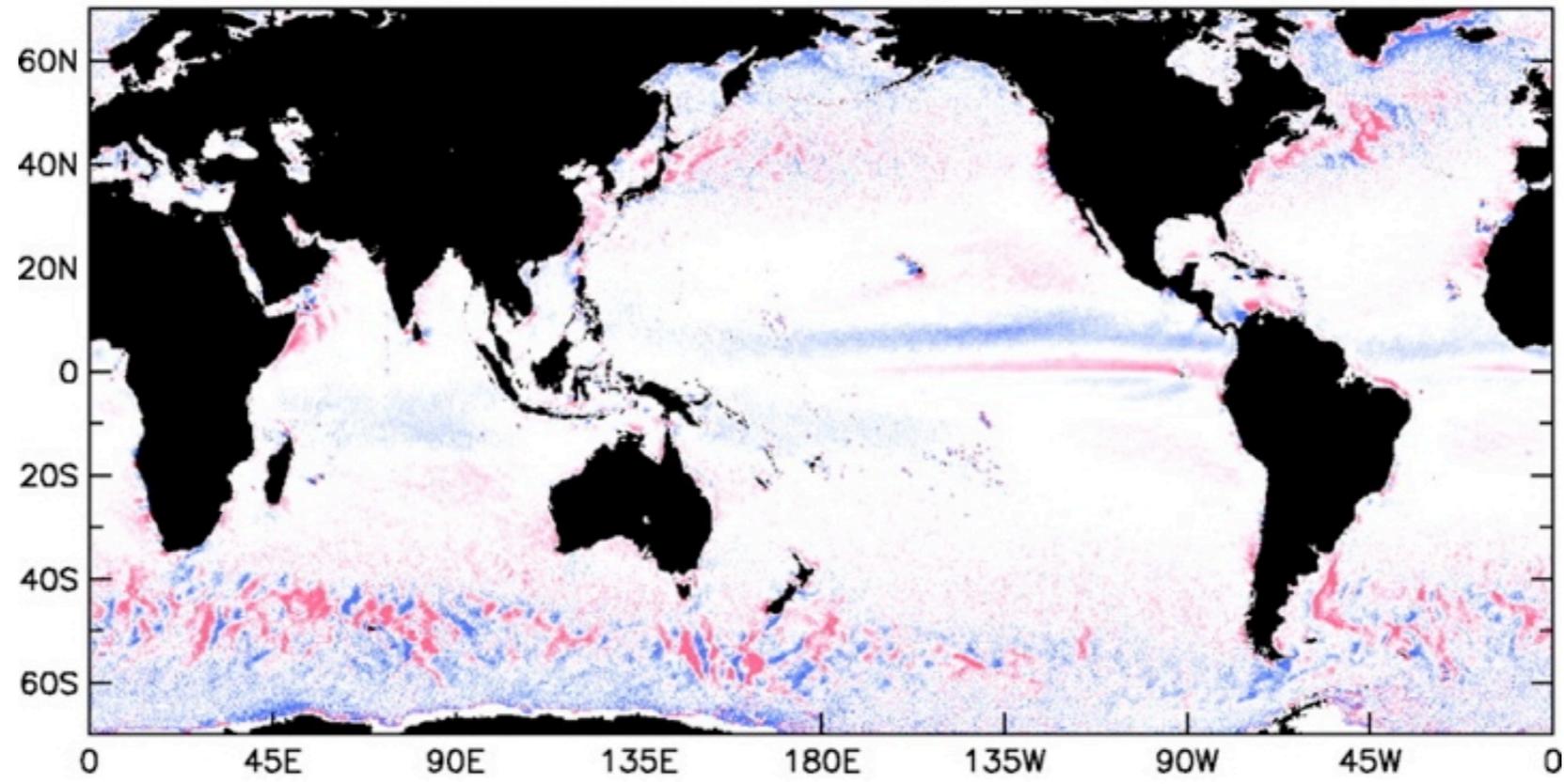




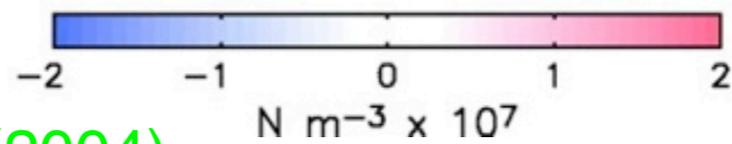
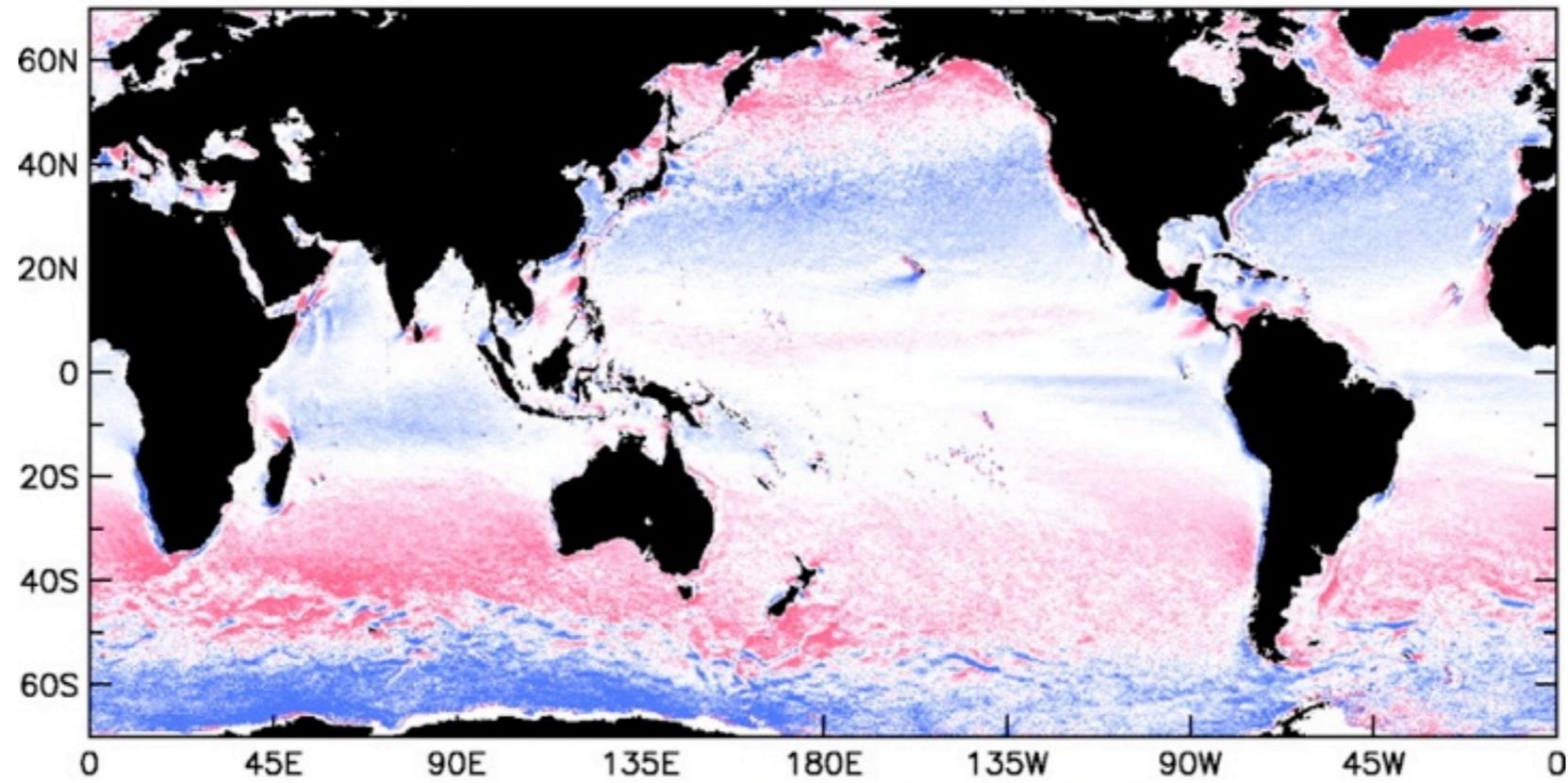
October 7, 2007 00:12 UTC

# Coriolis Force

Wind Stress Divergence



Wind Stress Curl



Chelton *et al.*, *Science* **303**, 978 (2004)

# Stratification

# Stratification

$$q = \nabla^2 \psi + f - \frac{\psi}{\ell_R^2}$$

# Stratification

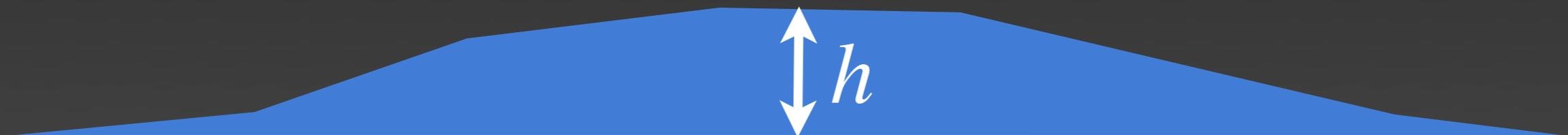
$$q = \nabla^2 \psi + f - \frac{\psi}{\ell_R^2}$$

$$\ell_R^2 = \frac{gh}{f^2}$$

# Stratification

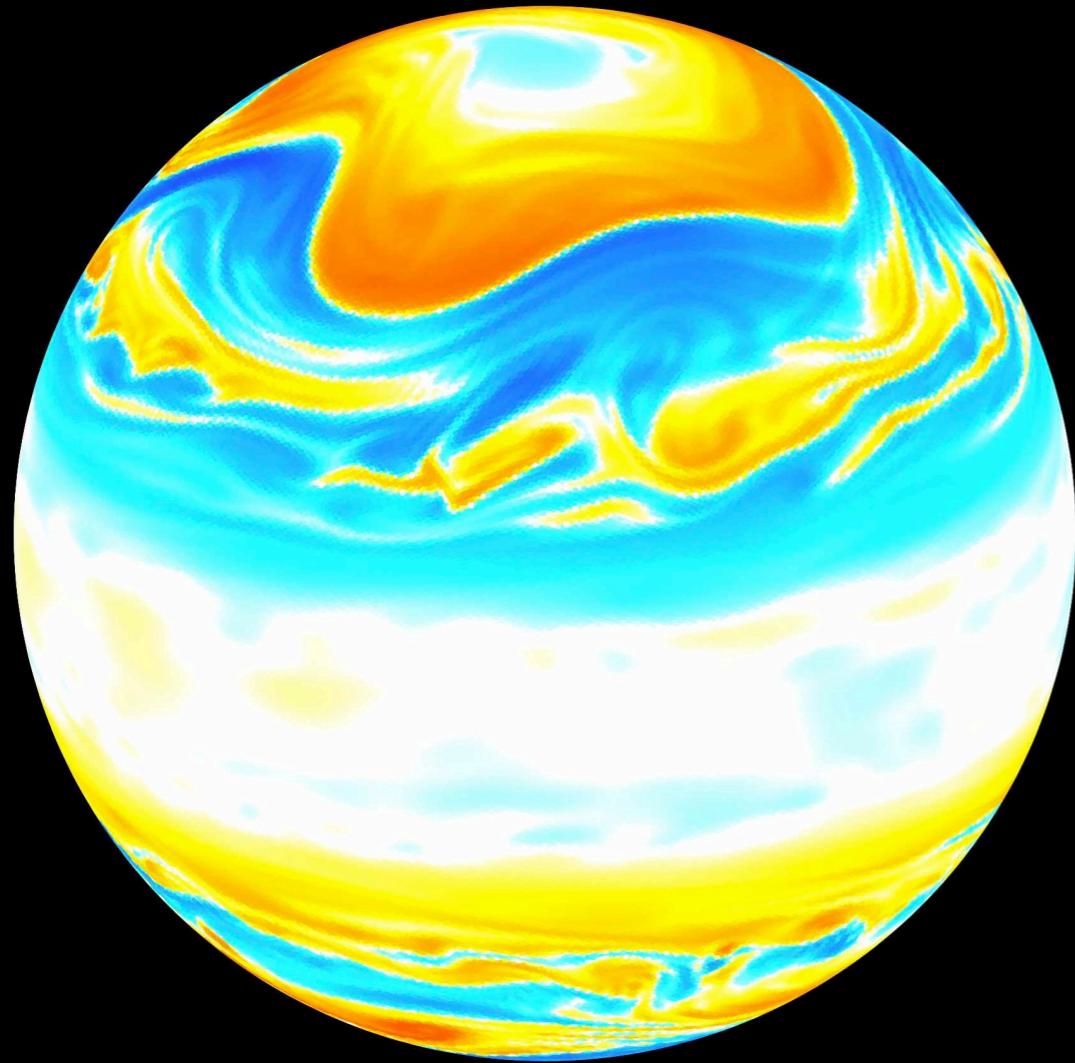
$$q = \nabla^2 \psi + f - \frac{\psi}{\ell_R^2}$$

$$\ell_R^2 = \frac{gh}{f^2}$$

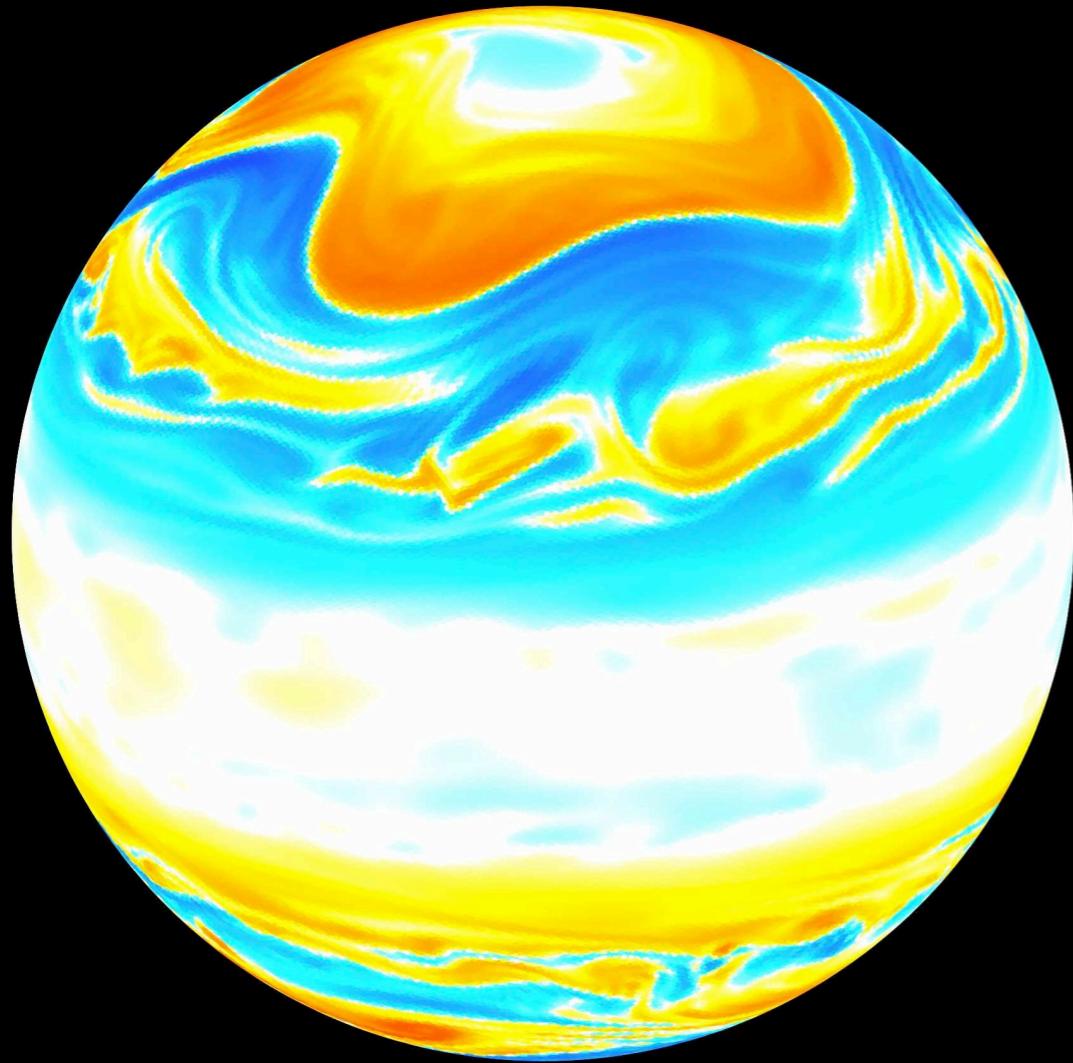


$$\ell_R = \mathcal{O}(1,000 \text{ km})$$

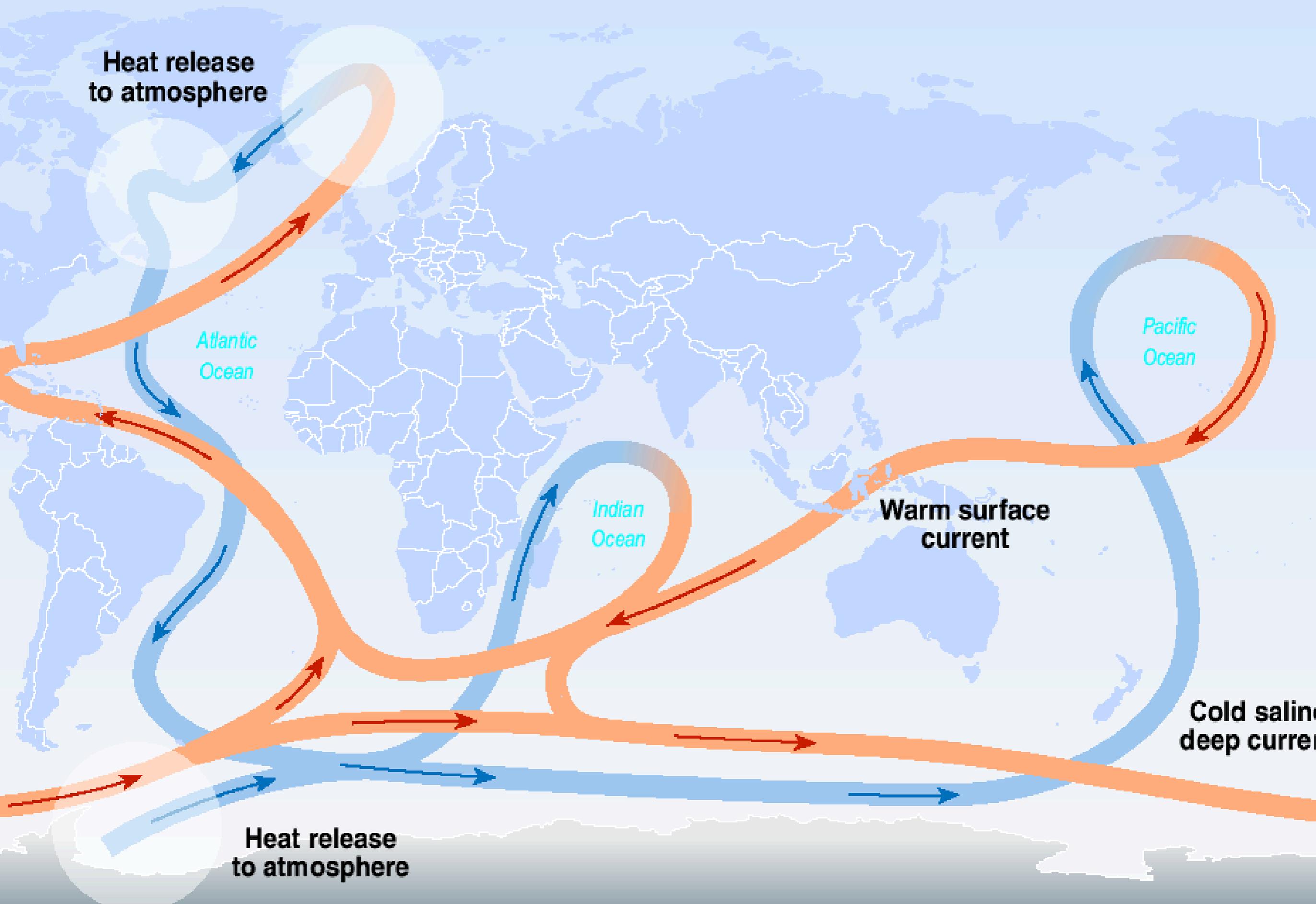
# Stratification Sets Synoptic Length Scale



# Stratification Sets Synoptic Length Scale



# Great ocean conveyor belt



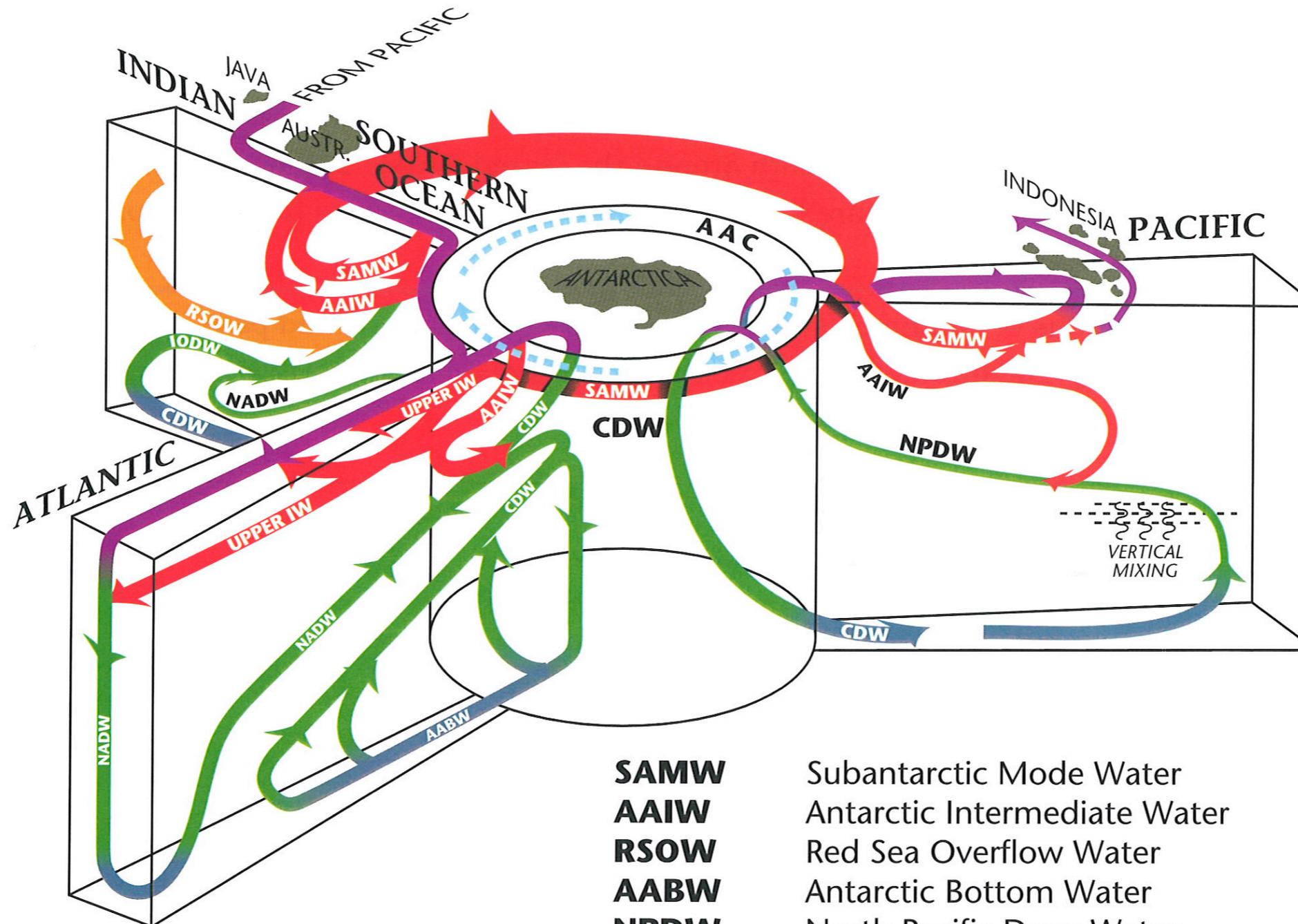
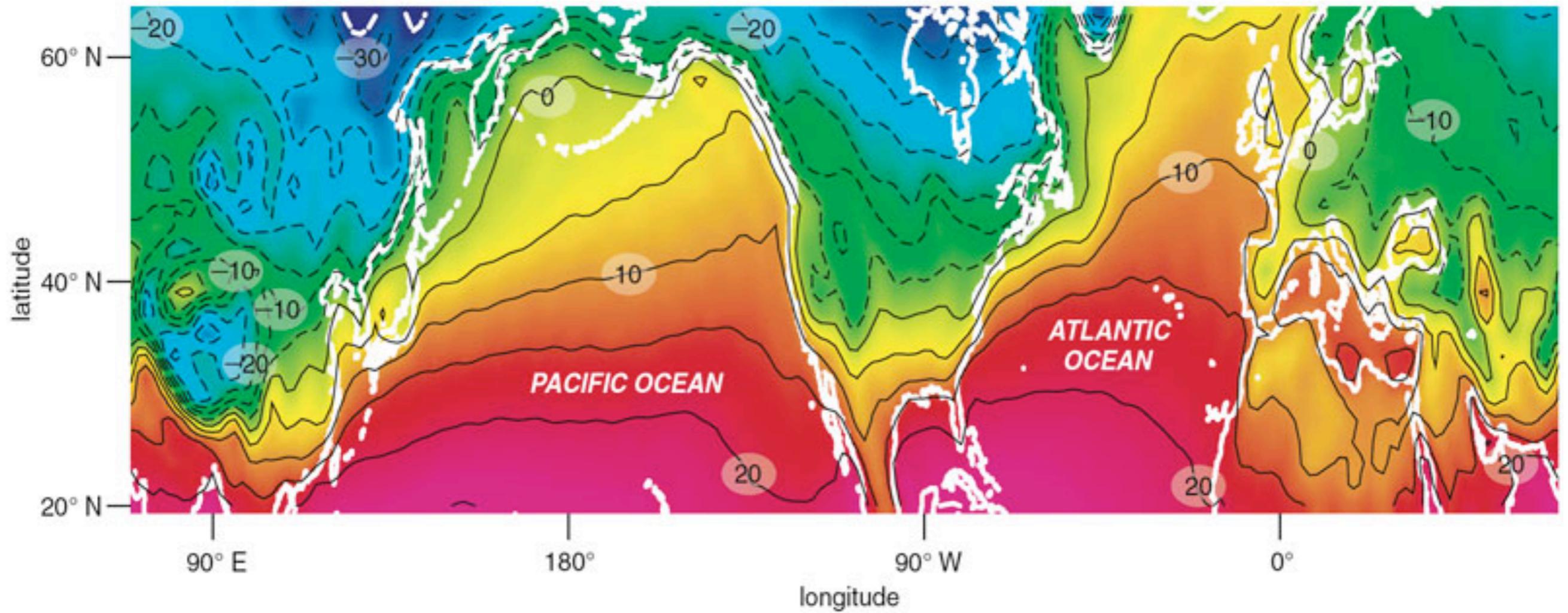
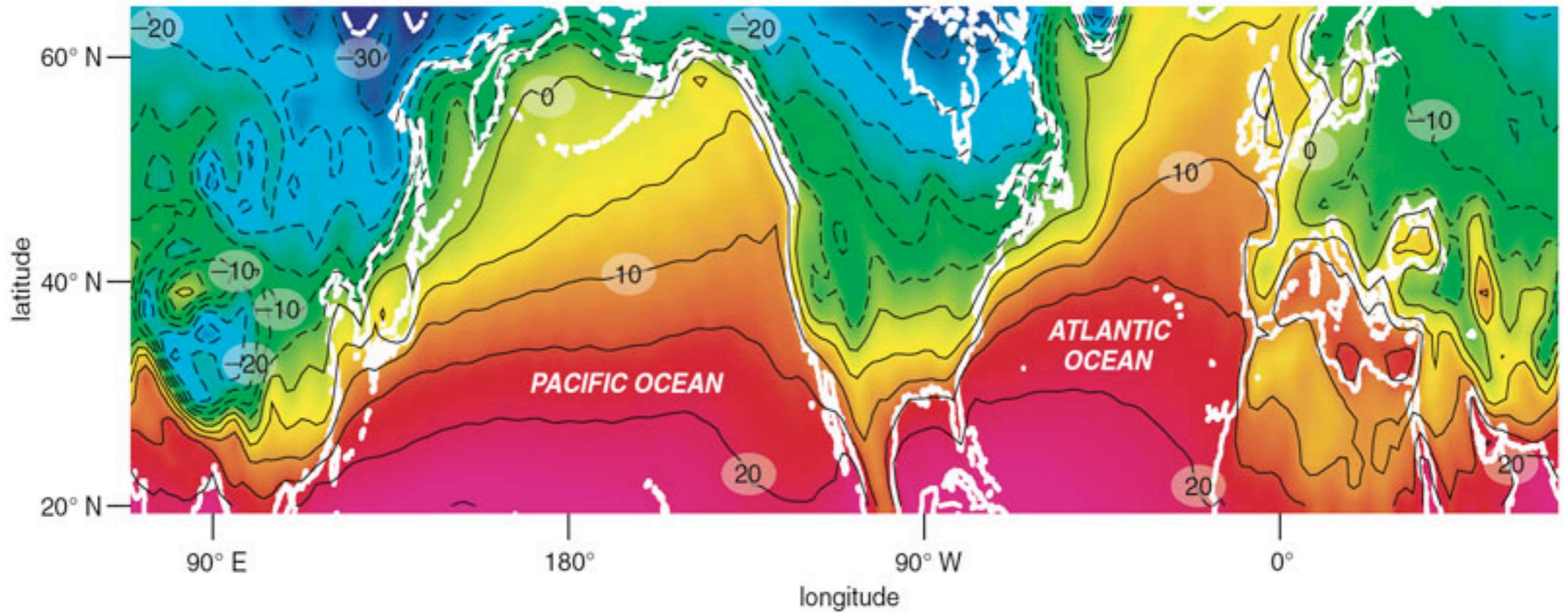


Figure I-91: A new version of Figure I-90 with schematic meridional sections of interbasin flow for each ocean with their global linkages.

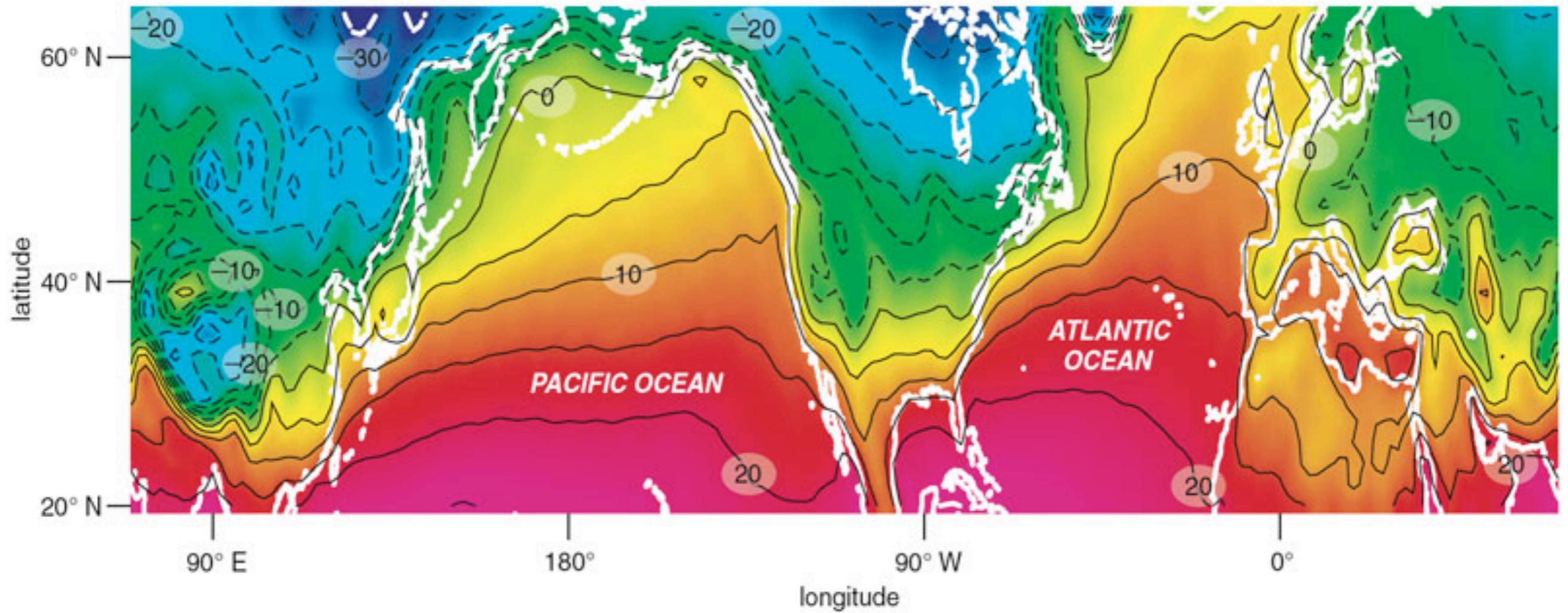
- SAMW** Subantarctic Mode Water
- AAIW** Antarctic Intermediate Water
- RSOW** Red Sea Overflow Water
- AABW** Antarctic Bottom Water
- NPDW** North Pacific Deep Water
- AAC** Antarctic Circumpolar Current
- CDW** Circumpolar Deep Water
- NADW** North Atlantic Deep Water
- UPPER IW**  $26.8 \leq \sigma_{\theta} \leq 27.25$
- IODW** Indian Ocean Deep Water



City	Latitude	January (°F)	August (°F)
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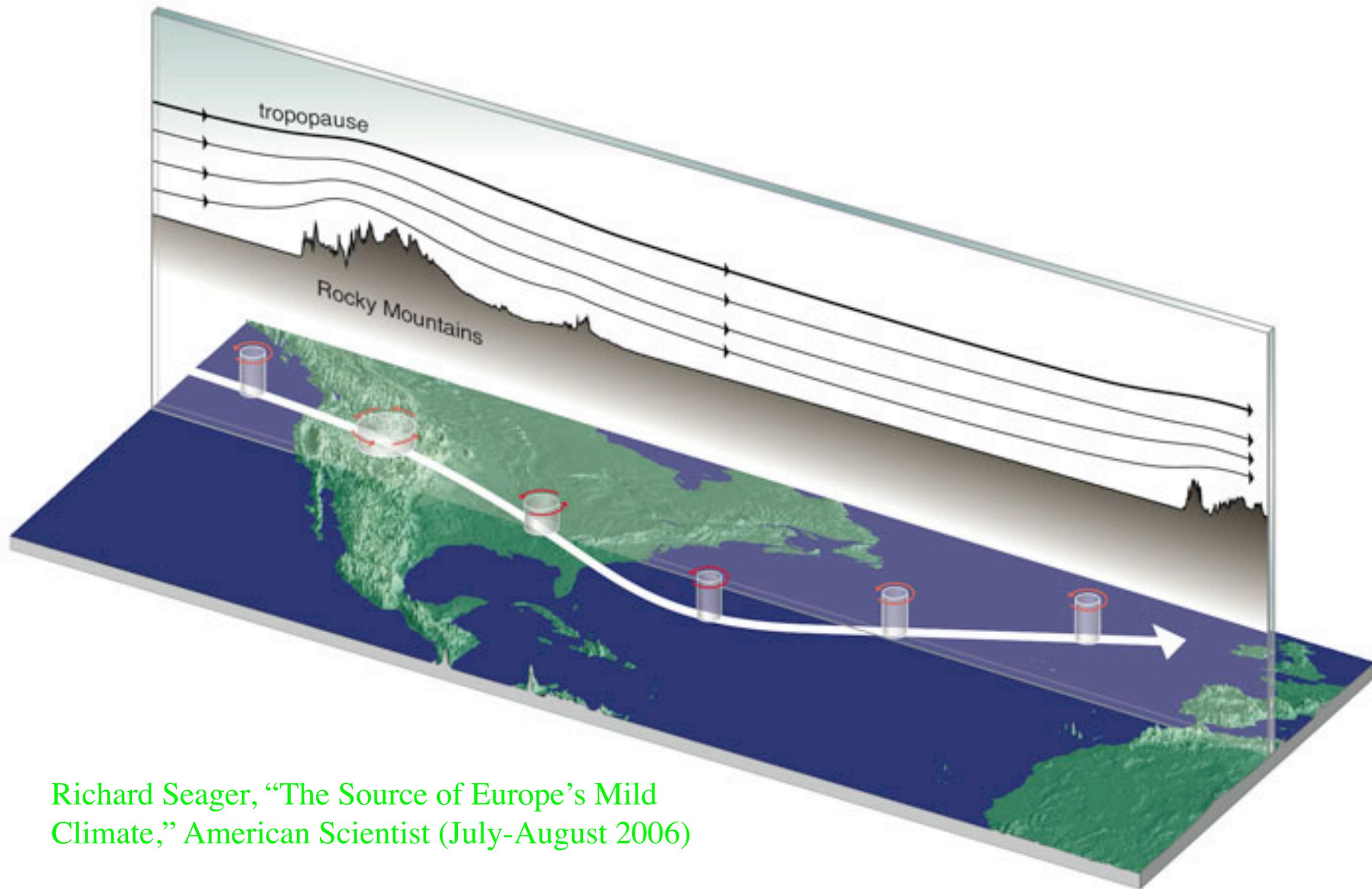


City	Latitude	January (°F)	August (°F)
Glasgow	56°	34 to 45	52 to 64



City	Latitude	January (°F)	August (°F)
Glasgow	56°	34 to 45	52 to 64
Sitka	57°	30 to 38	52 to 62

# Topography & Angular Momentum

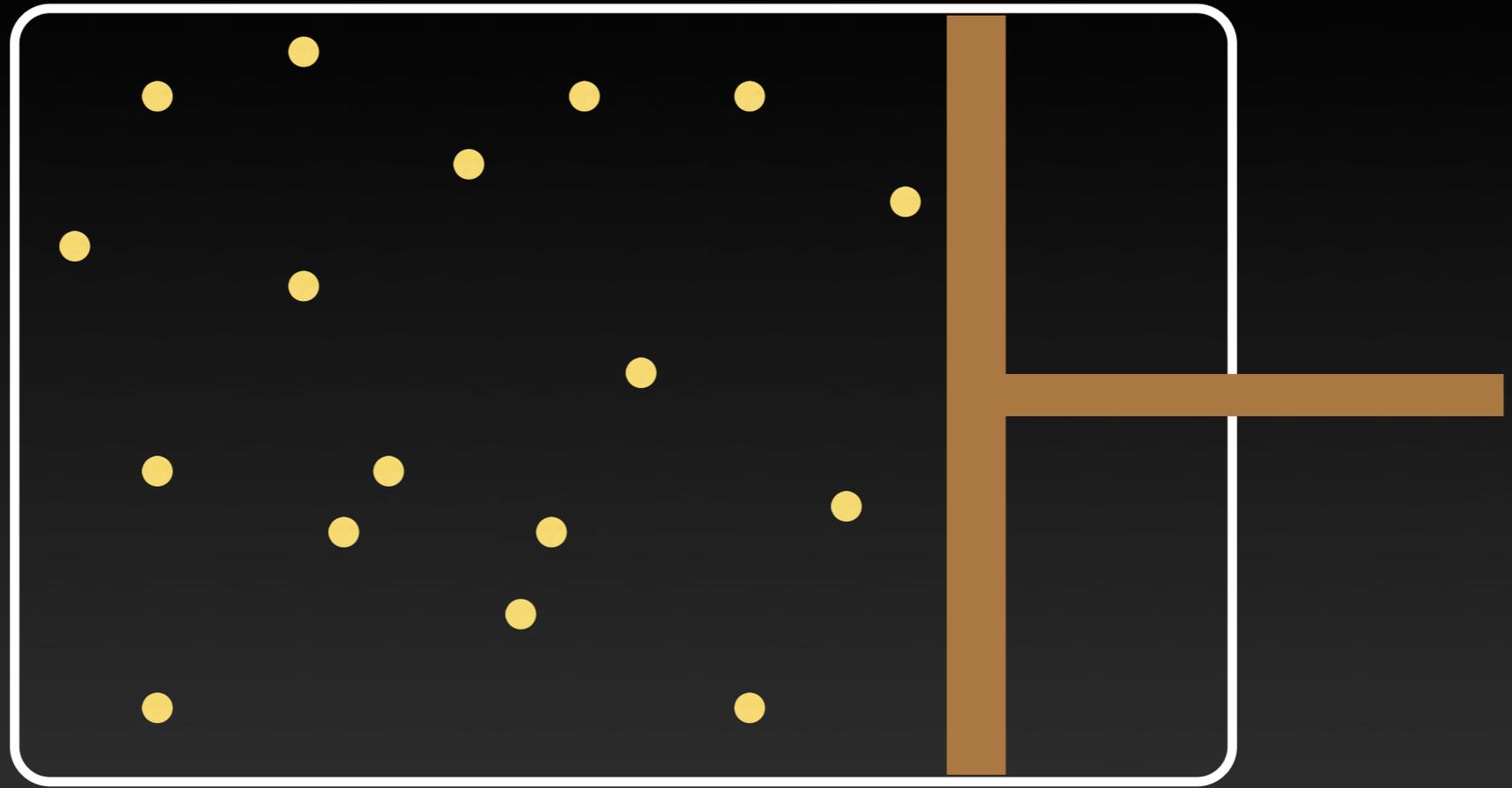


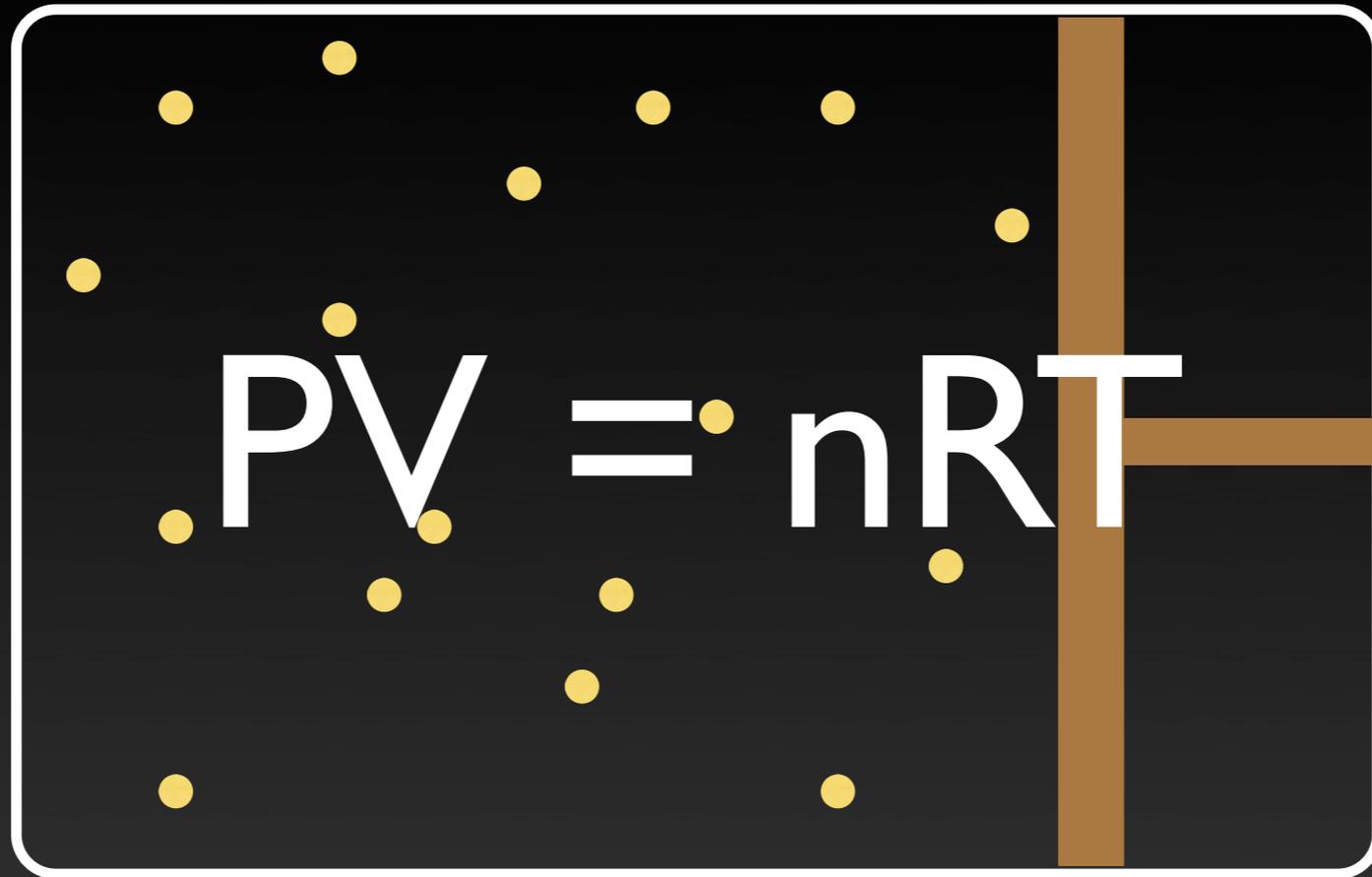
Richard Seager, "The Source of Europe's Mild Climate," *American Scientist* (July-August 2006)

# Quantum Field Theory of Global Warming?

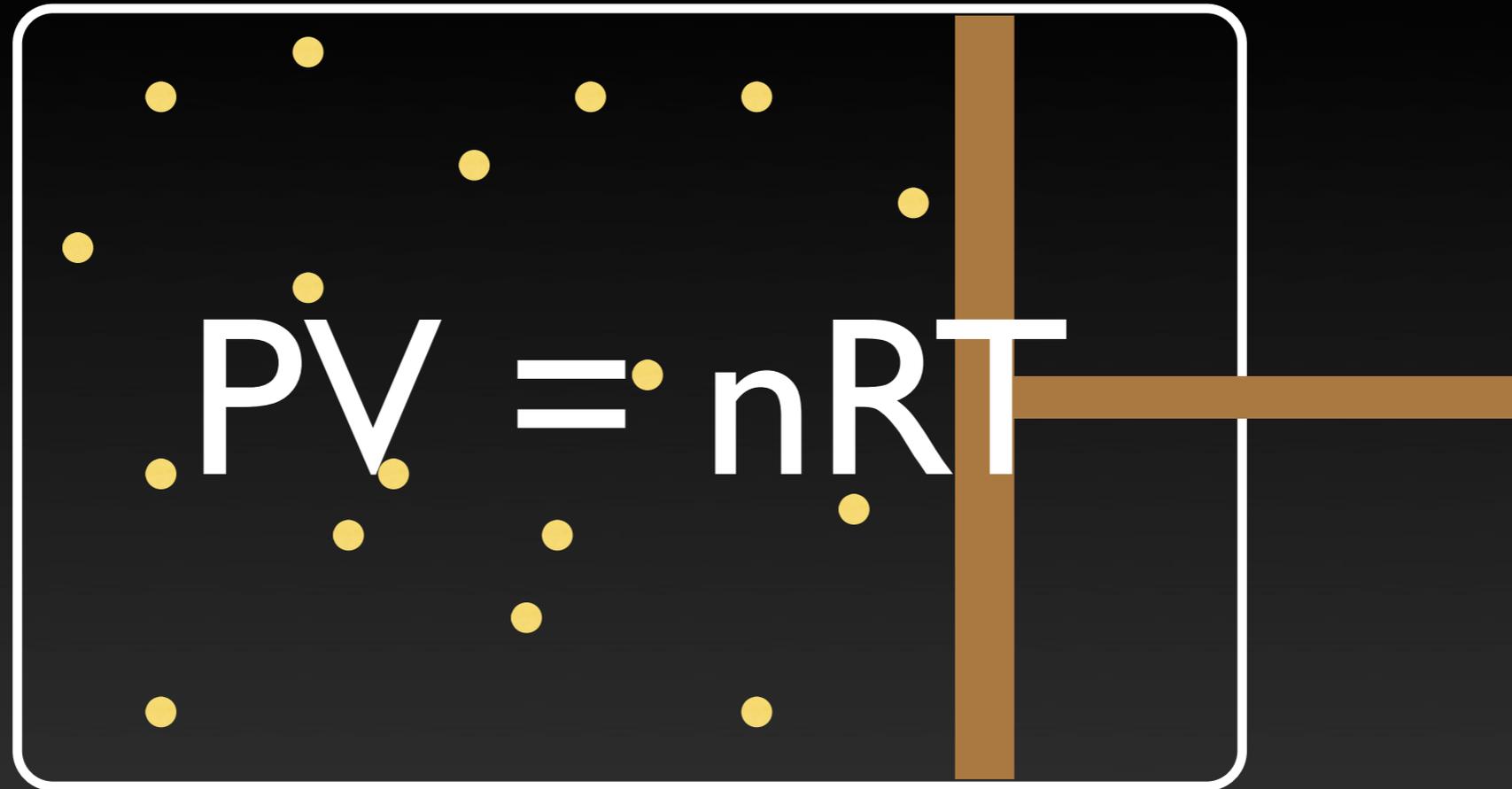
"More than any other theoretical procedure, numerical integration is also subject to the criticism that it yields little insight into the problem. The computed numbers are not only processed like data but they look like data, and a study of them may be no more enlightening than a study of real meteorological observations. An alternative procedure which does not suffer this disadvantage consists of deriving a new system of equations whose unknowns are the statistics themselves."

*Edward Lorenz, The Nature and Theory of the General Circulation (1967)*

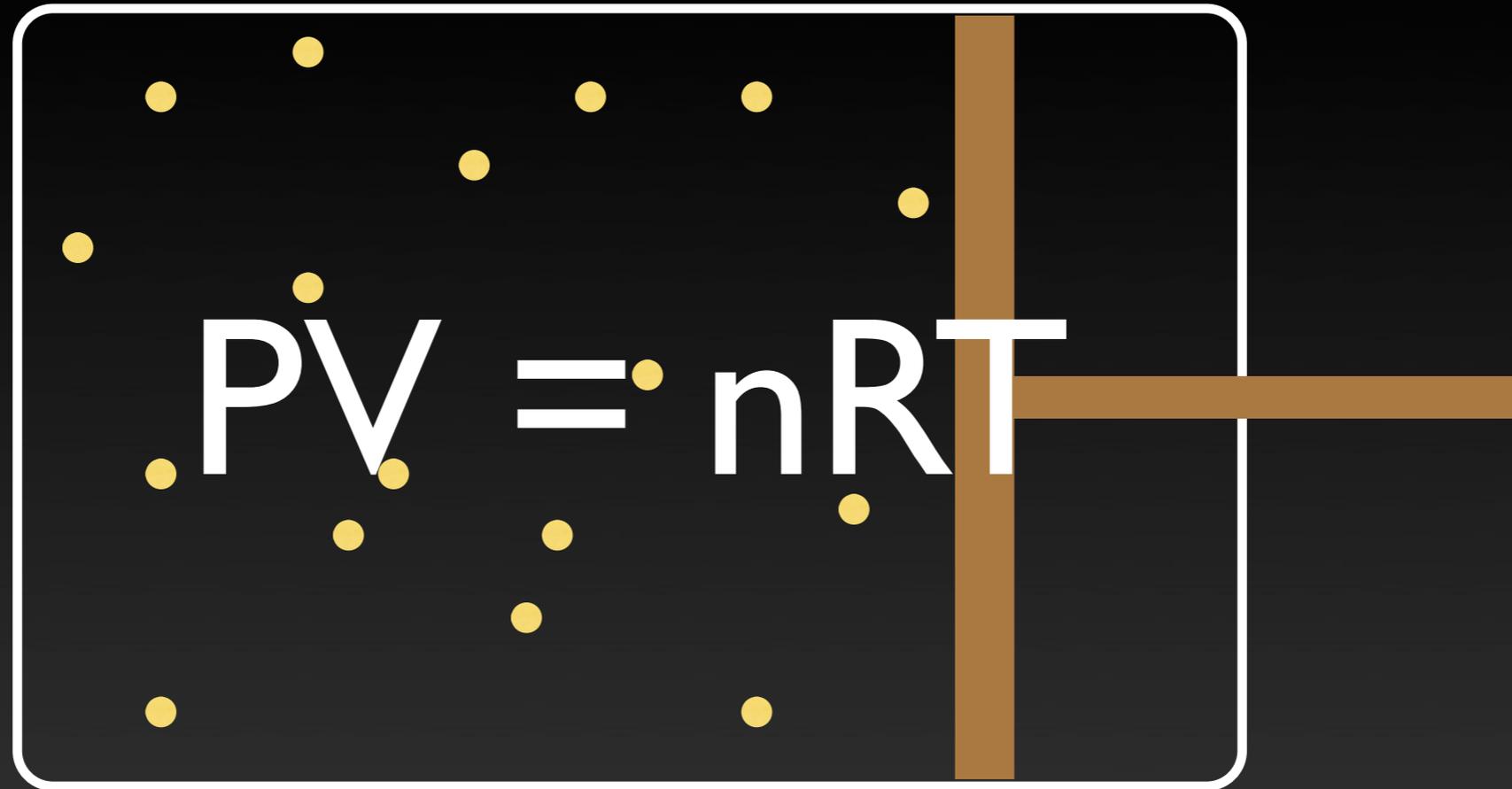




$$PV = nRT$$

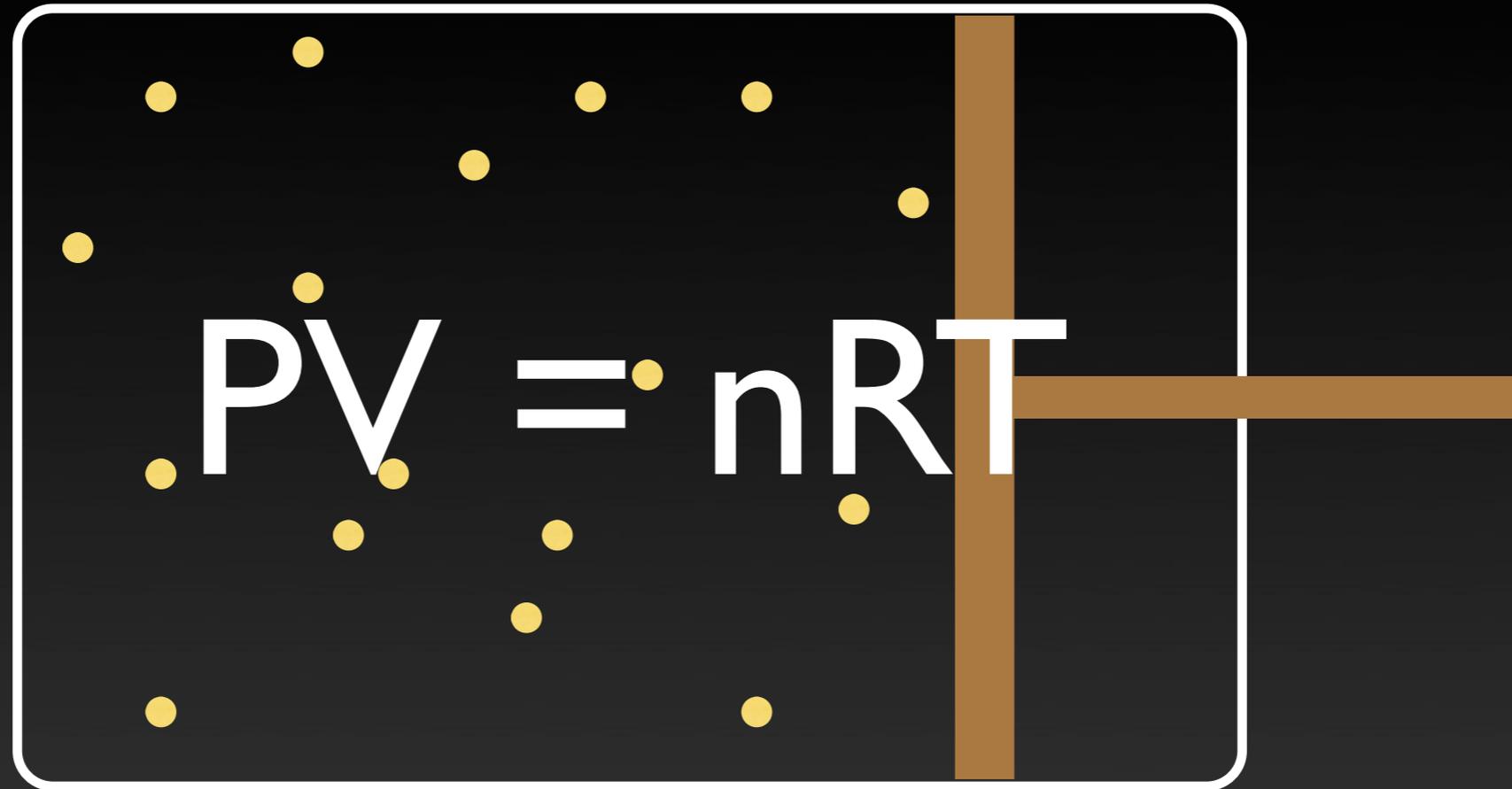


Thermodynamics vs. Statistical Mechanics



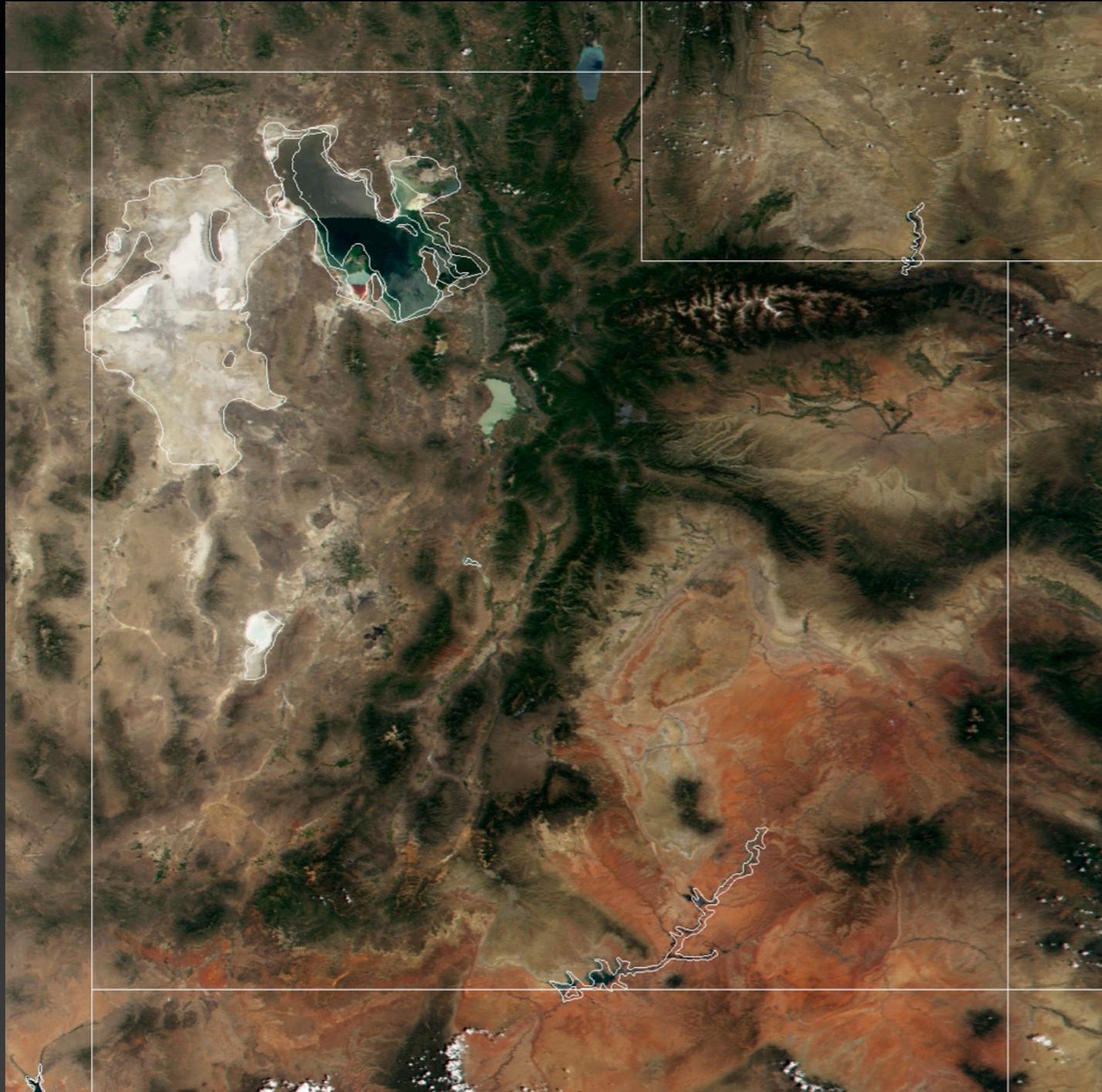
Thermodynamics vs. Statistical Mechanics

Equilibrium vs. Out-of-Equilibrium



Thermodynamics vs. Statistical Mechanics

Equilibrium vs. Out-of-Equilibrium





# Hopf Functional Approach

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$$\frac{dx}{dt} = x^2$$

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$$\Psi(t, u) \equiv e^{iux(t)}$$

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$$i \frac{\partial}{\partial t} \Psi = u \frac{\partial^2}{\partial u^2} \Psi$$

# Hopf Functional Approach

$$\frac{dx}{dt} = x^2$$

$$\Psi(t, u) \equiv e^{iux(t)}$$

$$i \frac{\partial}{\partial t} \Psi = u \frac{\partial^2}{\partial u^2} \Psi$$

$$i \frac{\partial}{\partial t} \bar{\Psi} = u \frac{\partial^2}{\partial u^2} \bar{\Psi}$$

# Hopf Functional Approach

$$\frac{dx}{dt} = x^2$$

$$\Psi(t, u) \equiv e^{iux(t)}$$

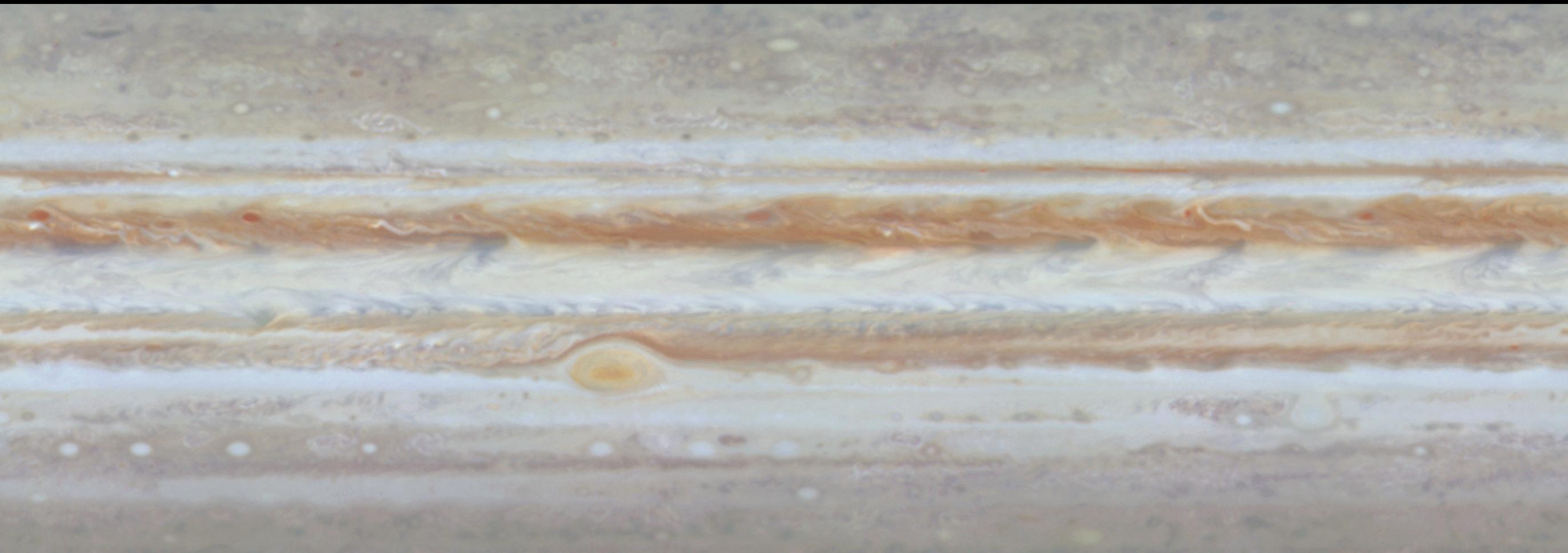
$$i \frac{\partial}{\partial t} \Psi = u \frac{\partial^2}{\partial u^2} \Psi$$

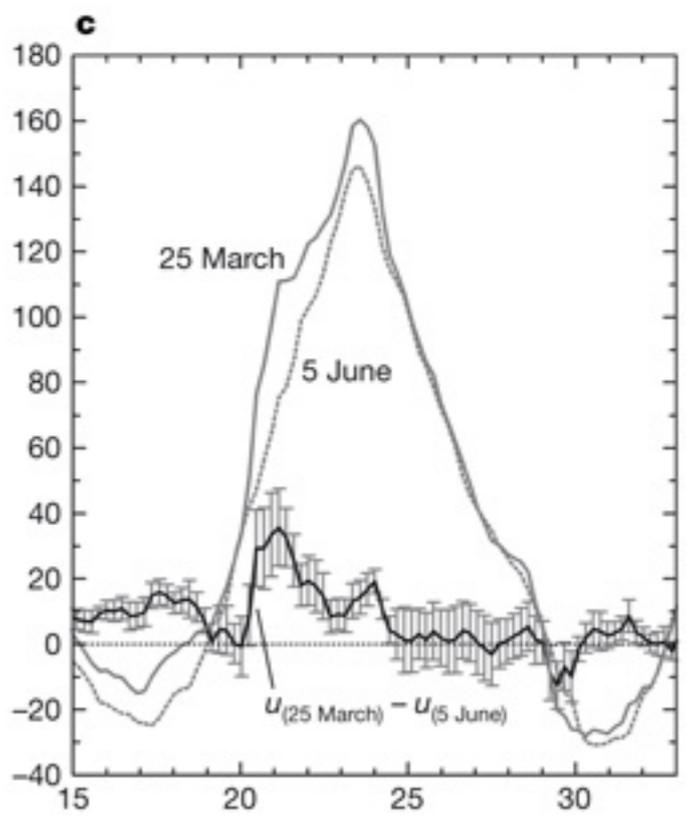
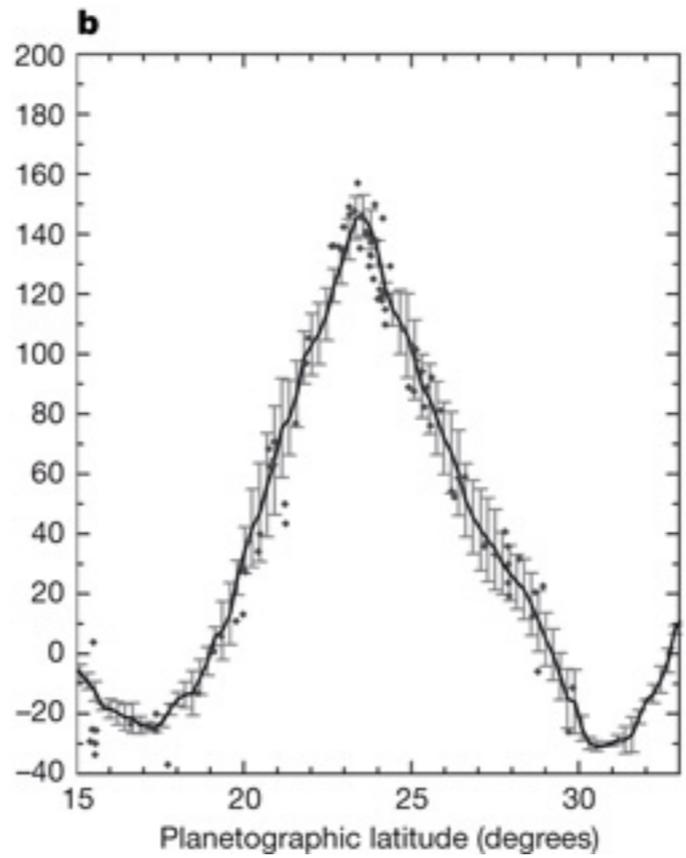
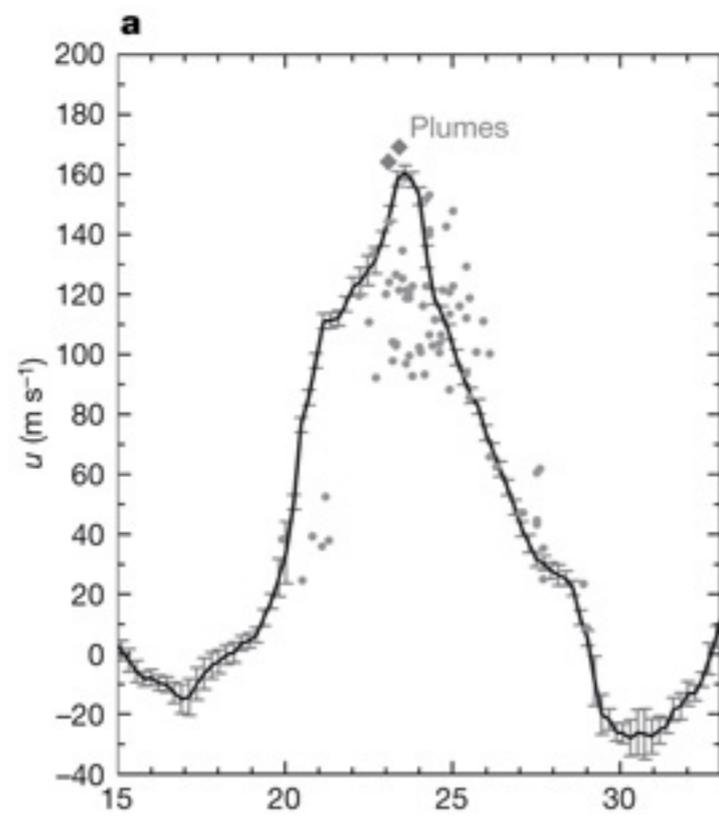
$$i \frac{\partial}{\partial t} \bar{\Psi} = u \frac{\partial^2}{\partial u^2} \bar{\Psi}$$

$$\hat{H} \bar{\Psi}_0 = 0$$

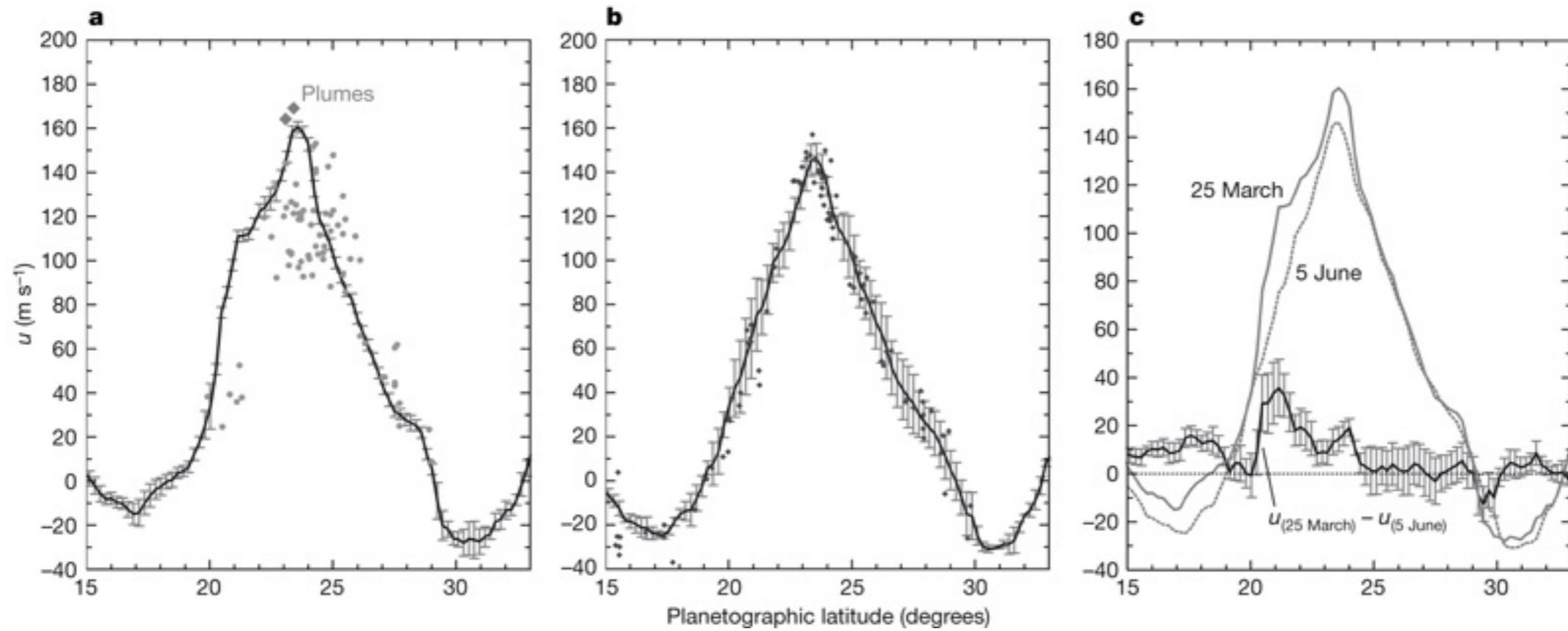
$$\bar{\Psi}_0(u) = \exp\left\{iu\langle x \rangle - \frac{1}{2!}u^2(\langle x^2 \rangle - \langle x \rangle^2) + \dots\right\}$$

$$\langle x \rangle = -i \left. \frac{\partial \bar{\Psi}_0(u)}{\partial u} \right|_{u=0}$$

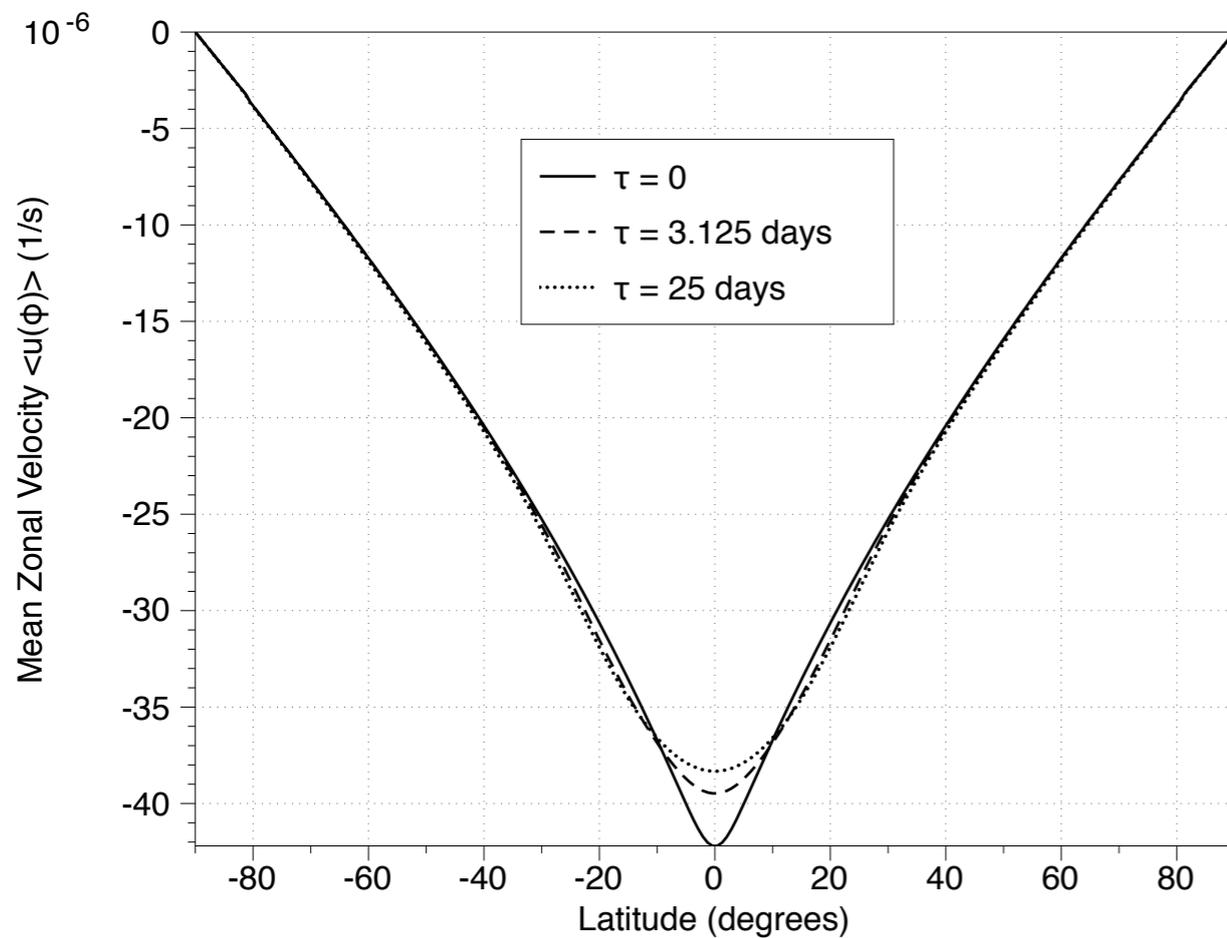


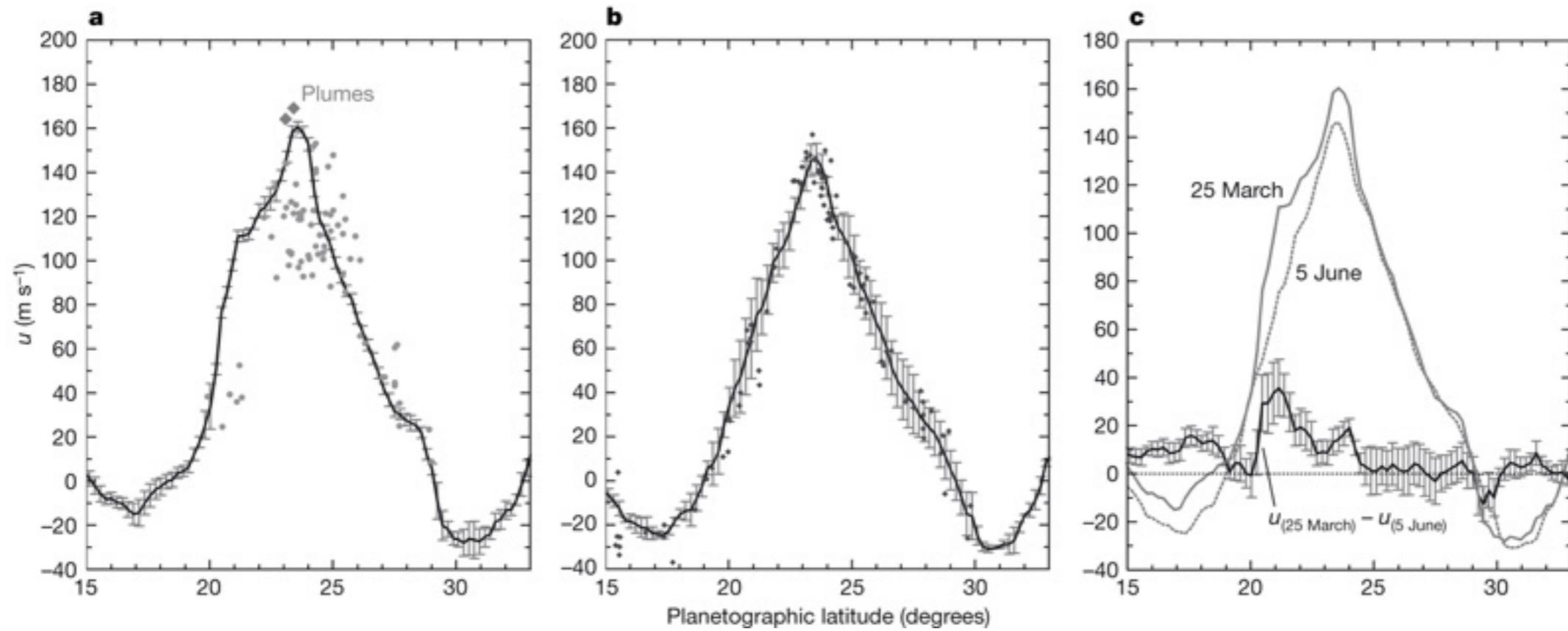


A. Sanchez-Lavega *et al.* Nature **451**, 437 (2008)

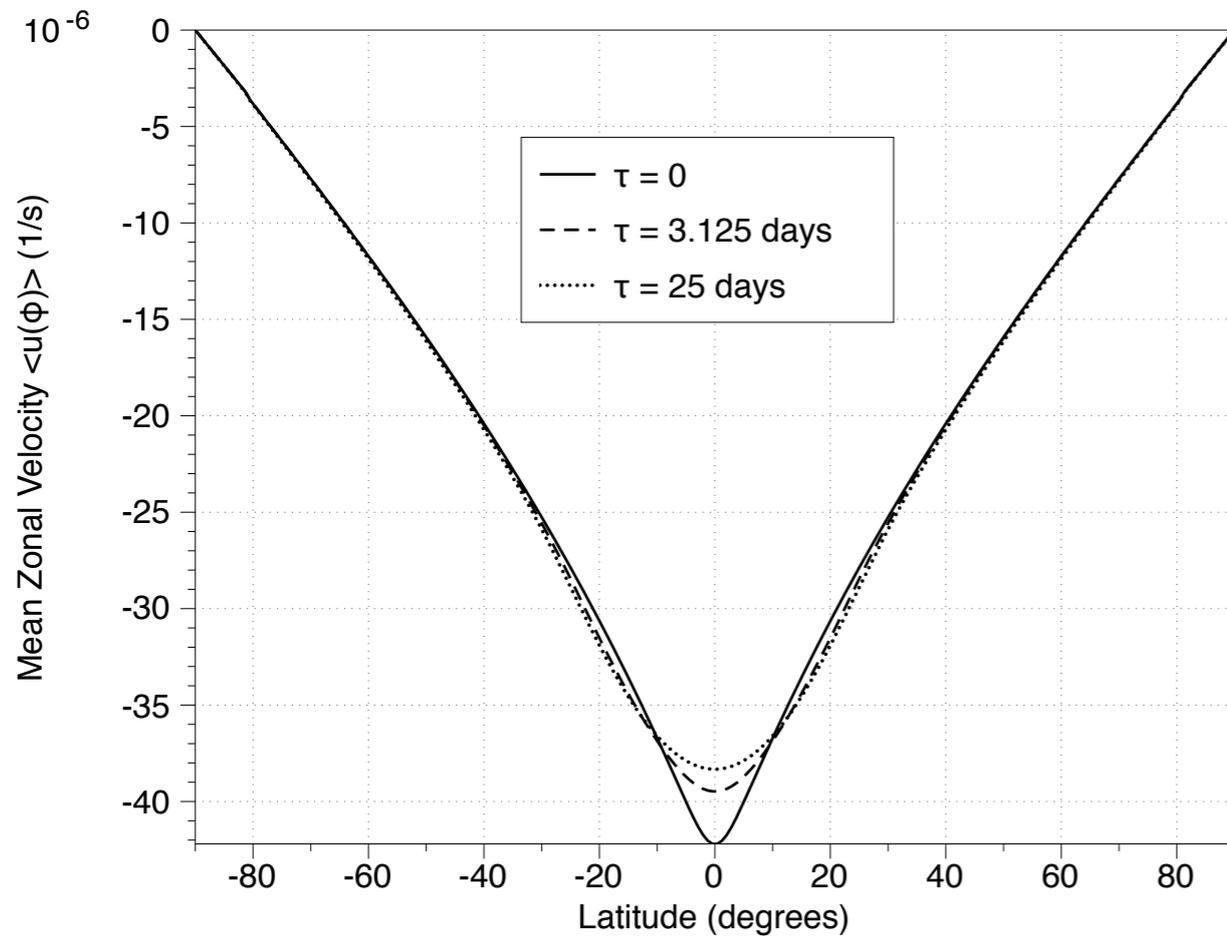


A. Sanchez-Lavega *et al.* Nature **451**, 437 (2008)





A. Sanchez-Lavega *et al.* Nature **451**, 437 (2008)



$$\frac{\partial q}{\partial t} + J[\psi, q] = \frac{q_{\text{jet}} - q}{\tau}$$

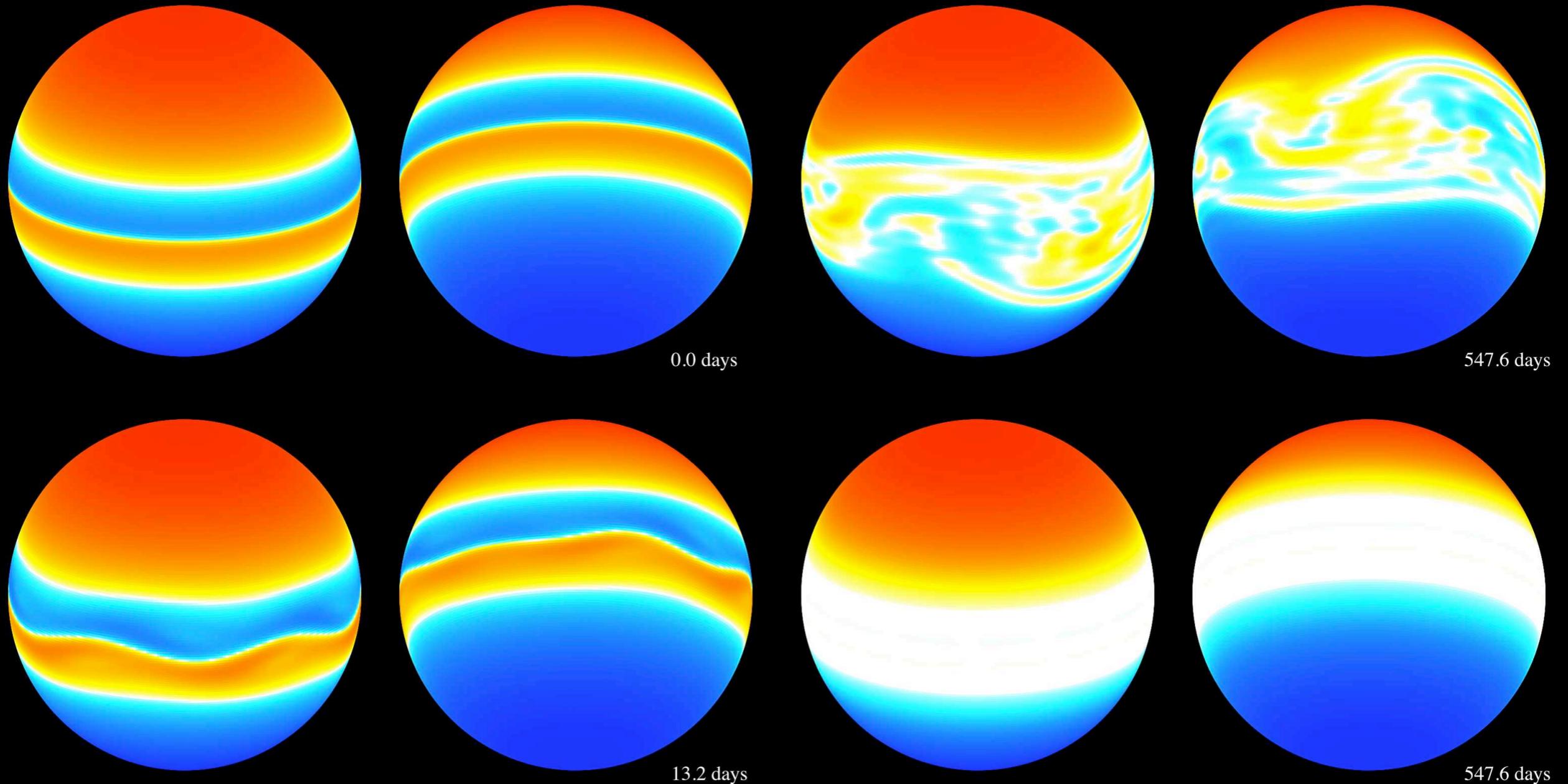
# Direct Numerical Simulation of Jet

jet relaxation time = 25 days

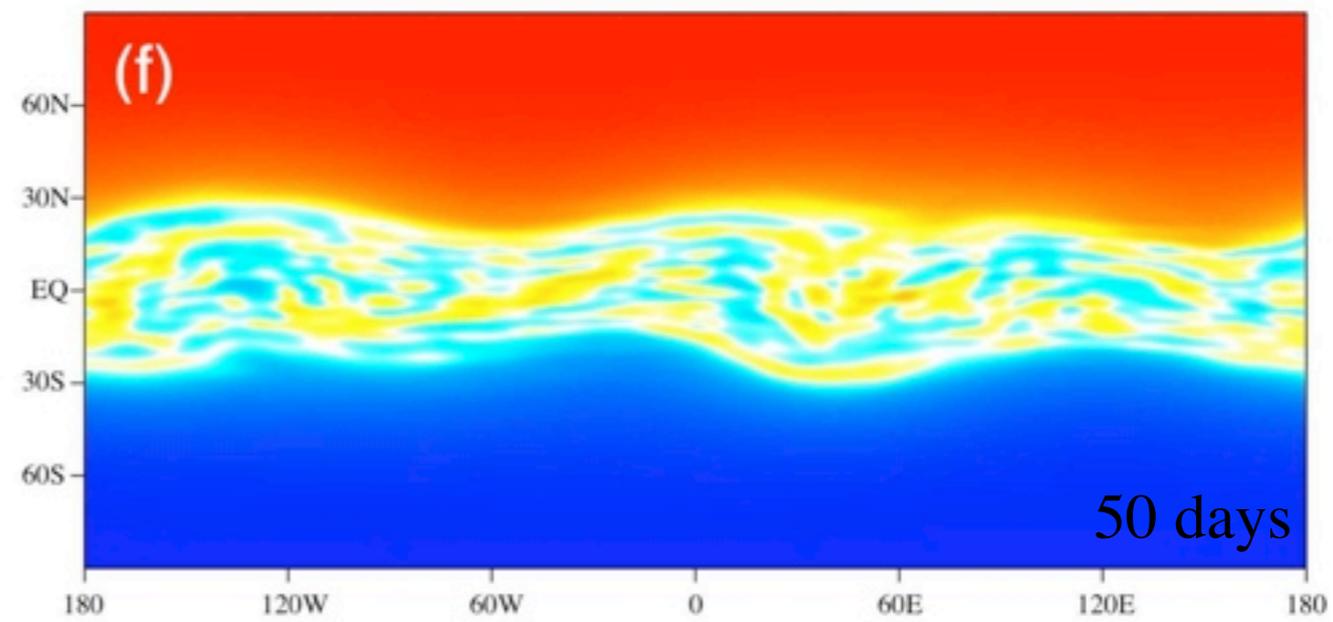
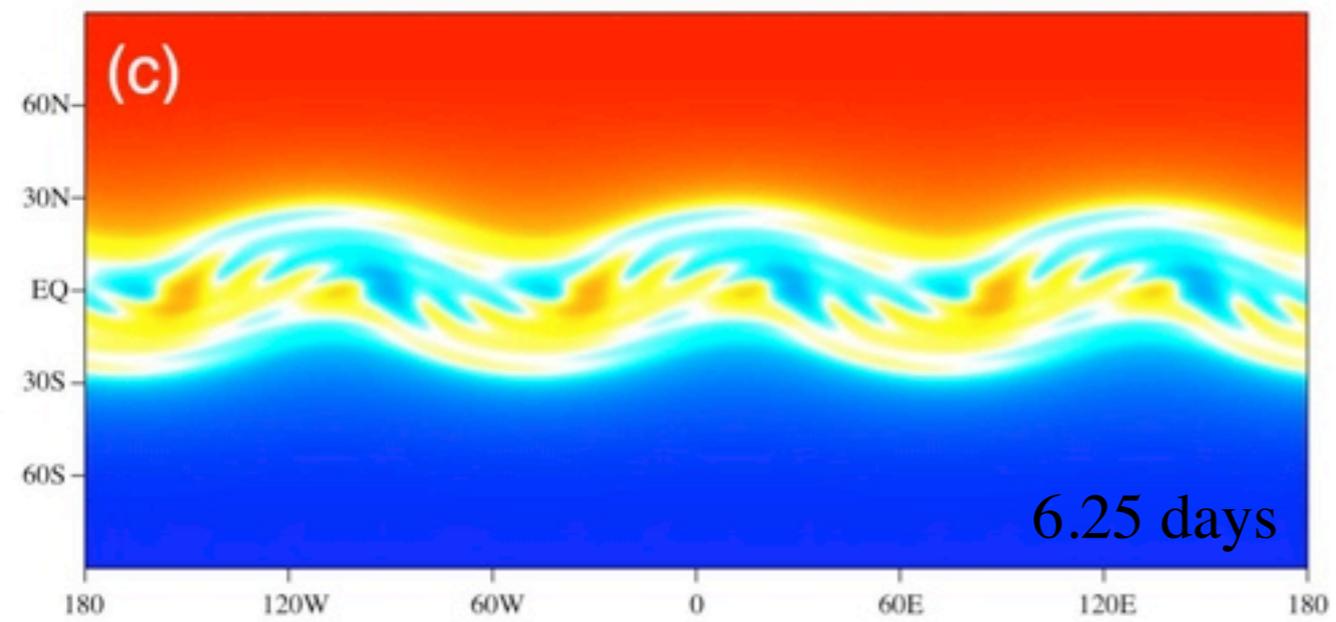
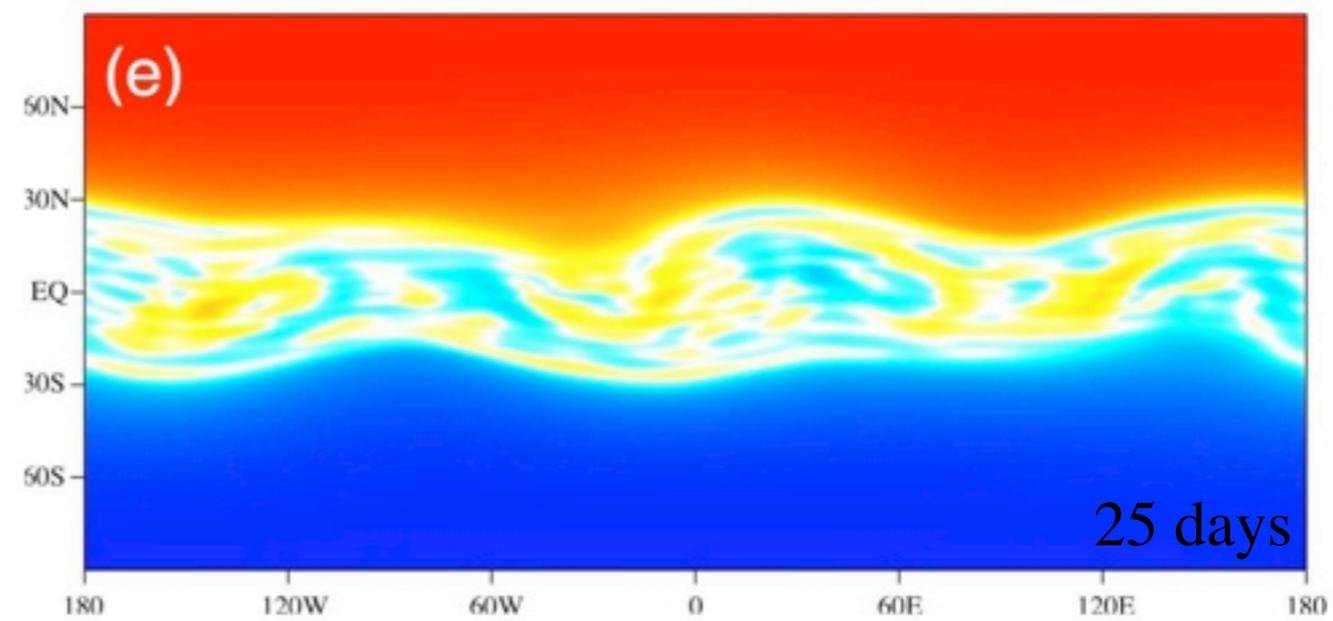
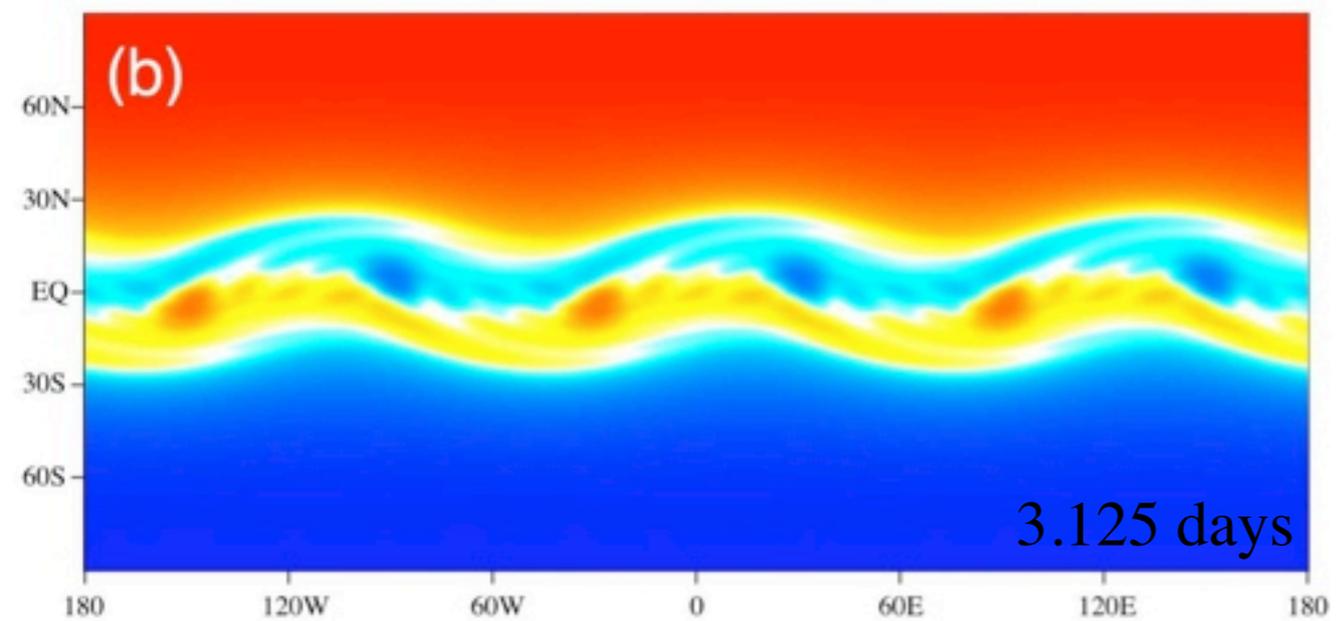
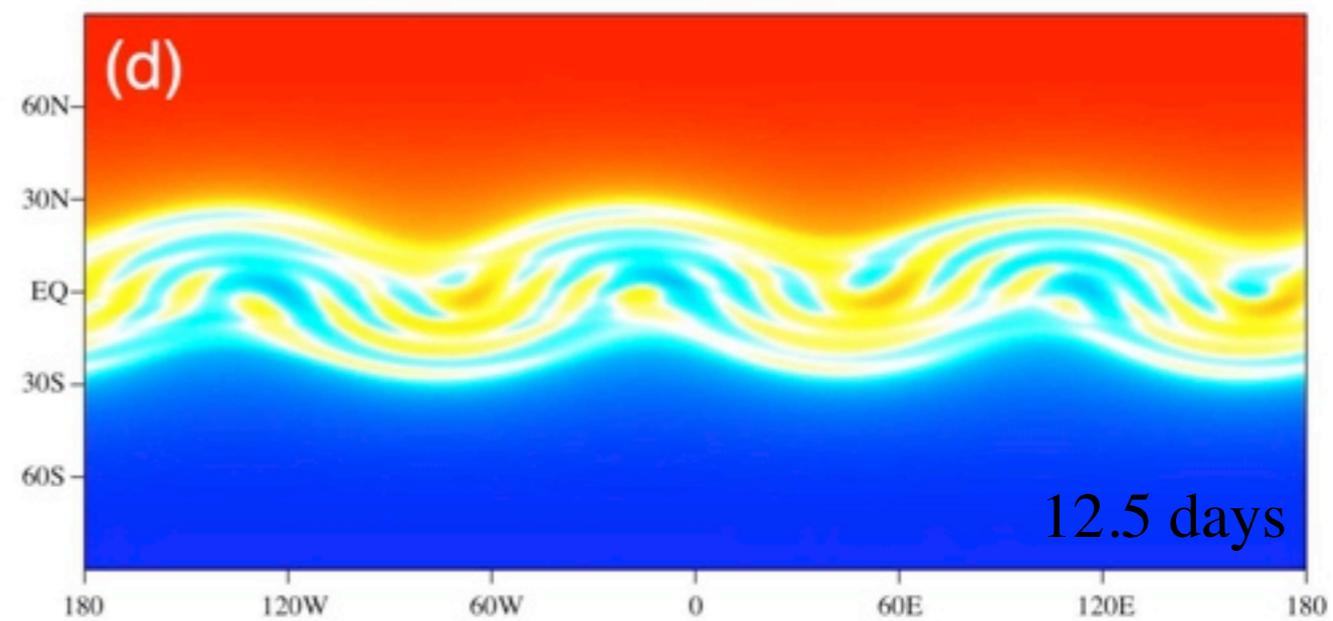
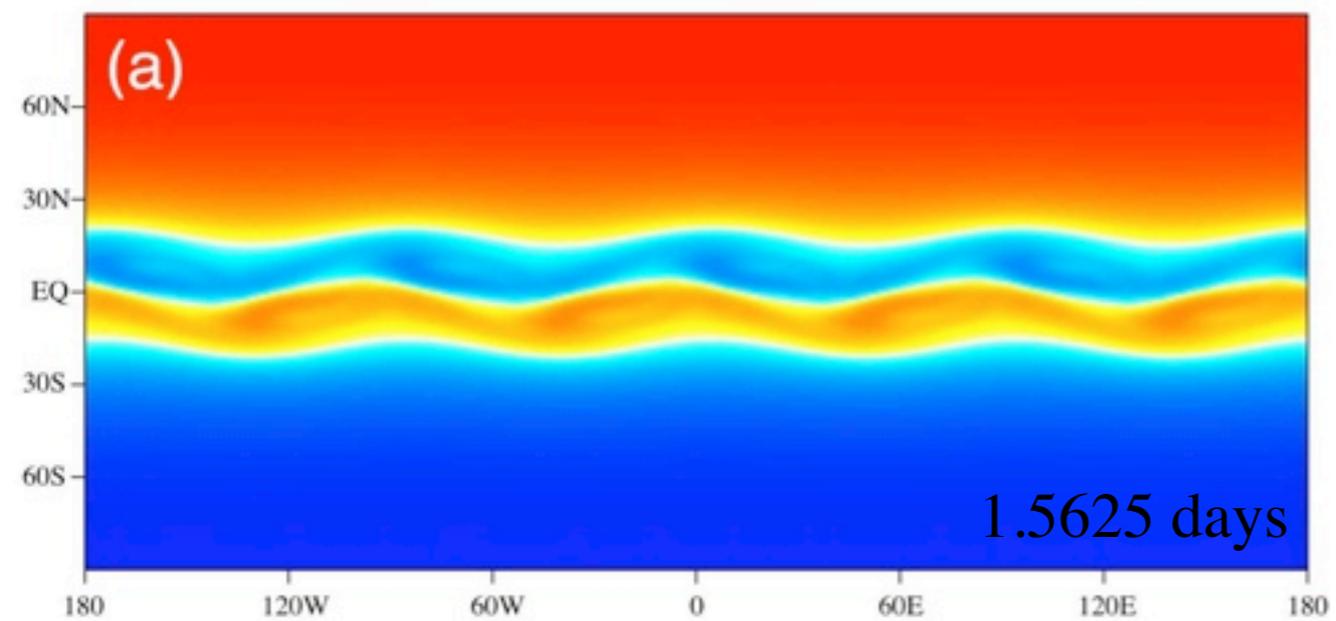
J. B. M, E. Conover, and T. Schneider, *J. Atmos. Sci.* **65**, 1955 (2008)

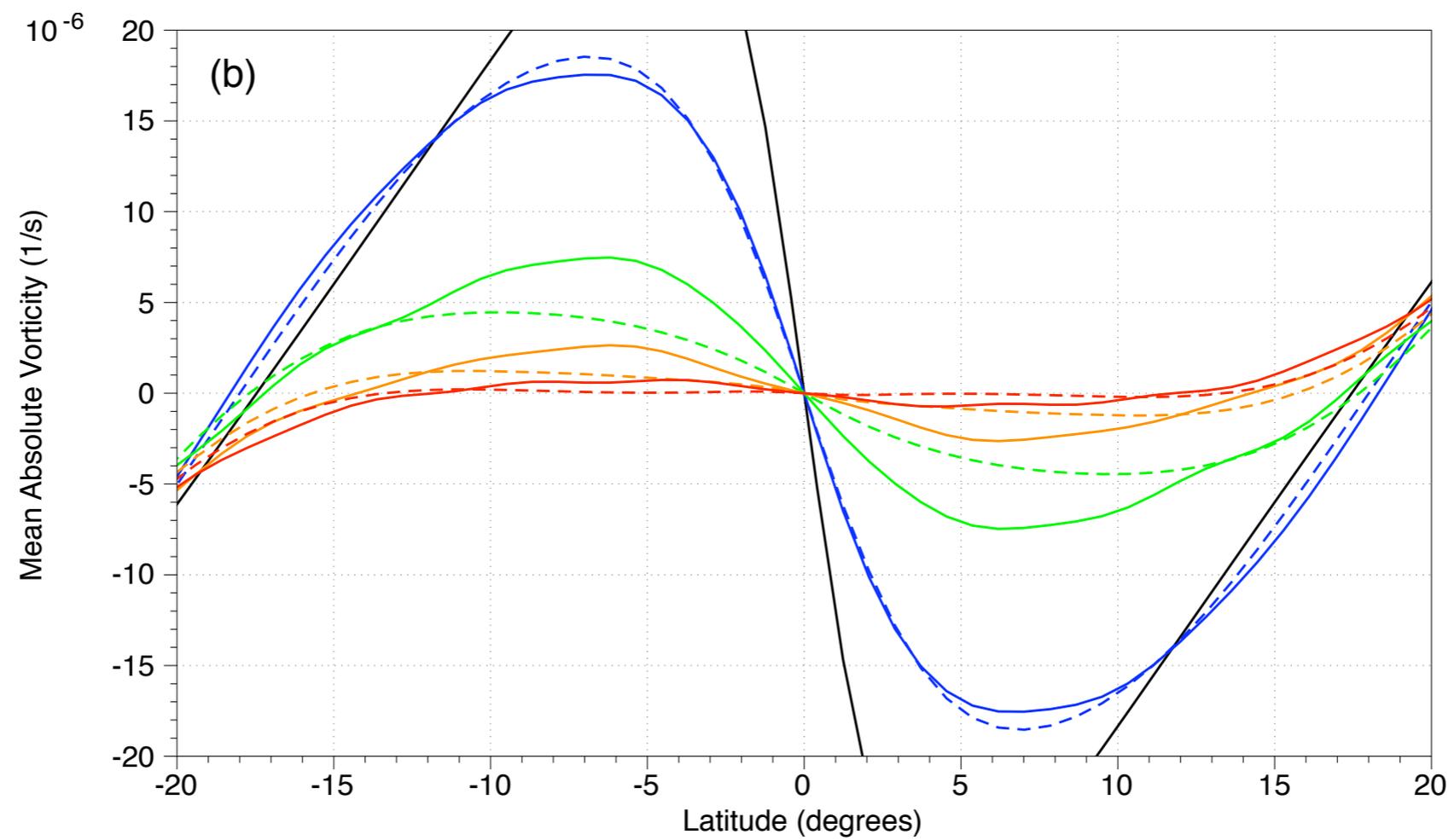
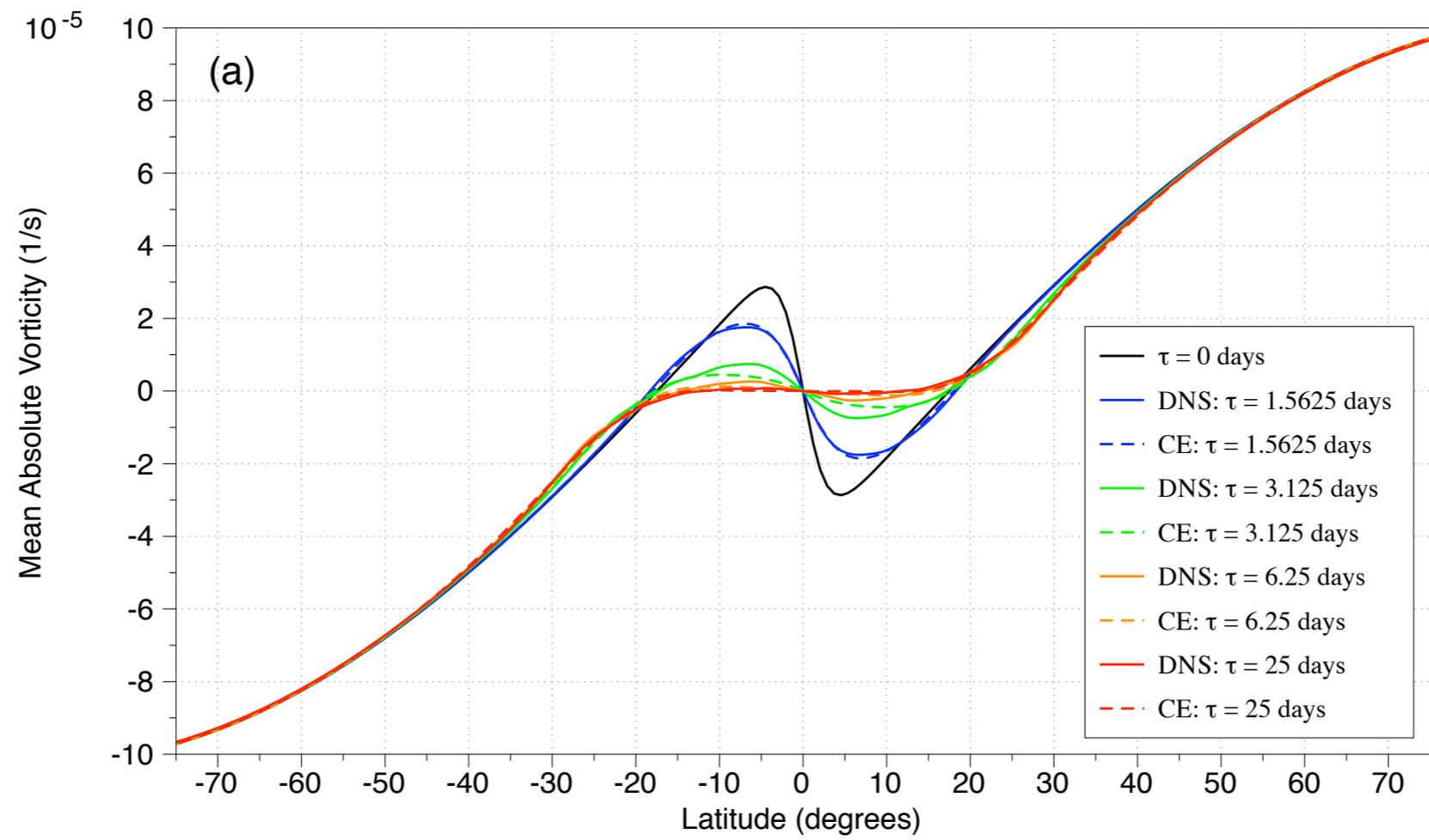
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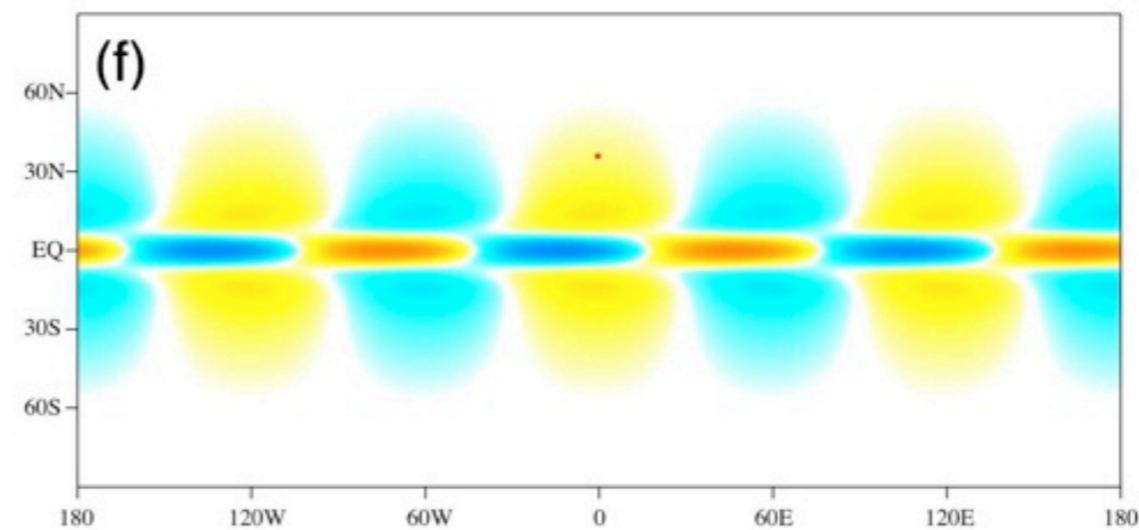
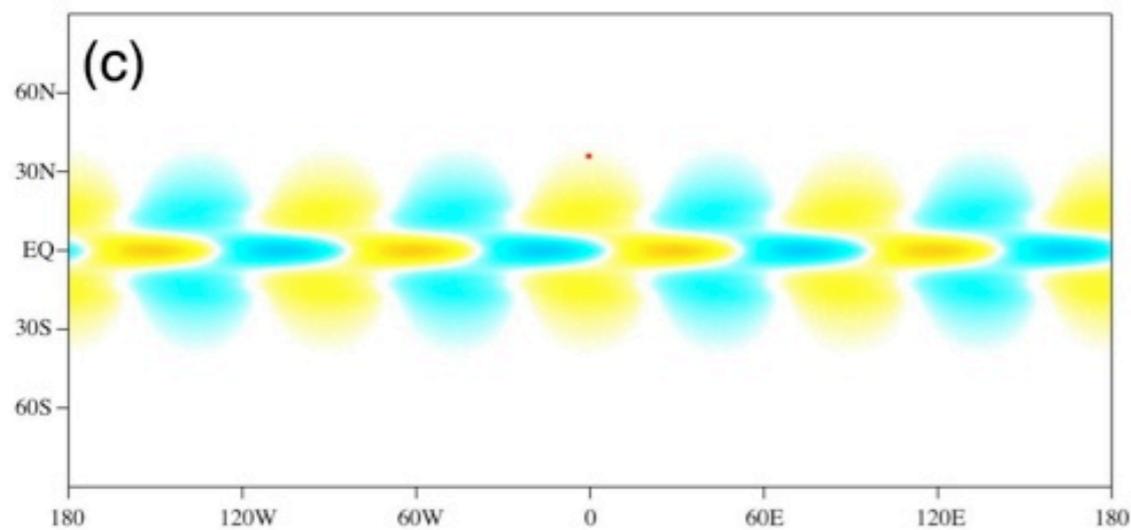
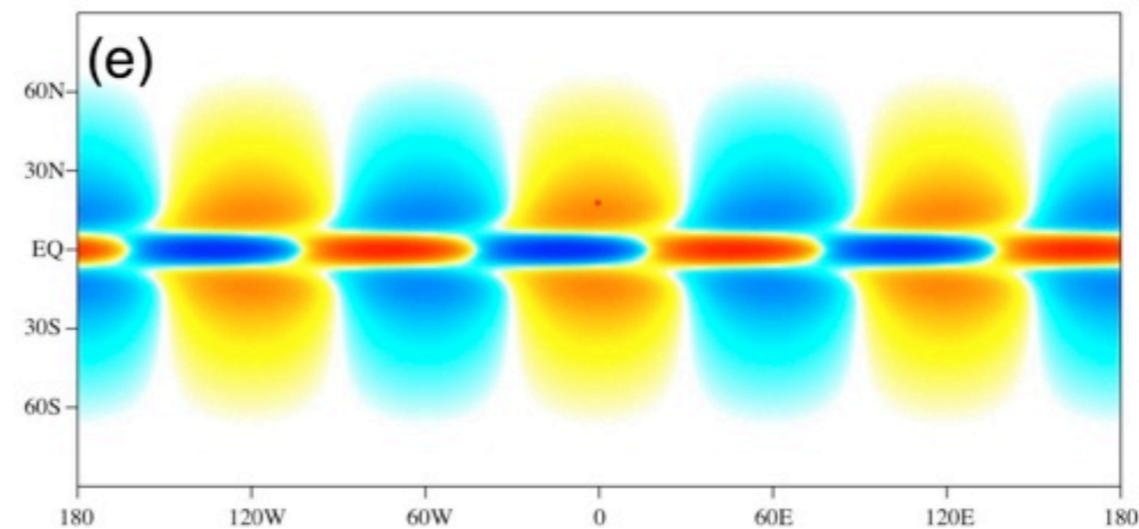
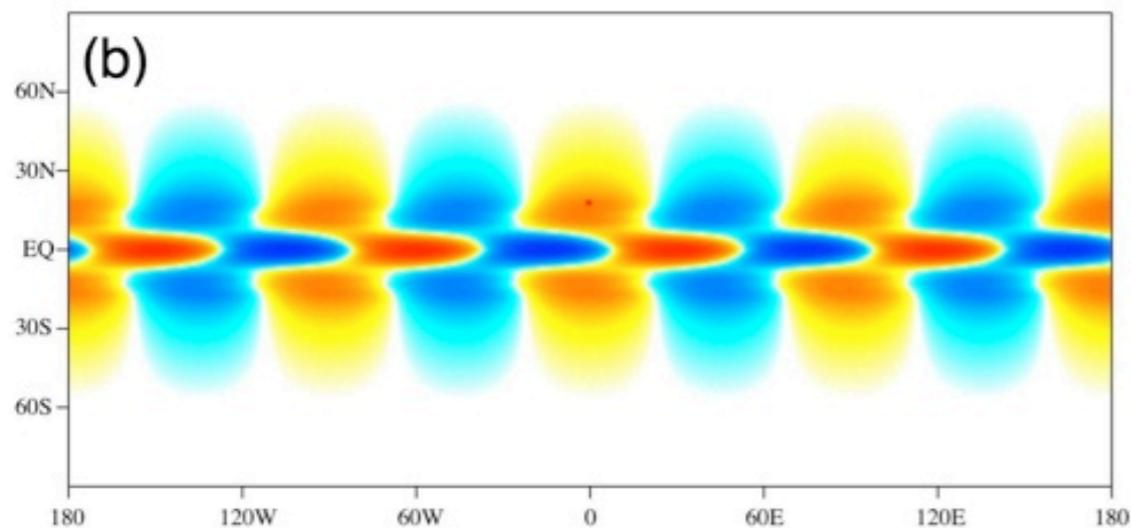
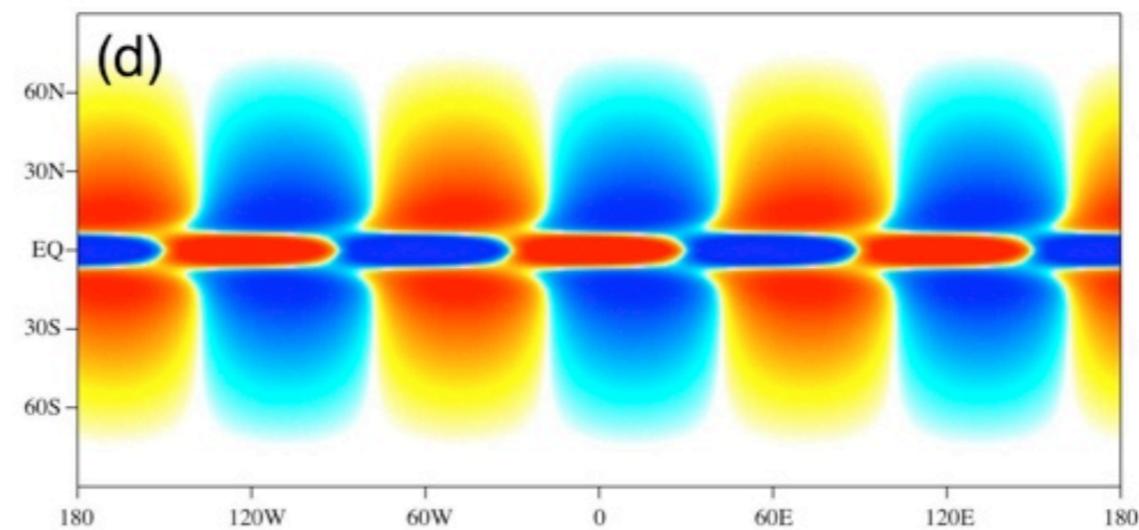
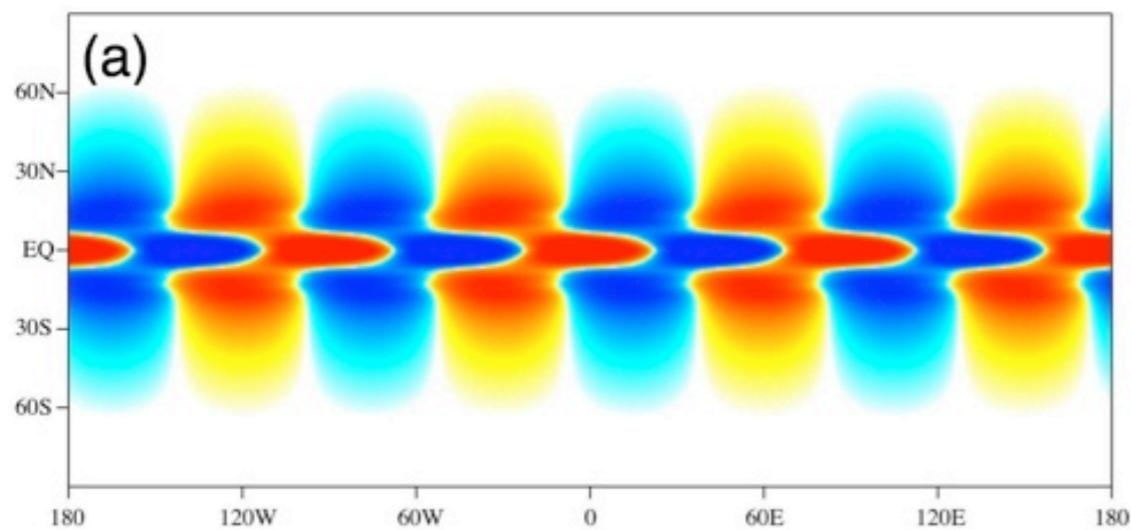


J. B. M, E. Conover, and T. Schneider, *J. Atmos. Sci.* **65**, 1955 (2008)

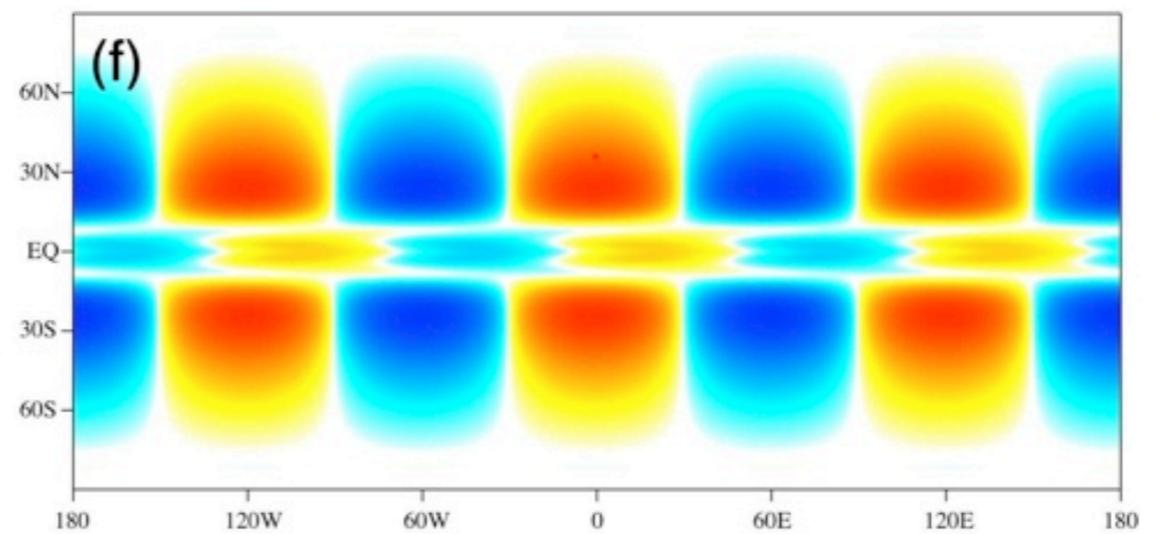
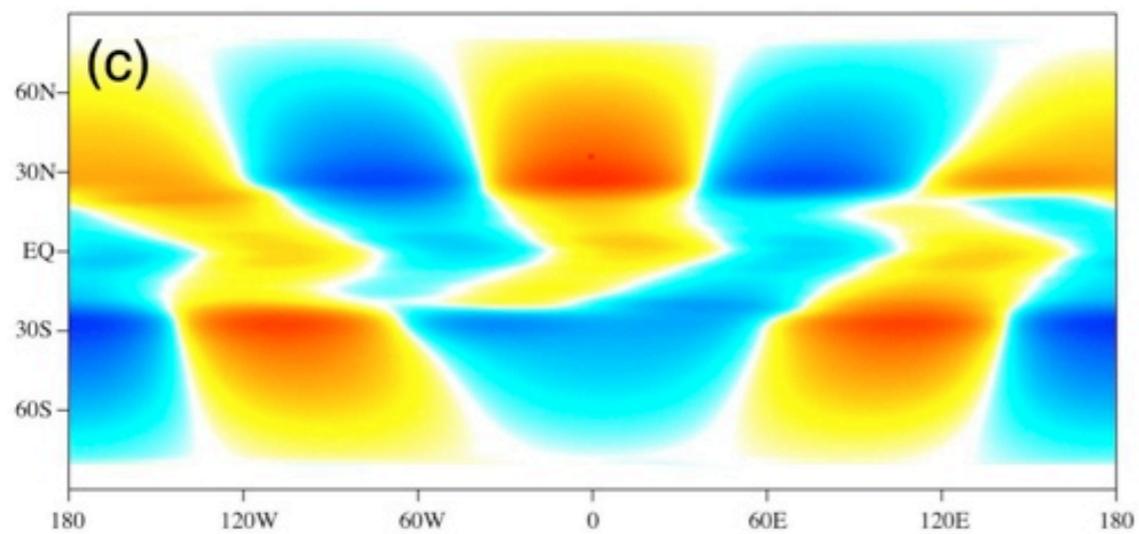
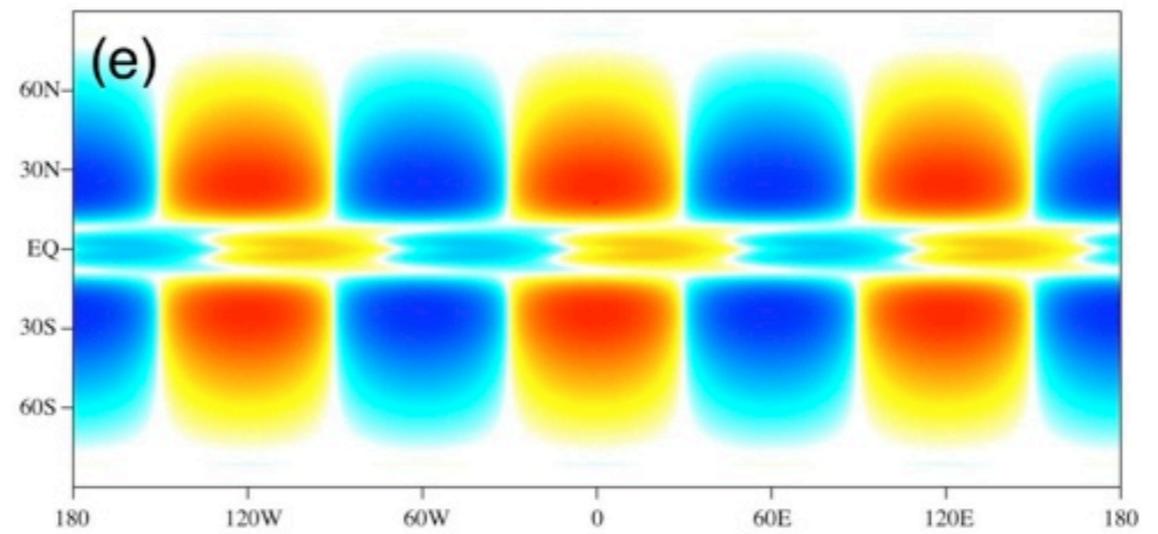
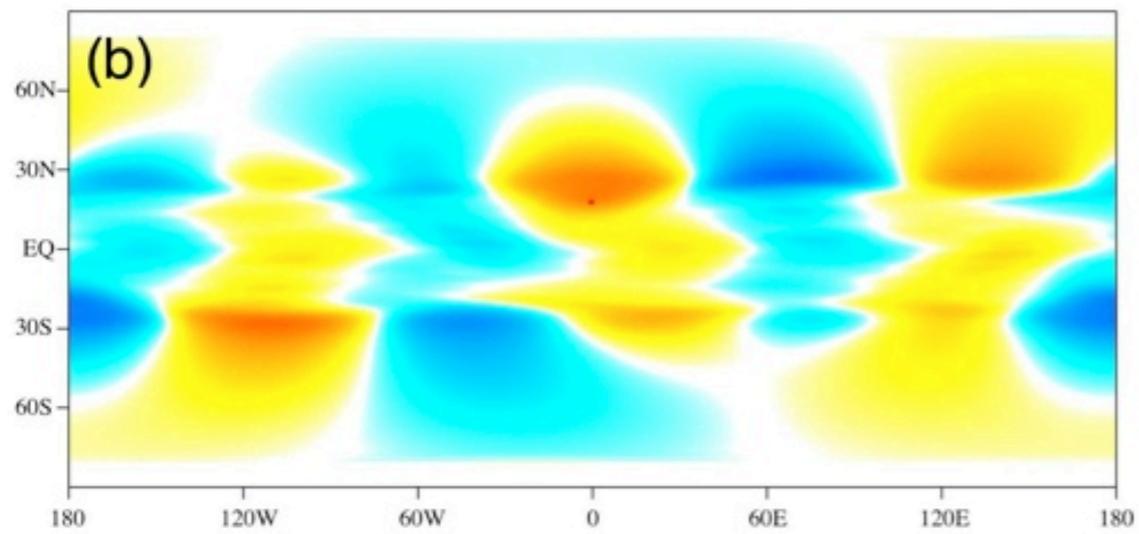
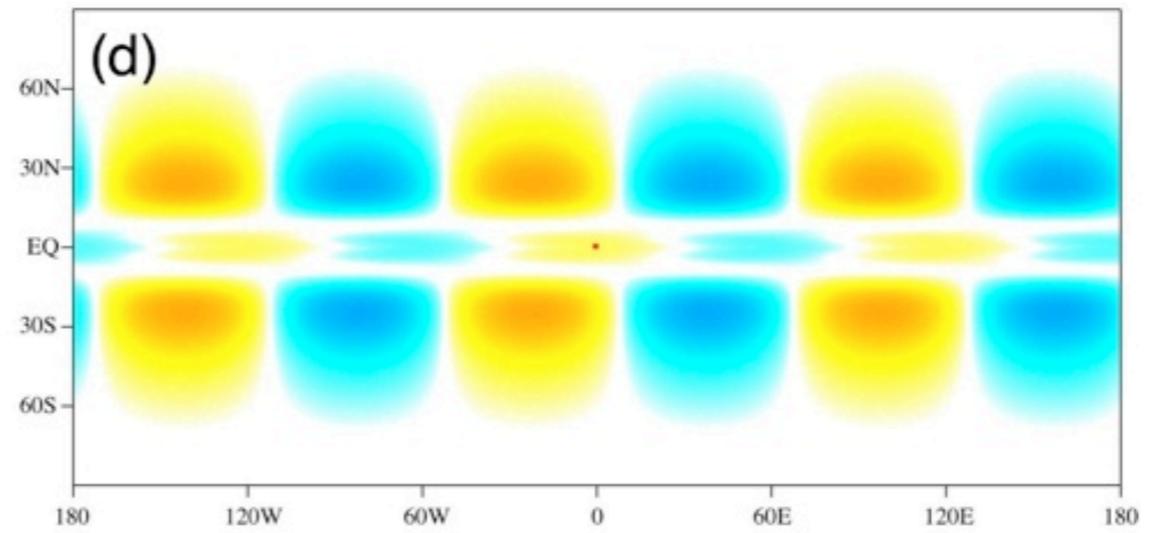
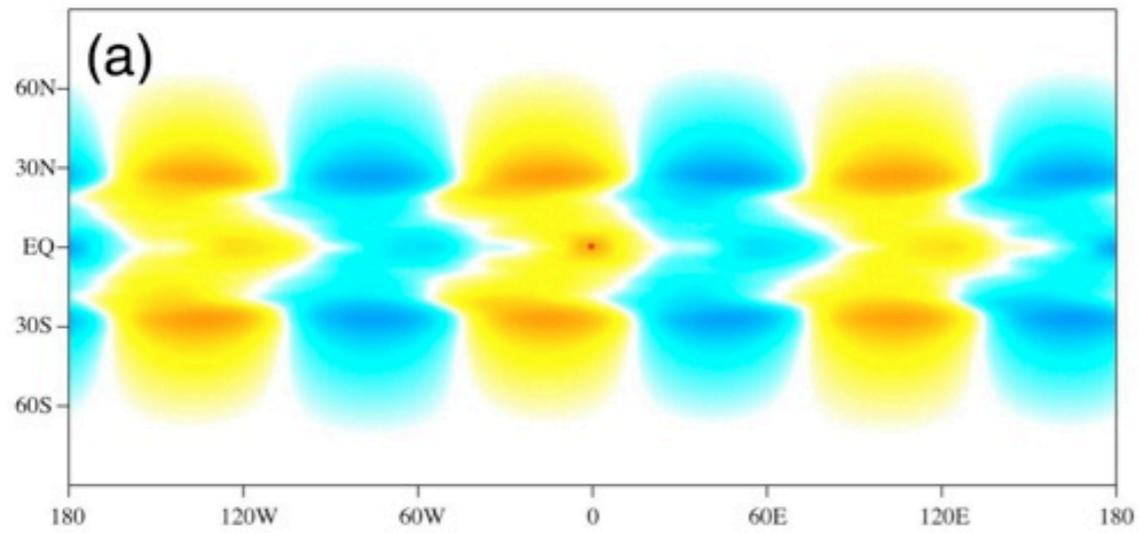




# 2nd Cumulant = 2-point Correlation Function

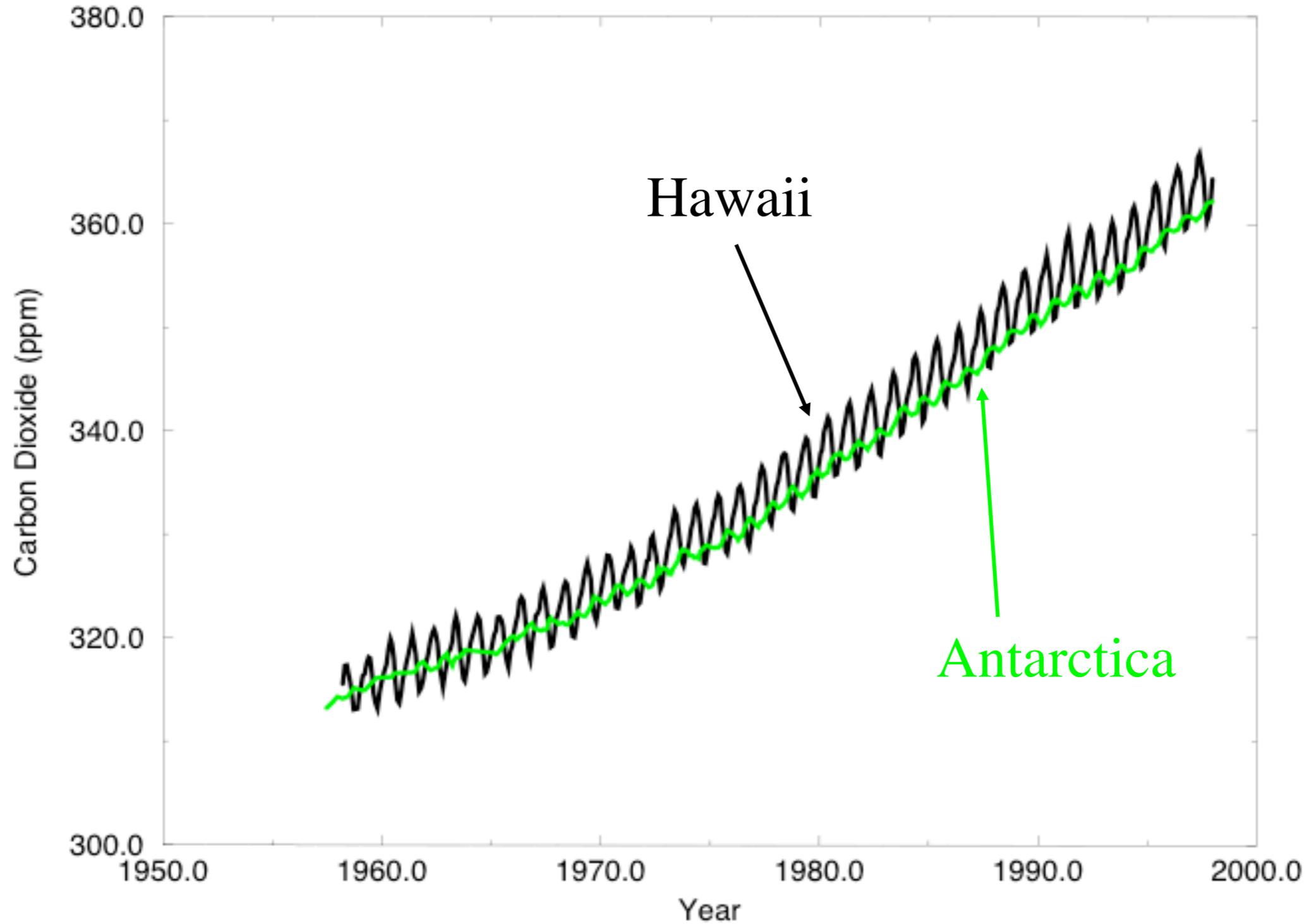


# 25 days



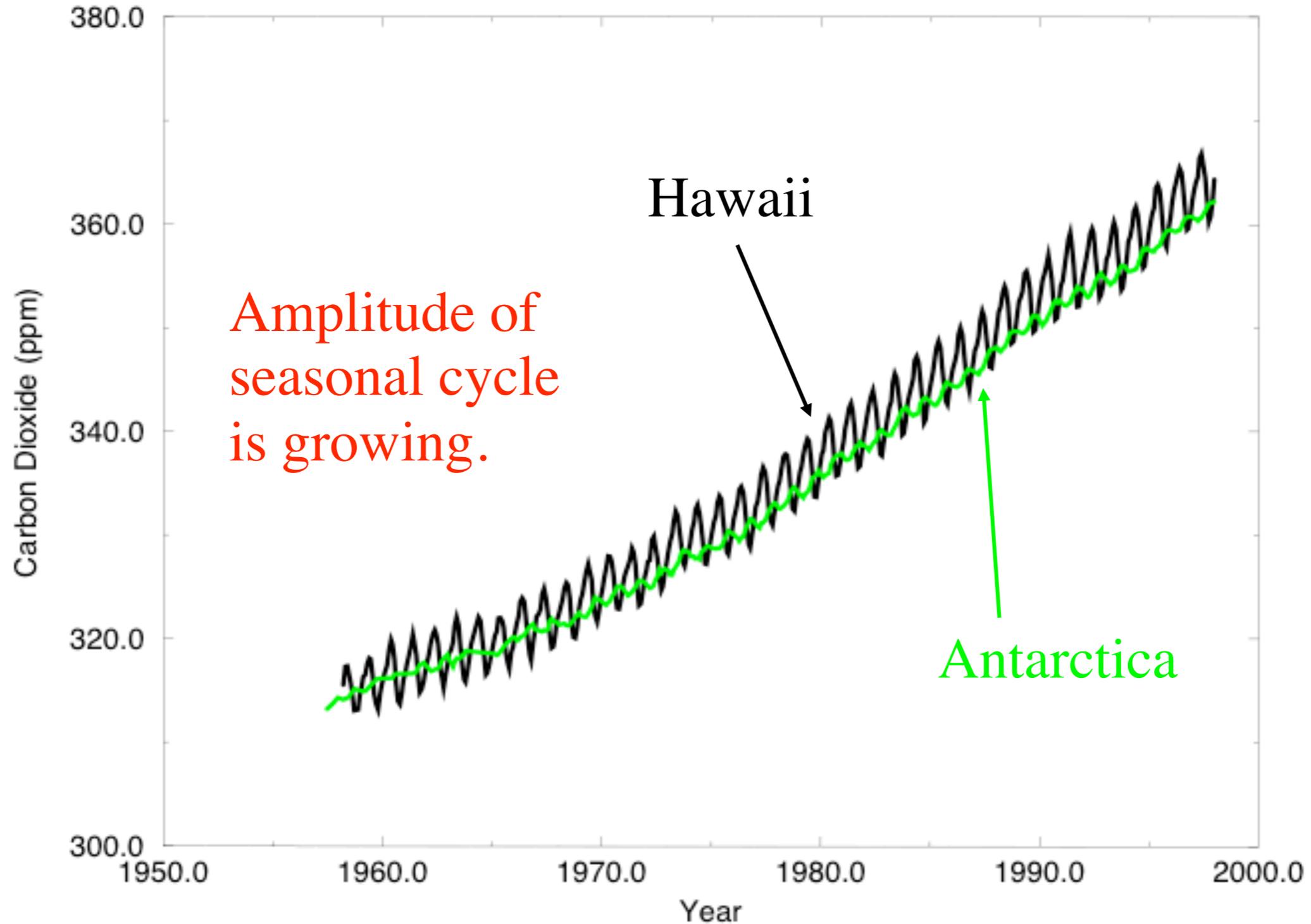
# Ecosystems & Feedbacks

## Mauna Loa and Antarctic Carbon Dioxide

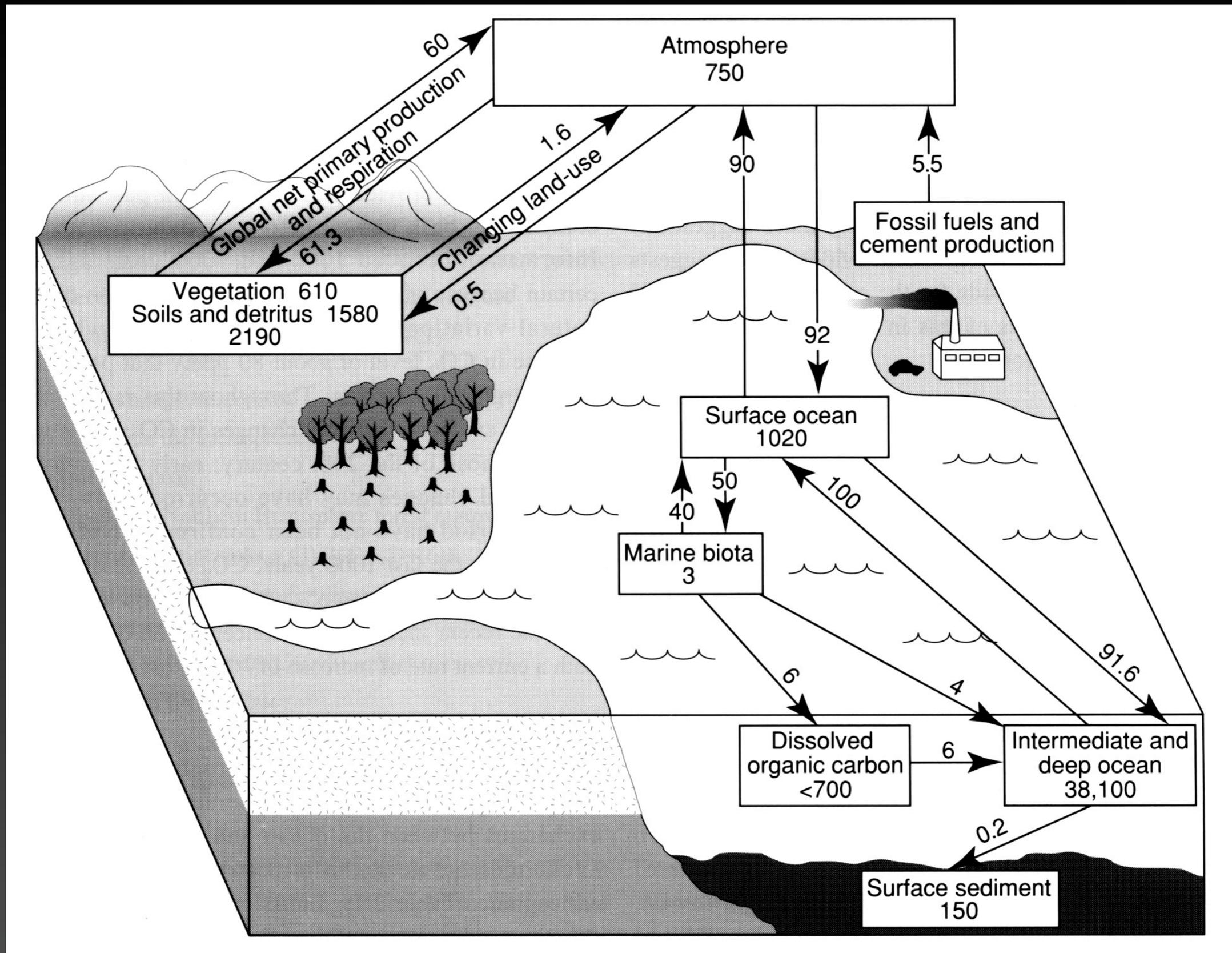


# Ecosystems & Feedbacks

## Mauna Loa and Antarctic Carbon Dioxide

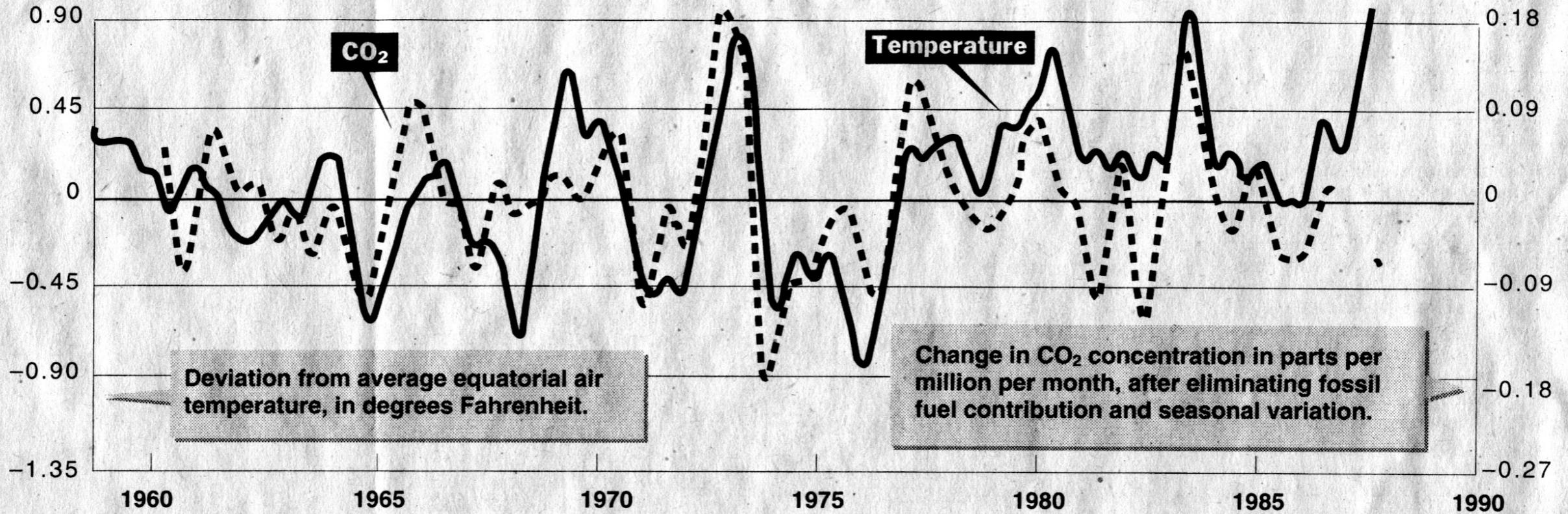


# Vast Reservoirs of Carbon & Enormous Fluxes



## Temperature and CO<sub>2</sub>, Moving in Tandem

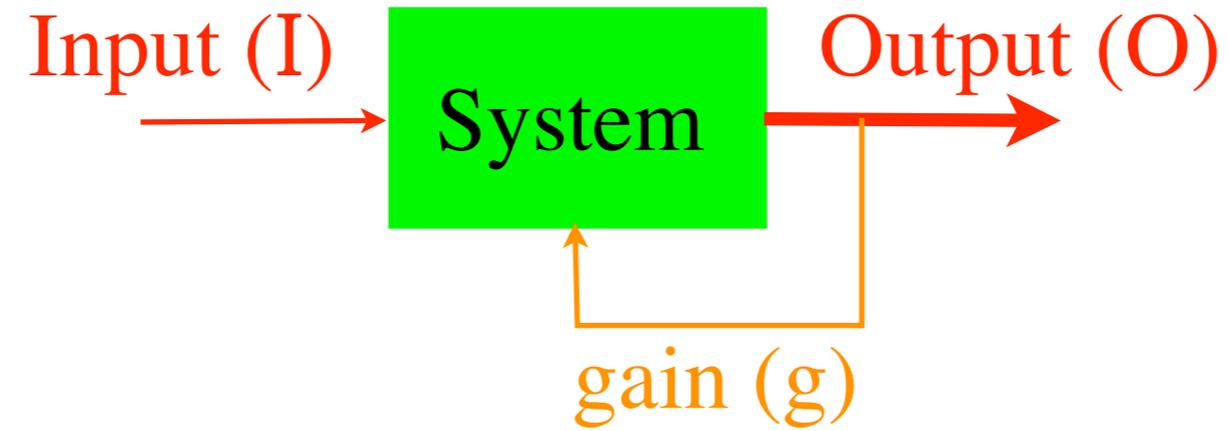
Research results indicate a strong correlation between variations in temperature and variations in atmospheric carbon dioxide concentration. Some scientists say that the temperature increases often precede the carbon dioxide rises, meaning that warming could build on itself to create further warming.



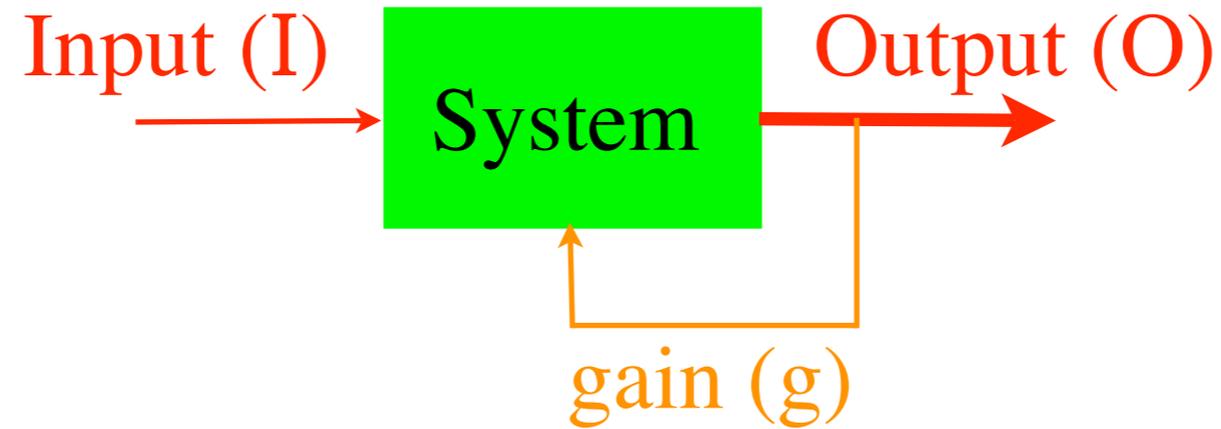
Sources: Dr. Michael Oppenheimer, and Dr. J.B. Marston

J. B. Marston, M. Oppenheimer, R. M. Fujita, and S. R. Gaffin, "CO<sub>2</sub> and temperature" *Nature* **349**, 573 (1991).

# Physics of Feedbacks



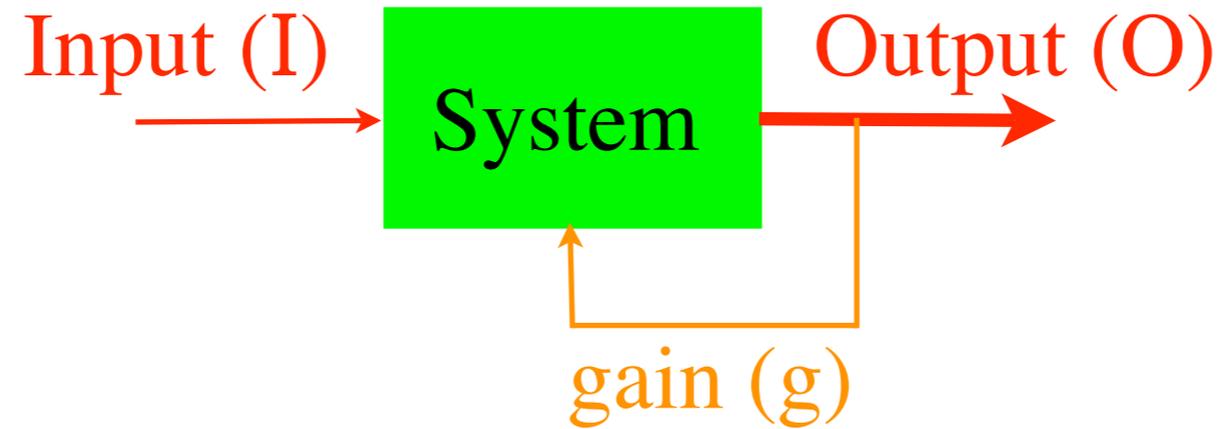
# Physics of Feedbacks



$$\begin{aligned} \mathcal{O} &= \mathcal{I} + g\mathcal{I} + gg\mathcal{I} + \dots \\ &= \frac{\mathcal{I}}{1 - g} \quad \text{if } g < 1 \end{aligned}$$

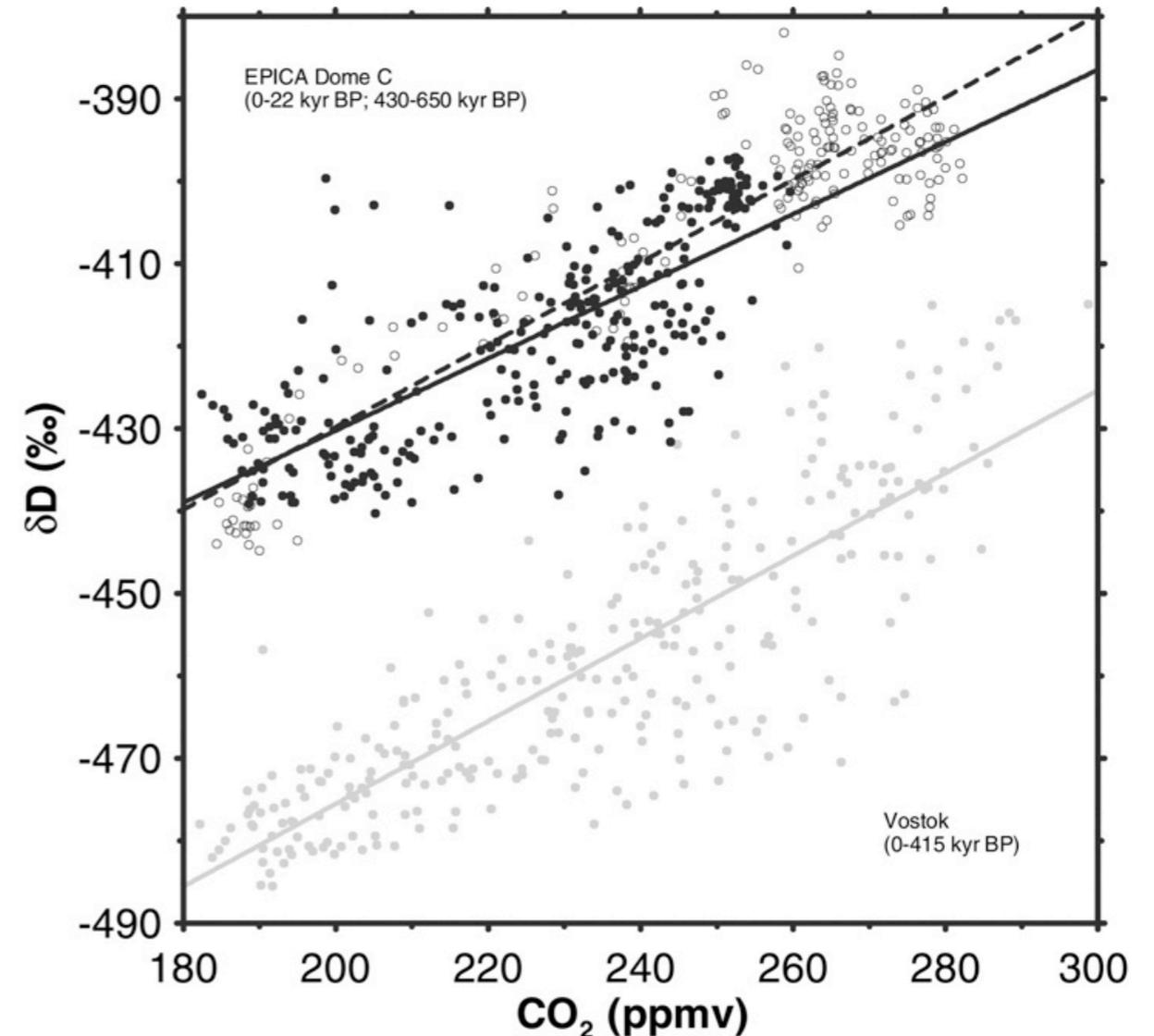
$$g = \sum_i \left( \frac{\partial T}{\partial p_i} \right) \left( \frac{\partial p_i}{\partial T} \right)$$

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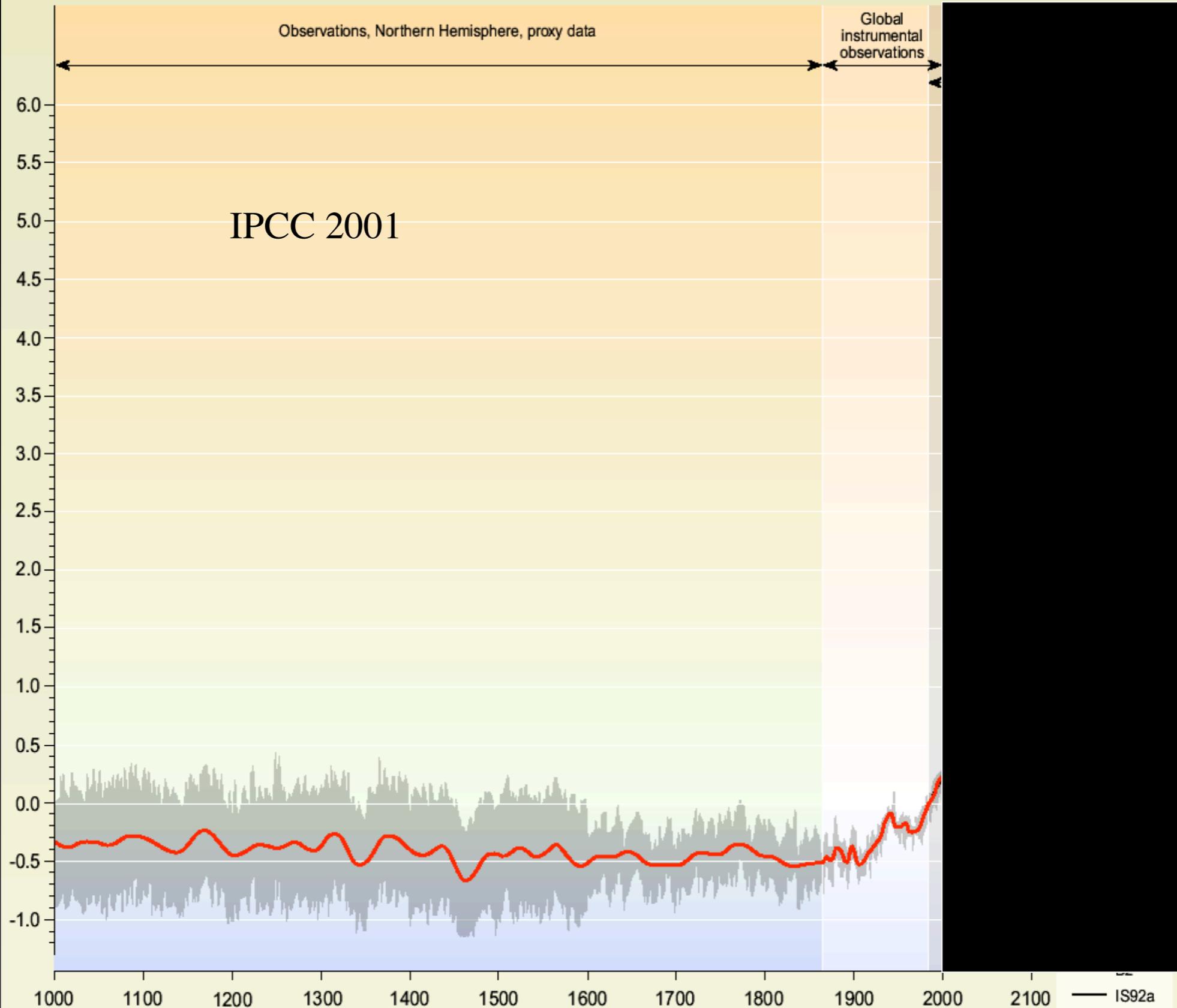
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$$\Delta T \approx 1^\circ C / (1 - 0.71) \approx 3.4^\circ C$$

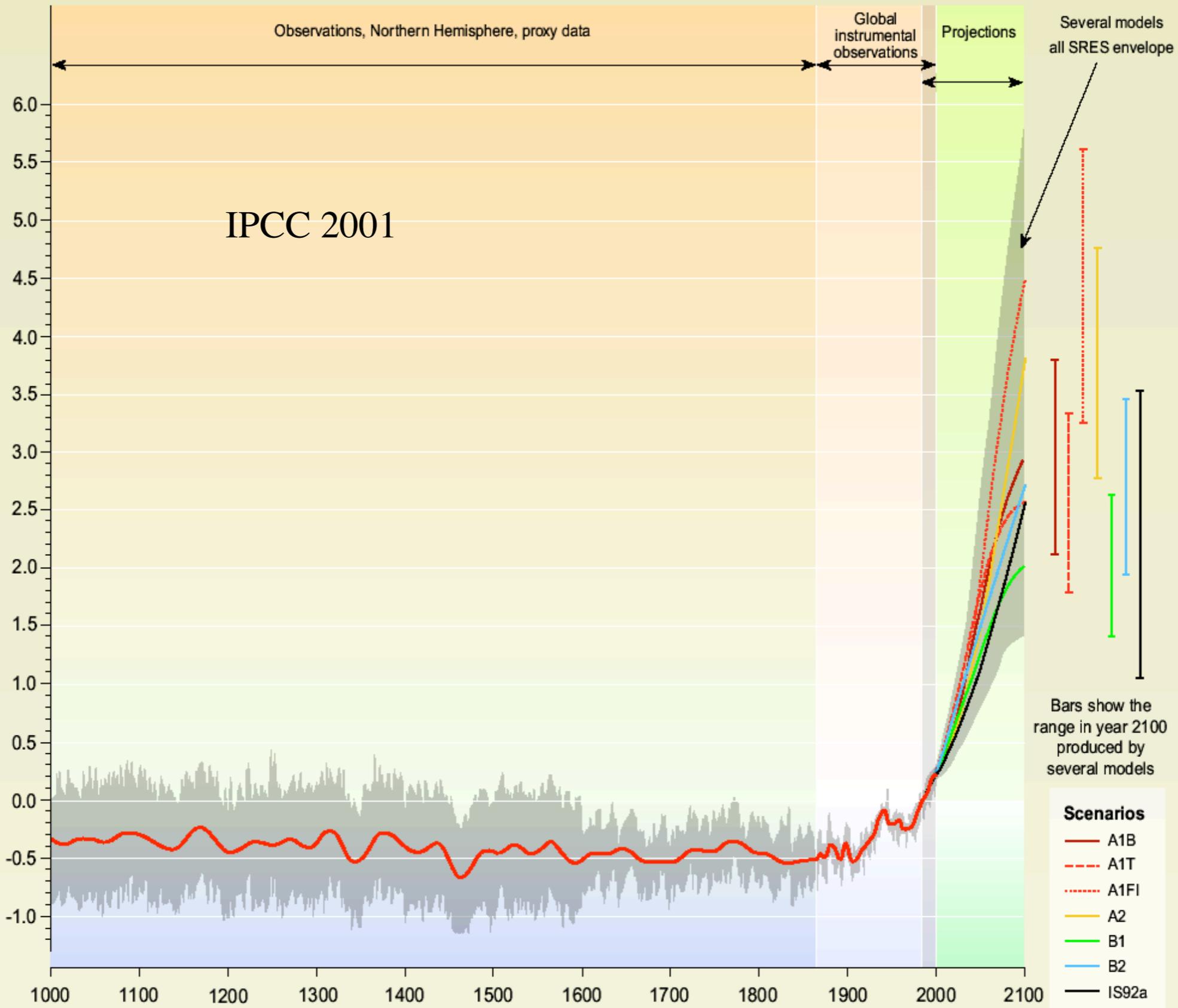
# Variations of the Earth's surface temperature: years 1000 to 2100

Departures in temperature in °C (from the 1990 value)



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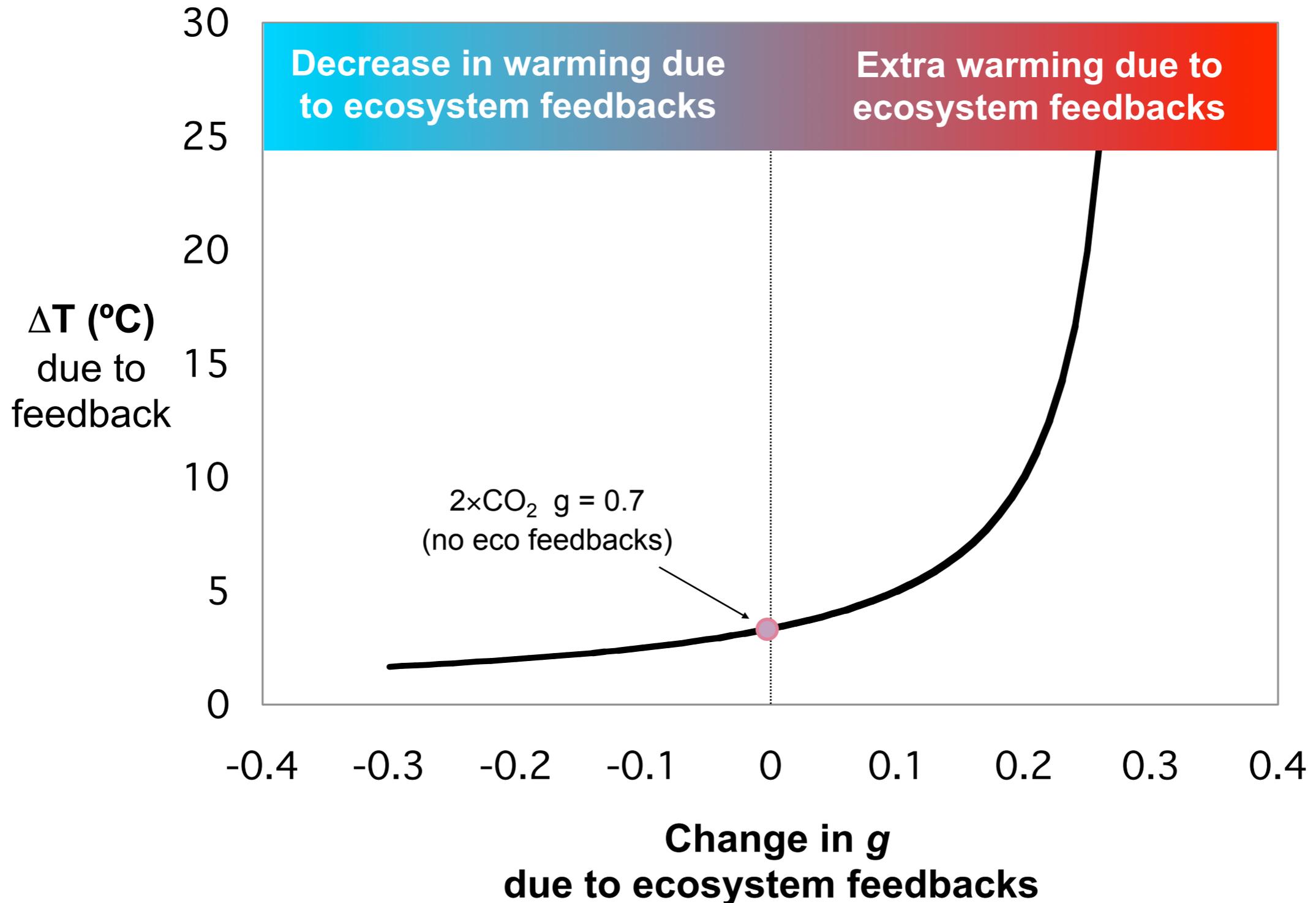
Departures in temperature in °C (from the 1990 value)



# Global Carbon Cycle

*Small change in  $g$  causes large  $\Delta T$*

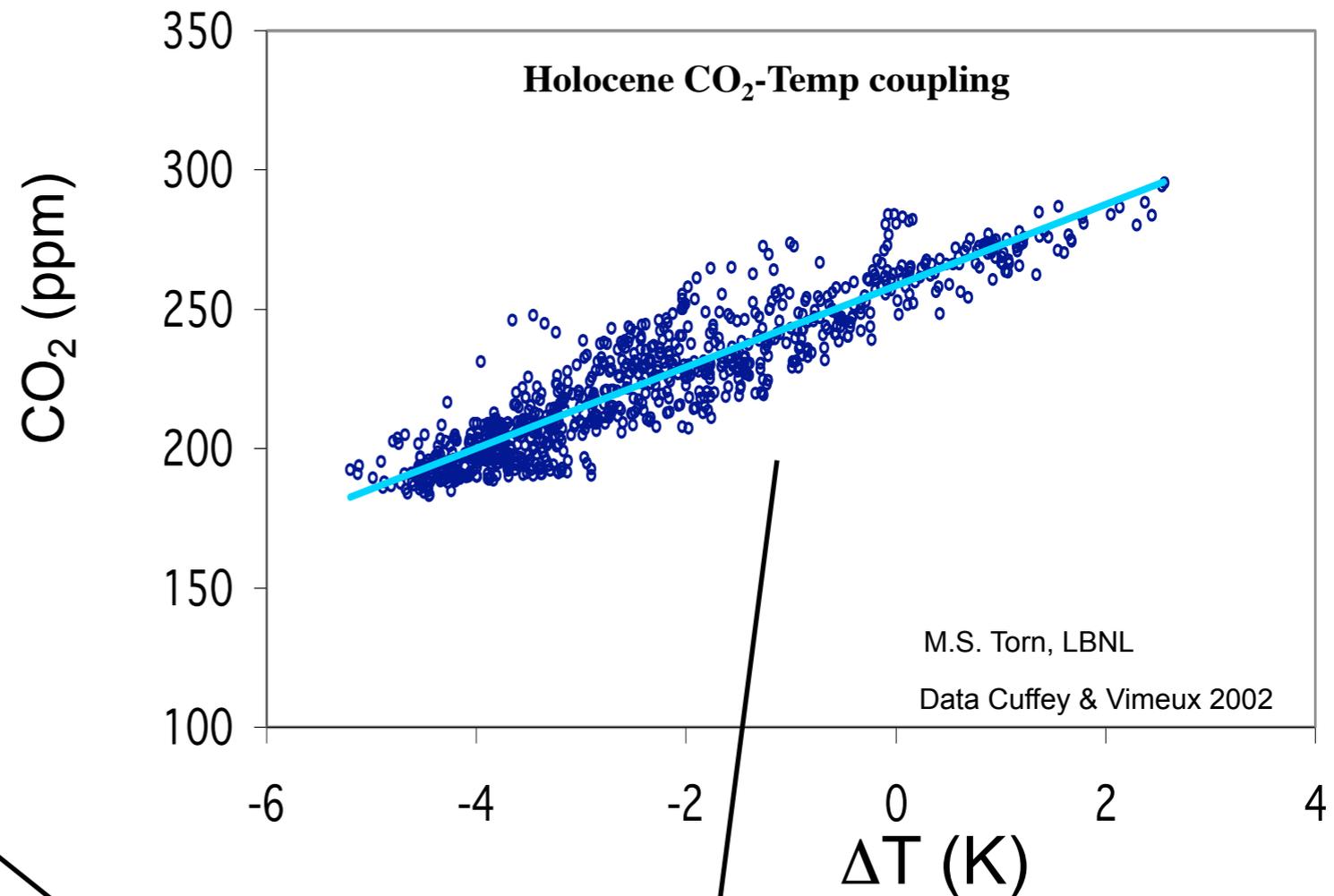
## Asymmetries



# An estimate of the contribution to g from Vostok core data:

Torn and Harte, GRL33, L10703 (2006)

General  
Circulation  
Models

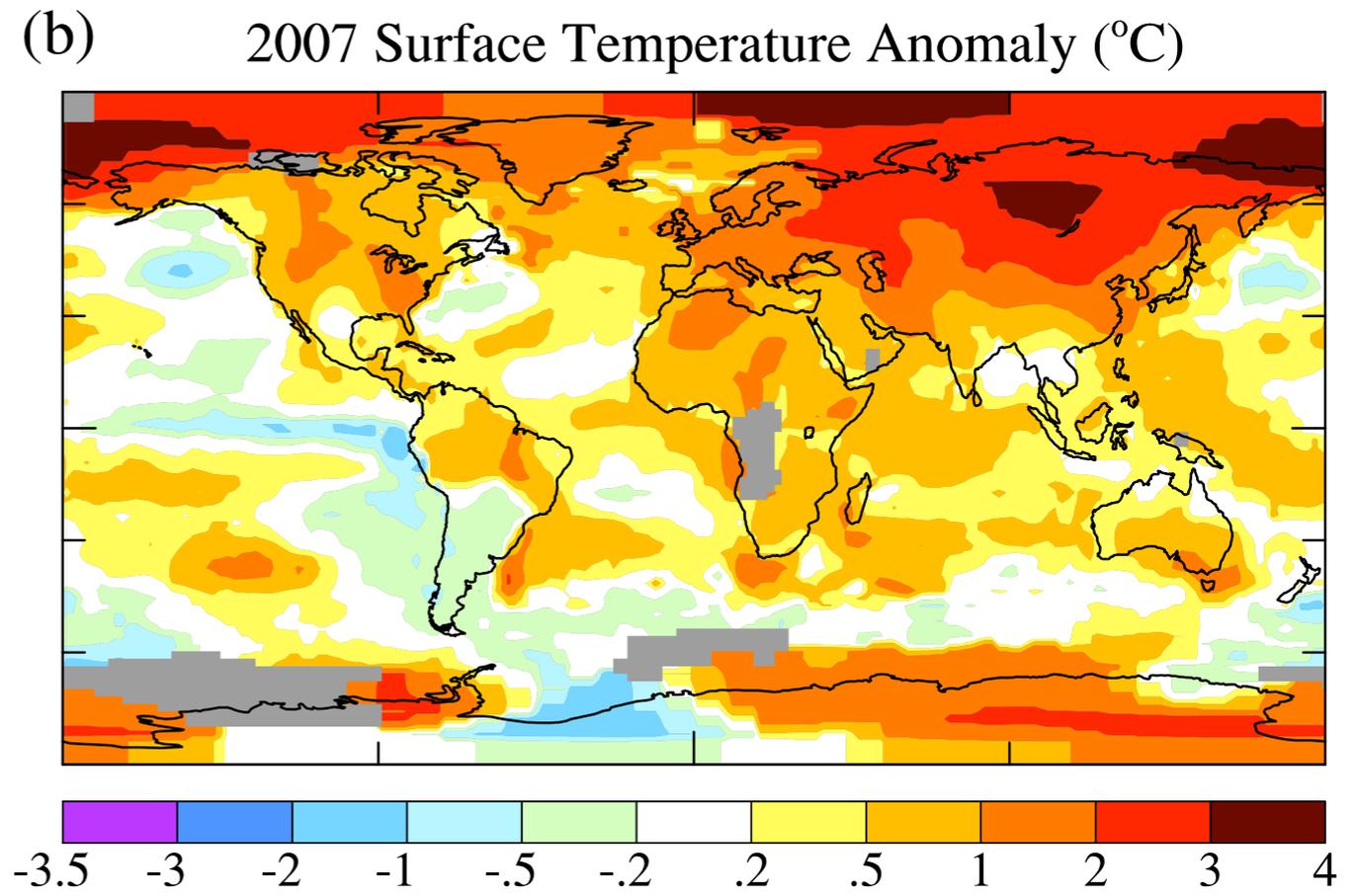
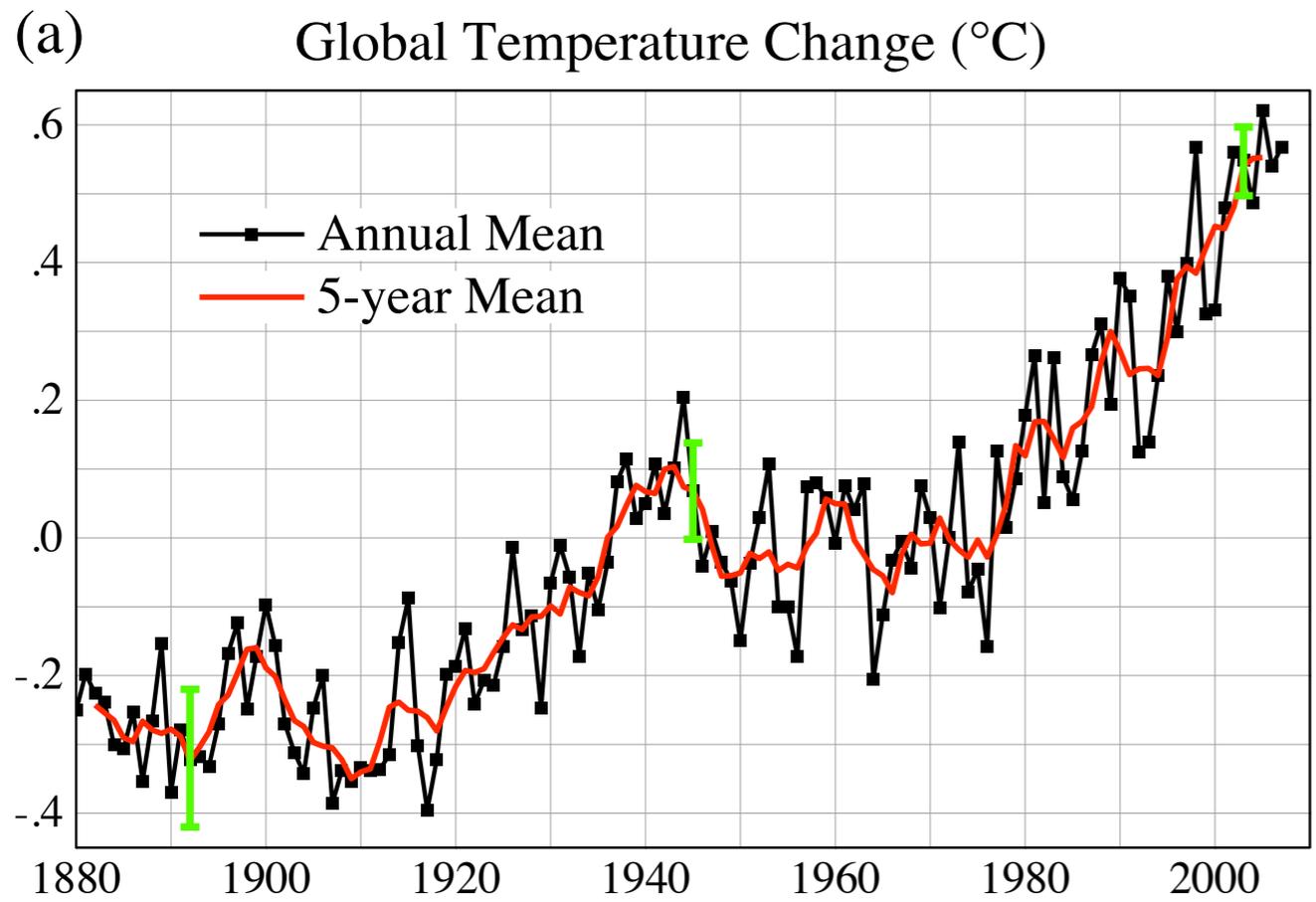


$$g_{CO_2} = \frac{\partial T}{\partial [CO_2]} \cdot \frac{\partial [CO_2]}{\partial T} = \frac{1^\circ C}{275 ppmv} \cdot \frac{14.6 ppmv}{1^\circ C} = 0.053$$

$$1^\circ C / (1 - .71) = 3.4^\circ C$$

$$1^\circ C / (1 - .71 - .05) = 4.2^\circ C$$

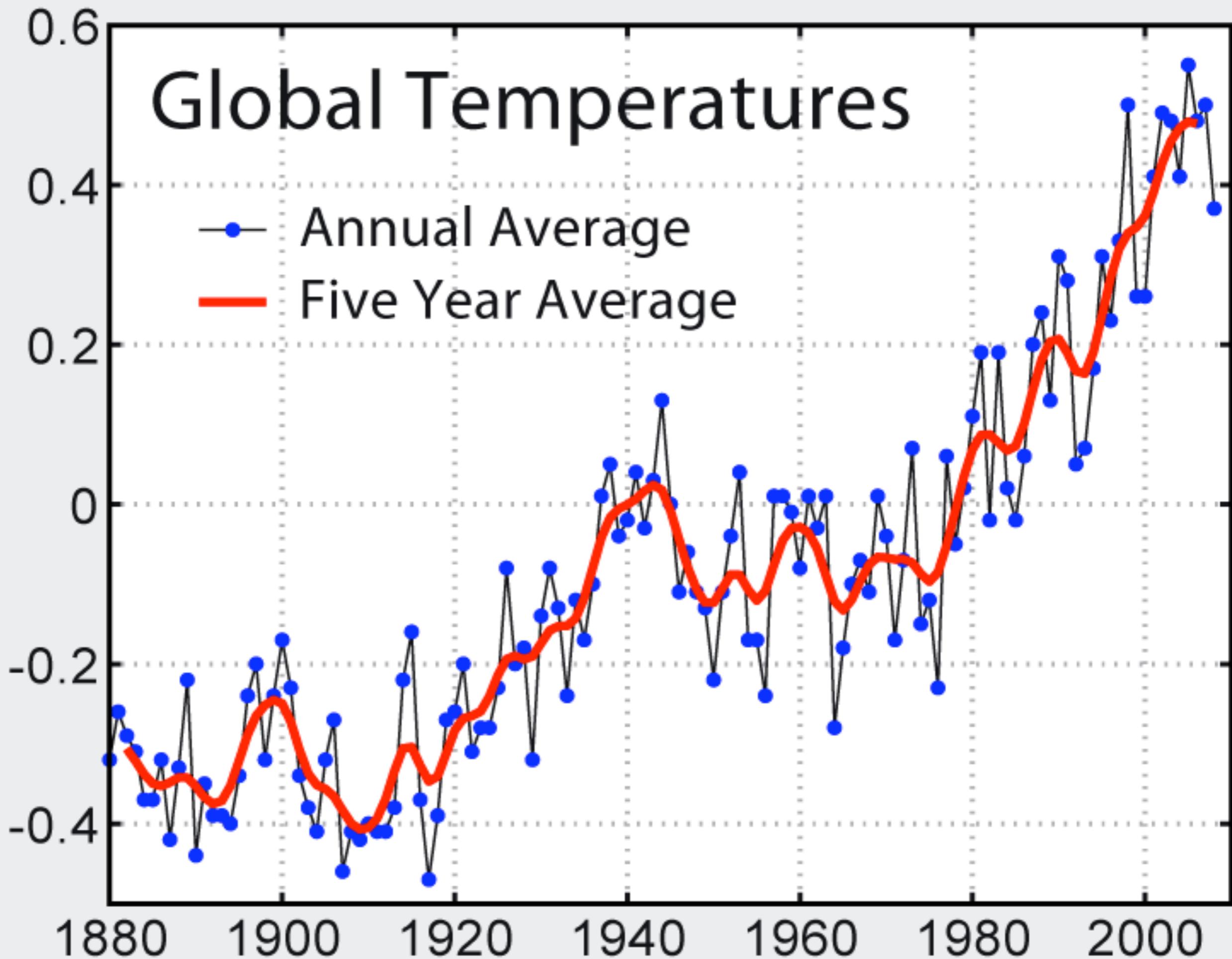
**But where is the carbon coming from?**



# Global Temperatures

Temperature Anomaly ( $^{\circ}\text{C}$ )

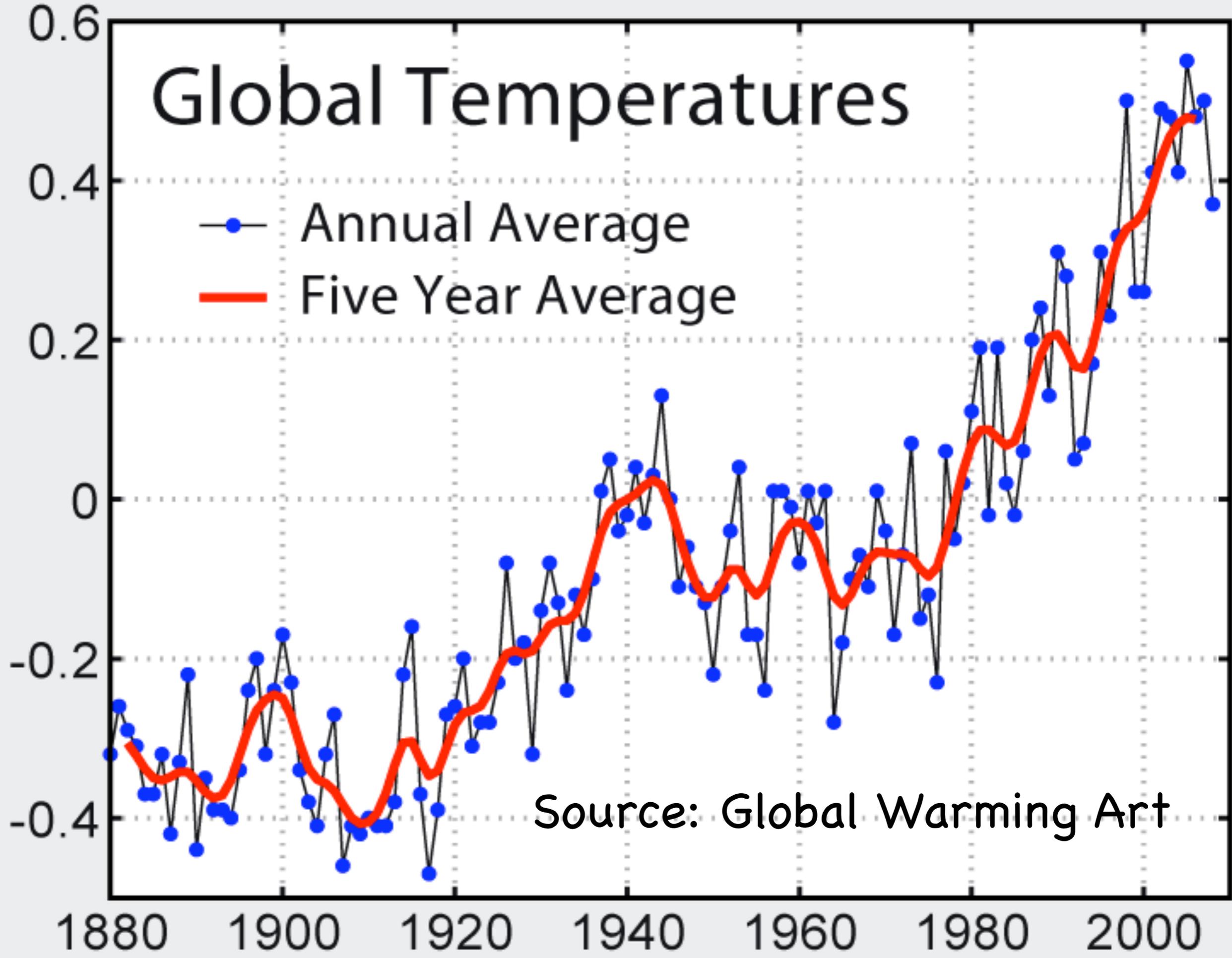
- Annual Average
- Five Year Average



# Global Temperatures

Temperature Anomaly ( $^{\circ}\text{C}$ )

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- Five Year Average



Source: Global Warming Art



# What Can Modern Physics Contribute?

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## Aspen Center for Physics

Summer 2005 Workshop

*Novel Approaches To Climate*

Funding: NSF, BP Research, & ICAM



John Harte's long-term ecosystem heating experiment at the Rocky Mountain Biological Laboratory near Aspen.

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## Kavli Institute for Theoretical Physics

*Physics of Climate Change*  
April 28 -- July 11, 2008  
*Frontiers of Climate Science*  
*& Engineering the Earth*  
May 6 -- 10, 2008



Co-organizers:  
J. Carlson, G. Falkovich, J. Harte,  
J. B. Marston, and R. Pierrehumbert

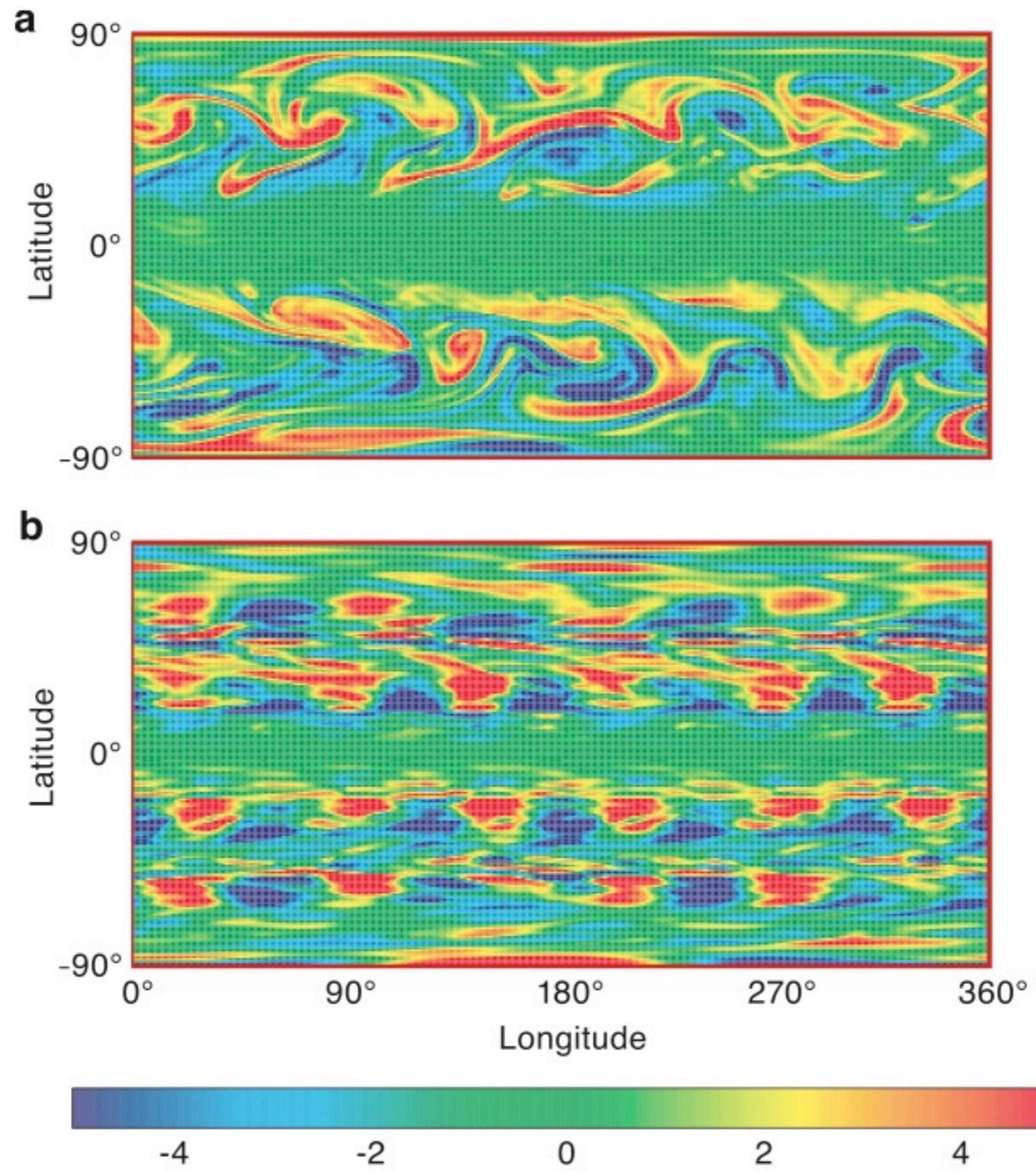




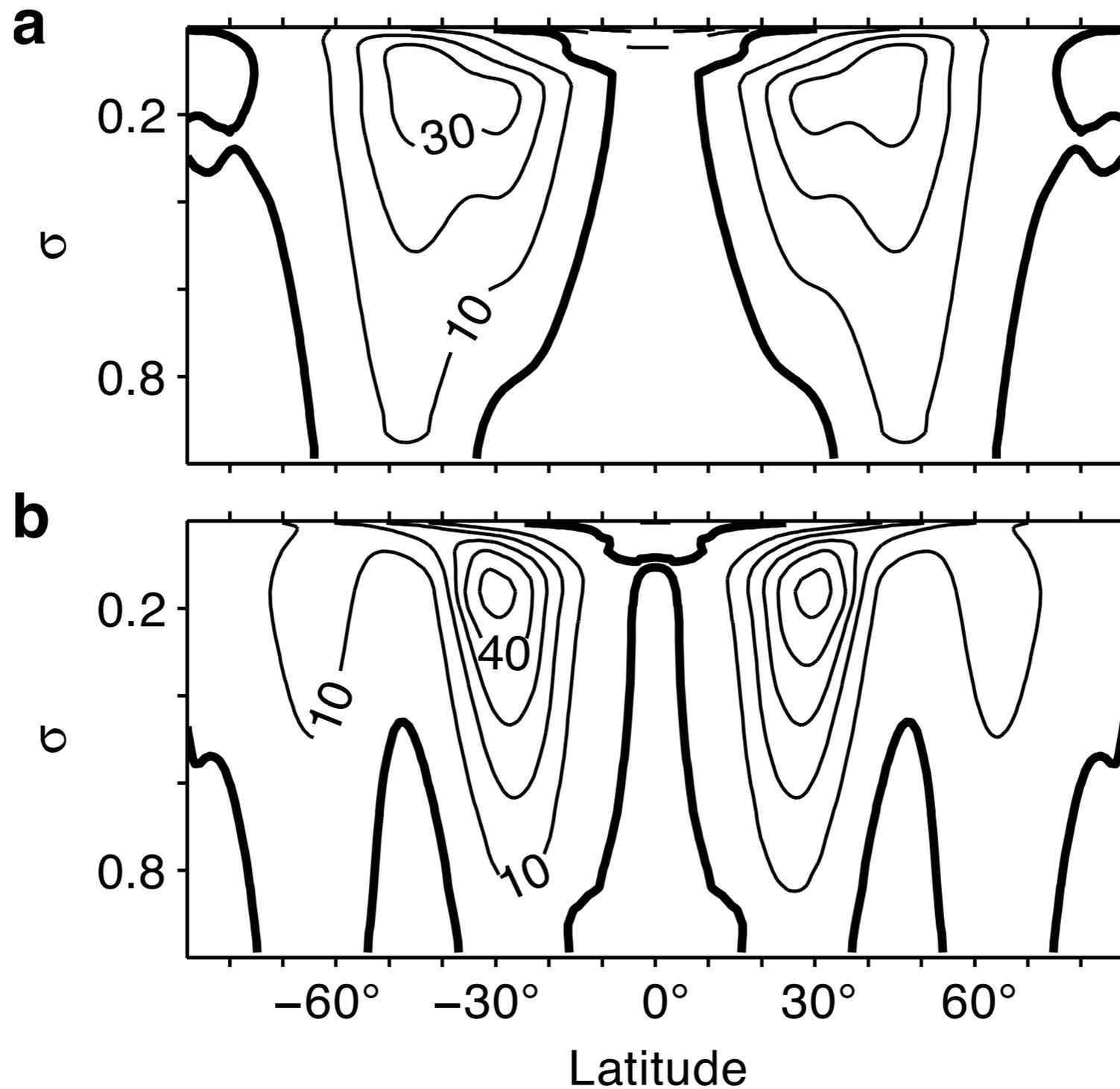


“Human beings are now carrying out a large scale geophysical experiment of a kind that could not have happened in the past nor be reproduced in the future. Within a few centuries we are returning to the atmosphere and oceans the concentrated organic carbon stored in sedimentary rocks over hundreds of millions of years. (Revelle and Suess, 1957)



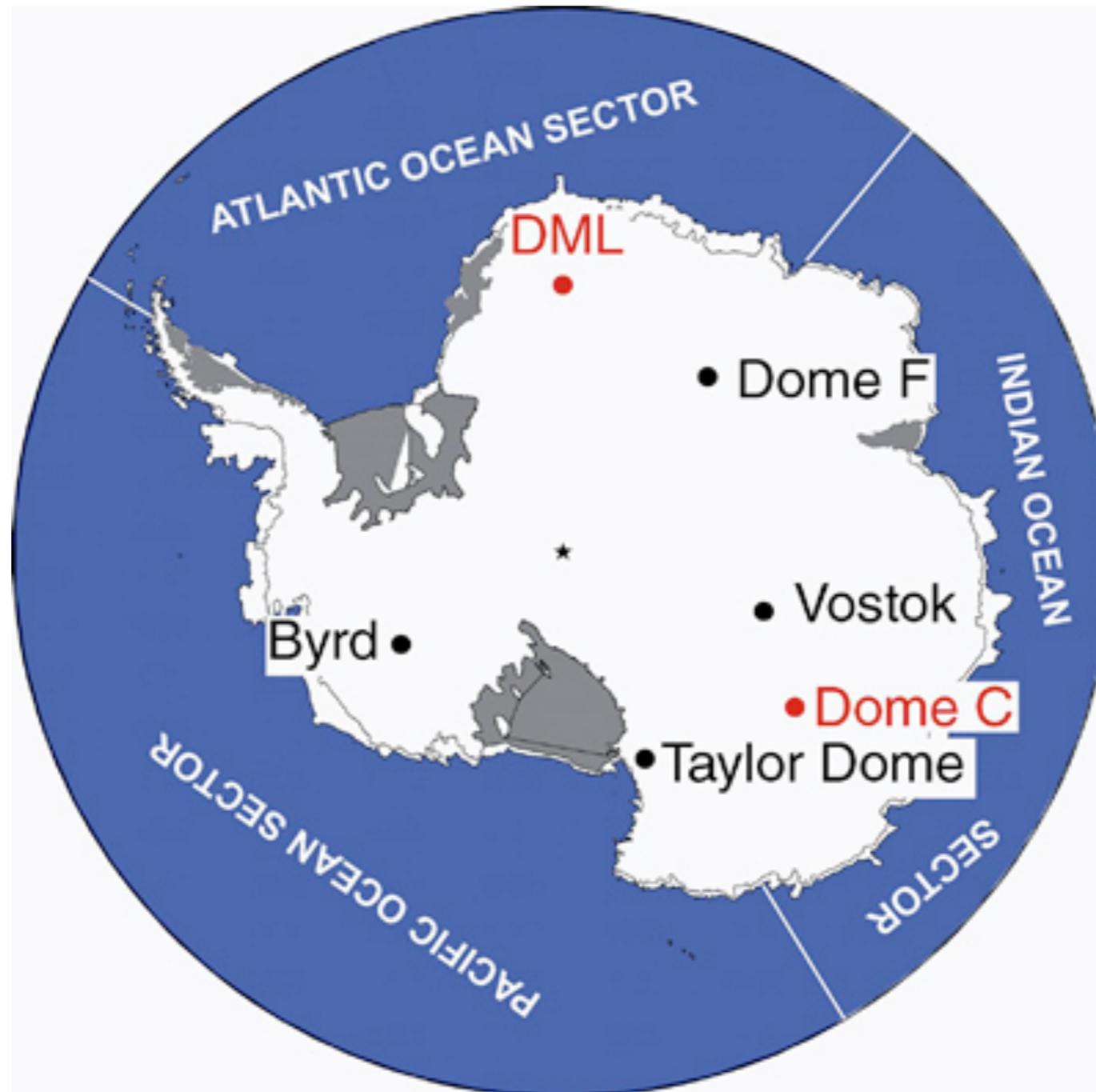


**Figure 1.** Typical instantaneous vorticity fields ( $10^{-5} \text{ s}^{-1}$ ) in (a) the full simulation and (b) the simulation without eddy-eddy interactions. The horizontal surface shown is in the mid-troposphere at  $\sigma = 0.5$ . The fields are shown at times after the simulations have reached statistically steady states.



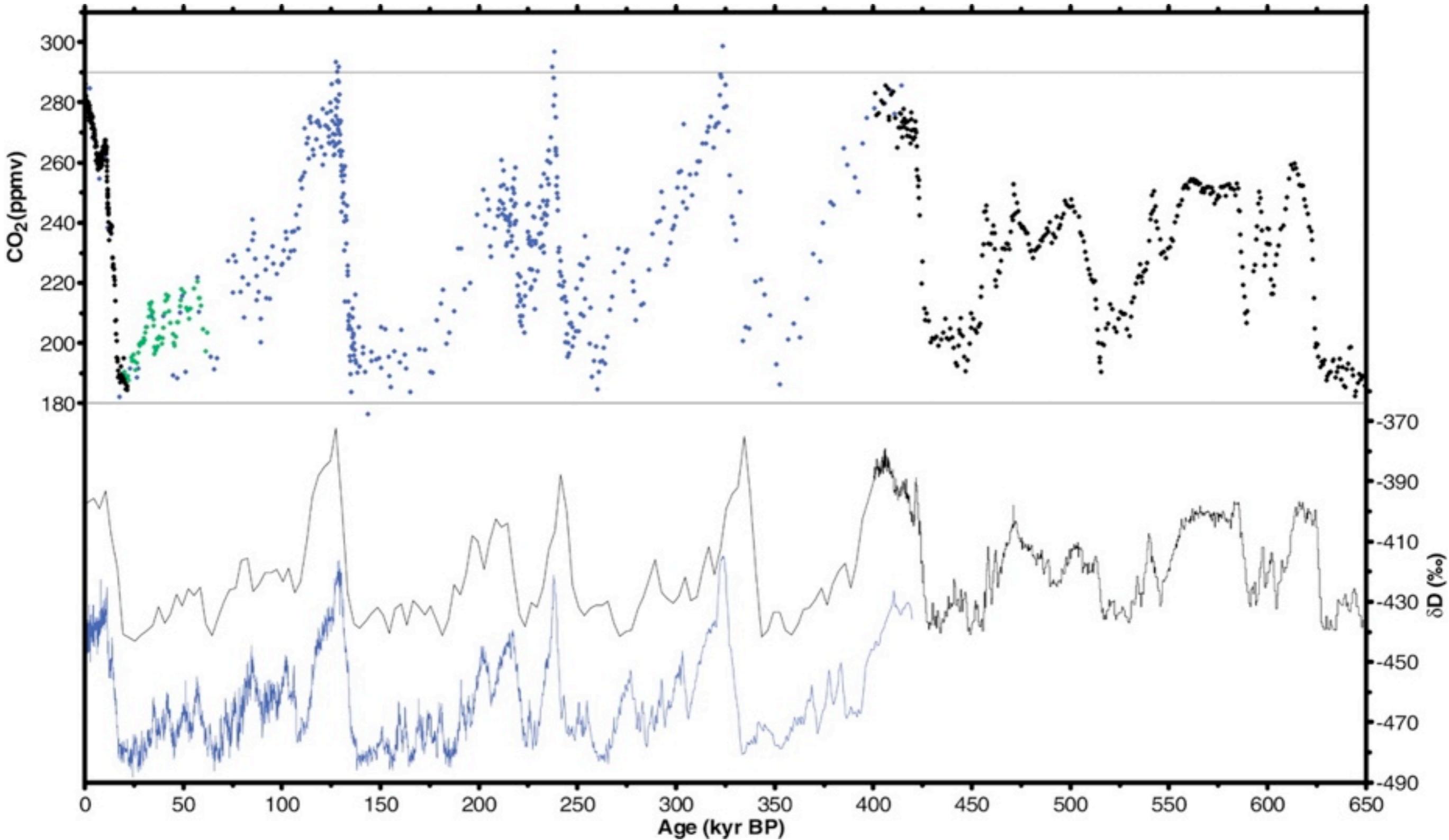
**Figure 3.** Mean eastward wind ( $\text{m s}^{-1}$ ) in the meridional plane in (a) the full simulation and (b) the simulation without eddy-eddy interactions. The mean is a zonal, time, and interhemispheric average with mass weighting. The thick solid lines are the zero-wind lines.

# Antarctic Dome C



Siegenthaler *et al.* Science **310**, 1313 (2005)

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Siegenthaler *et al.* Science **310**, 1313 (2005)



Lake Mead