University of Virginia

Condensed Matter Seminar

November 19, 2009

Counting statistics of electron transport in nanostructures

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Theory collaborators:

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Antti-Pekka Jauho (Copenhagen/Helsinki)

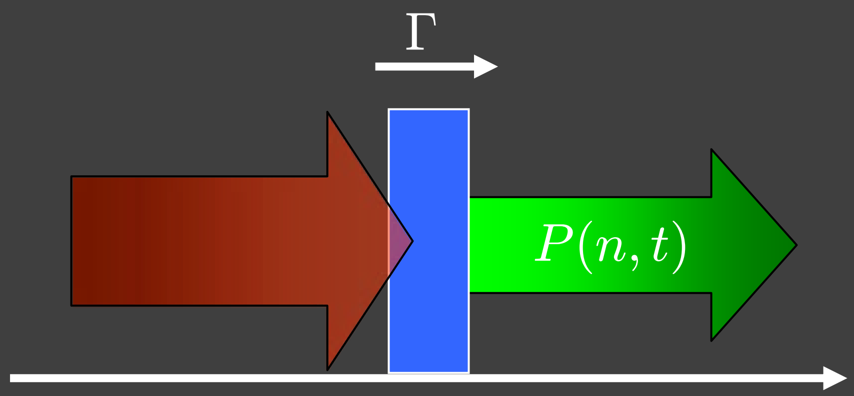
Experiments:

Christian Fricke (Hannover)
Frank Hohls (Hannover)
Rolf J. Haug (Hannover)

Recent papers:

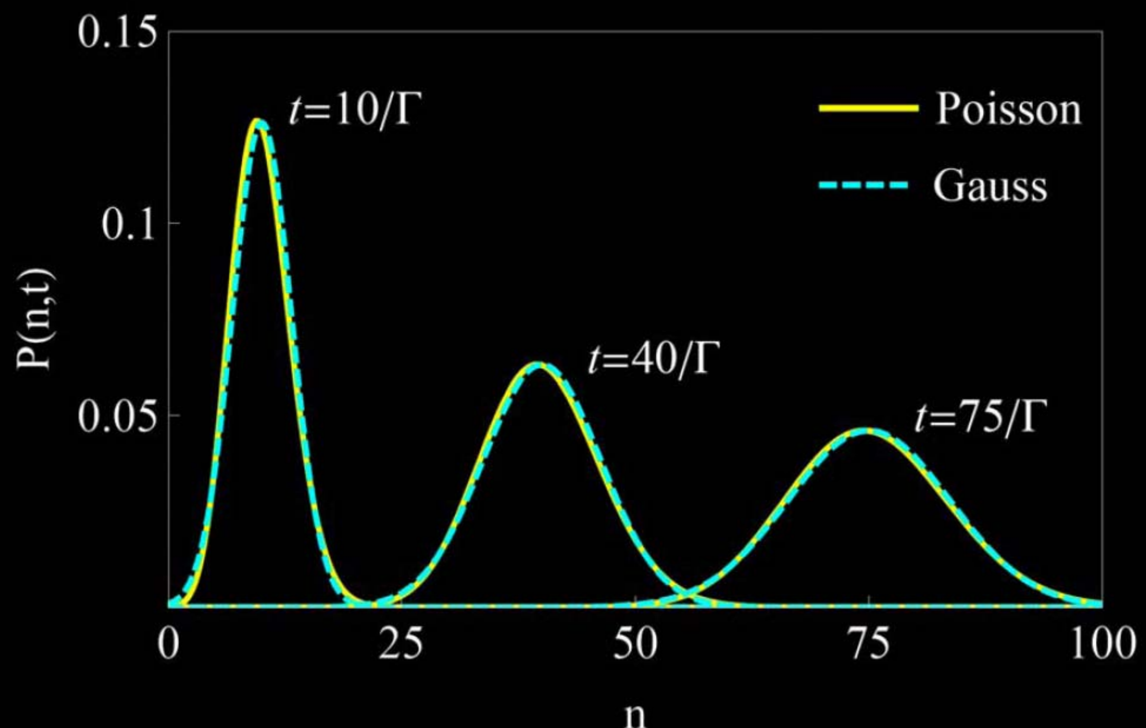
C. Flindt *et al.*, PRL **100**, 150601 (2008)
C. Flindt *et al.*, PNAS **106**, 10116 (2009)
C. Flindt *et al.*, in prep. (2009)

Introduction to counting statistics



Uncorrelated, classical particles transmitted through a barrier with rate Γ $\Rightarrow$ Poisson process

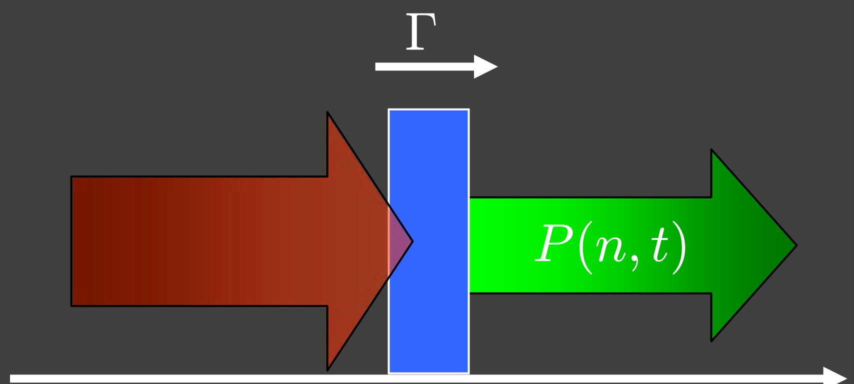
$$P(n,t) = \frac{(\Gamma t)^n}{n!} e^{-\Gamma t}$$



Looks Gaussian...

... *but look closer!!*

Introduction to counting statistics



Uncorrelated, classical particles transmitted through a barrier with rate Γ ⇒ Poisson process

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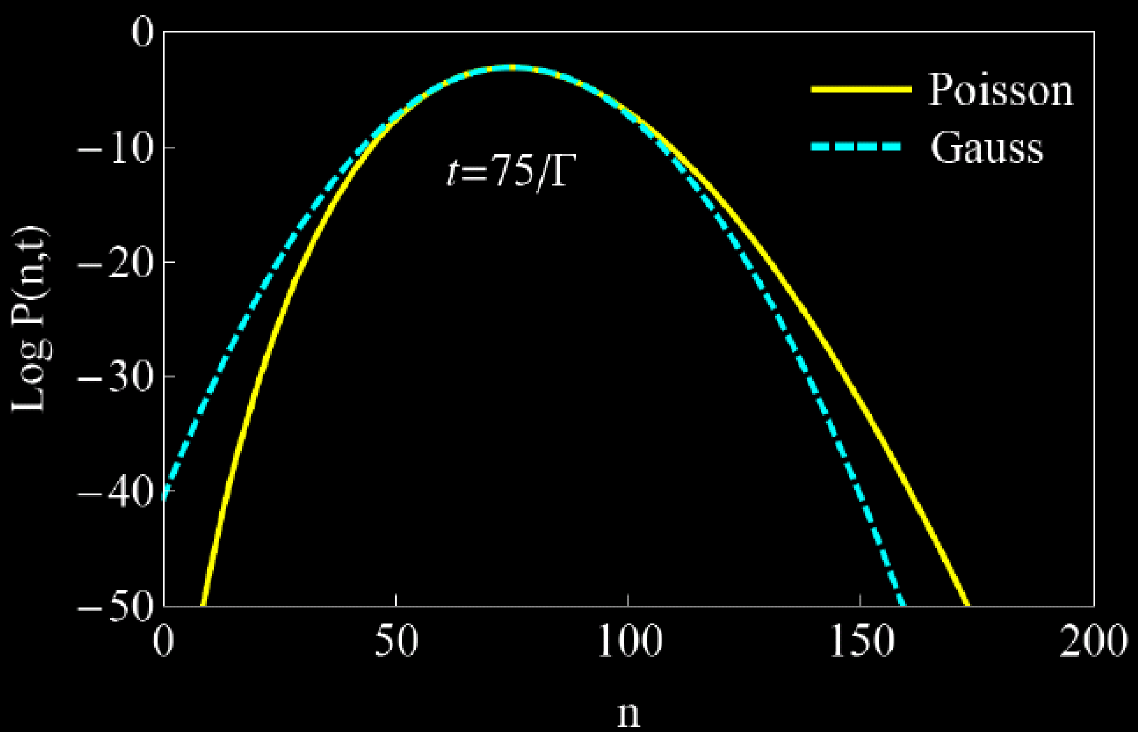
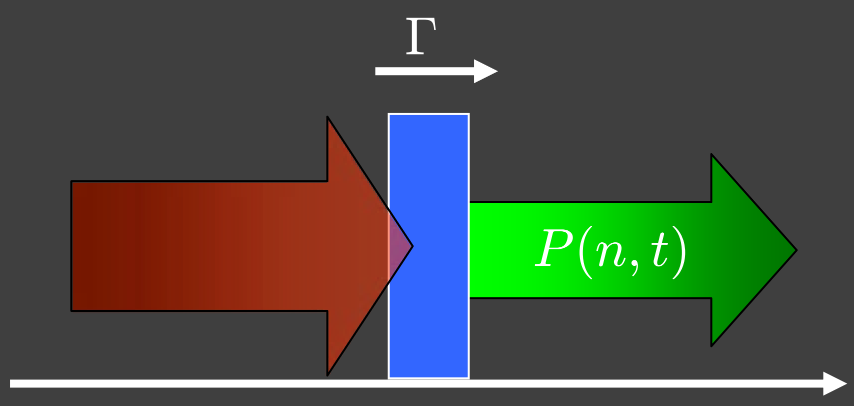


- Peak well-described
- Tails not captured by Gaussian approximation

⇒ Consider cumulants

Introduction to counting statistics



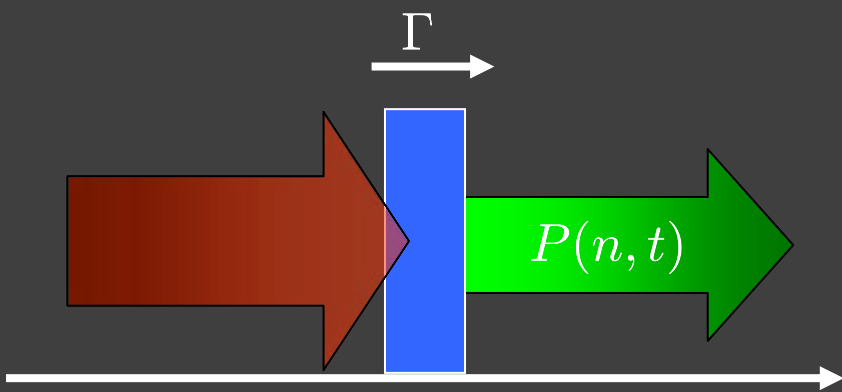
Uncorrelated, classical particles transmitted through a barrier with rate Γ $\Rightarrow$ Poisson process

$$P(n,t) = \frac{(\Gamma t)^n}{n!} e^{-\Gamma t}$$

Cumulants:

- 1st cumulant – mean: $\langle\langle n \rangle\rangle = \langle n \rangle$
- 2nd cumulant – variance: $\langle\langle n^2 \rangle\rangle = \langle (n - \langle n \rangle)^2 \rangle$
- 3rd cumulant – skewness: $\langle\langle n^3 \rangle\rangle = \langle (n - \langle n \rangle)^3 \rangle$
- General definition: $\langle\langle n^m \rangle\rangle = \frac{\partial^m \mathcal{S}(\chi,t)}{\partial (i\chi)^m}|_{\chi\to 0}$ (χ: “counting field”)
- Cumulant generating function (CGF): $e^{\mathcal{S}(\chi,t)} \equiv \sum_n P(n,t) e^{in\chi}$
- Poisson process: $\langle\langle n^m \rangle\rangle = \Gamma t$. Gauss distribution: $\langle\langle n^m \rangle\rangle = 0, m > 2$

Introduction to counting statistics



Uncorrelated, classical particles transmitted through a barrier with rate Γ ⇒ Poisson process

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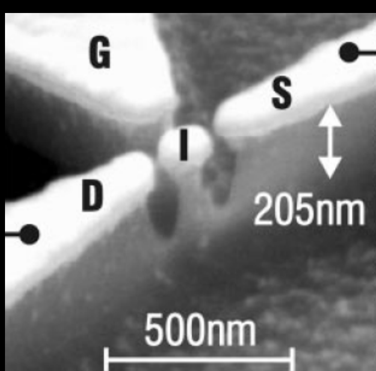
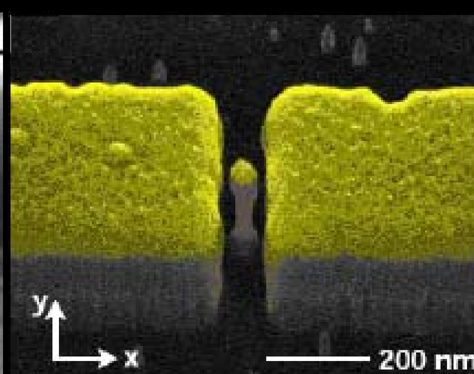
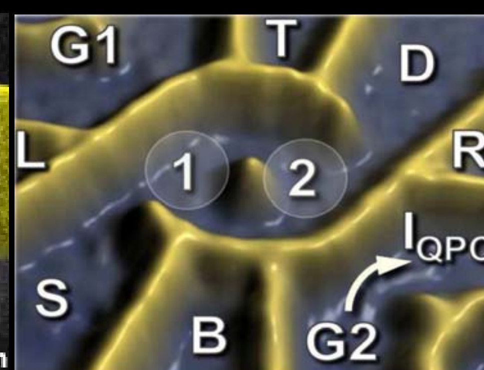
Non-zero higher order cumulants reveal non-Gaussian behavior!

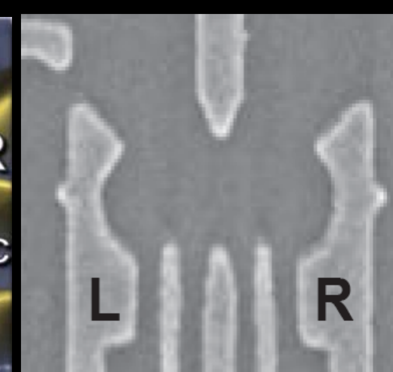
This talk – Outline

- Introduction to counting statistics
- General theory (open-quantum-systems approach)
  Example: a nano-electromechanical system (NEMS)
- Memory effects (due to environment)
  Example: dissipative double quantum dot (open spin-boson model)
- High-order cumulants
  Example: single quantum dot (experiment)
- Summary and outlook



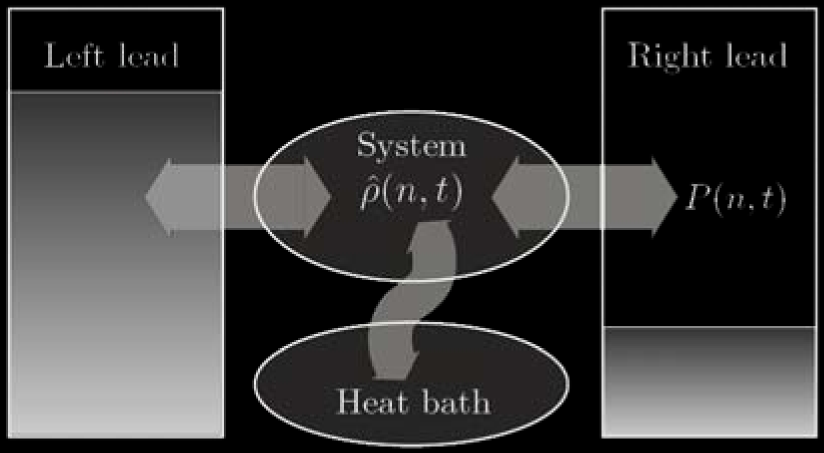






Pictures from Münich, Wisconsin, ETH-Zürich, and Harvard

General theory (open-quantum-systems approach)



n-resolved reduced density matrix:
- R. J. Cook, PRA **23**, 1243 (1981)
- Yu. Makhlin *et al.*, RMP **73**, 357 (2001)

$$\hat\rho(n,t) \Rightarrow P(n,t) = \mathrm{Tr}\{\hat\rho(n,t)\}$$

$$\Rightarrow e^{\mathcal{S}(\chi,t)} = \mathrm{Tr}\{\hat\rho(\chi,t)\}$$

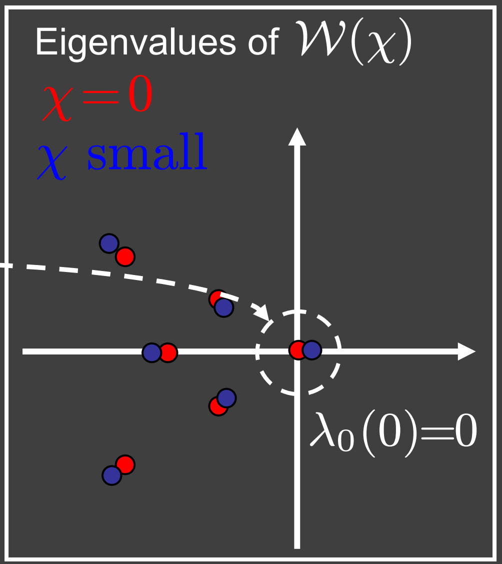
Assuming Markovian dynamics (e.g. weak coupling):

$$\frac{d}{dt}\hat\rho(\chi,t) = \mathcal{W}(\chi)\hat\rho(\chi,t)$$

$$\Rightarrow \hat\rho(\chi,t) = e^{\mathcal{W}(\chi)t}\hat\rho(\chi,0)$$

$$\Rightarrow e^{\mathcal{S}(\chi,t)} = \mathrm{Tr}\{e^{\mathcal{W}(\chi)t}\hat\rho(\chi,0)\} \to e^{\lambda_0(\chi)t} \text{ for large } t$$

$$\Rightarrow \langle\!\langle n^m \rangle\!\rangle = t\frac{\partial^m \lambda_0(\chi)}{\partial(i\chi)^m}\Big|_{\chi\to 0} \Rightarrow \langle\!\langle I^m \rangle\!\rangle = \frac{\partial^m \lambda_0(\chi)}{\partial(i\chi)^m}\Big|_{\chi\to 0}$$



D. A. Bagrets and Yu. V. Nazarov, PRB **67**, 085316 (2003), C. Flindt *et al.*, EPL **69**, 475 (2005)

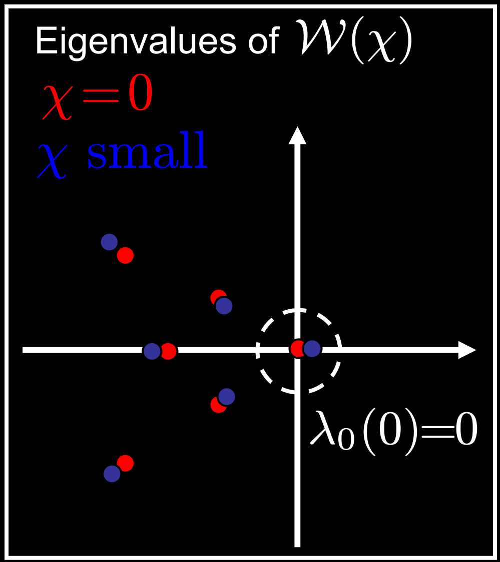
General theory (open-quantum-systems approach)

Technical issue: How do I solve the eigenproblem?

$$\mathcal{W}(\chi)|0(\chi)\rangle\!\rangle = \lambda_0(\chi)|0(\chi)\rangle\!\rangle$$

Solution: Perturbation theory in χ !

- Cumulants: $\lambda_0(\chi) = \sum_n \frac{(i\chi)^n}{n!}\langle\!\langle I^n\rangle\!\rangle$
- Unperturbed problem: $\mathcal{W}(0)|0(0)\rangle\!\rangle = \overbrace{\lambda_0(0)}^{=0}|0(0)\rangle\!\rangle$
- Perturbation: $\mathcal{W}'(\chi) \equiv \mathcal{W}(\chi) - \mathcal{W}(0) = \sum_n \frac{(i\chi)^n}{n!}\mathcal{W}^{(n)}$



Perturbation theory can be done recursively using the building blocks:

$$\mathcal{P} = \mathcal{P}^2 \equiv |0(0)\rangle\!\rangle\langle\!\langle\tilde{0}(0)|,\quad \mathcal{Q} = \mathcal{Q}^2 \equiv 1 - \mathcal{P},\quad \mathcal{R} \equiv \mathcal{Q}\frac{1}{\mathcal{W}(0)}\mathcal{Q}$$

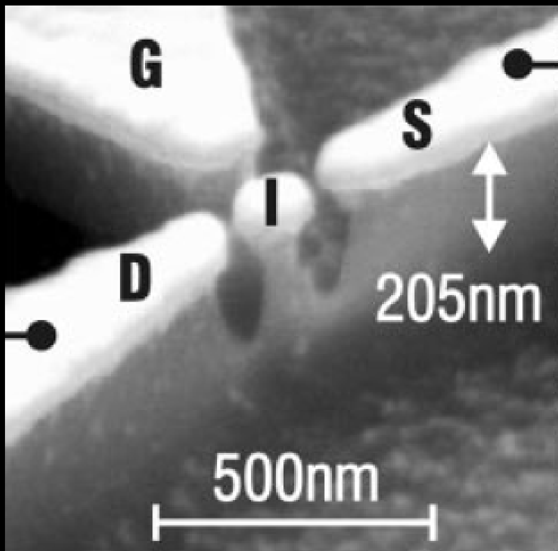
Recursive scheme:
(numerically stable up to ~100'th cumulant)

$$\langle\!\langle I^n\rangle\!\rangle = \sum_{m=1}^{n}\binom{n}{m}\langle\!\langle\tilde{0}|\mathcal{W}^{(m)}|0^{(n-m)}\rangle\!\rangle$$

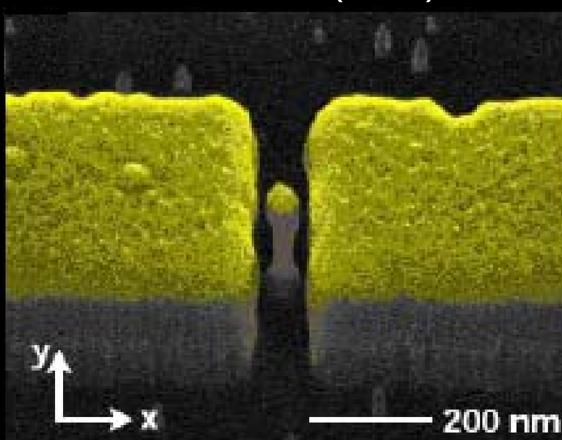
$$|0^{(n)}\rangle\!\rangle = \mathcal{R}\sum_{m=1}^{n}\binom{n}{m}\left[\langle\!\langle I^m\rangle\!\rangle - \mathcal{W}^{(m)}\right]|0^{(n-m)}\rangle\!\rangle$$

C. Flindt *et al.*, PRL **100**, 150601 (2008) + in prep. (2009)

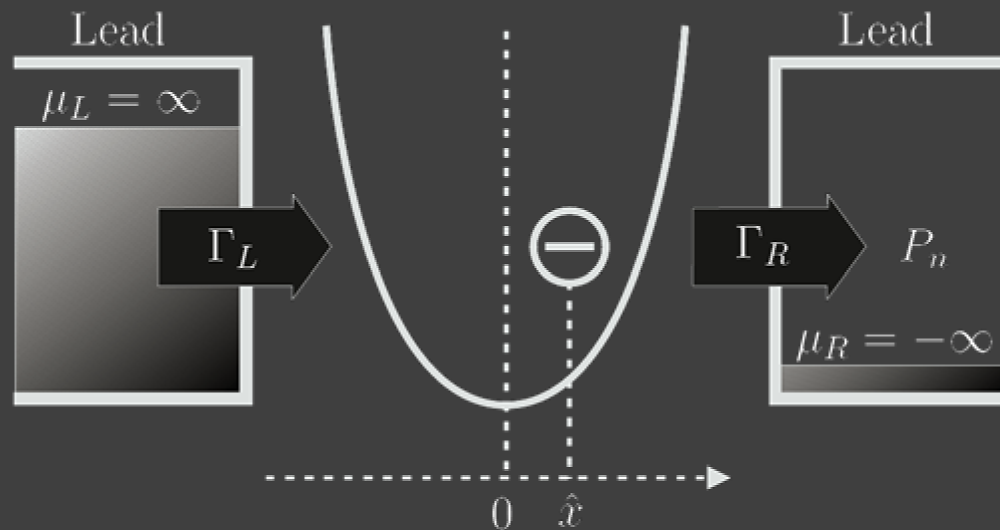
Example: A nano-electromechanical system (NEMS)



D. V. Scheible *et al.*,
APL **84**, 4632 (2004)



H. S. Kim *et al.*,
Nanotech. **18**, 065201 (2007)



$$\hat{H} = \epsilon_0 \hat{c}_0^\dagger \hat{c}_0 + \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega_0^2 \hat{x}^2 - eE\hat{x}\hat{c}_0^\dagger \hat{c}_0$$

$$+\hat{H}_{\text{tunnel}}(\hat{x}) + \hat{H}_{\text{leads}} + \hat{H}_{\text{damp}}$$

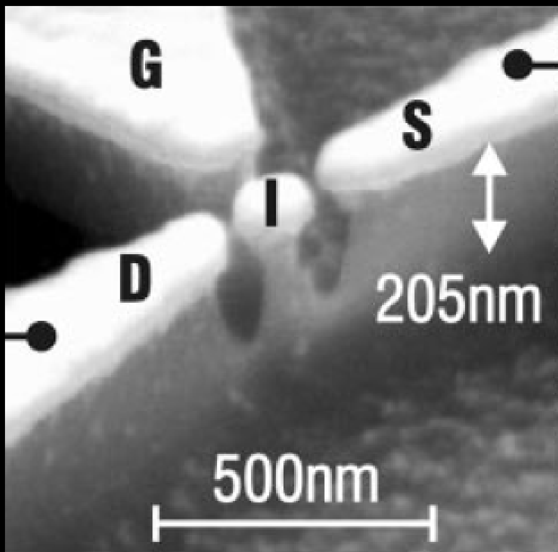
Trace out external reservoirs

⇒ Generalized master equation

$$\tfrac{d}{dt}\hat{\rho}(\chi) = [\mathcal{W}_{\text{coh}} + \mathcal{W}_{\text{damp}} + \mathcal{W}_{\text{leads}}(\chi)]\hat{\rho}(\chi)$$

Charge shuttle: L. Y. Gorelik *et al.*, PRL **80**, 4526 (1998)
T. Novotný *et al.*, PRL **90**, 256801 (2003)

Example: A nano-electromechanical system (NEMS)



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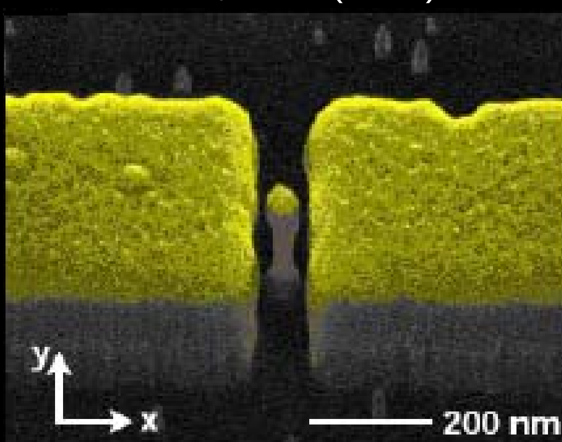

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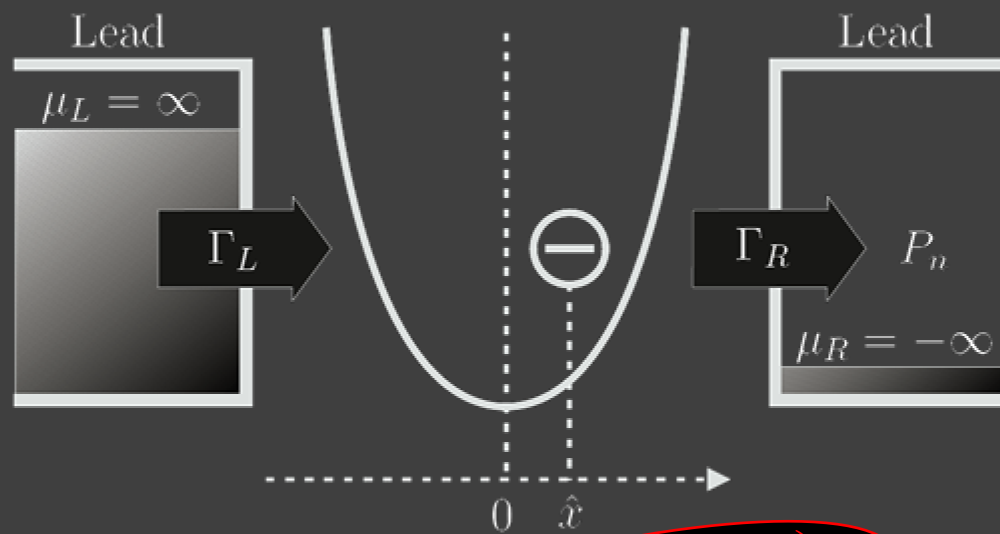

$$\hat{H} = \epsilon_0 \hat{c}_0^\dagger \hat{c}_0 + \ldots \hat{p}^2 \ldots x \hat{c}_0^\dagger \hat{c}_0$$

$$\ldots \text{leads} + \hat{H}_{\text{damp}}$$

With 100 oscillator states W(χ) is a 20.000x20.000 matrix!

Trace out external reservoirs

⇒ Generalized master equation

$$\frac{d}{dt}\hat{\rho}(\chi) = [\mathcal{W}_{\text{coh}} + \mathcal{W}_{\text{damp}} + \mathcal{W}_{\text{leads}}(\chi)]\hat{\rho}(\chi)$$

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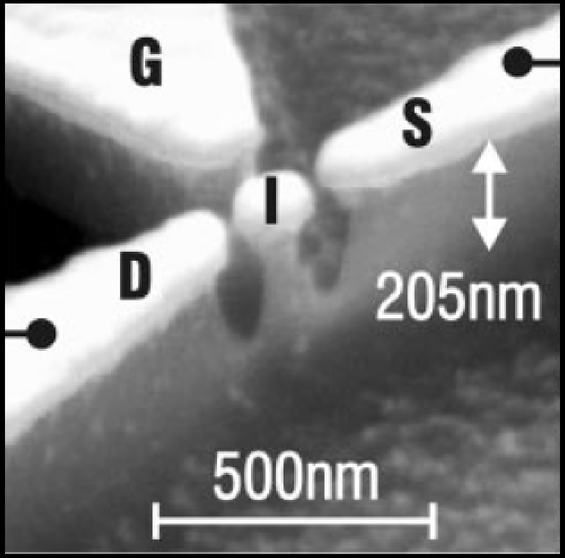

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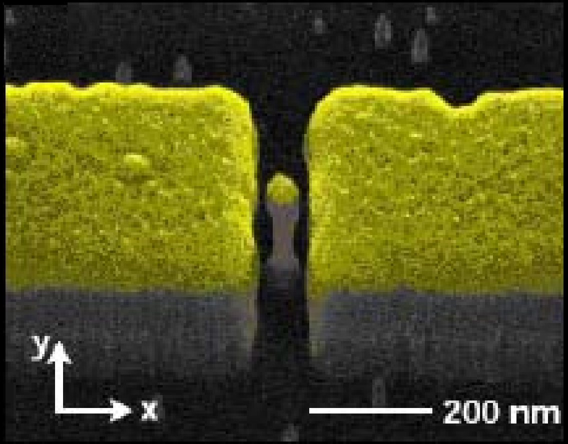

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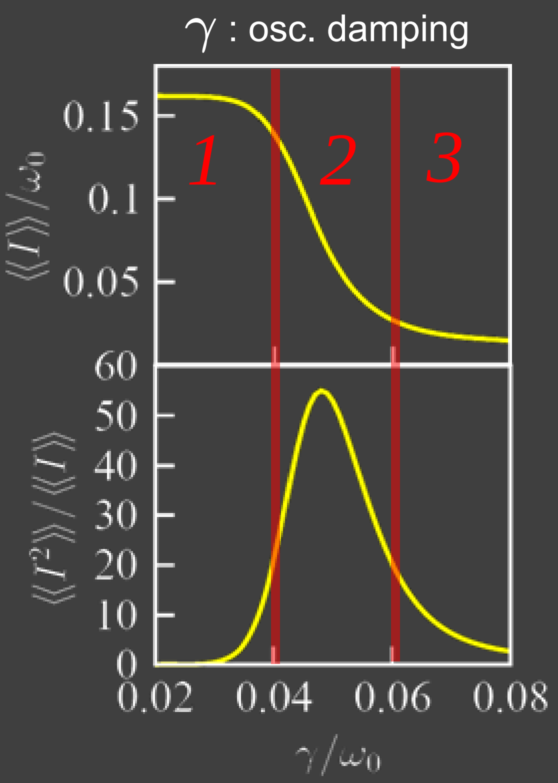
Nanotech. **18**, 065201 (2007)

Mean current $\langle\langle I^1 \rangle\rangle$ & zero-frequency noise $\langle\langle I^2 \rangle\rangle$



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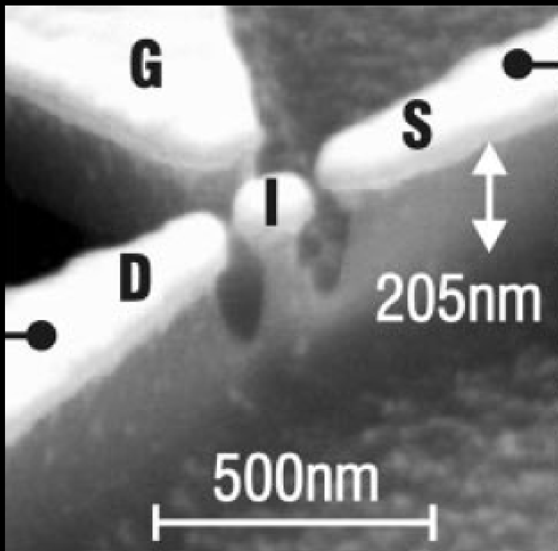

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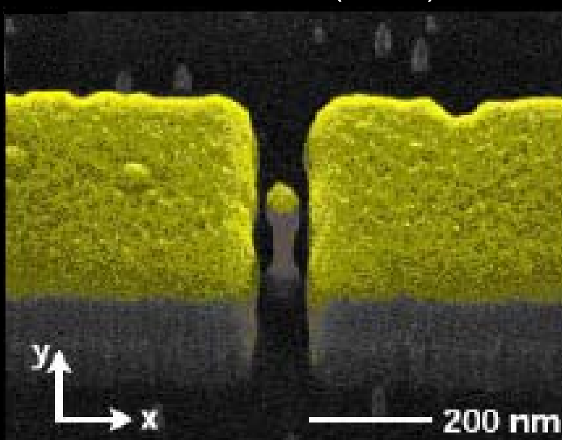
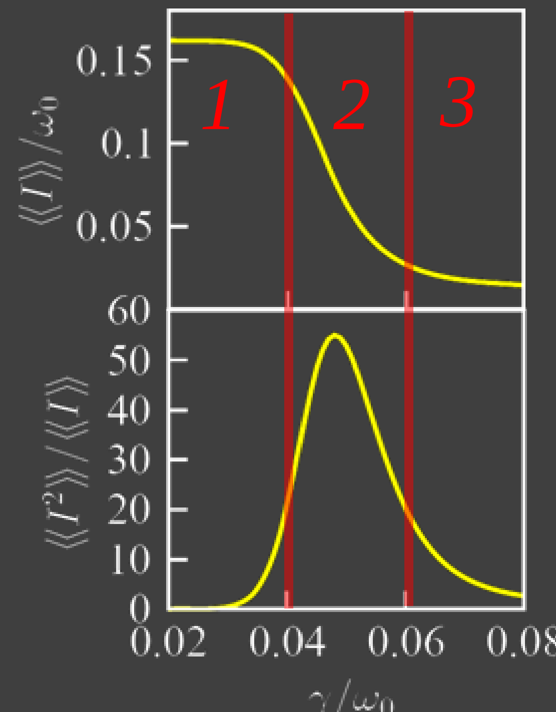


H. S. Kim *et al.*,
Nanotech. **18**, 065201 (2007)

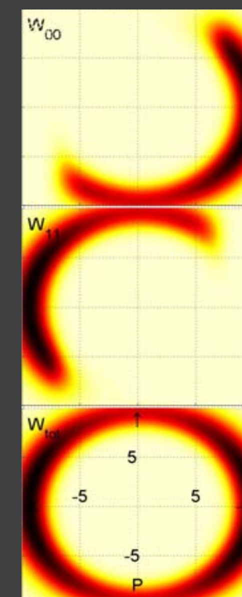
Mean current $\langle\langle I^1 \rangle\rangle$ & zero-frequency noise $\langle\langle I^2 \rangle\rangle$

1. *Shuttle regime:* $\langle\langle I^1 \rangle\rangle \simeq \frac{\omega_0}{2\pi}, \ \langle\langle I^2 \rangle\rangle \simeq 0$

γ : osc. damping



Osc. phase space



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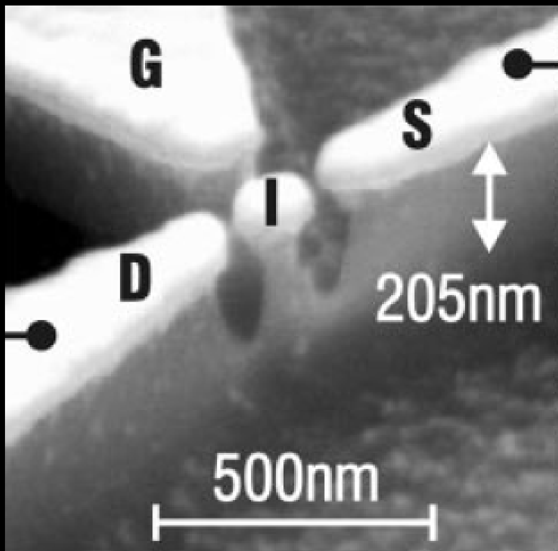

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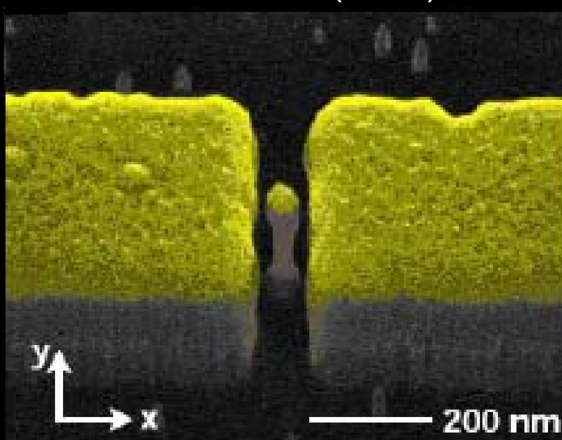
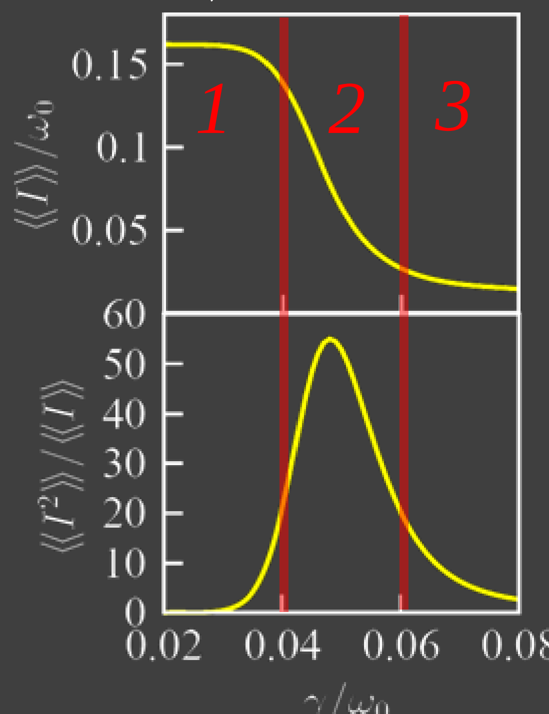


H. S. Kim *et al.*,
Nanotech. **18**, 065201 (2007)

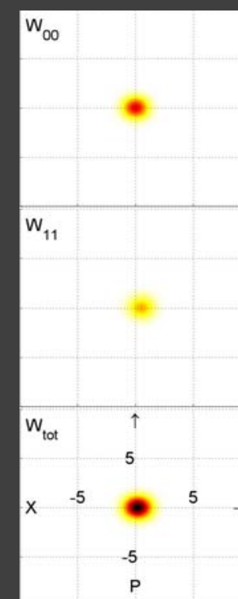
Mean current $\langle\langle I^1 \rangle\rangle$ & zero-frequency noise $\langle\langle I^2 \rangle\rangle$

3. "Static dot" regime

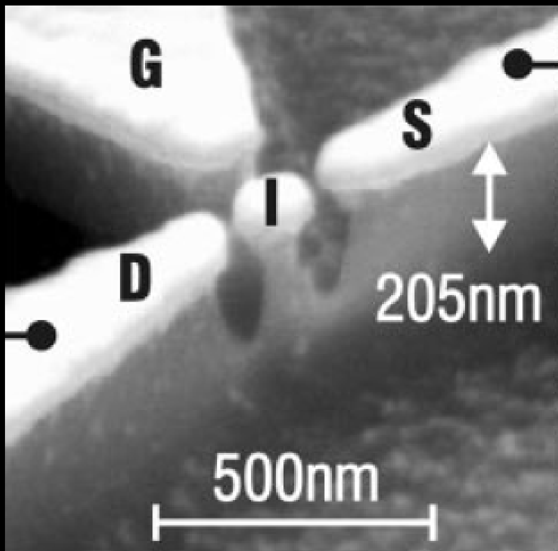
γ : osc. damping



Osc. phase space



Example: A nano-electromechanical system (NEMS)



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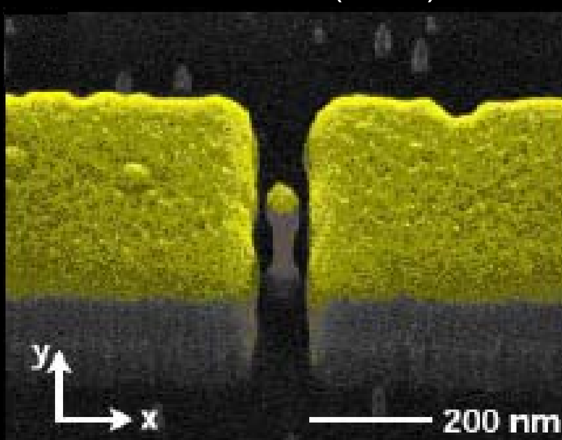

H. S. Kim *et al.*,
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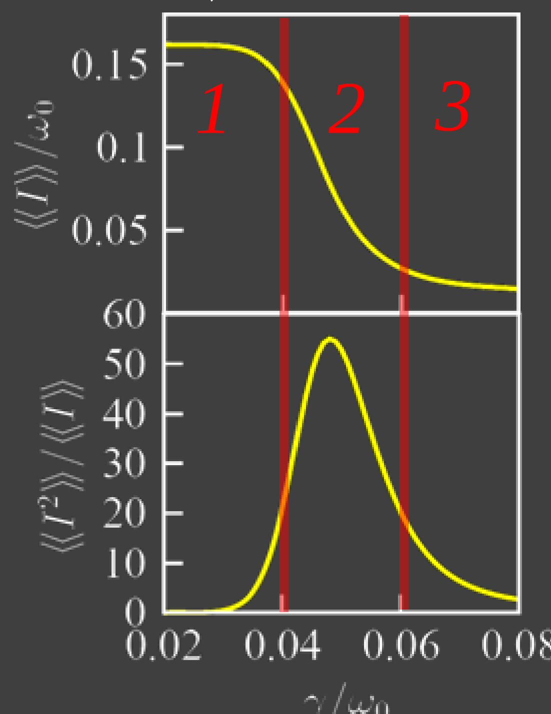
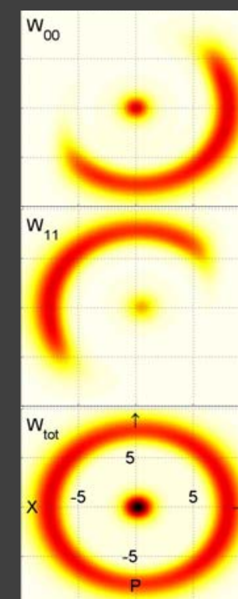
Mean current $\langle\langle I^1 \rangle\rangle$ & zero-frequency noise $\langle\langle I^2 \rangle\rangle$

2. "Co-existence" regime – enhanced noise

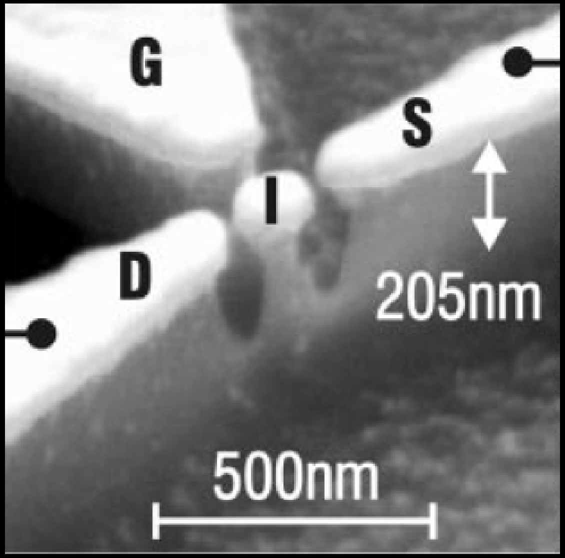
γ : osc. damping



Osc. phase space



Example: A nano-electromechanical system (NEMS)



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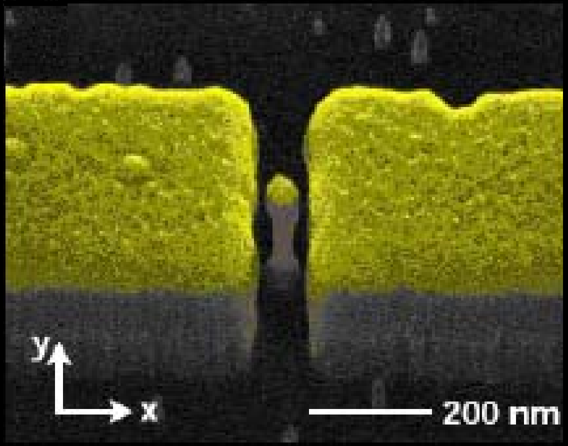
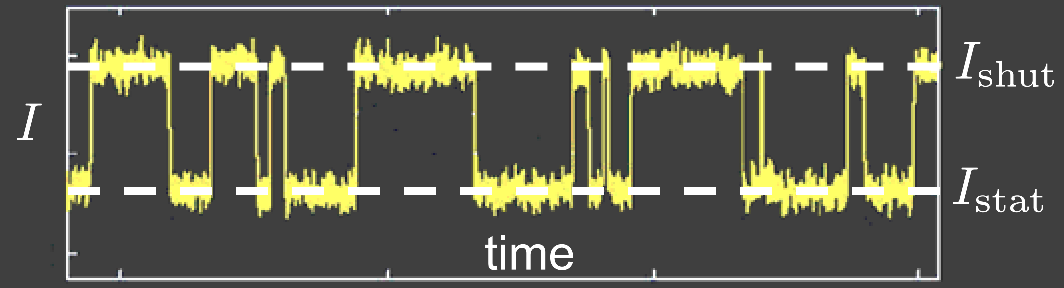


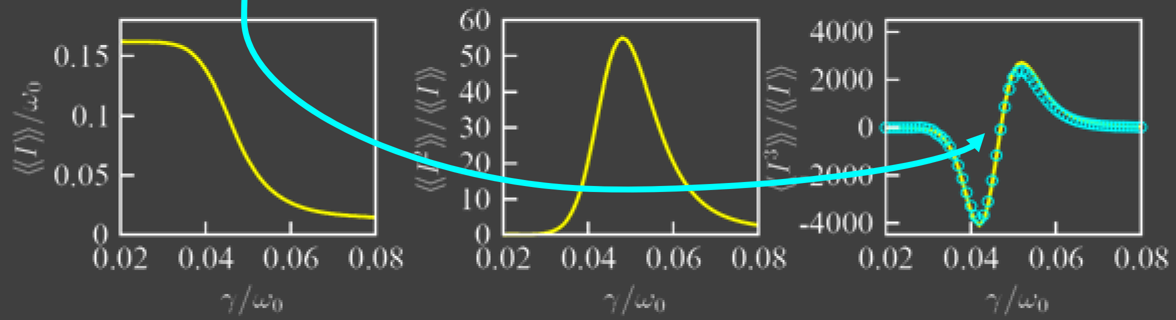
H. S. Kim *et al.*,
Nanotech. **18**, 065201 (2007)

Noise enhancement due to mechanical bistability causing random telegraph noise (RTN)?



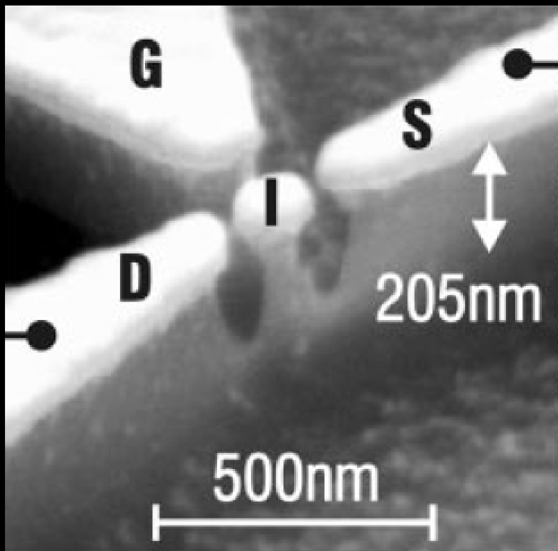
For RTN:

$$\langle\!\langle I^3 \rangle\!\rangle = 3 \langle\!\langle I^2 \rangle\!\rangle \frac{(I_{\rm shut} + I_{\rm stat})/2 - \langle\!\langle I^1 \rangle\!\rangle}{(I_{\rm shut} - \langle\!\langle I^1 \rangle\!\rangle)(\langle\!\langle I^1 \rangle - I_{\rm stat} \rangle)}$$

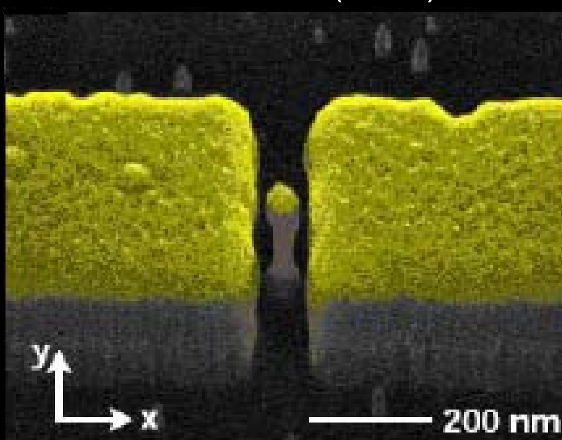


C. Flindt *et al.*, EPL **69**, 475 (2005)

Example: A nano-electromechanical system (NEMS)

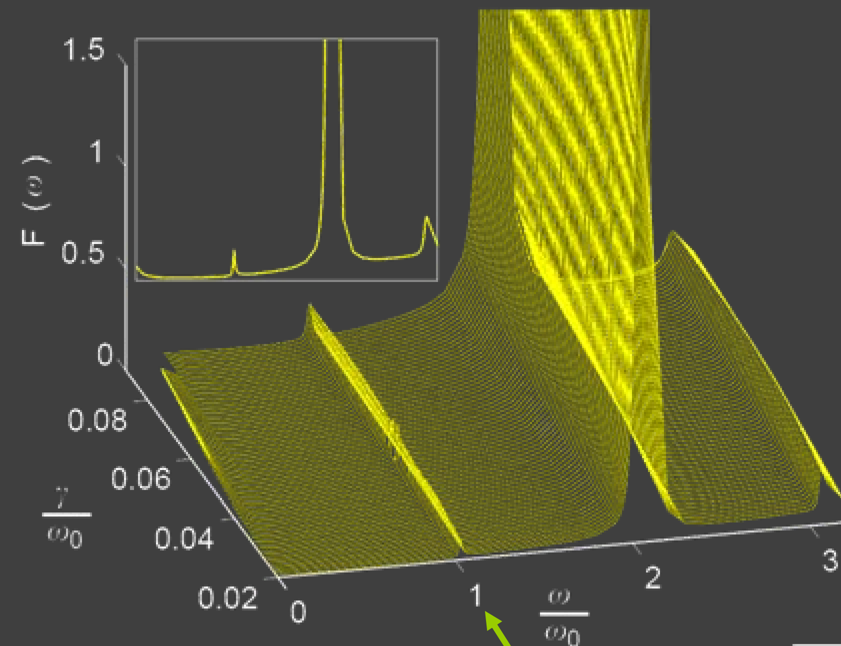


D. V. Scheible *et al.*,
APL **84**, 4632 (2004)



H. S. Kim *et al.*,
Nanotech. **18**, 065201 (2007)

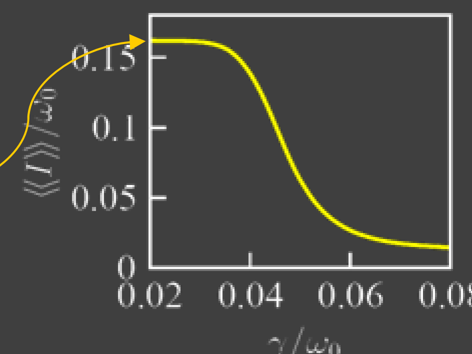
Finite-frequency noise $F(\omega)$:



$$\omega_{\text{res}} = 1.03\omega_0$$

$$I_{\text{shut}} = \frac{\omega_{\text{res}}}{2\pi} = 0.164\omega_0$$

$$\left(\frac{1}{2\pi} \simeq 0.159\right)$$



C. Flindt *et al.*, Physica E **29**, 411 (2005)

Memory effects (due to environment)

Non-Markovian generalized master equation (à la Nakajima-Zwanzig)

$$\frac{d}{dt}\hat{\rho}(n,t) = \sum_{n'}\int_0^t dt'\mathcal{W}(n-n',t-t')\hat{\rho}(n',t') + \hat{\gamma}(n,t)$$

electron counting memory kernel initial correlations (important at finite freq.)

t=*0* (counting begins)

In Laplace space: $\hat{\rho}(\chi,z) = \mathcal{G}(\chi,z)\left[\hat{\rho}(\chi,t=0) + \hat{\gamma}(\chi,z)\right]$
and χ-space

$$\mathcal{G}(\chi,z) \equiv \left[z - \mathcal{W}(\chi,z)\right]^{-1}$$

A. Braggio, *et al.,* PRL **96**, 026805 (2006)
C. Flindt *et al.*, PRL **100**, 150601 (2008)

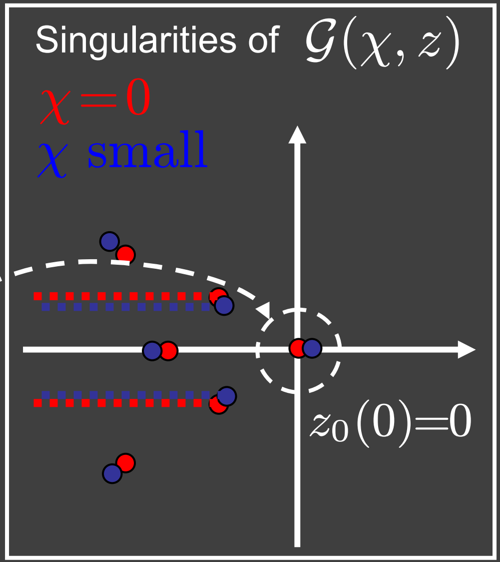
Memory effects (due to environment)

$$\hat{\rho}(\chi, z) = \mathcal{G}(\chi, z)\left[\hat{\rho}(\chi, t=0) + \hat{\gamma}(\chi, z)\right]$$

$$\mathcal{G}(\chi, z) \equiv \left[z - \mathcal{W}(\chi, z)\right]^{-1}$$

Long-*t* limit given by pole z_0 of $\mathcal{G}(\chi, z)$ close to 0:

$$\mathcal{S}(\chi, t) \to z_0(\chi)t$$



<u>Markovian case:</u>

$$z_0 - \lambda_0(\chi) = 0$$

Generalization

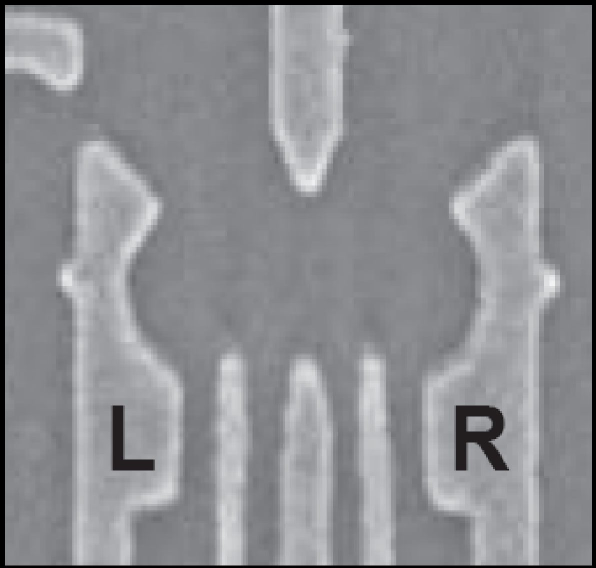
<u>Non-Markovian case:</u>

$$z_0 - \lambda_0(\chi, z_0) = 0$$

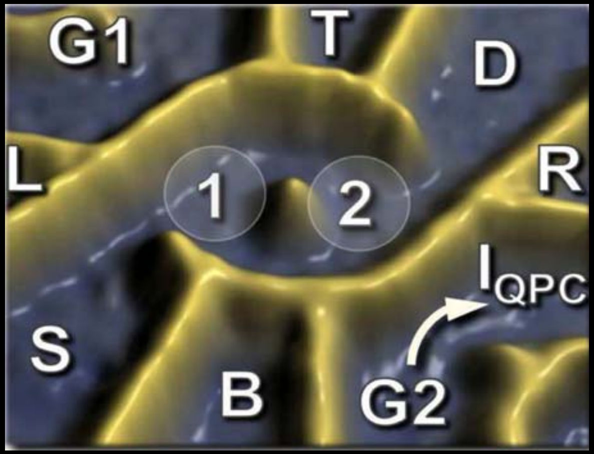
Can also be solved recursively using perturbation theory in χ *and* z.

A. Braggio *et al.*, PRL **96**, 026805 (2006)
C. Flindt *et al.*, PRL **100**, 150601 (2008) + in prep. (2009)

Example: dissipative double quantum dot

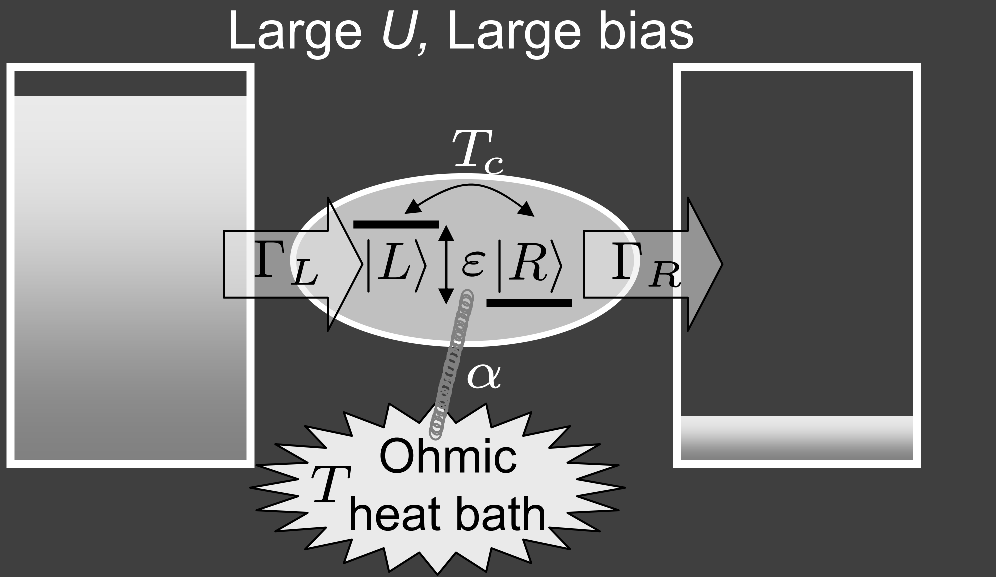


Harvard 2005



ETH-Zürich 2007

Open spin-boson model



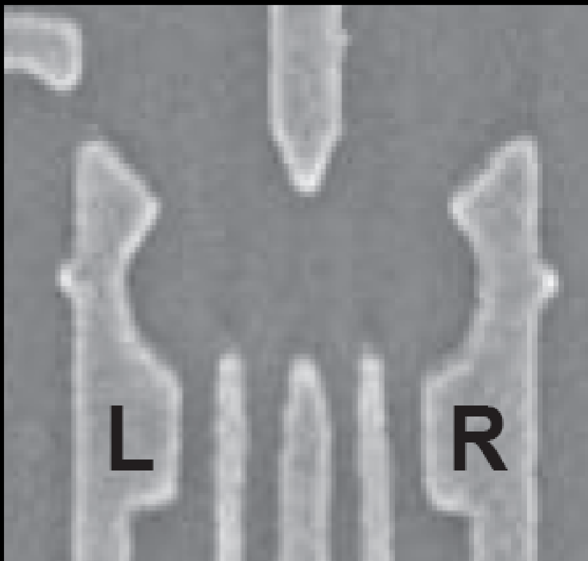
$$\hat{H} = \varepsilon \hat{s}_z + T_c \hat{s}_x + \hat{H}_T + \hat{H}_{\text{leads}} + \hat{s}_z \hat{V}_B + \hat{H}_B$$

$$\hat{s}_z = |L\rangle\langle L| - |R\rangle\langle R|$$

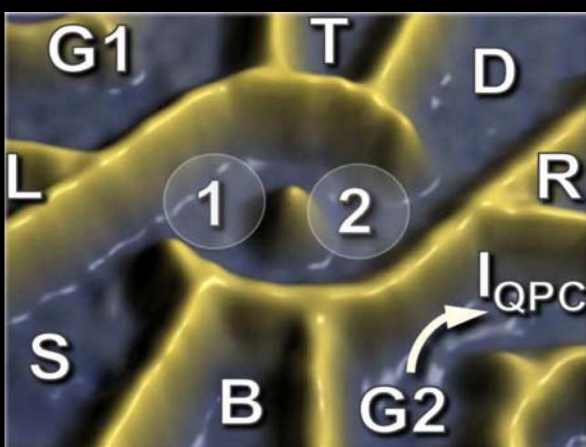
$$\hat{s}_x = |L\rangle\langle R| + |R\rangle\langle L|$$

T. Brandes *et al.*, PRL **83**, 3021 (1999)
R. Aguado *et al.*, PRL **92**, 206601 (2004)

Example: dissipative double quantum dot



Harvard 2005



ETH-Zürich 2007

Open spin-boson model

Large *U*, Large bias

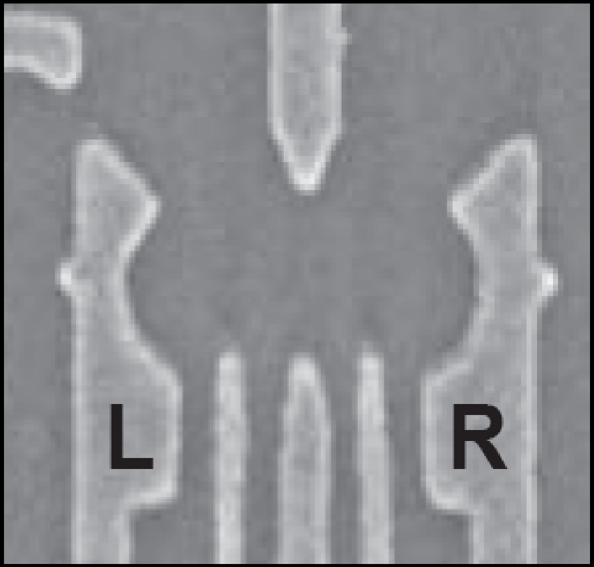
Quantum dot occupations: $\hat{\rho} = (\rho_0, \rho_L, \rho_R)^T$

$$\mathcal{W}(\chi, z) = \begin{pmatrix} -\Gamma_L & 0 & \Gamma_R e^{i\chi} \\ \Gamma_L & -\Gamma_B^{(+)}(z) & \Gamma_B^{(-)}(z) \\ 0 & \Gamma_B^{(+)}(z) & -\Gamma_B^{(-)}(z) - \Gamma_R \end{pmatrix}$$

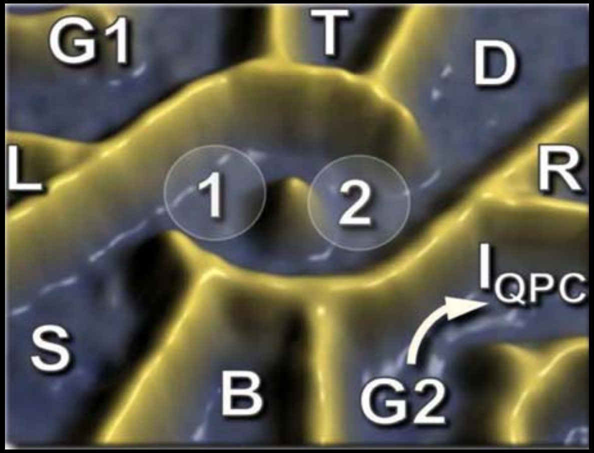
Bath-assisted hopping rates

C. Flindt *et al.*, PRL **100**, 150601 (2008) + in prep. (2009)

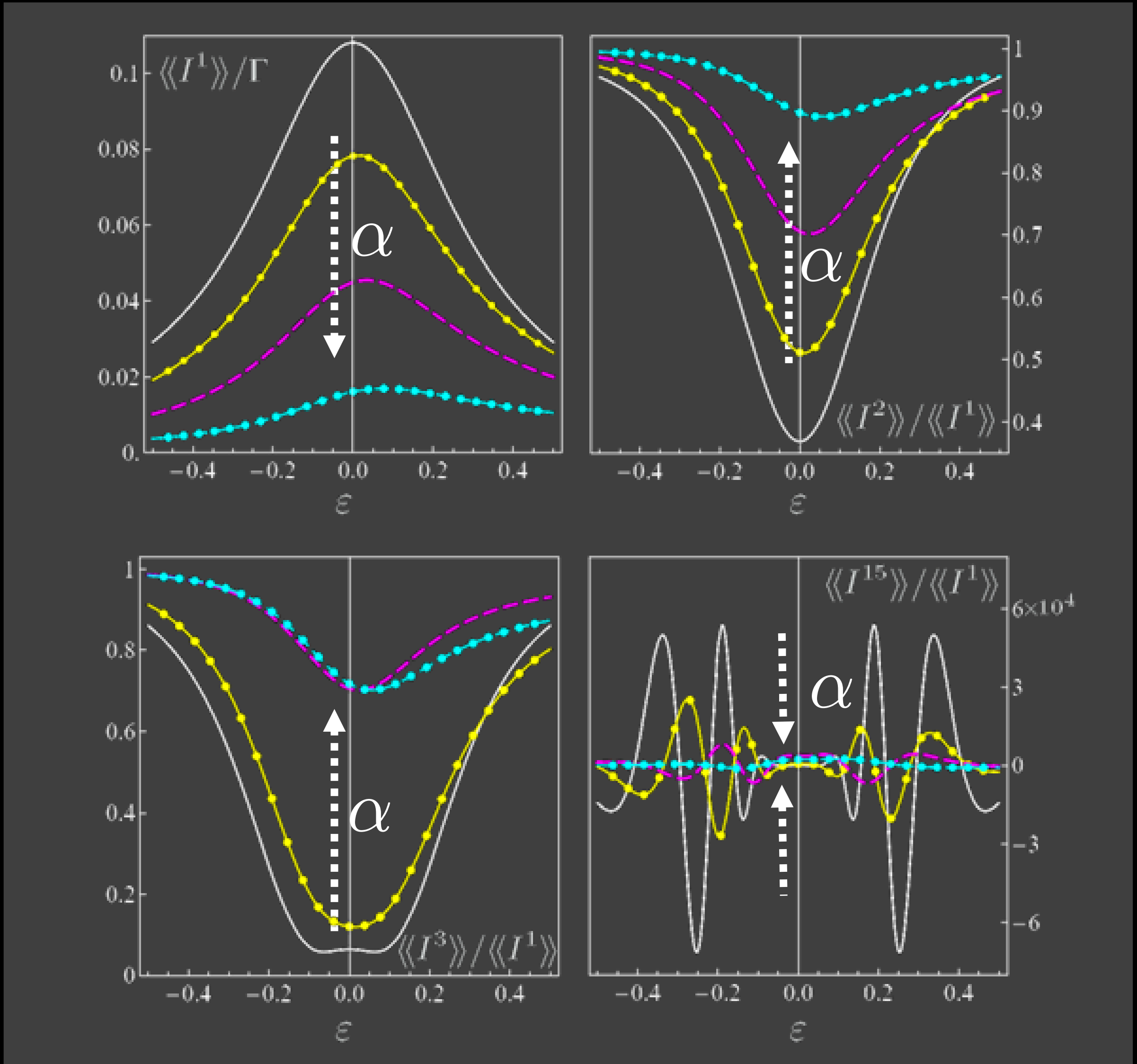
Example: dissipative double quantum dot



Harvard 2005

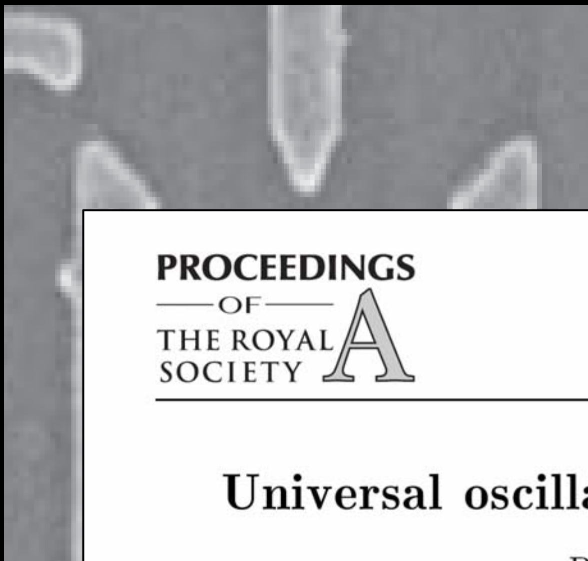


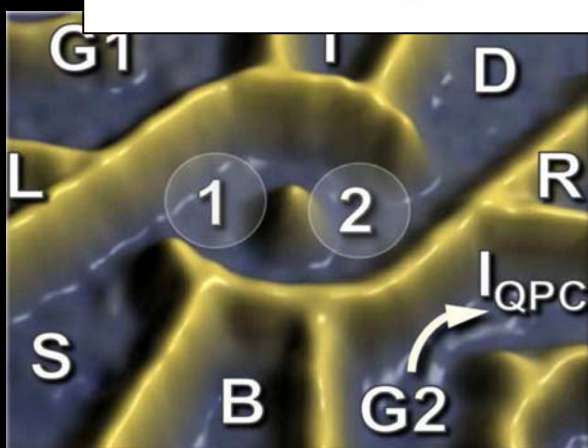
ETH-Zürich 2007



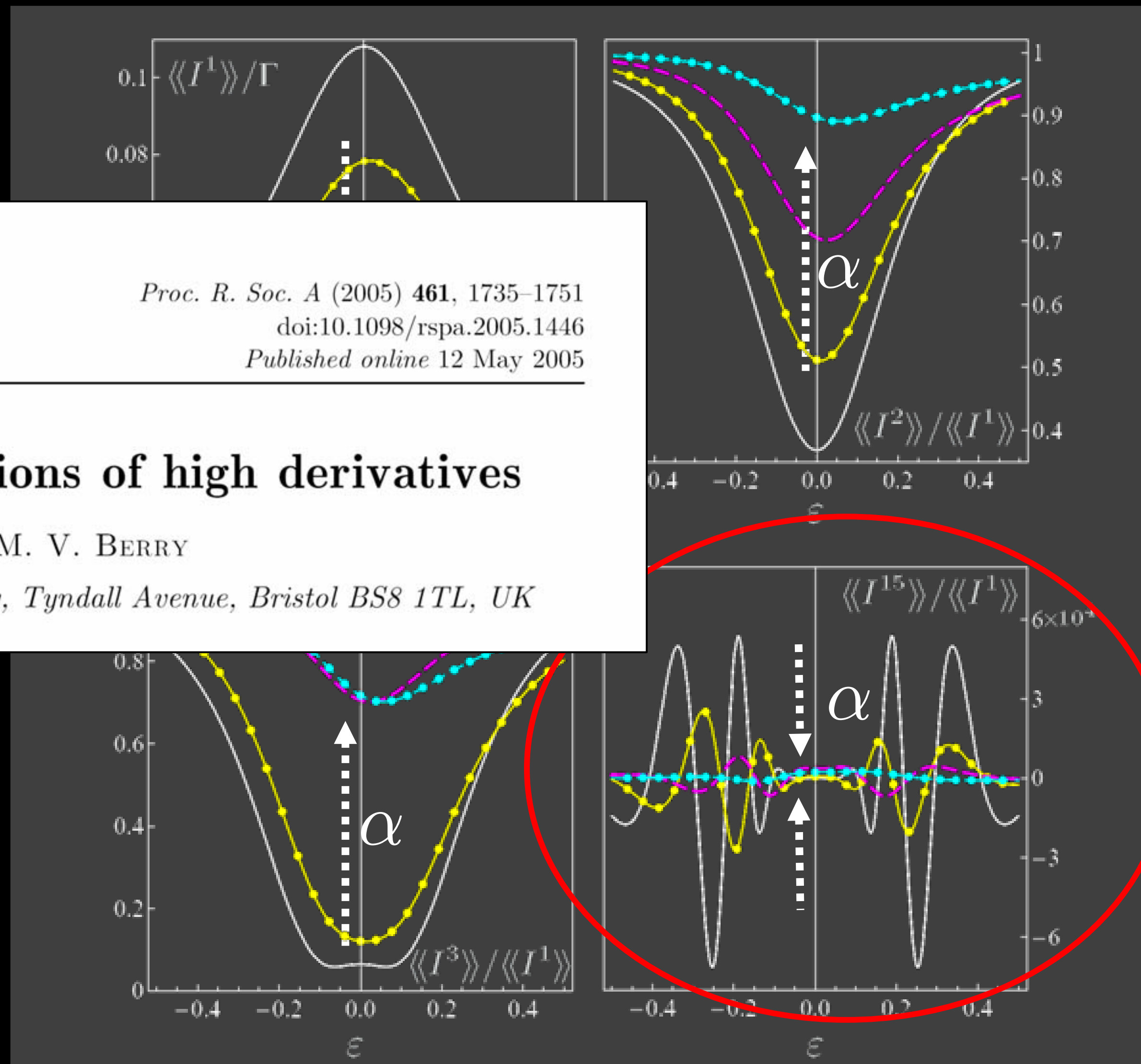
C. Flindt *et al.*, PRL **100**, 150601 (2008) + in prep. (2009)

Example: dissipative double quantum dot





ETH-Zürich 2007



PROCEEDINGS OF THE ROYAL SOCIETY A

Proc. R. Soc. A (2005) **461**, 1735–1751
doi:10.1098/rspa.2005.1446
Published online 12 May 2005

Universal oscillations of high derivatives

By M. V. Berry

H.H. Wills Physics Laboratory, Tyndall Avenue, Bristol BS8 1TL, UK

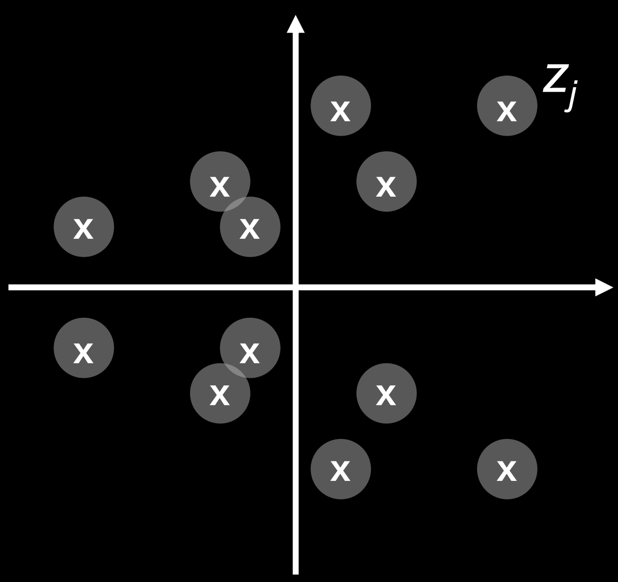
C. Flindt *et al.*, PRL **100**, 150601 (2008) + in prep. (2009)

High-order cumulants

• Cumulants are *derivatives* of the cumulant generating function (CGF)

• High-order derivatives of a function are determined by its singularities in the complex plane [M. V. Berry, Proc. R. Soc. A **461**, 1735 (2005)]

• Consider a general CGF $S(\overset{i\chi}{z},\lambda)$ with singularities at $z = z_j$, $j=1,2,3,\ldots$
Here λ denotes all relevant system parameters, including time
(entire functions are excluded – e.g. the Poisson process)



Close to a singularity z_j we can write

$$S(z,\lambda) \simeq A_j/(z-z_j)^{\mu_j}$$

for some A_j and μ_j. We then have

$$\partial_z^m S(z,\lambda) \simeq (-1)^m A_j B_{m,\mu_j}/(z-z_j)^{m+\mu_j}$$

with

$$B_{m,\mu_j} \equiv (m+\mu_j-1)(m+\mu_j-2)\ldots\mu_j$$

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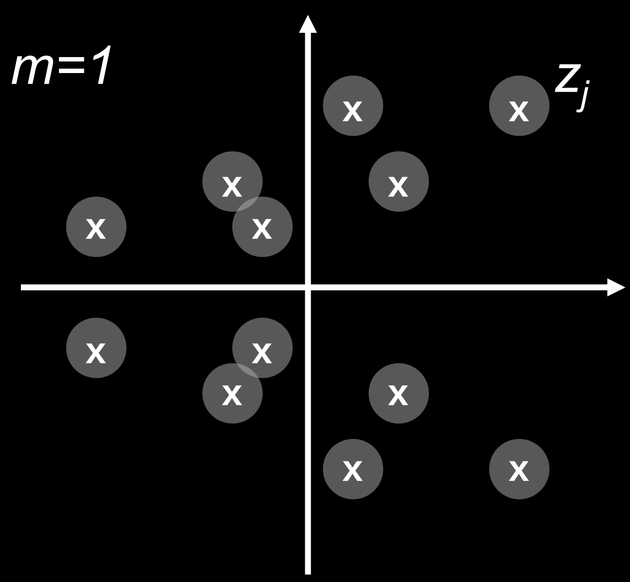

The approximation

$$\partial_z^m S(z,\lambda) \simeq (-1)^m A_j B_{m,\mu_j}/(z-z_j)^{m+\mu_j}$$

becomes increasingly better away from z_j as *m* is increased (Darboux Theorem).

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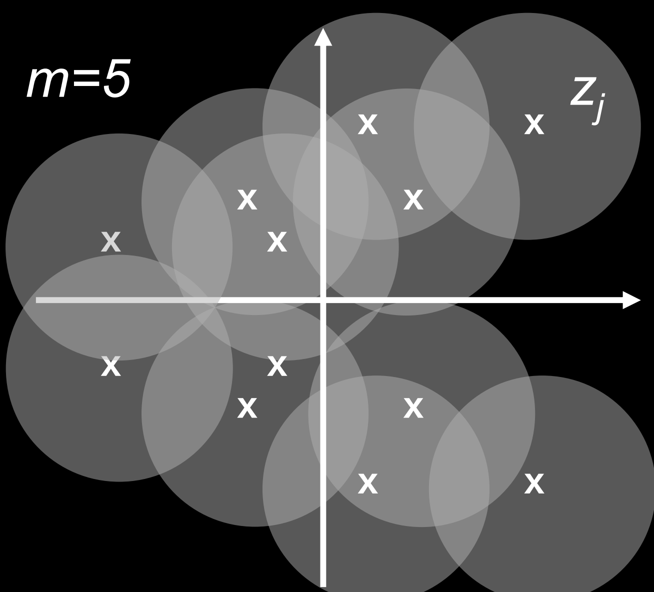

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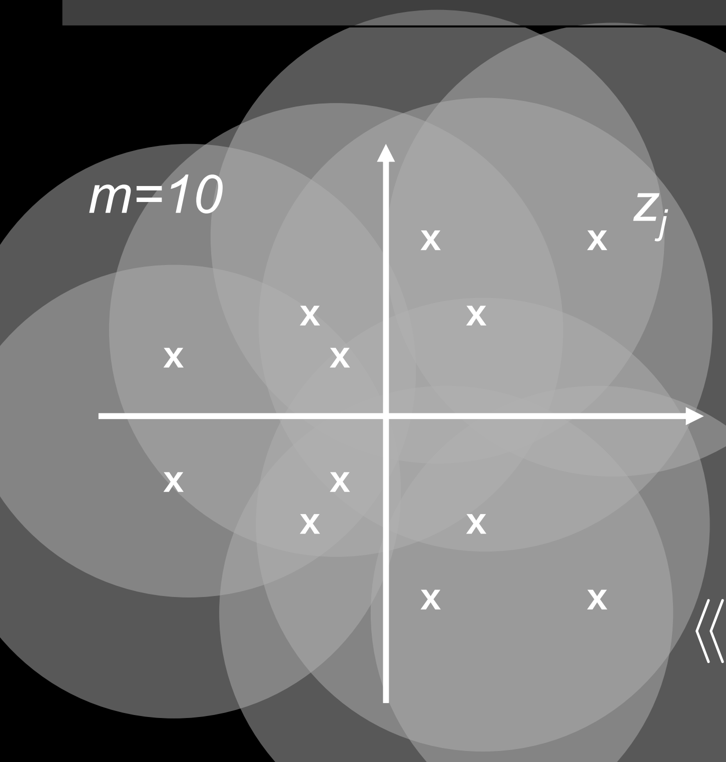
- Consider a general CGF *S(z,λ)* with singularities at $z = z_j$, *j=1,2,3,...*
Here *λ* denotes all relevant system parameters, including time (entire functions are excluded – e.g. the Poisson process)



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We thus find

$$\langle\langle n^m \rangle\rangle = \partial_z^m S(z,\lambda)|_{z\to 0} \to \sum_j \frac{(-1)^{\mu_j} A_j B_{m,\mu_j}}{z_j^{m+\mu_j}}$$

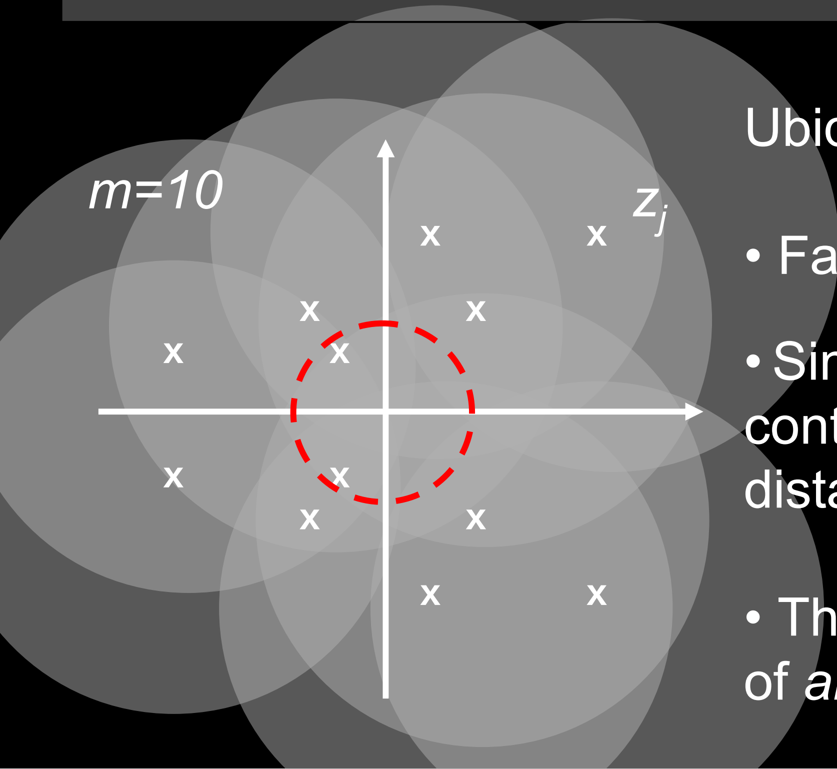
High-order cumulants

The general result

$$\langle\langle n^m \rangle\rangle \simeq \sum_j \frac{(-1)^{\mu_j} A_j B_{m,\mu_j}}{z_j^{m+\mu_j}} = \sum_j \frac{(-1)^{\mu_j} A_j B_{m,\mu_j}}{|z_j|^{m+\mu_j}} e^{-i(m+\mu_j)\phi_j}$$

shows several ubiquitous features. Here z_j=$|z_j|$exp$(i\varphi_j)$.

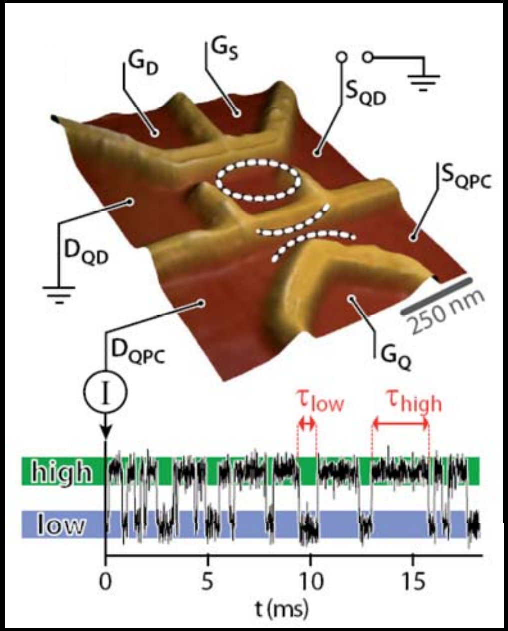
C. Flindt *et al.*, PNAS **106**, 10116 (2009)

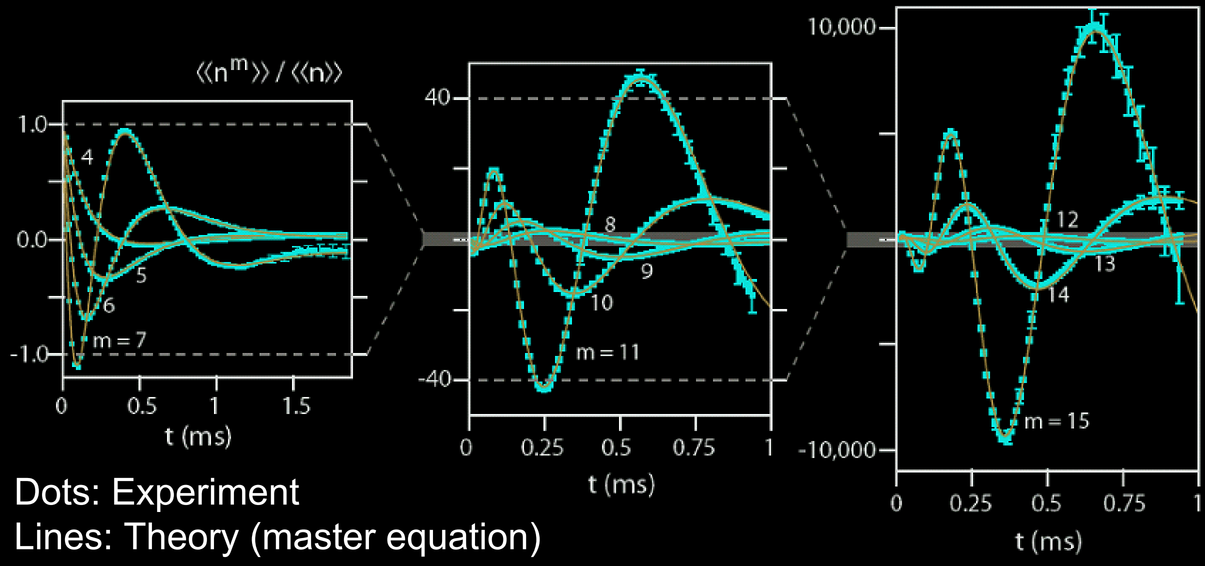


Ubiquitous features:

- Factorial growth of cumulants due to B_{m,μ_j}
- Singularities closest to 0 dominate. Other contributions are suppressed with relative distance to 0 and order *m*
- The cumulants become oscillatory functions of *any* parameter among λ that changes φ_j

Example: single quantum dot (experiment)





Dots: Experiment
Lines: Theory (master equation)

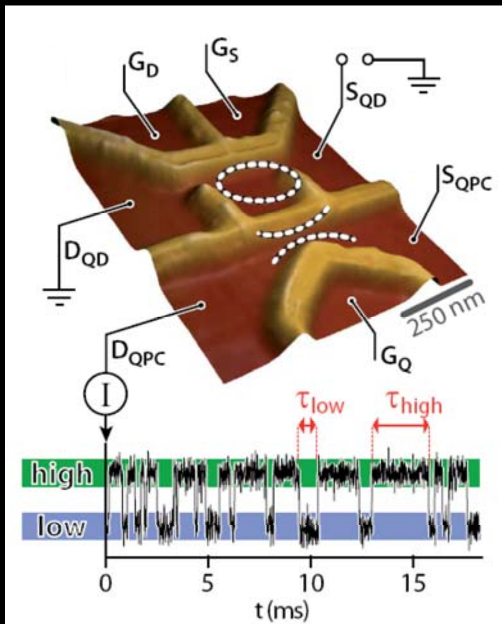
- Single electron counting using nearby quantum point contact [experiment by Hannover group (C. Fricke, F. Hohls, R. J. Haug)]

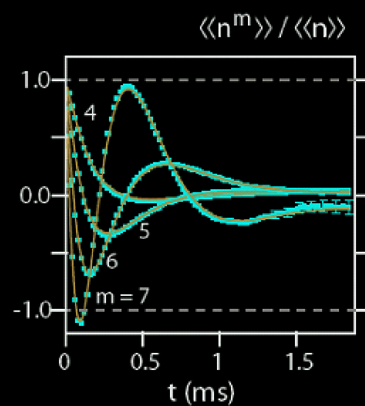
- Similar experiments at ETH-Zürich (2006) and NTT-Tokyo (2006)

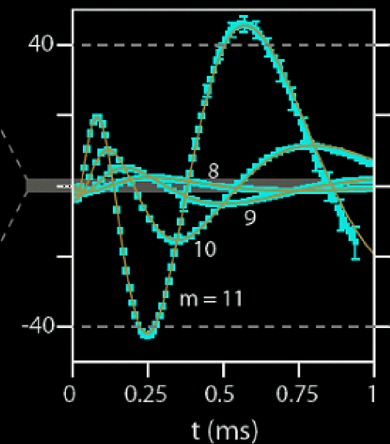
- Cumulants oscillate with time (before reaching linear-in-time regime) and grow factorially in magnitude with the cumulant order

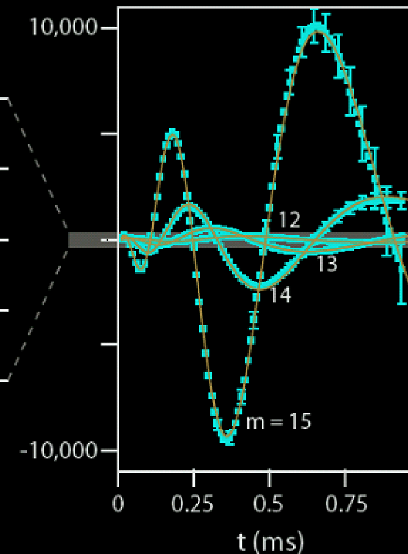
C. Flindt *et al.*, PNAS **106**, 10116 (2009)

Example: single quantum dot (experiment)

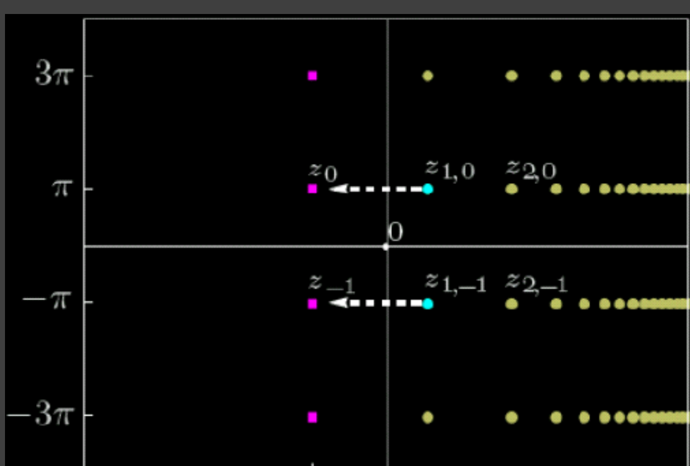






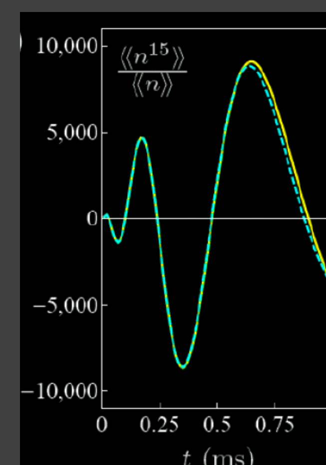


Dots: Experiment
Lines: Theory (master equation)



Full line: Master equation
Dashed line: Asymptotic exp.

$$\langle\langle n^m \rangle\rangle \simeq \sum_j \frac{(-1)^{\mu_j} A_j B_{m,\mu_j}}{z_j^{m+\mu_j}}$$



C. Flindt *et al.*, PNAS **106**, 10116 (2009)

High-order cumulants: Possible applications

PRL **100**, 086602 (2008) PHYSICAL REVIEW LETTERS week ending 29 FEBRUARY 2008

Allowed Charge Transfers between Coherent Conductors Driven by a Time-Dependent Scatterer

A. G. Abanov[1,2] and D. A. Ivanov[2]

[1]*Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794-3800, USA*
[2]*Institute of Theoretical Physics, Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland*
(Received 26 September 2007; published 28 February 2008)

We derive constraints on the statistics of the charge transfer between two conductors in the model of arbitrary time-dependent instant scattering of noninteracting fermions at zero temperature. The constraints are formulated in terms of analytic properties of the generating function: its zeros must lie on the negative real axis. This result generalizes existing studies for scattering by a time-independent scatterer under time-dependent bias voltage.

analytic properties of the generating function

- Abanov & Ivanov have determined the locations of singularities for *non-interacting* electrons [PRL **100**, 086602 (2008)]

- Oscillations of high-order cumulants reveal locations of dominating singularities and may be useful to characterize the effects of interactions (work in progress...)

High-order cumulants: Possible applications

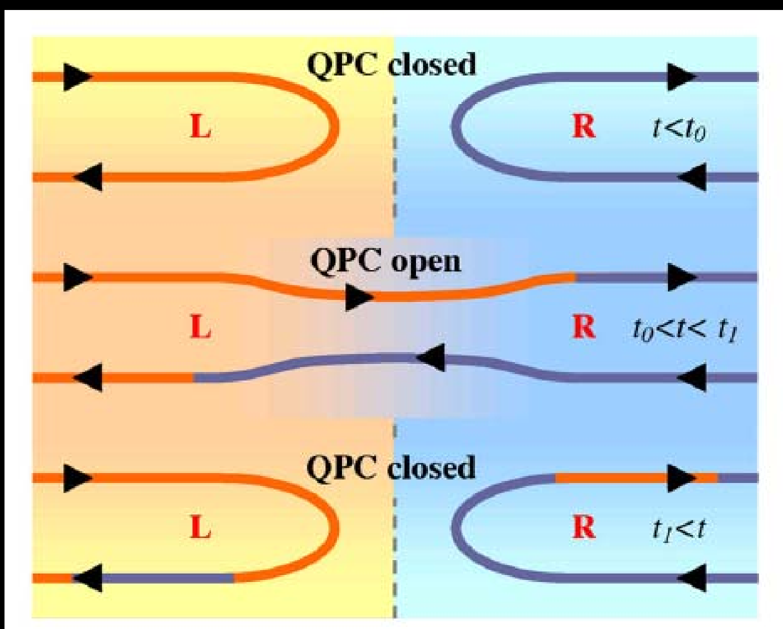
PRL **102**, 100502 (2009) PHYSICAL REVIEW LETTERS week ending 13 MARCH 2009

Quantum Noise as an Entanglement Meter

Israel Klich[1] and Leonid Levitov[1,2]

[1]*Kavli Institute for Theoretical Physics, University of California Santa Barbara, Santa Barbara, California 93106, USA*
[2]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

(Received 10 June 2008; published 9 March 2009)



Entanglement entropy generated by opening and closing a quantum point contact (QPC) can be related to cumulants:

Only even cumulants are shown to contribute to entropy:

$$\mathcal{S} = \sum_{m>0} \frac{\alpha_m}{m!} C_m, \qquad \alpha_m = \begin{cases} (2\pi)^m |B_m|, & m \text{ even} \\ 0, & m \text{ odd} \end{cases}, \quad (2)$$

where B_m are Bernoulli numbers [27] ($B_2 = \frac{1}{6}$, $B_4 = -\frac{1}{30}$, $B_6 = \frac{1}{42} \ldots$). The first few contributions are

$$\mathcal{S} = \frac{\pi^2}{3} C_2 + \frac{\pi^4}{15} C_4 + \frac{2\pi^6}{945} C_6 + \ldots. \quad (3)$$

What does the behavior of high-order cumulants imply for the entanglement entropy?

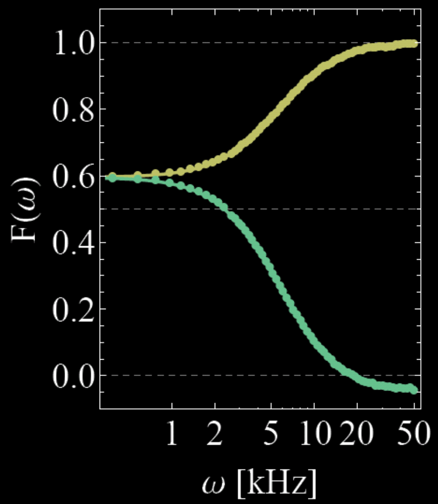
Summary

- Introduction to counting statistics
- General theory (open-quantum-systems approach)
  Example: a nano-electromechanical system (NEMS)
- Memory effects (due to environment)
  Example: dissipative double quantum dot (open spin-boson model)
- High-order cumulants
  Example: single quantum dot (experiment)

[Papers: C. Flindt *et al.,* PRL **100**, 150601 (2008), PNAS **106**, 10116 (2009), in prep. (2009)]

Outlook

- High-order cumulants, interacting vs. non-interacting
  systems, entanglement entropy
- Finite-frequency fluctuations, noise, skewness, ...
  (measurements and calculations in progress)



Thank you for your attention!