Strings and QCD

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Motivation: Use string theory to study strong interaction physics.

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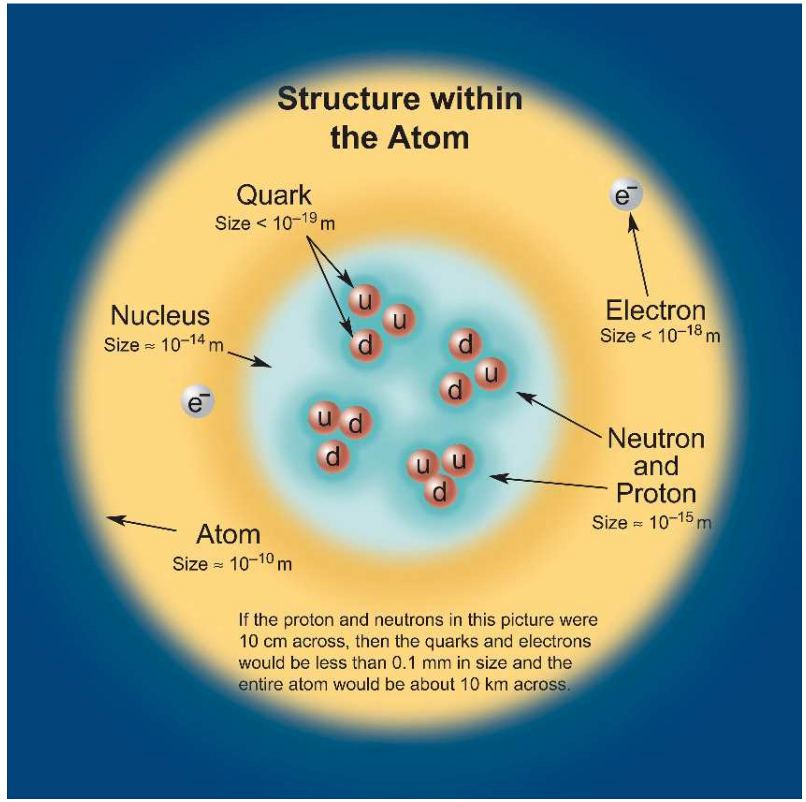
What is difficult about strong interaction physics?

Standard Model of Particle Physics



Subatomic physics

Standard Model of Particle Physics

More systematically,

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

BOSONS force carriers spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Matter

Force mediators

Properties of the Interactions

The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at 10^{-18} m	10^{-41}	0.8	1	25
Strength at 3×10^{-17} m	10^{-41}	10^{-4}	1	60

Strong interactions and QCD: why QCD is hard at low energies

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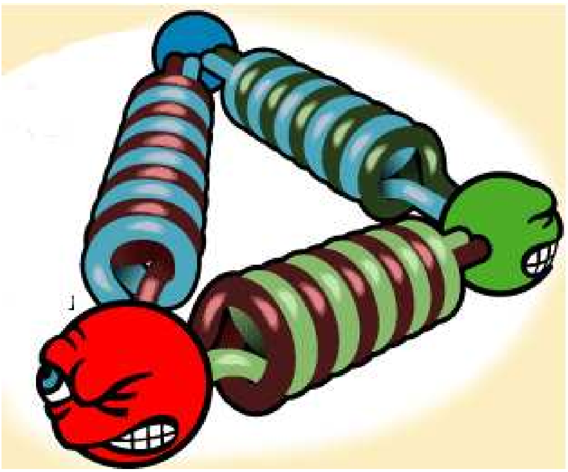

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The fundamental theory of strong interactions is called Quantum Chromodynamics (QCD). It is a gauge theory (of gluons) coupled with matter (quarks).

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The fundamental theory of strong interactions is called Quantum Chromodynamics (QCD). It is a gauge theory (of gluons) coupled with matter (quarks). Gauge theories have a built-in local symmetry:

-the gauge boson and the matter fields may transform with a local parameter

$$A^{ab}_{\mu}(X) \longrightarrow A^{ab}_{\mu}(X) + \partial_{\mu}\epsilon^{ab}(X) + \ldots, \qquad \psi^{a}(X) \longrightarrow \psi^{a}(X) + i\epsilon^{ab}(X)\psi^{b}(X)$$

but all local observables remain the same.

The Standard Model is a gauge theory based on the group

$$\underline{SU(3) \times SU(2) \times U(1).}$$

The gauge theory is described by a Lagrangian

$$L = \frac{1}{2g_{YM}^2}(\partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu])^2$$

which contains all the information about the interactions between the quanta (gauge bosons).

The coupling constant g_{YM} measures the strength of the interaction. It is not actually a constant, and it varies with the scale at which it is measured.

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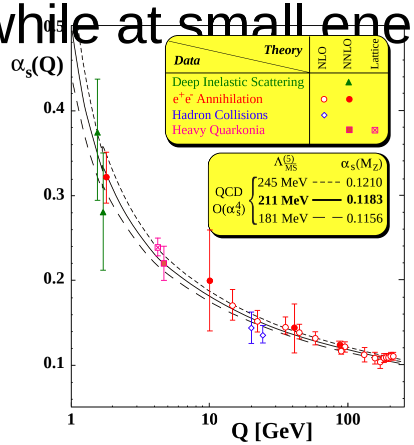

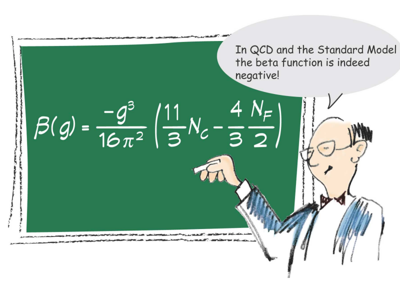

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String theory to the rescue!

Theme: Hidden inside any nonabelian gauge (Yang-Mills) theory lies a theory of quantum gravity in one extra dimension.

Holography and AdS/CFT correspondence

The starting point is a duality between a 4d conformal field theory and a theory of quantum gravity on a curved 5d background (Holography).

The meaning attributed to the extra dimension is that of energy scale.

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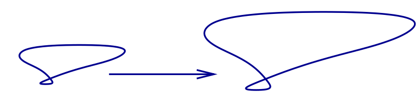
$$\mathrm{Scale\ \ Transformations:} \qquad \mathrm{X}^{\mu} \quad \longrightarrow \quad \lambda X^{\mu}$$



$$\Downarrow$$

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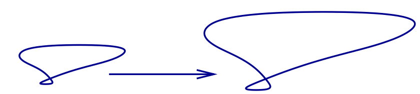

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The emerging 5d geometry which encodes the requirement of scale invariance is:

$$\boxed{ds_5^2 = r^2 dX^\mu dX_\mu + \frac{1}{r^2} dr^2 \quad \text{(anti-de Sitter or AdS)!}}\,.$$

The symmetries of the QFT dictated the symmetries of the corresponding gravity dual:

Lorentz group $SO(3,1)$ + scale transform $\longrightarrow$ conformal group $SO(4,2)$

$$\Downarrow$$

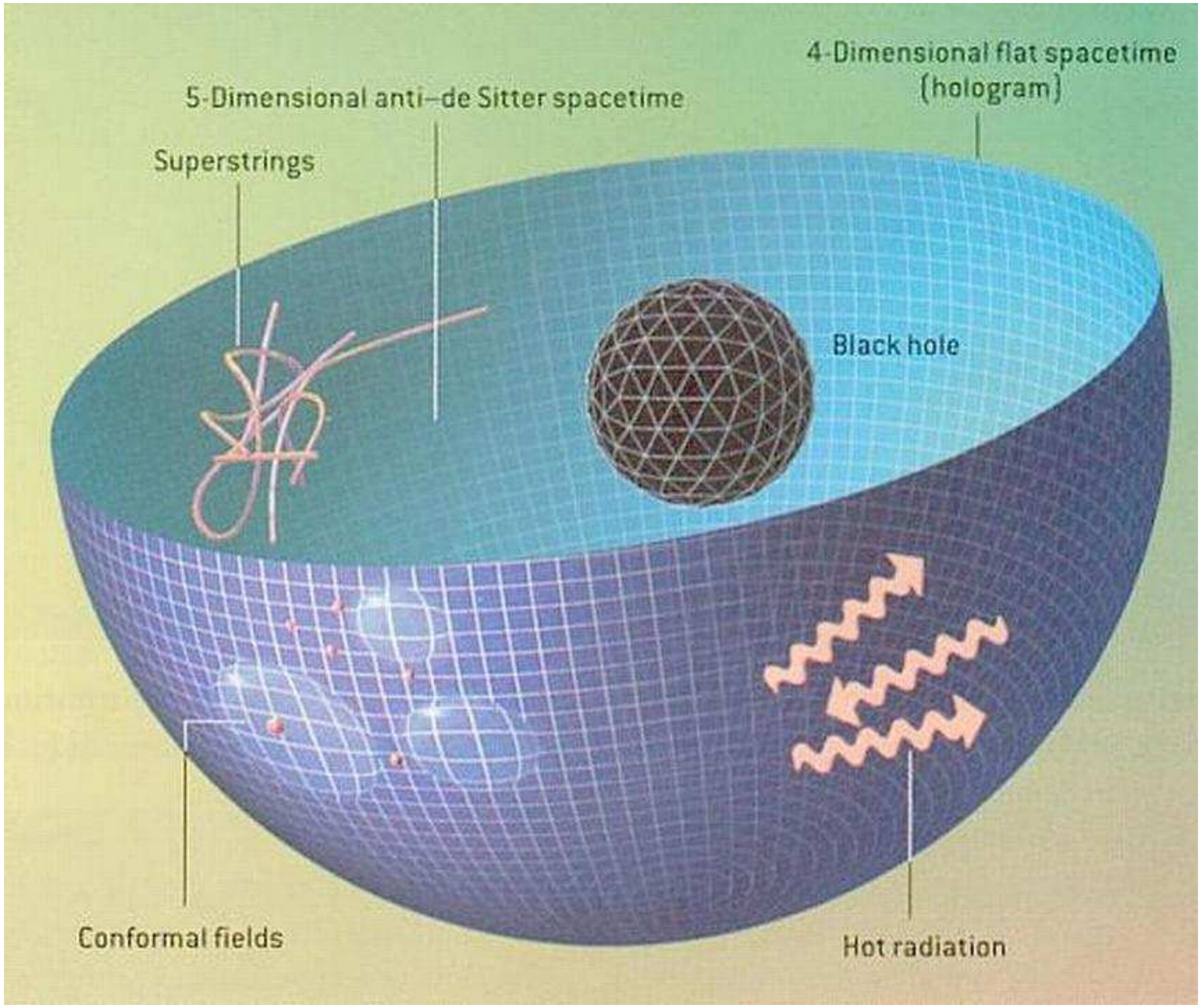
AdS space: $-U^2 - X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 =$ - R^2 $\longleftarrow$ $SO(4,2)$ invariant

$$\Downarrow$$

AdS/CFT duality

(yet to evolve into “AdS”/QCD duality)

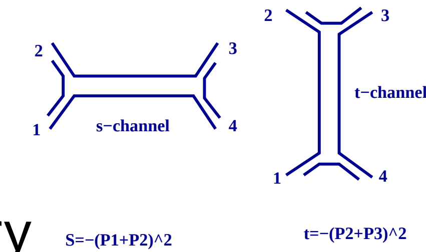
Holography


4-Dimensional flat spacetime
(hologram)
5-Dimensional anti–de Sitter spacetime
Superstrings
Black hole
Conformal fields
Hot radiation

To highlight the importance of this curved 5d geometry, let us review some earlier relations between string models and strong interaction physics.

Previous attempts and new perspectives

In the pre-QCD days, the experimental data required that the theory of strong interactions must have



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-generate Regge trajectories ($J = \alpha' E^2 + \alpha_0$).

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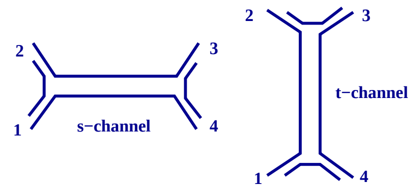

-crossing $s - t$ symmetry S=−(P1+P2)^2 t=−(P2+P3)^2 ;

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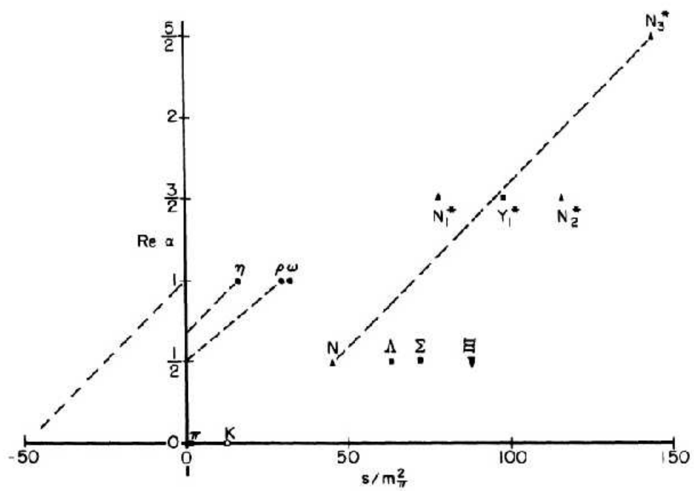

FIG. 1. The spin of particles of baryon number less than two, plotted against the square of their mass in units of m_π^2. In order to give a rough indication of slopes, the dashed lines connect pairs of points supposedly on the same trajectories, as explained in the text, but a strict linear behavior of the trajectories is not to be inferred.

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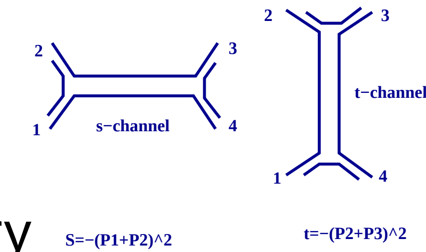

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Dual resonance models (4d flat space string models) had these properties. What went wrong?

String scattering is soft (exponential tail): $A_{dual}(p) \sim e^{-\# \cdot p}$ What is actually observed in QCD is a power law behaviour $A(p) \sim p^{4-\Delta}$ (hard scattering on partons).

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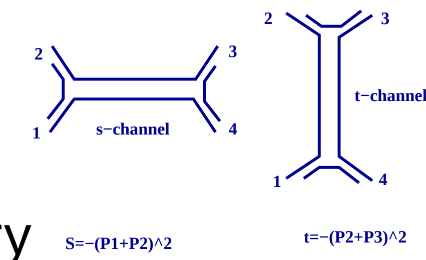

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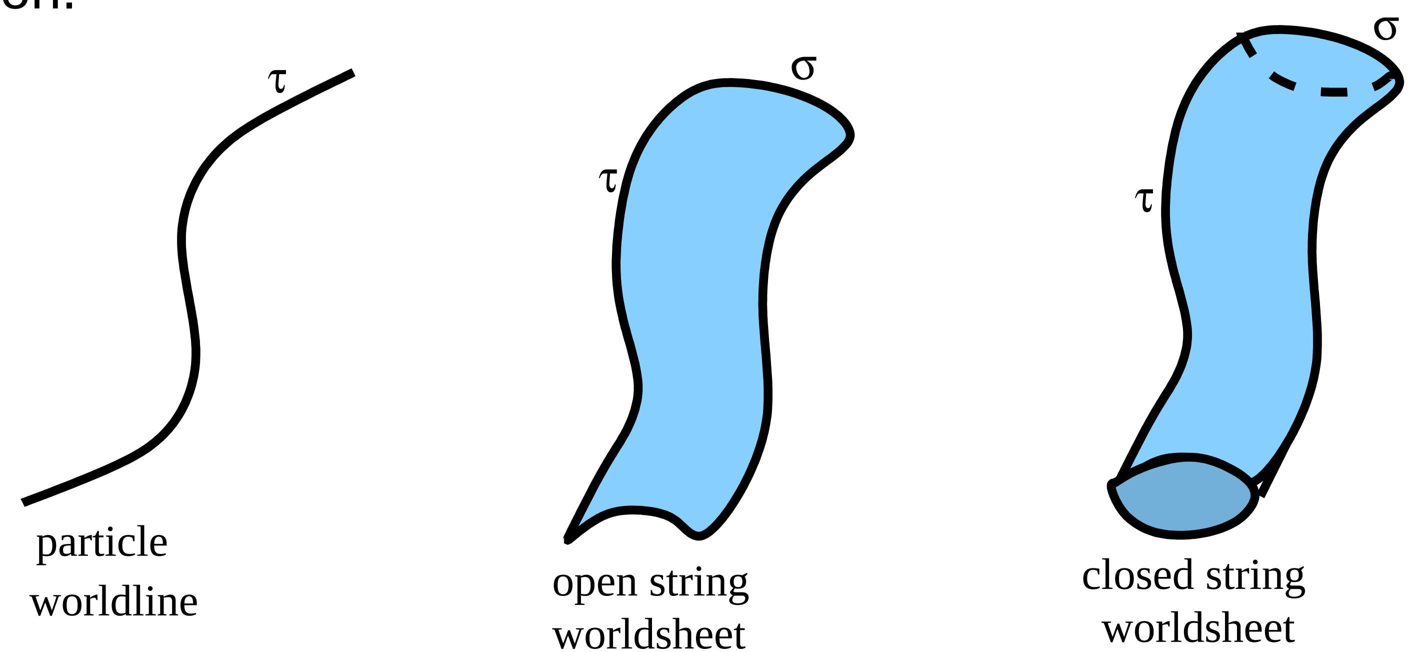
A crucial role is played by the curved AdS geometry:

$$A(p) = \int dr\, r^3 A_{dual}(\frac{p}{r}) \left(\frac{r_{min}}{r} \right)^{\Delta} \sim p^{4-\Delta}. \qquad \surd$$

String theory in 5 minutes or less.

Strings, branes and supersymmetry: living in 10d

String theory: quantum theory of objects extended in one spatial dimension.



String modes of vibrations give rise to the spectrum of particles.

Open string: massless spin 1 particle$\equiv$ gauge boson, tower of massive particles

Closed string: massless spin 2 particle$\equiv$ graviton, etc...

To introduce fermions in a theory of strings, one uses supersymmetry.
Supersymmetry is a global symmetry which pairs bosons and fermions:

$$\text{supermultiplet} \equiv (b, f): \qquad \delta f = \epsilon \partial b, \qquad \delta b = \epsilon f$$

$$\Rightarrow \text{"}\delta^2\text{"}\, b \;=\; \text{"}\epsilon^2\text{"}\, \partial b$$

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Quantum consistency requires that <u>superstrings live in 10d</u>.

The low energy limit of superstrings

-a supersymmetric gauge theory ${A_\mu}^1, {\lambda_\alpha}^{1/2} \equiv \text{gaugino}$, for open strings;

-a supersymetric generalization of Einstein gravity (supergravity)

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Particles $\longrightarrow$ Strings $\longrightarrow$ more?

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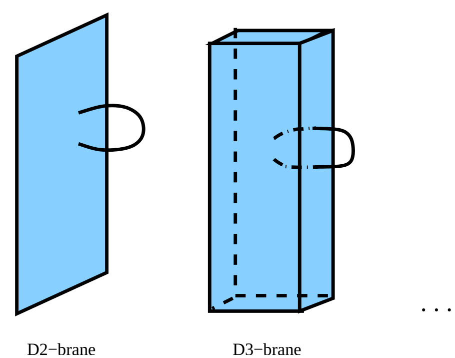
Particles $\longrightarrow$ Strings $\longrightarrow$ Branes:



Holography: from 5d to 10d

A more refined statement of the AdS/CFT duality:

The 4d maximally susy gauge theory with gauge group $SU(N)$,
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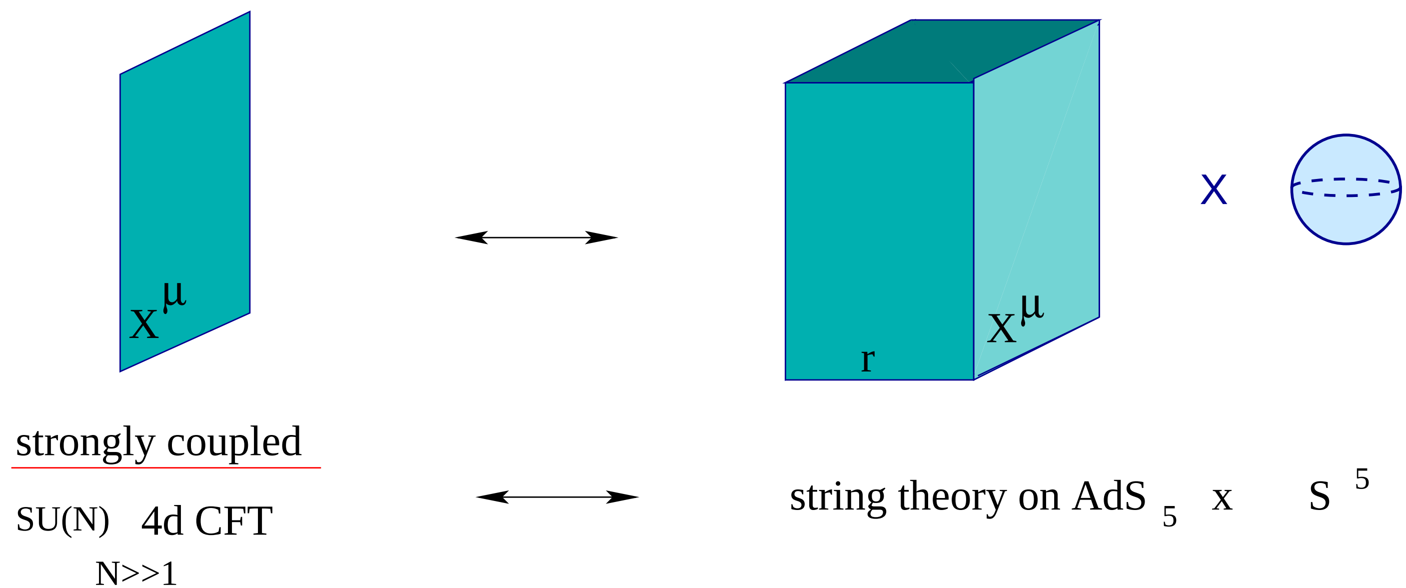
The duality proceeds with the identifications:

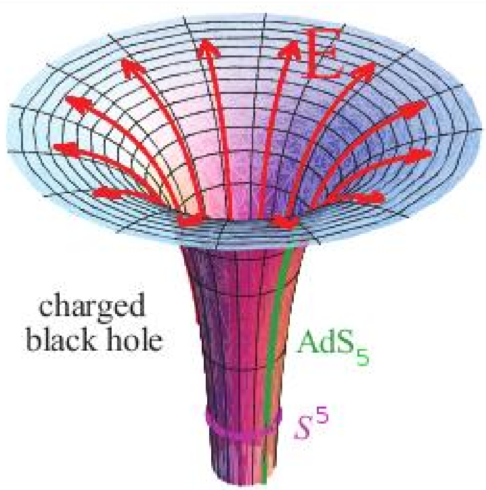
$$\boxed{\frac{g_{YM}^2}{4\pi} = g_s, \quad g_{YM}^2 N \alpha'^2 = R^4 \quad \text{where } R_{AdS} = R_{S^5} \equiv R \gg 1}$$

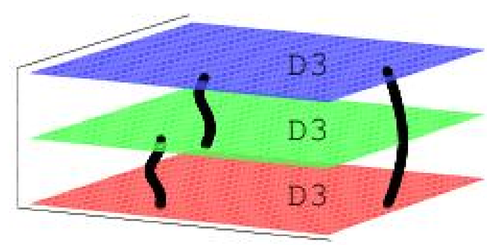


Argument leading to the AdS/CFT duality:

Consider a stack of N D3 branes:

Solutions of supergravity	Dynamical objects
$ds^2_{10} = h(r)^{-1/2} dX^\mu dX_\mu + h(r)^{1/2}(dr^2 + r^2 d\Omega_5^2)$ $F_{(5)} = dh(r)^{-1} dX^0 \wedge dX^2 \cdots \wedge dX^4$ $h(r) = 1 + \frac{R^4}{r^4}$	gauge fields A_μ, 6 scalars $\Phi^{m=1,2\ldots 6}$, 4 spin 1/2 fermions, all in the adjoint rep of SU(N)





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Closed strings on $AdS_5 \times S^5$ $\equiv$ 4d maximally susy Yang Mills theory

$SO(4,2) \times SO(6)$ conformal symmetry $\times$ global SO(6)

What does the AdS/CFT equivalence imply?

It has to be a one-to-one correspondence: the complete set of data characterizing $\mathcal{N} = 4$ sYM (operators, correlators) must have a a counterpart on the $AdS_5 \times S^5$ sugra/string side.

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-correlators of chiral primary operators can be computed using the supergravity partitition function $Z = \exp(S_{sugra})$;

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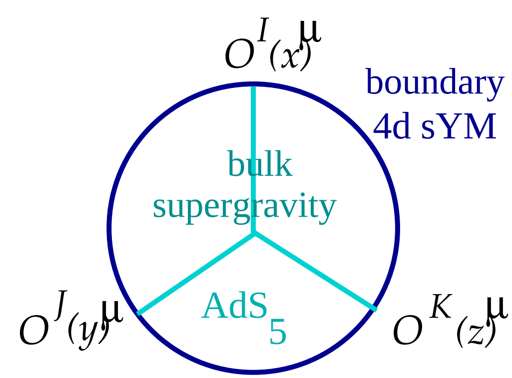
4d sYM $\quad O^I(x^\mu) = C^I_{i_1 i_2 \cdots i_n} \, Tr(\Phi^{i_1}\Phi^{i_2}\dots\Phi^{i_n})(x^\mu)$

$AdS_5 \times S^5 \quad \delta h(x,y) = h^I(x) \, C^I_{i_1 i_2 \cdots i_n} \, Y^{i_1} Y^{i_2} \dots Y^{i_n}(y)$

$S^5: \quad (Y_1)^2 + (Y_2)^2 + \dots + (Y_6)^2 = R^2$

$$Z_{sYM} = Z_{sugra}[h_0] = \langle e^{\int h_0^I O^I(x^\mu)} \rangle_{sYM}$$

$$h^I(x) \longrightarrow h_0^I(x^\mu)$$



Nontrivial checks: the plane wave limit

The states of $\mathcal{N} = 4$ sYM include more than just supergravity modes.

Since it is not known how to find the string spectrum in curved backgrounds, it is difficult to prove the conjecture.

There is the additional challenge of extrapolating from weak to strong coupling.

Nontrivial checks: the plane wave limit

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However, we fare better in the limit of semiclassical approximation.

-identify classical string configurations in the AdS background, with large quantum numbers;
-expand in fluctuations around the classical configuration
-identify the CFT operators corersponding to the various string modes
-proceed to check the AdS/CFT correspondence

The geometry seen by the fluctuations of a string with large angular momentum on S^5 is a plane wave. The full spectrum of string modes can be found.

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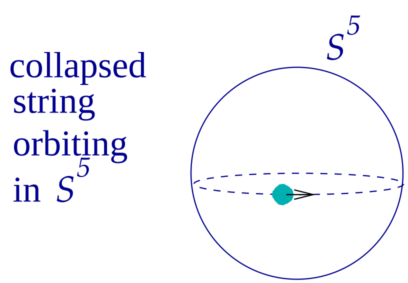
It leads to a non-susy protected match of the string states in the plane wave geometry and CFT operators $Tr(Z^l X Z^{J-l} X) e^{2\pi i\, l\, n/J}$ and their correlators.

Classical configuration:



Geometry as seen by the fluctuations:

Plane wave

$$ds^2 = -4\, dx^+ dx^- - \mu^2\, dx^{+2}\, \vec{z}^{\,2} + d\vec{z}\, d\vec{z}$$

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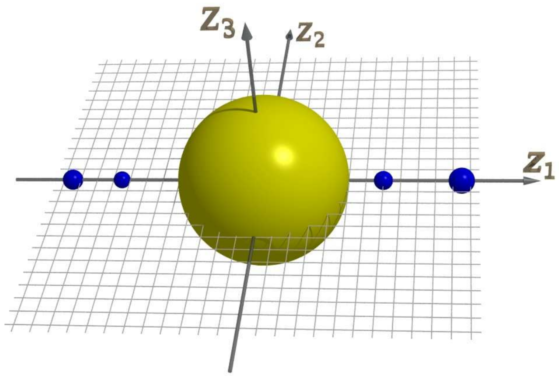
-correlators of chiral primary operators can be computed using the supergravity partititon function $Z = \exp(S_{sugra})$;

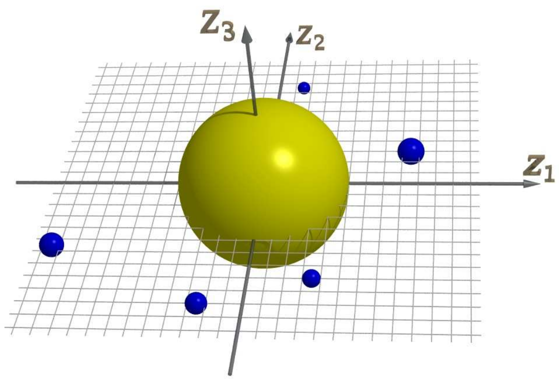
-for heavy 1/2 BPS states, the correspondence involves a class 1/2 BPS supergravity backgrounds ("bubbling AdS") which asymptote to AdS.

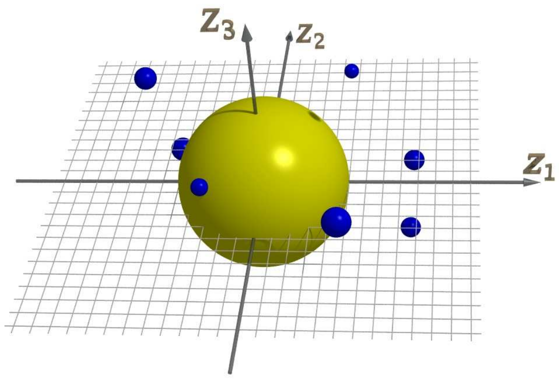
$$\Downarrow$$

A growing set of evidence that the conjectured duality is correct.

Gravity Duals to N=4SYM States







To state the equivalence slightly differently:

4d maximally susy $SU(N)$ sYM re-organizes itself, <u>in the limit $N \gg 1$ and at strong coupling</u>, as a theory of strings.

From AdS/CFT towards “AdS”/QCD. . .

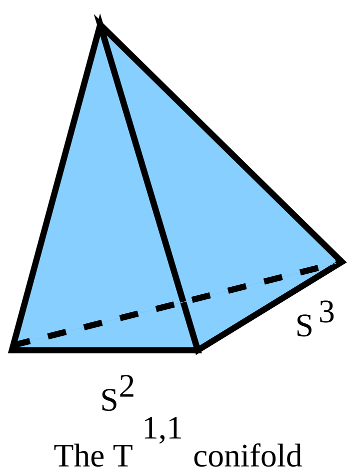
Towards a theory of confinement

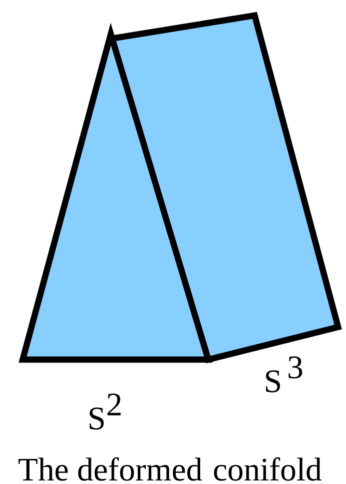
To obtain more realistic gauge models via the gauge/string duality, one needs to "tweak" the $AdS_5 \times S^5$ geometry:

-break some supersymmetry by placing the D3 branes at orbifold singularities (tip of the conifold);

-add wrapped D5 branes to further break the conformal symmetry;

-deform the conifold to excise some unwanted singularities: this theory confines!





-or, consider finite temperature D4-branes compactified on a circle: below the KK scale, the dual 4d gauge theory is non-supersymmetric and confines.

What are the criteria for a supergravity background dual to a confining gauge theory?

Confining gauge theory: the area law for the Wilson loop

$$< W[C] = Tr\mathcal{P}\exp\int_C dx^\mu A_\mu > \ \sim \ \ \exp(-T Area_{\ C})$$

For a rectangular loop of width L and length t, the $Q\bar{Q}$ potential is $V(L) = -(\ln W[C])/t \sim L$.

A sufficient condition for confinement in sugra:

With the 5d bulk $ds^2 = h(r)^{1/2} dX^\mu dX_\mu + h^{-1/2} dr^2$, then there is a value r_0 of the bulk radial coordinate s.t.

$$\partial_r h(r)\Big|_{r_0} = 0, \qquad h(r_0) \neq 0$$

$r = r_0$ $\equiv$ confining wall

What can we actually compute in the dual string theory?

E.g. Regge trajectories for glueballs and mesons, hadronic density of states,...

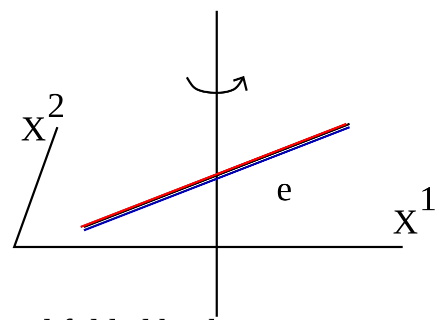
Disclaimer: since no actual dual to large $SU(N)$ QCD theory was identified, rather we have a class of backrounds which are dual to confining gauge theories, for the time being we can use the outcome of these calculations only to build up a common trend.

Glueball Regge trajectories from AdS/QCD

Glueballs $\equiv$ closed string configurations

Glueballs with large spin $\equiv$ closed spinning string configurations, with $J \gg 1$

Flat space picture:



Closed folded back spinning string

$$X^0_{cls} = e\tau, \qquad X^1_{cls} = e\cos(\tau)\cos(\sigma), \qquad X^2_{cls} = e\sin(\tau)\cos(\sigma)$$

The Regge trajectories:

$$E_{cls} = \int_0^{2\pi} d\sigma P^0_{cls} = \frac{1}{2\pi\alpha'}\int_0^{2\pi} d\sigma \partial_\tau X^0_{cls} = \frac{1}{\alpha'} e$$

$$J_{cls} = \int_0^{\pi} d\sigma (X^1 P^2 - X^2 P^1)_{cls} = \frac{1}{2\alpha'} e^2 = \frac{1}{2}\alpha' E^2_{cls}$$

The glueball Regge trajectories can be extracted using the gauge/string duality: flat space no longer appropriate! need a curved geometry: the confining supergravity background.

Use the semiclassical quantization method:
-classical spinning string configurations:
glueball $\equiv$ a folded back closed spinning string sitting at $r = r_0$
-find linear Regge trajectory: $J = \frac{1}{2}\alpha'_{eff}E^2, \;\; \alpha'_{eff} = \alpha'/g_{00}(r_0)$;
-compute quantum corrections to the Regge trajectories.

In any confining backgrounds: $\boxed{J = \alpha_1 E^2 + \alpha_0 + \alpha_{1/2}E}$

Pomeron trajectory $\alpha(t) = 1.10 + 0.25GeV^{-2}t + 0.079GeV^{-4}t^2$; qualitative agreement: $\alpha_0 > 0$ and positive curvature.

So far, we’ve learned how to deal with a strongly coupled theory of pure glue (plus some exotic scalars which were part of the supersymmetric package), using a theory of strings.

Can quarks (color charged spin 1/2 fermions) be accounted for?

Where are the quarks?

The AdS/CFT correspondence was derived by decoupling the open strings.

Quarks (transforming in the fundamental rep of SU(N)) are represented by open strings with one end on the D3 branes whose backreaction yielded the AdS geometry.
To bring back the open strings, one adds probe branes which extend in radial AdS direction.

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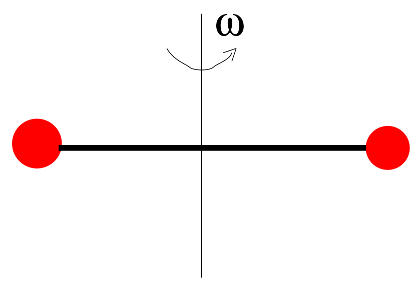
A phenomenological model of the meson: a spinning open string with massive end-points.



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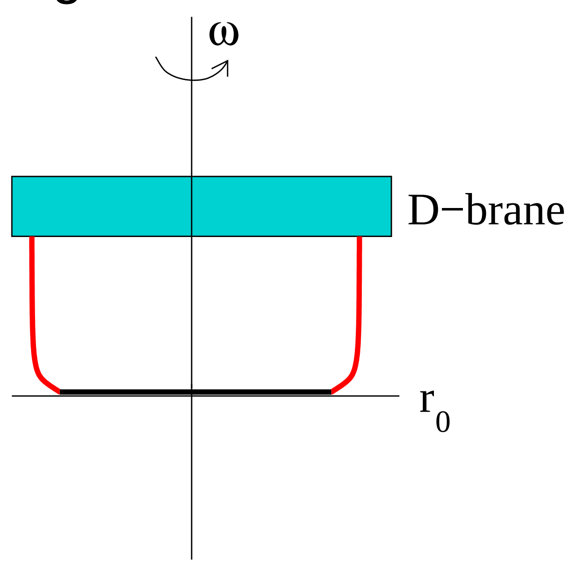
For a string derivation of this model, introduce probe branes in the confining supergravity background of choice.



Solve the classical string eom and find a U-shaped spinning string configuration. The corresponding Regge trajectories $J(E^2)$ get a correction due to the “quark masses” (“vertical arms”)

$$
\begin{aligned}
E &= \frac{2T_g}{\omega}\left(\arcsin x + \frac{1}{x}\sqrt{1-x^2}\right) \\
J &= \frac{2T_g}{\omega^2}\left(\arcsin x + \frac{3}{2}x\sqrt{1-x^2}\right), \quad x = \text{speed of the endpoints}
\end{aligned}
$$

⇒: the mass-loaded Chew-Frautschi formula.

$$
\text{Regge regime } x \to 1: \qquad J = \frac{1}{\pi T_g}E^2\left(1 + \frac{\sqrt{2}}{\pi}\left(\frac{m_Q}{E}\right)^{3/2} + \frac{\pi-1}{\pi}\frac{m_Q}{E} + \ldots\right)
$$

where $m_Q = (1-x^2)T_g/(\omega x)$.

Limitations:

With probe branes we are forced to consider only cases $N_f \ll N_c$. From the dual gauge theory this limit corresponds to neglecting loops of virtual quarks (quenched approximation).

Can one do better?

Yes, include the backreaction of the probe branes.

Idea: modify the AdS background by including the supergravity fields that are sourced by the probe branes.

Any Dp-brane is charged under a certain supergravity field which is a $p+1$ form.

Program:

-construct D3-D7 brane solutions which have non-trivial 5-form fields and non-trivial axion-dilaton (which is dual to an 8-form);

-supersymmetry implies that the space transverse to the D3's "X" is Kahler: flat, conifold (modified by the D7 branes),...

$$ds^2 = h^{-1/2}(r)(-dt^2 + d\vec{x}^2) + h^{1/2}(r)ds_X^2$$

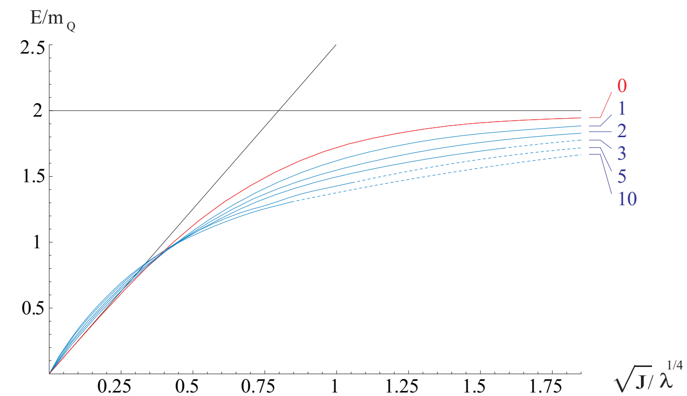
and solve the Einstein equations for $h(r)$.

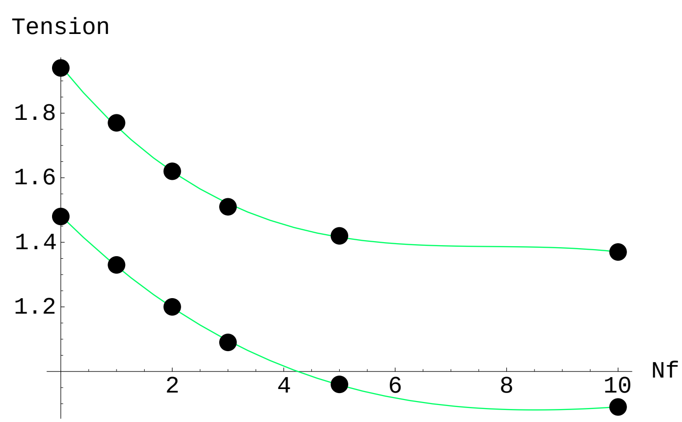
-take the decoupling limit: $\alpha' \to 0, \qquad g_{Dp}^2 = g_s \alpha'^{(p-3)/2}$

$\Rightarrow$ The D7 brane dynamics decouples: only the D3-D7 ("quarks") and D3-D3 ("glue") survive.

Next consider spinning strings in the backreacted geometries.

The "meson" Regge trajectories show a screening of the color charge (a modified flux tube tension) due to virtual quark effects:





Finite temperature AdS/CFT

Zero temperature -> Finite temperature in AdS/CFT

Field theory side

$\exp(-\beta H)$: time $\rightarrow$ euclidean time $\rightarrow$ β= period of euclidean time = 1/T

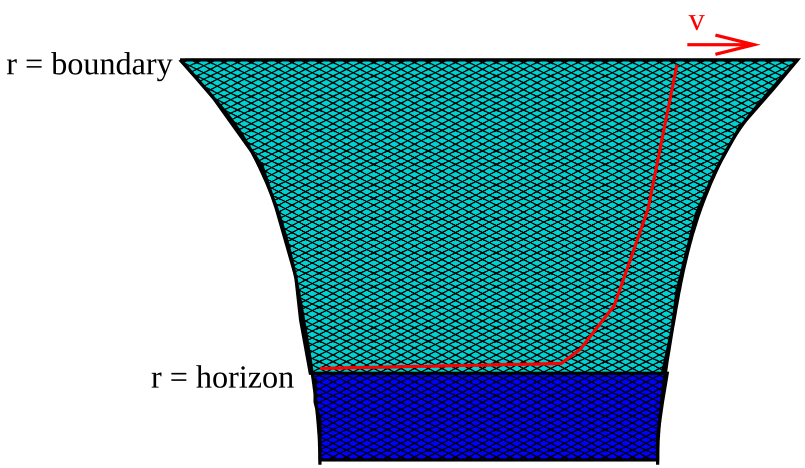
Gravity side: time $\rightarrow$ euclidean time

Two geometries, both with the same asymptotics: Euclidean AdS (low temperature phase) and AdS-Schwarzschild black hole (high temperature phase).

Horizon radius $\rightarrow$ Hawking temperature (same as that of the QFT)

Zero temperature: AdS $\leftrightarrow$ vacuum state in the QFT.

High temperature AdS-S $\leftrightarrow$ thermal vacuum state in the QFT (think strongly coupled quark-gluon plasma).



Perturbative QCD can also benefit from string based methods.

String theory and perturbative QCD

An unexpected connection between tree level QCD and string theory:

-susy string theory on twistor space can be used to evaluate tree level QCD amplitudes;

-the string calculation reduces to localizing the amplitude onto curves of a certain degree in twistor space;

-the Feynman perturbative expansion can be reorganized in a more efficient way, using MHV vertices;

-on-shell recurrence relations were also found as a bonus:

$$\boxed{\mathcal{A}(P,\{P_i\},Q,\{Q_j\}) = \sum_{i,j} \mathcal{A}_L(\hat{P},\{P_i\},K)\frac{1}{(P+\sum_i P_i)^2}\mathcal{A}_R(\hat{Q},\{Q_j\},K)\,,}$$

where $\mathcal{A}_L, \mathcal{A}_R$ are lower n-point functions obtained by isolating two reference gluons with shifted momenta, $\hat{P} = P - z\eta$, $\hat{Q} = Q + z\eta$ with $\eta^2 = \eta \cdot P = \eta \cdot Q = 0$, on the two sides of the cut.

Recurrence relations in QCD

With hindsight, the on-shell recurrence relations could have been found directly within the framework of QFT.

Recurrence relations in QCD

Useful set-up: space-cone gauge (a generalization of the more standard light-cone gauge): $\eta^\mu A_\mu = 0$, with a space-like complex η^μ;

The recurrence relation origin lies in the cutting rules in the QFT. Another crucial role is played by the existence of a an ordering (largest time equations) among a sequence of space-time points.

Recurrence relations in QCD

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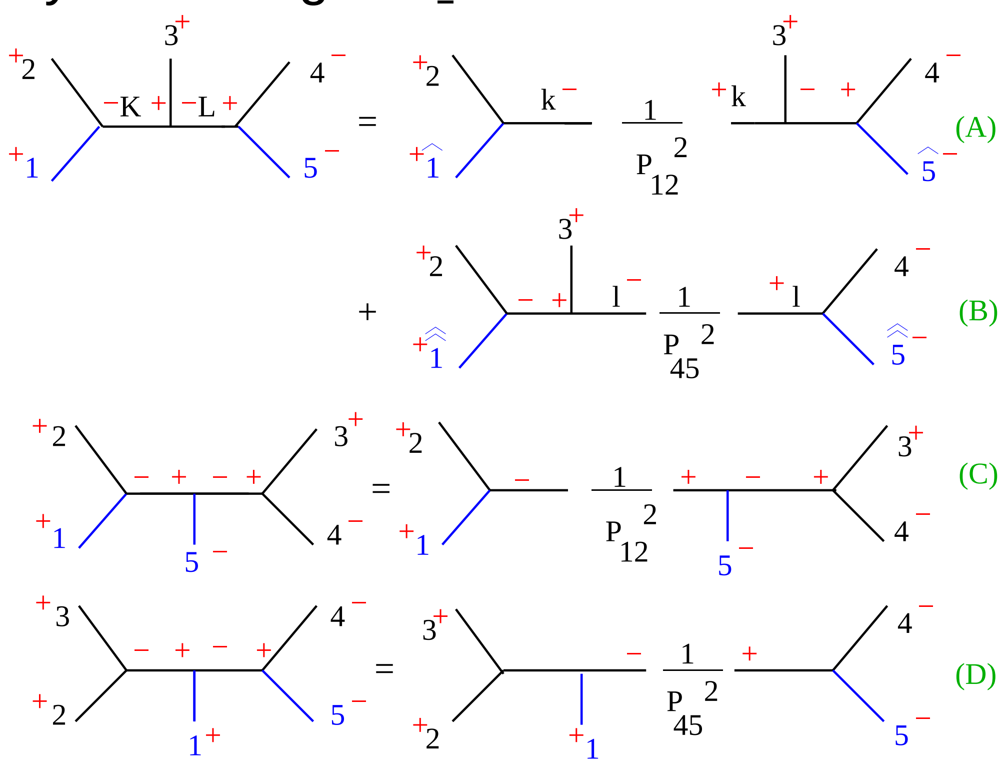
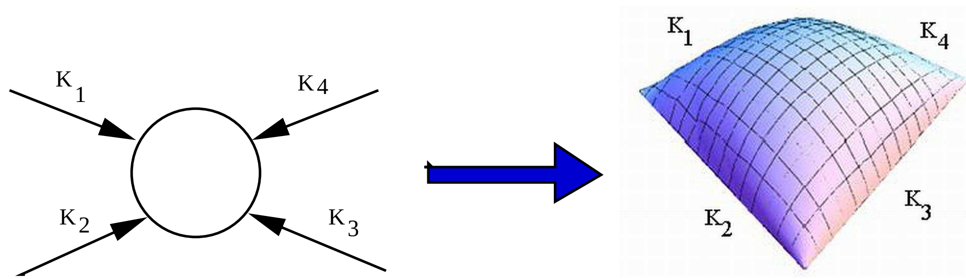
The recurrence relation origin lies in the cutting rules in the QFT. Another crucial role is played by the existence of a an ordering (largest time equations) among a sequence of space-time points. Then it follows that each individual Feynman diagram factorizes:

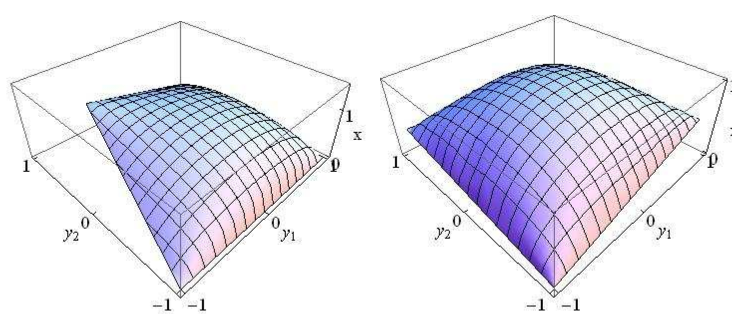


Scattering amplitudes at strong coupling

Alday&Maldacena: Map the scattering data (momenta of external particles) into boundary data for a minimal area string worldsheet. For 2→2 gluon ($gg \to gg$) scattering one recovers the BDS ansatz obtained in perturbation theory, from extrapolating the first few lowest loop amplitudes.



For massive particle scattering, $qq \to gg$ and $qq \to qq$, the boundary polygon lines are tilted in the bulk of AdS.



Perspectives and future directions

String theory on AdS-like geometries emerges as an alternative description of strongly coupled gauge theories ($N \gg 1$).

This offers the computational means to address hadronic physics (QCD at low energies, in the limit when $N = 3$). We saw that it is possible to derive some of the strong interaction features from the string dual.

String inspired methods to address perturbative QCD are also a valid alternative to standard Feynman diagramatic expansion. This is important for LHC, where the QCD background must be identified first.

Based on

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- **Bit strings from N=4 gauge theory**, with H. Verlinde, JHEP 0311: 041,2003, hep-th/0209215
- **Tracing the string: BMN correspondence at finite** J^2/N, with J. Pearson, M. Spradlin, H. Verlinde, A. Volovich , JHEP 0305:022,2003, hep-th/0210102
- **Hadronic density of states from string theory**, with L.A. Pando Zayas, Phys.Rev.Lett.91: 111602, 2003, hep-th/0306107
- **Holographic duals of flavored N=1 super Yang-Mills: beyond the probe approximation**, with B.A. Burrington, J. T. Liu, L.A. Pando Zayas, JHEP 0502:022,2005, hep-th/0406207
- **Regge trajectories for mesons in the holographic dual of large** N_c **QCD**, with M. Kruczenski, L.A. Pando Zayas, J. Sonnenschein, JHEP 0506: 046, 2005, hep-th/0410035
- **Bubbling 1/4 BPS solutions in type IIB and supergravity reductions on** $S^N \times S^N$, with J.T. Liu, W.Y. Wen, Nucl.Phys.B739: 285, 2006, hep-th/0412043
- **Bubbling 1/2 BPS solutions of minimal six-dimensional supergravity**, with J.T. Liu, Phys.Lett.B642: 411, 2006, hep-th/0412242.
- **The D3 / D7 background and flavor dependence of Regge trajectories**, with I. Kirsch, Phys.Rev.D72: 026007, 2005, hep-th/0505164
- **QCD recursion relations from the largest time equation**, with Y.P. Yao, JHEP 0604:030,2006, hep-th/0512031
- **Supersymmetric branes on** $AdS_5 \times Y^{P,Q}$ **and their field theory duals**, with F. Canoura, J.D. Edelstein, L.A.Pando Zayas , A. V. Ramallo, JHEP 0603:1 01, 2006, hep-th/0512087
- **Bubbling AdS and droplet descriptions of BPS geometries in IIB supergravity**, with B. Chen, S. Cremonini, A. Donos, F.L. Lin, H. Lin, J.T. Liu, W.Y. Wen, JHEP 0710:003, 2007, arXiv:0704.2233 [hep-th]
- **“Analytic Scattering Amplitudes for QCD”**, with Y. P. Yao, Mod. Phys. Lett. A **23**, 847 (2008), arXiv:0805.3351 [hep-th]
- **“The Space-Cone Gauge, Lorentz Invariance and On-Shell Recursion for One-Loop Yang-Mills amplitudes”**, with Y. P. Yao, arXiv:0805.2645 [hep-th]
- **“Massive quark scattering at strong coupling from AdS/CFT”**, with E. Barnes, arXiv:0911.0010 [hep-th], submitted JHEP

The hadronic density of states

In the '60s Hagedorn inferred that the hadronic density of states grows like $n(E) = exp(\beta_H E)$, where $\beta_H \sim 1/T_H$ ($T_H \equiv$ Hagedorn temperature). The free energy in the confined phase is given by a one-loop string computation: $Z = Tr(e^{-\beta P^0}) = \int dE\, n(E) e^{-\beta E}$.

Any confining sugra background allows for a winding string classical configuration at the confining wall:

$$X^0 = X^0_{cls} + X^0_{qu} = \beta(m\sigma_1 + n\sigma_2) + X^0_{qu}$$

The asymptotic density of states, as read from the string dual, is

$$n(E) = e^{\#E/\sqrt{T_g}} \qquad \surd$$

where $T_g = \frac{g_{00}(r_0)}{2\pi\alpha'}$ is the tension of the $Q\bar{Q}$ flux tube.

At T_H the system undergoes a confinement/deconfinement phase transition.