Novel magnetism in ultracold atomic gases

Austen Lamacraft

UNIVERSITY of VIRGINIA

Virginia, February 2010

faculty.virginia.edu/austen/

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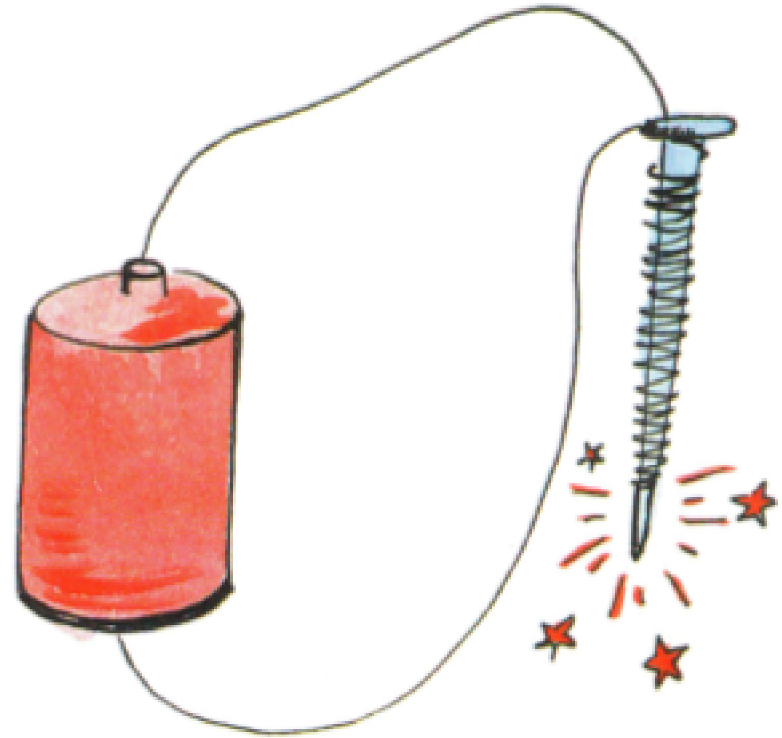

Electromagnet

Novel magnetism in ultracold atomic gases

Austen Lamacraft

Dynamics and Statistical Mechanics of Multicomponent Quantum Fluids

Virginia, February 2010

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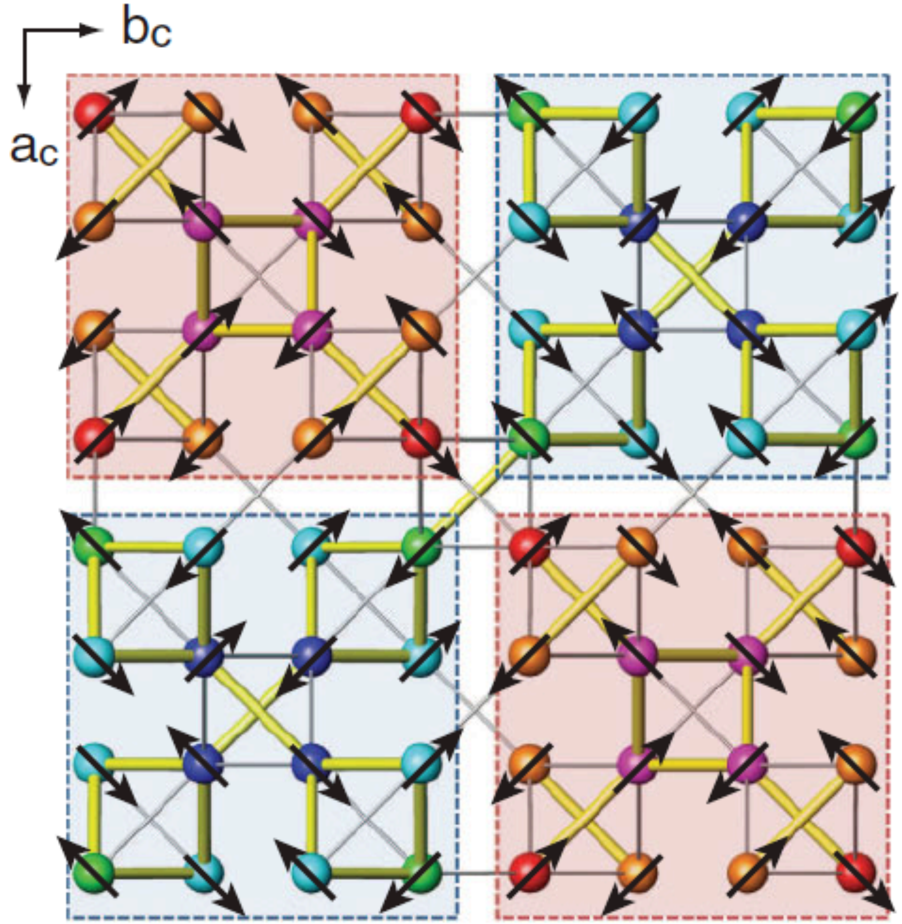
Outline

- Magnetism in Bose condensates
  - Phases of spin 1 bosons
  - Ferromagnetic and polar states
- The dynamics of the Bose ferromagnet
  - Superfluid flow
  - Equations of motion
  - Dipolar interactions
- Statistical mechanics of the polar state
  - Vortices, domain walls, and phase transitions

Exotic magnetism - solid state

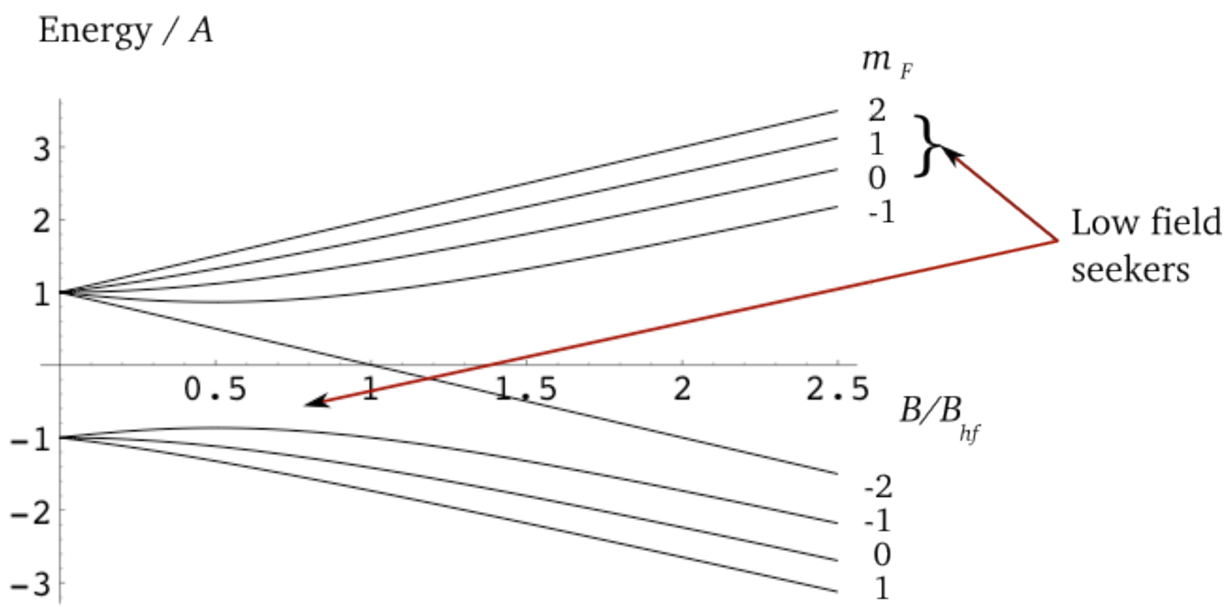


$ZnCr_2O_4$

– Ji *et al.*, PRL (2009)

Exotic magnetism - atomic gas

- ^{87}Rb is a *boson* with I=3/2, S=1/2
  - Possible total spin F=1 or 2



- What are magnetic properties of F=1 or 2 Bose gas?

Recent observation of *Fermi* magnet

Itinerant Ferromagnetism in a Fermi Gas of Ultracold Atoms

Gyu-Boong Jo,[1*] Ye-Ryoung Lee,[1] Jae-Hoon Choi,[1] Caleb A. Christensen,[1] Tony H. Kim,[1] Joseph H. Thywissen,[2] David E. Pritchard,[1] Wolfgang Ketterle[1]

– Science 325, 1521-1524 (2009)

Ultracold atomic gases

- Fe becomes ferromagnetic at T=1043 K
- Ultracold atomic physics takes place at <10^{-6} K

Quantum effects determine collective (i.e. material) properties when

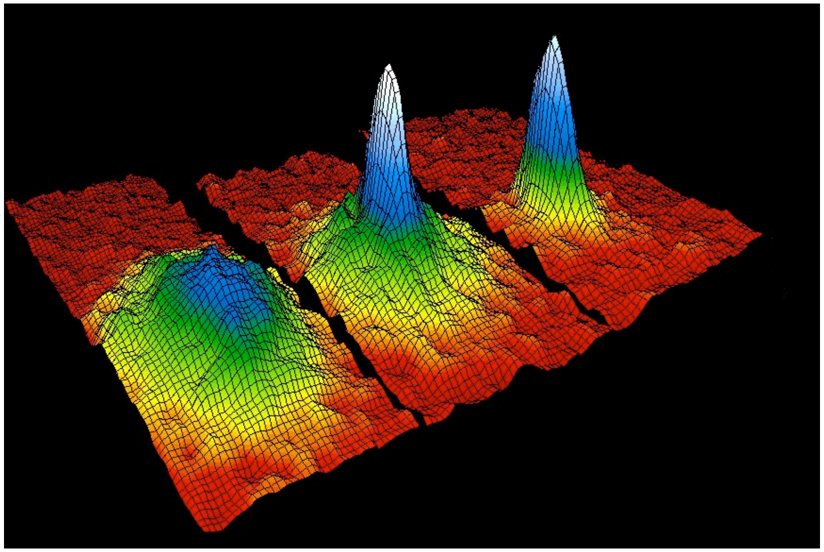
thermal wavelength ≈ interparticle separation

$$k_B T \sim \frac{\hbar^2 n^{2/3}}{m}$$

- Atomic gases are *heavier* and *less dense* than gas of electrons in Fe

So what's new?

- In the solid state we (mostly) care about the quantum mechanics of electrons. These are *fermions*
- By contrast, atoms (considered as particles) may be *bosons* or *fermions*
- Possibility of *Bose-Einstein condensation* - bosons accumulate in lowest energy state



Nobel prize 2001

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Magnetism in Bose gases

- BEC: (nearly) all atoms sit in same quantum state
  - This state ϕ is called the *condensate wavefunction*
- But what if lowest energy state is degenerate?

Condensate wavefunction is a spin vector (*spinor)* and *must* pick a direction in spin space

Bose condensates with spin are *always* magnets

Why higher spin is fun

- Spin 1/2 (e.g. of electron) points in some direction

$$\phi = \begin{pmatrix} e^{-i\phi/2}\cos\theta/2 \\ e^{i\phi/2}\sin\theta/2 \end{pmatrix}$$

  – To make electron magnetism more interesting can invoke non-trivial arrangements on lattice (e.g. Néel)

- Spin 1 doesn’t necessarily “point” anywhere

$$\phi = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

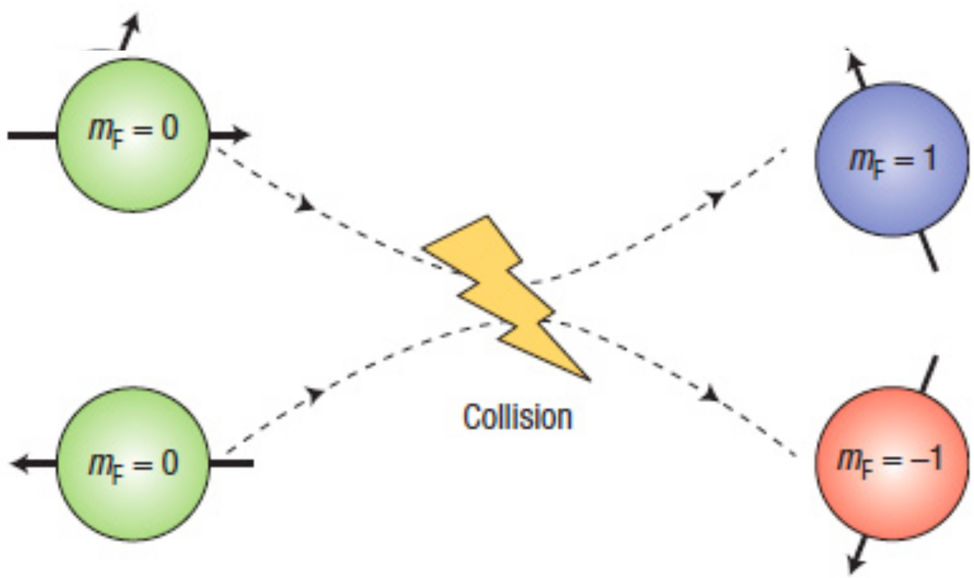
$$\phi^\dagger(\hat{\mathbf{m}}\cdot\mathbf{S}^{(1)})\phi_1 = 0$$

– $\mathbf{S}^{(1)}$ spin-1 matrices

  – Yet evidently there is still a *director* or *nematic* axis!

Which spin state wins?

- Must consider interatomic interactions
- Atoms can collide with total spin 0 or 2
  - Total spin 1? Antisymmetric and blocked by Bose statistics



spin 0

Spin dependent interactions

$$
\begin{aligned}
H_{\text{int}} &= \sum_{i<j} \delta(\mathbf{r}_i - \mathbf{r}_j)(g_0 \mathcal{P}_0 + g_2 \mathcal{P}_2) \\
&= \sum_{i<j} \delta(\mathbf{r}_i - \mathbf{r}_j)(c_0 + c_2 \mathbf{S_i} \cdot \mathbf{S_j})
\end{aligned}
$$

$$
\begin{aligned}
c_0 &= (g_0 + 2g_2)/3 \\
c_2 &= (g_2 - g_0)/3
\end{aligned}
$$

- Energy of state $\phi_{m_1}(\mathbf{r}_1)\cdots\phi_{m_N}(\mathbf{r}_N)$ includes a piece

$$
\boxed{\langle H_{\text{spin}} \rangle = \frac{Nnc_2}{2}(\phi^\dagger \mathbf{S} \phi) \cdot (\phi^\dagger \mathbf{S} \phi)}
$$

  - For $c_2 < 0$ (e.g. ^{87}Rb): maximize $\phi^\dagger \mathbf{S}^{(1)} \phi$ ***Ferromagnet***
  - For $c_2 > 0$ (e.g. ^{23}Na): minimize $\phi^\dagger \mathbf{S}^{(1)} \phi$ ***Polar state***

Mean field ground states: spin 1

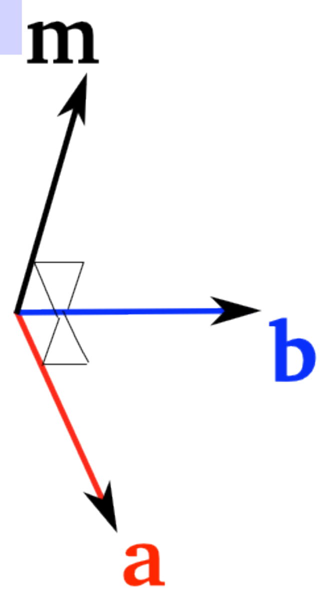
- Work in *cartesian* components where

$$\left(S_i^{(1)}\right)_{jk} = -i\epsilon_{ijk}$$

$$\phi = \mathbf{a} + i\mathbf{b}$$

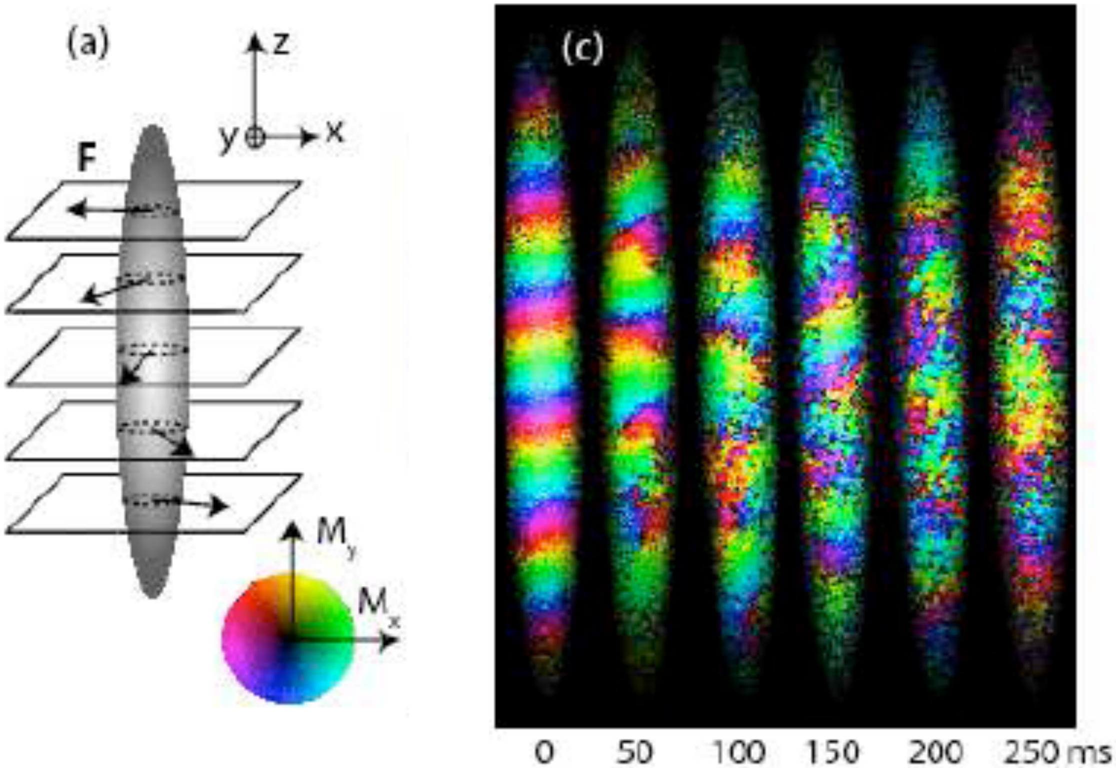
$$\mathbf{m} = \phi^\dagger \mathbf{S}^{(1)} \phi = 2\mathbf{a} \times \mathbf{b}$$

- **Ferromagnet**
  - $\phi^\dagger \mathbf{S}^{(1)} \phi$ maximal for $\mathbf{a} \perp \mathbf{b}$
  - $\mathbf{m}, \mathbf{a}, \mathbf{b}$ form orthonormal triad



- **Polar state**
  - $\phi^\dagger \mathbf{S}^{(1)} \phi$ minimal for $\mathbf{a} \parallel \mathbf{b}$

The Bose ferromagnet: ^{87}Rb



– Stamper-Kurn group, Berkeley

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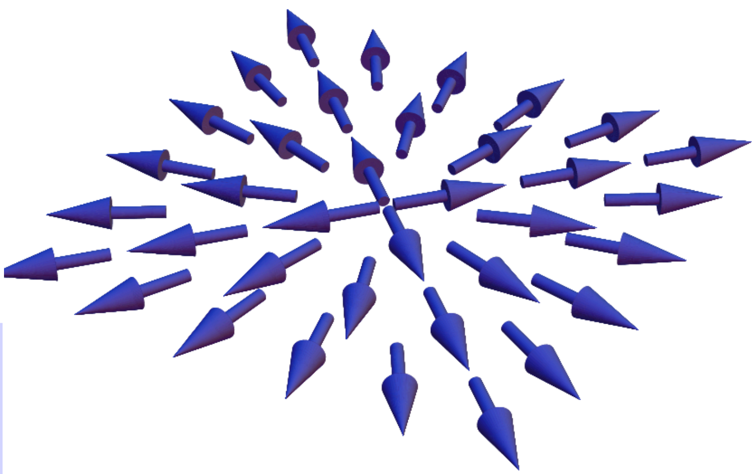
Circulation quantized in normal superfluids

$$\phi = \sqrt{\rho} e^{i\theta}$$

$$\mathbf{j} = \frac{\hbar}{m} \mathrm{Im}\, \phi^* \nabla \phi = \frac{\hbar \rho}{m} \nabla \theta$$

$$\mathbf{v}_s = \frac{\hbar}{m} \nabla \theta$$

$$\oint \mathbf{v}_s \cdot d\mathbf{l} = \frac{\hbar}{m} \Delta\theta = \frac{h}{m} \times \text{ Integer}$$

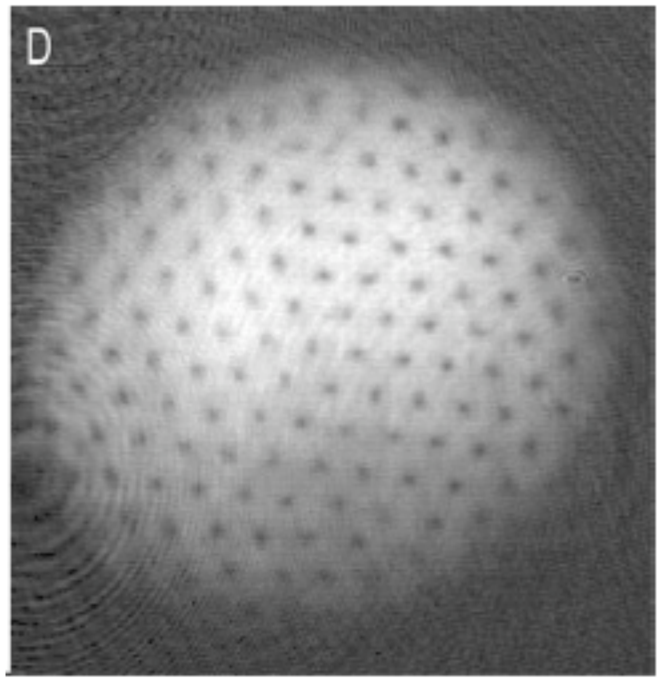


– Onsager-Feynman relation

$$\nabla \times \mathbf{v}_s = 0, \text{ Except at vortex core where } \rho = 0$$

Observation of Vortex Lattices in Bose-Einstein Condensates

J. R. Abo-Shaeer, C. Raman, J. M. Vogels, W. Ketterle



20 APRIL 2001 VOL 292 SCIENCE

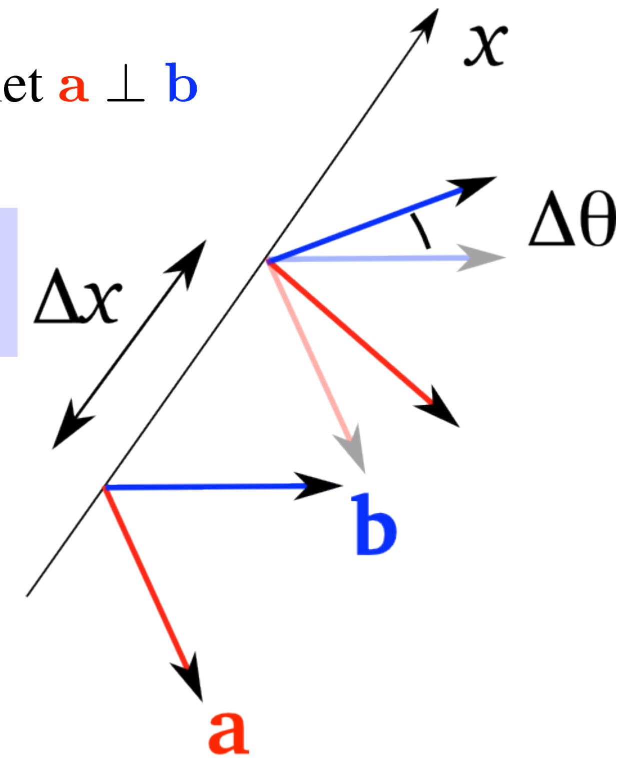
Superfluid velocity in the Ferromagnet

$$\mathbf{v}_s = \frac{\hbar}{m}\mathrm{Im}\,\phi^\dagger\nabla\phi$$

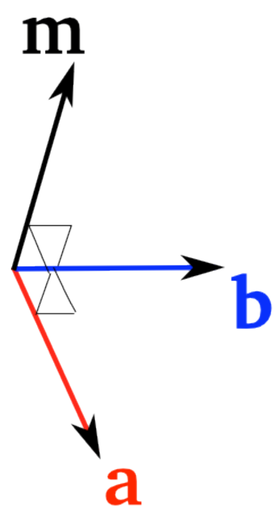
$$\phi = \frac{1}{\sqrt{2}}\left[\mathbf{a} + i\mathbf{b}\right], \text{ and in the Ferromagnet } \mathbf{a} \perp \mathbf{b}$$

$$\boxed{\mathbf{v}_s = \frac{\hbar}{2m}\left[a_i\nabla b_i - b_i\nabla a_i\right]}$$

$$(\mathbf{v}_s)_x = \frac{\hbar}{m}\frac{\Delta\theta}{\Delta x}$$



The Mermin-Ho relation



$$\mathbf{v}_s = \frac{\hbar}{2m}\left[a_i \nabla b_i - b_i \nabla a_i\right]$$

$$\nabla \times \mathbf{v}_s = \frac{\hbar}{m} \nabla a_i \times \nabla b_i = \frac{\hbar}{2m} \epsilon_{abc} m_a \nabla m_b \times \nabla m_c$$

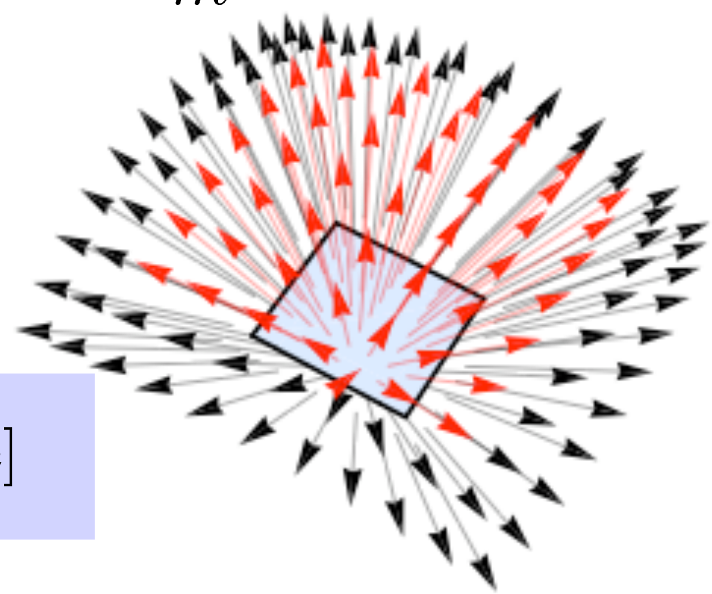
– Mermin & Ho (1976)

Geometrical meaning

$$\nabla \times \mathbf{v}_s = \frac{\hbar}{2m}\epsilon_{abc} m_a \nabla m_b \times \nabla m_c = \frac{\hbar}{m}\nabla\varphi \times \nabla \cos\theta$$

- Stokes' theorem ➔

$$\oint \mathbf{v}_s \cdot d\mathbf{l} = \frac{\hbar}{m}\left[2\pi n \pm \text{solid angle}\right]$$

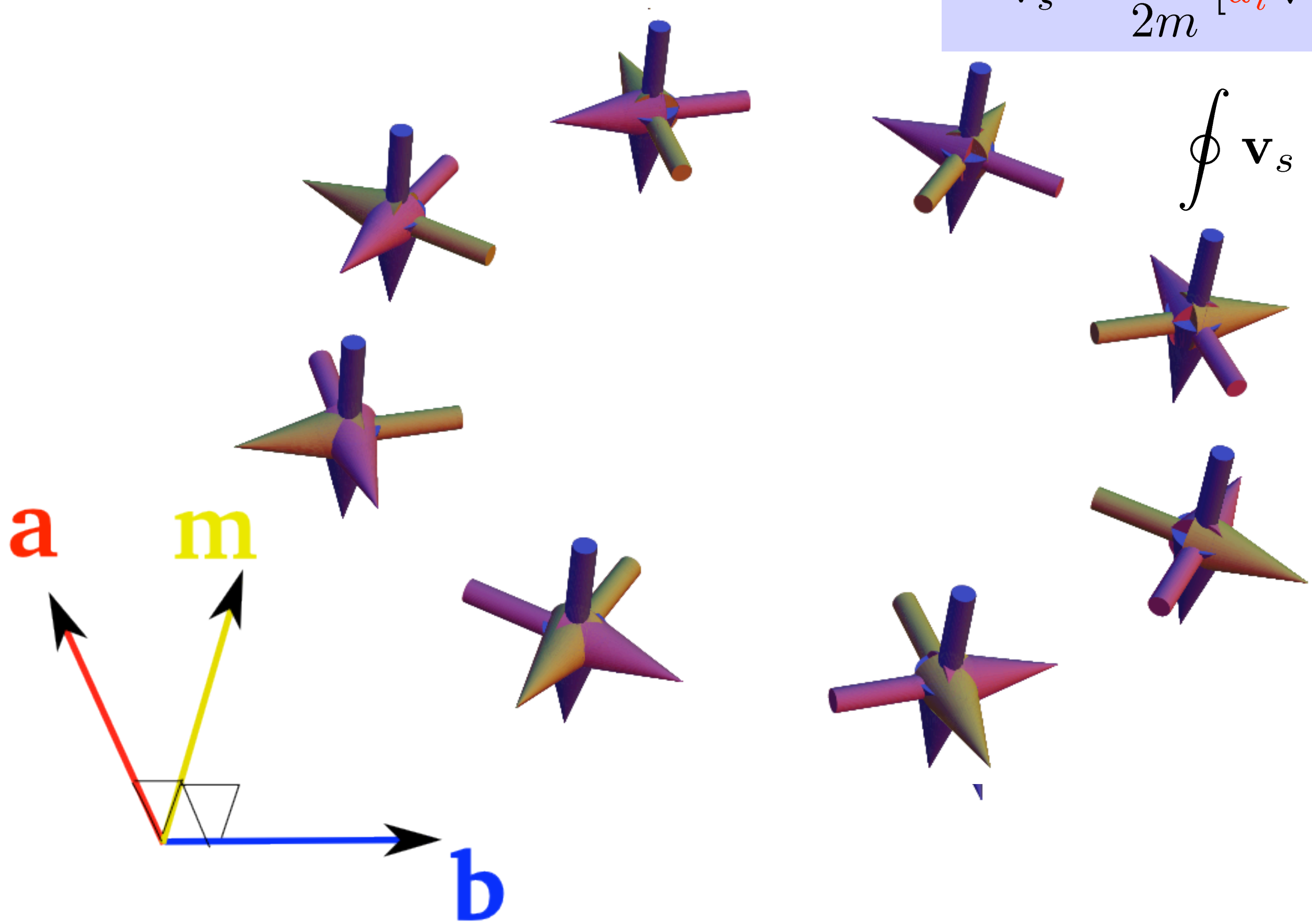


"…a result apparently due to Gauss himself"

Unwinding a vortex

$$\mathbf{v}_s = \frac{\hbar}{2m}\left[a_i \nabla b_i - b_i \nabla a_i\right]$$

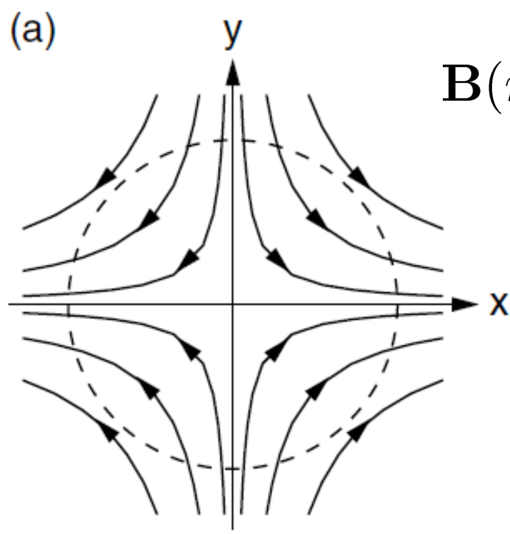
$$\oint \mathbf{v}_s \cdot d\mathbf{l} = 0 \frac{\hbar}{n} 2\pi$$



VOLUME 90, NUMBER 14 PHYSICAL REVIEW LETTERS week ending 11 APRIL 2003

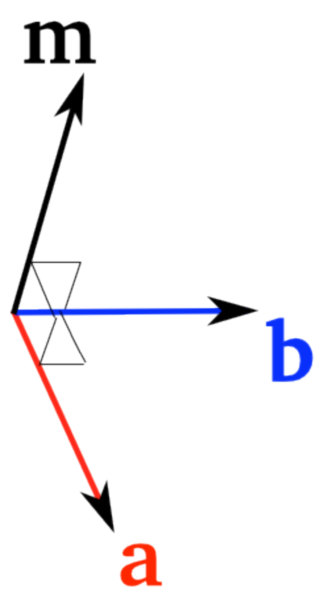
Coreless Vortex Formation in a Spinor Bose-Einstein Condensate

A. E. Leanhardt, Y. Shin, D. Kielpinski, D. E. Pritchard, and W. Ketterle*
Department of Physics, MIT-Harvard Center for Ultracold Atoms, and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 20 December 2002; published 9 April 2003)

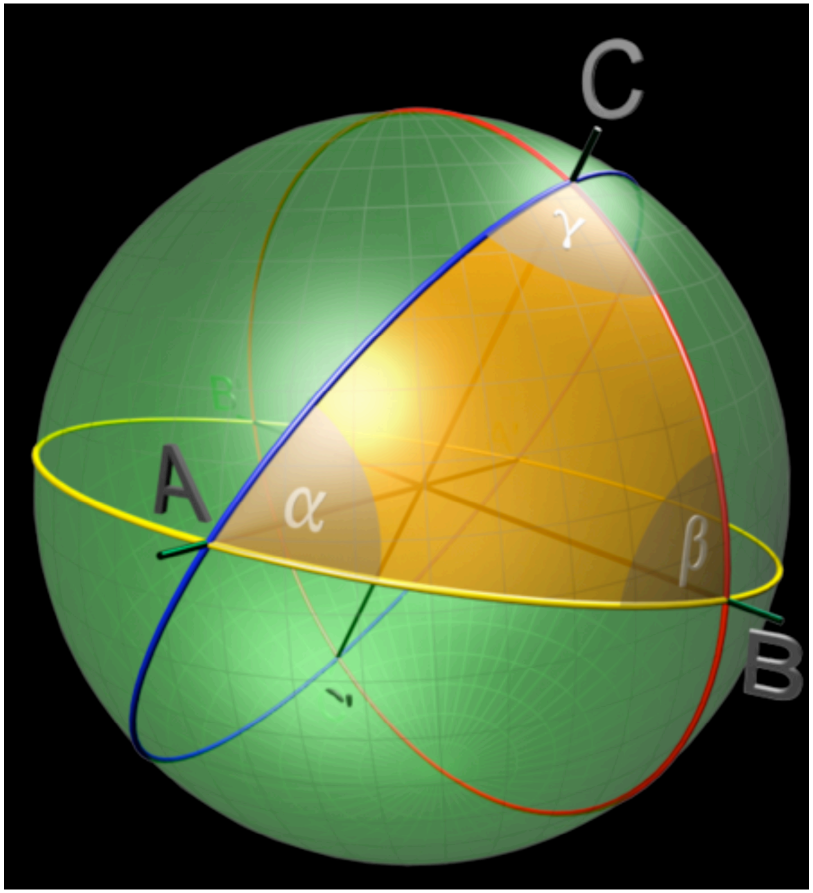


$$\mathbf{B}(r,\phi,z) = B_z\hat{\mathbf{z}} + B'r\left[\cos(2\phi)\hat{\mathbf{r}} - \sin(2\phi)\hat{\boldsymbol{\phi}}\right]$$

Superfluids and bicycle wheels



Spherical triangle



$$\text{Area} = \text{Angle excess}$$
$$= \alpha + \beta + \gamma - \pi$$

Incompressible flow

Normal fluids ➔

approximately *incompressible* at low Mach number

$$\partial_t + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{\nabla P}{\rho}$$

$$(\nabla \cdot \mathbf{v} = 0)$$

Scalar *superfluids* ➔

$$\nabla \cdot \mathbf{v} = \nabla \times \mathbf{v} = 0$$

Leaves only possibility of isolated vortex lines

In the spinor case this limit is non-trivial!

Equations of motion of Bose Ferromagnet

$$\frac{D\mathbf{m}}{Dt} - \frac{\hbar^2}{2m}\mathbf{m} \times \nabla^2 \mathbf{m} = 0$$

$$(D/Dt = \partial_t + \mathbf{v} \cdot \nabla)$$

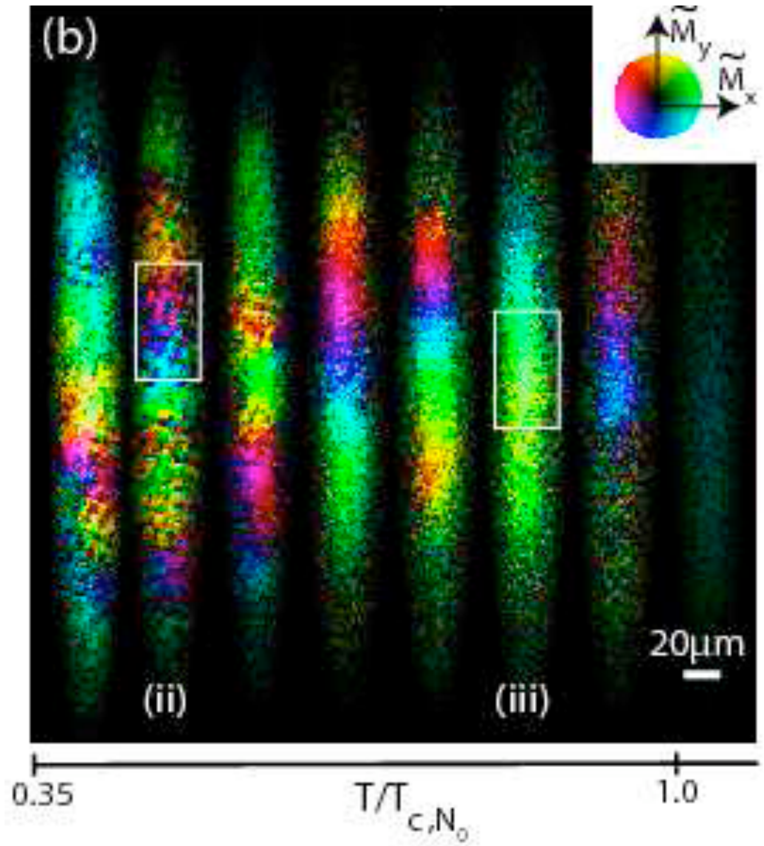
$$\nabla \cdot \mathbf{v} = 0$$

$$\nabla \times \mathbf{v} = \frac{\hbar s}{2m} \epsilon_{abc} m_a \nabla m_b \times \nabla m_c$$

AL, PRA **77** 63622 (2008)

Spinwaves have quadratic dispersion around uniform state

Relevance of dipolar forces?

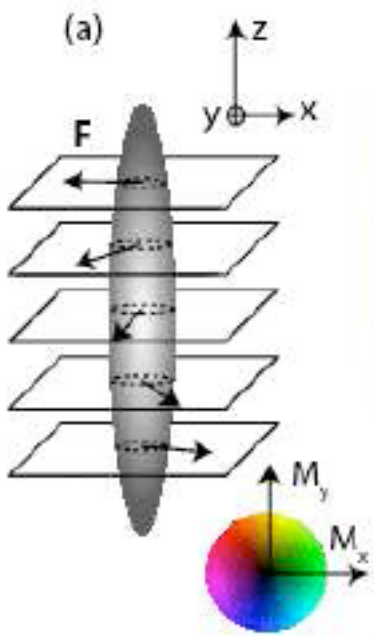


M. Vengalttore *et al.* arXiv:0901.3800

Easily include dipolar forces

- Larmor frequency dwarfs other scales
  - Average dipole-dipole energy over rapid precession

$$H_{\rm dip} = \frac{\omega_d \bar{\rho} d}{8\pi} \int \frac{d^2 q}{(2\pi)^2} \left(m^z_{\mathbf{q}} m^z_{-\mathbf{q}} - \frac{1}{2} \left[m^x_{\mathbf{q}} m^x_{-\mathbf{q}} + m^y_{\mathbf{q}} m^y_{-\mathbf{q}} \right] \right) \left[\frac{q_z^2 d}{q} - \frac{4\pi}{3} \right]$$



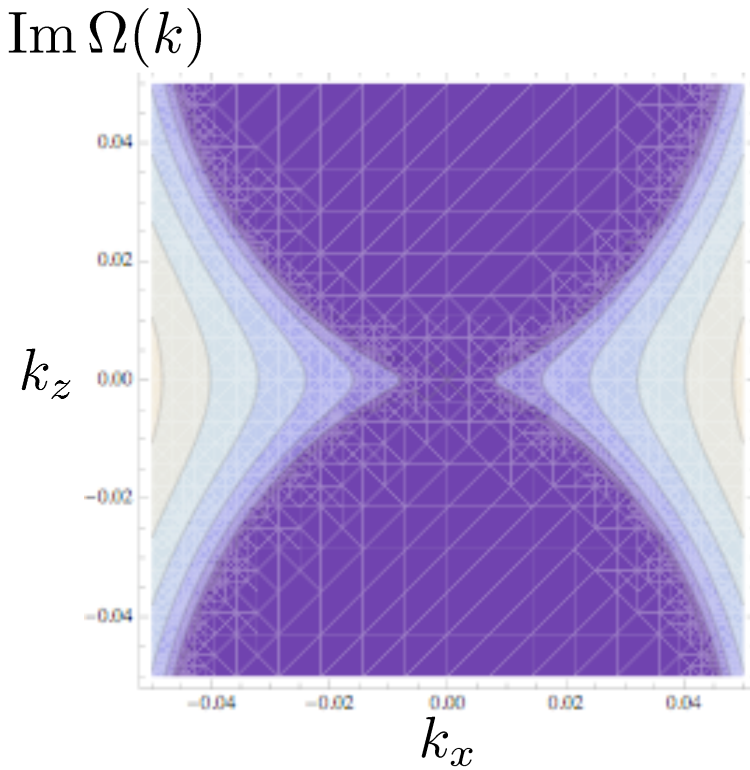
$$\omega_d = \mu_0 (g_F \mu_B)^2 \bar{\rho}$$

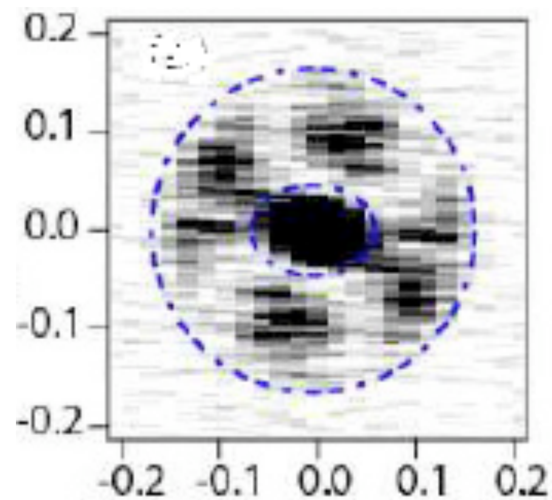
q=0 part is easy axis anisotropy

(exercise in demagnetizing factors)

Effect on spinwaves

$$\Omega(k) = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \frac{\omega_d d}{4}\frac{k_z^2}{k}\right)\left(\frac{\hbar^2 k^2}{2m} - \frac{\omega_d}{2} + \frac{\omega_d d}{2}\frac{k_z^2}{k}\right)}$$





Boundary between stability and instability

$$|k_z| \propto |k_x|^{3/2}$$

Other kinds of magnetic order

- Can also have *Antiferromagnetism* (e.g. MnO)

$$\langle \mathbf{S}_i \rangle \propto (-1)^{i+j}$$

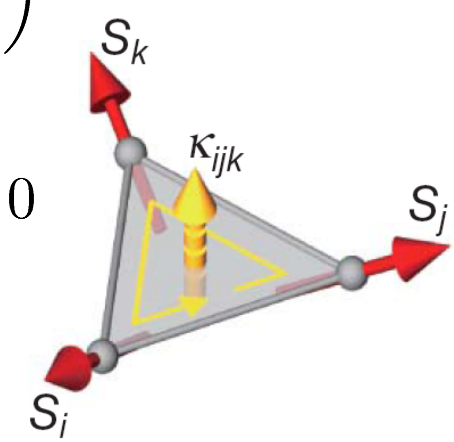
Néel order

- *Nematic* order a.k.a. *"Moment free magnetism"*

$$\langle S^a S^b \rangle \propto \left(\frac{1}{3}\delta_{ab} - n_a n_b \right)$$

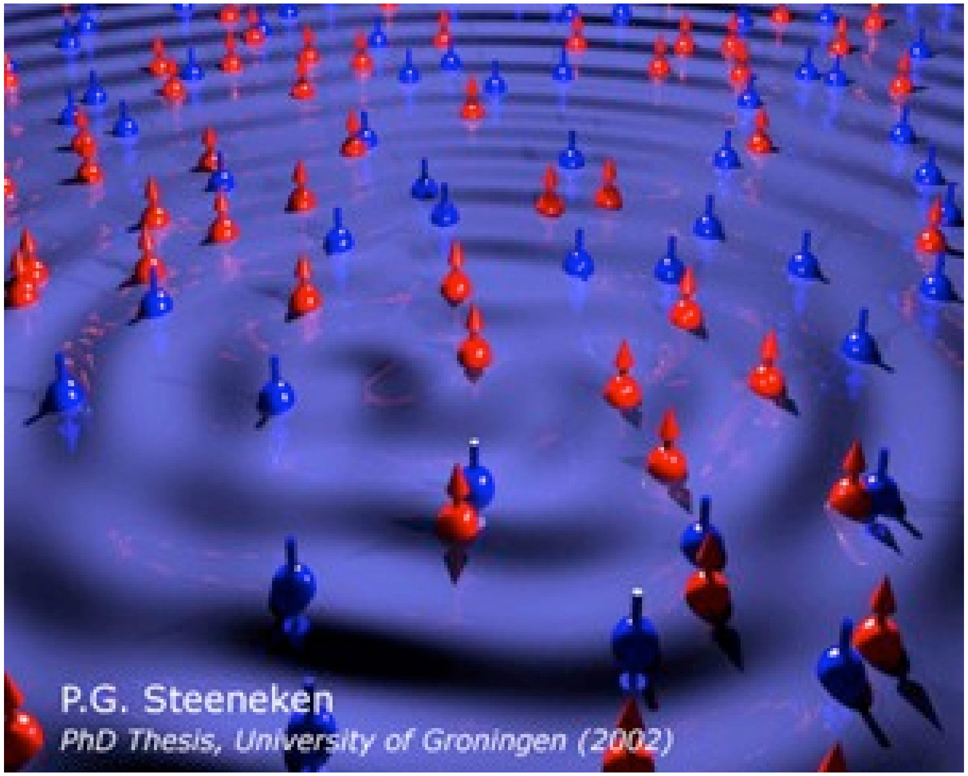
- *Chiral order*

$$\langle \mathbf{S}_i \cdot \mathbf{S}_j \times \mathbf{S}_k \rangle \neq 0$$



The holy grail: *no order at all!*

- Quantum fluctuations keep spins disordered at $T=0$



Quantum spin liquid *[Artist's impression]*

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Polar condensate

- Recall spin dependent interactions

$$\langle H_{\text{spin}} \rangle = \frac{Nnc_2}{2} (\phi^\dagger \mathbf{S} \phi) \cdot (\phi^\dagger \mathbf{S} \phi)$$

$$\phi = \mathbf{a} + i\mathbf{b}$$

$$\mathbf{m} = \phi^\dagger \mathbf{S}^{(1)} \phi = 2\mathbf{a} \times \mathbf{b}$$

- **Polar state** $c_2 > 0$
  - $\phi^\dagger \mathbf{S}^{(1)} \phi$ minimal for $\mathbf{a} \parallel \mathbf{b}$
  - Convenient to write

$$\phi = \mathbf{n} e^{i\theta}, \quad \mathbf{n}^2 = 1$$

Polar condensate - a spin nematic

$$\phi = \mathbf{n}e^{i\theta}, \quad \mathbf{n}^2 = 1$$

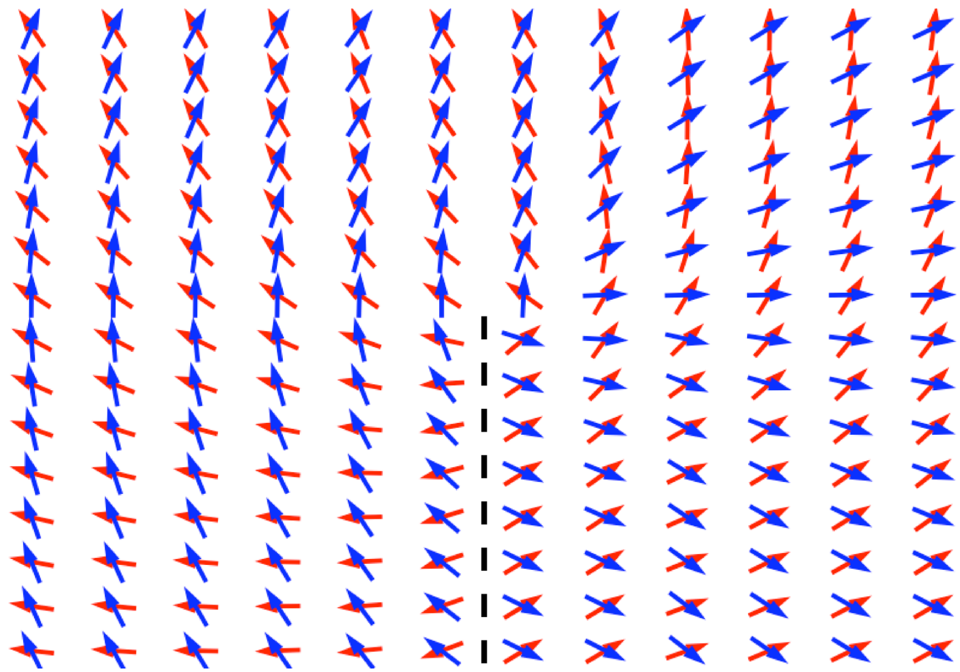
$$\langle S^a S^b \rangle \propto \left(\frac{1}{3}\delta_{ab} - n_a n_b \right)$$

- As far as spin is concerned $\pm\mathbf{n}$ equivalent

Half vortices and disclinations

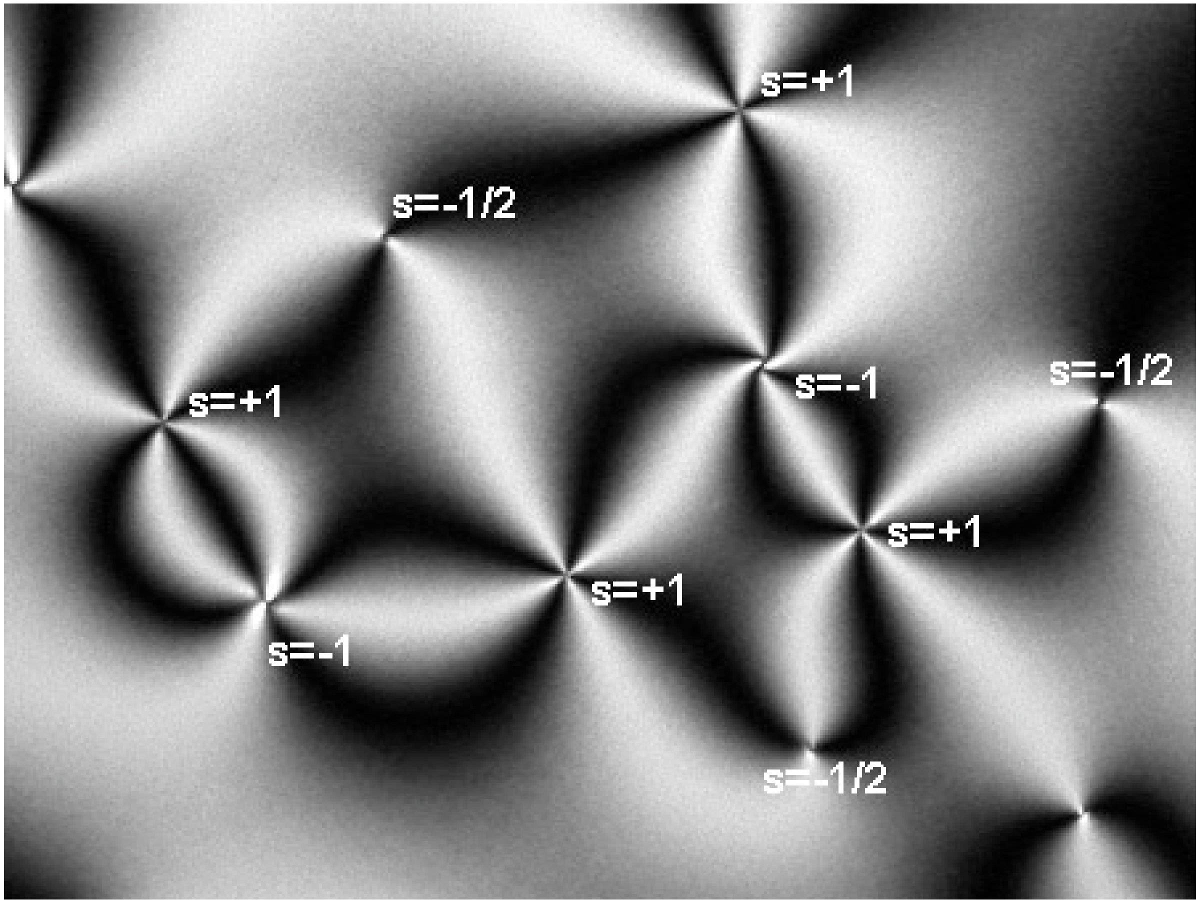
$$\phi = \mathbf{n} e^{i\theta}, \quad \mathbf{n}^2 = 1$$

- Notice that $(\mathbf{n}, \theta)$ and $(-\mathbf{n}, \theta + \pi)$ *are the same!*

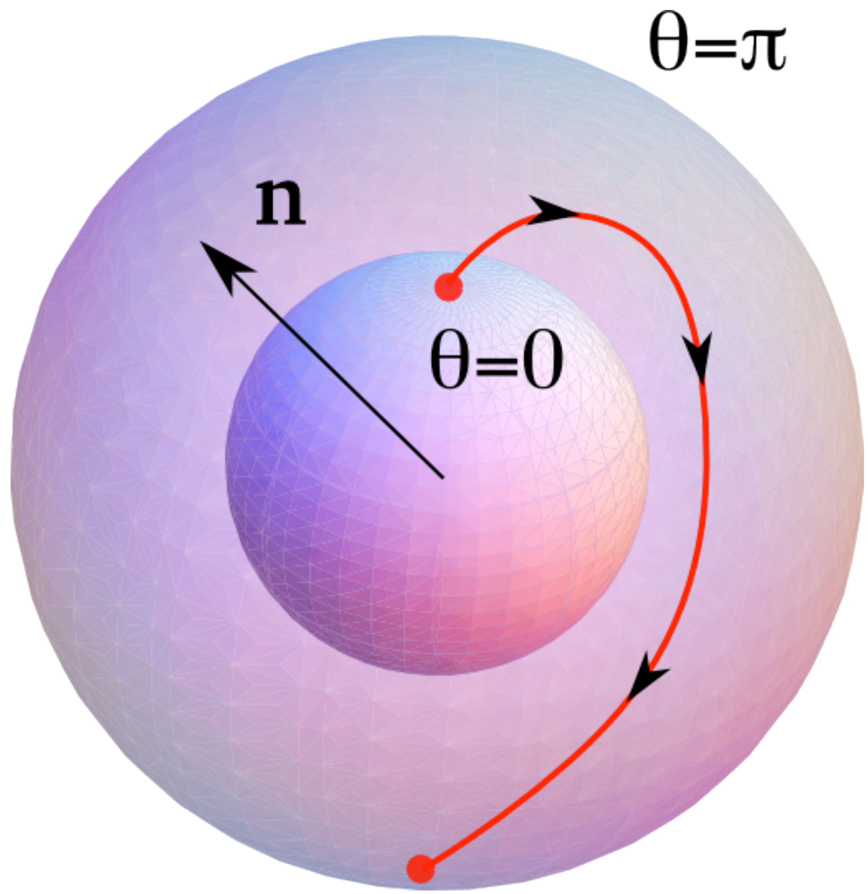


  – Possibility of half vortex / disclinations

Disclinations in a nematic liquid crystal

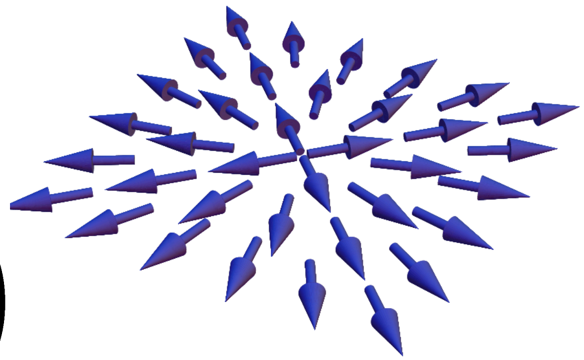


Picturing the space



Kosterlitz-Thouless transition

$$\mathbf{v} = \frac{\hbar}{m}\nabla\theta = \frac{\hbar}{m}\frac{\hat{\mathbf{e}}_\theta}{r}$$



$$\frac{n_s}{2m}\int d^2\mathbf{r}(\nabla\theta)^2 = \frac{n_s\hbar^2}{m}\pi\ln\left(\frac{R}{\xi}\right)$$

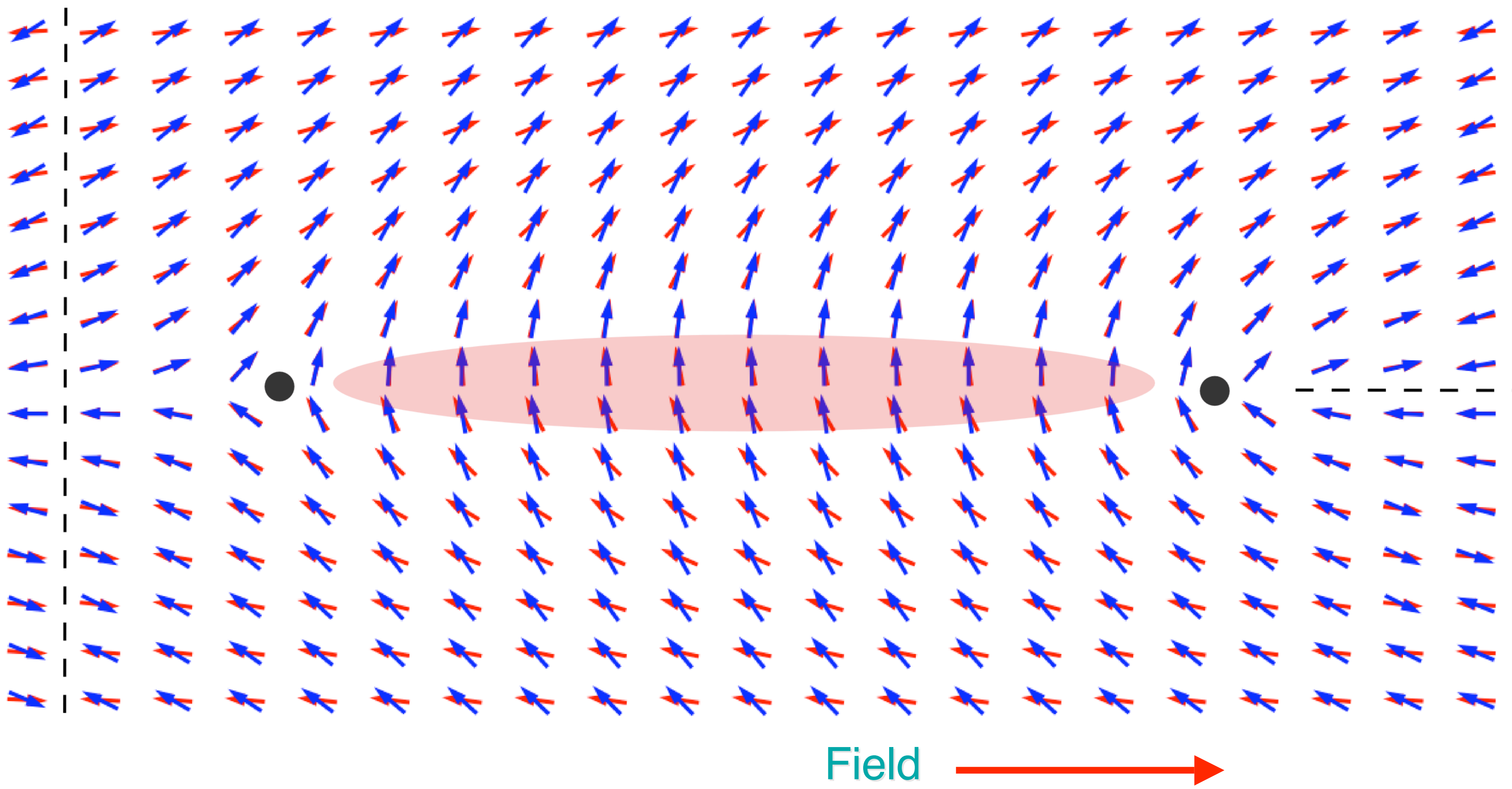
- Free energy to add a vortex

$$\begin{aligned} F &= U - TS \\ &= \frac{n\hbar^2}{m}\pi\ln\left(\frac{R}{\xi}\right) - k_B T\ln\left(\frac{L^2}{\xi^2}\right) \end{aligned}$$

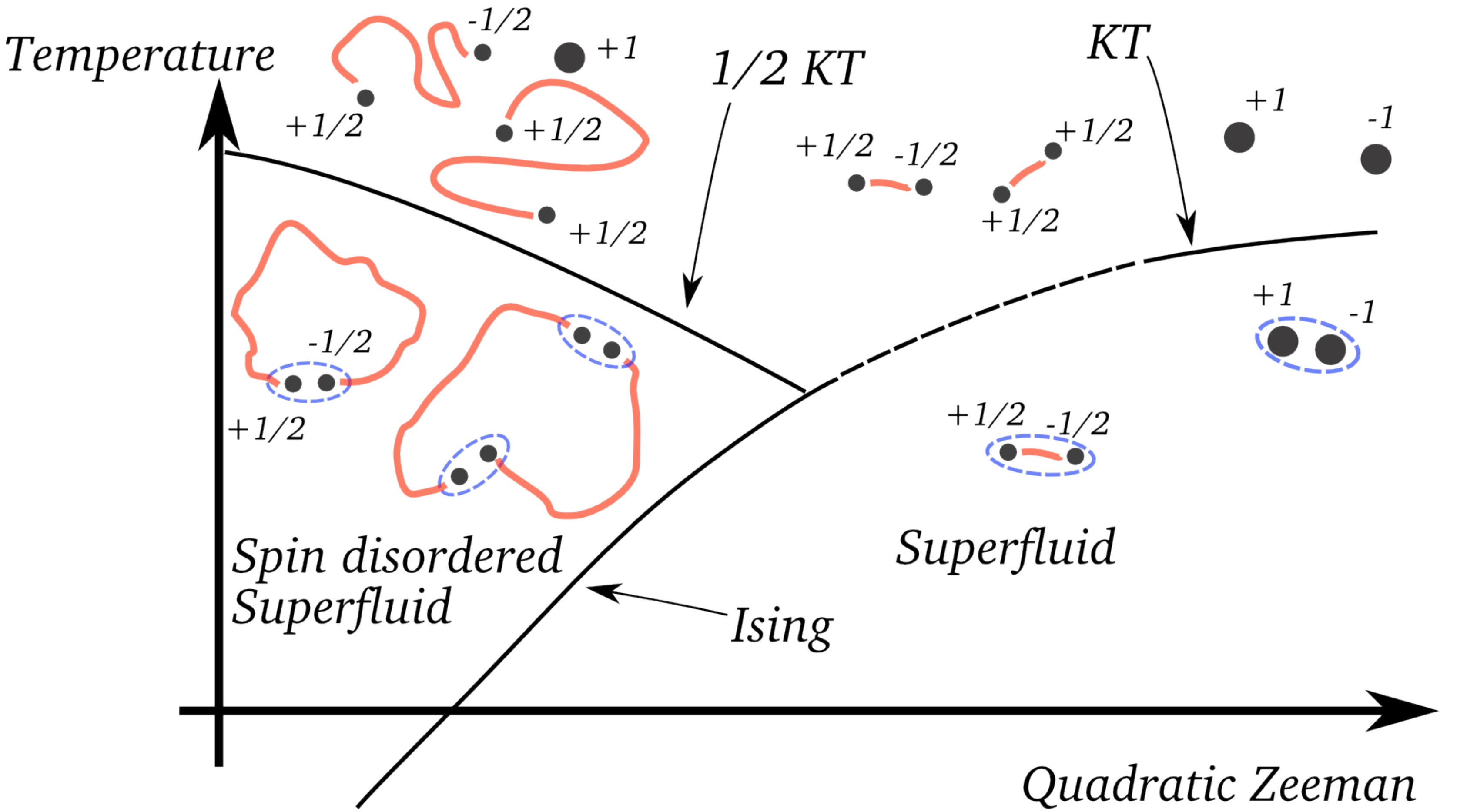
$$\boxed{n_{s,\mathrm{crit}} = \frac{2}{\pi}\frac{m k_B T}{\hbar^2}}$$

Domain Walls

- Quadratic Zeeman effect aligns **n** parallel to field



Conjectured phase diagram



Summary

- *Dynamics* of spinor condensates
  - Instabilities, dipole-dipole interactions, Berkeley experiment
    - Phys. Rev. Lett. **98**, 160404 (2007)
    - Phys. Rev. A **77**, 063622 (2008)
    - arXiv:0909.5620 (2009) *[concerns higher spin]*
- *Statistical mechanics* of polar condensates
  - Novel transitions driven by 1/2 vortices and domain walls
    - Current work with Andrew James