AdS/CFT and Consistent Massive Truncations of IIB Supergravity

Phillip Szepietowski

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1003.5374, 1009.4210 [Liu, PS, Zhao] and 1103.0029 [Liu, PS]

Outline

"Real World" Motivations – gauge/gravity duality

Theoretical Motivations – (for this work)

Consistent Truncations of IIB Supergravity on Sasaki-Einstein Manifolds

Final Comments

About Me

Phillip Szepietowski (can call me Phil, but not Dr. Phil)

- Graduated May 2011 - University of Michigan, advisor - Jim Liu
- Started at UVA: last month!

Research Interests

Gauge/gravity duality – both applications and conceptual questions

My Work

- Higher derivative corrections in the AdS/CFT correspondence
- Consistent truncations of IIB supergravity $\leftarrow$ most of this talk

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What's a theorist to do when coupling constants get large?

- Perturbative expansion in Feynman diagrams breaks down
- How to compute?
- Lattice? non-perturbative methods? How does one extract dynamics?

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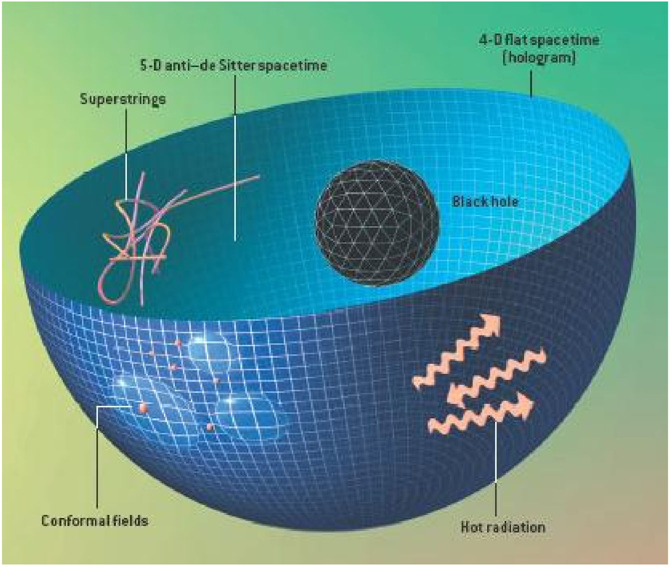
Other examples exist, present feeling is that $AdS_{d+1} \cong CFT_d$ – how general is this?

Pictorial View of Holography



[image from Scientific American (Alfred T. Kamajian)]

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- Bound saturated for any system with an Einstein gravity dual [Buchel, Liu]
- Perhaps a “universal” feature of strongly coupled plasmas
- Higher curvature terms known to violate bound $\eta/s = 1/4\pi[1-8\alpha]$ for Gauss-Bonnet corrections
- Computed perturbative effects of addition of $U(1)$ chemical potential – charged black hole – in higher derivative gauged supergravity: [Cremonini, Hanaki, Liu, PS]

$$\eta/s = 1/4\pi[1-8\alpha(1+q)]$$

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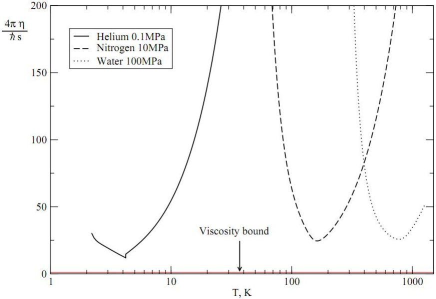

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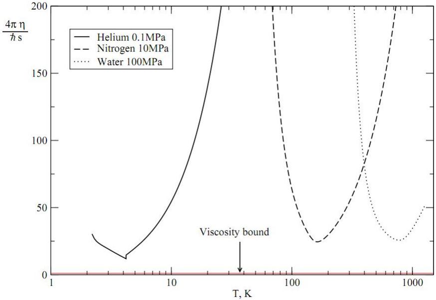

What is measured at RHIC? ($\sqrt{s} = 200\text{GeV}$)

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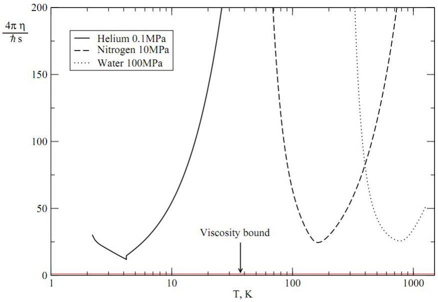

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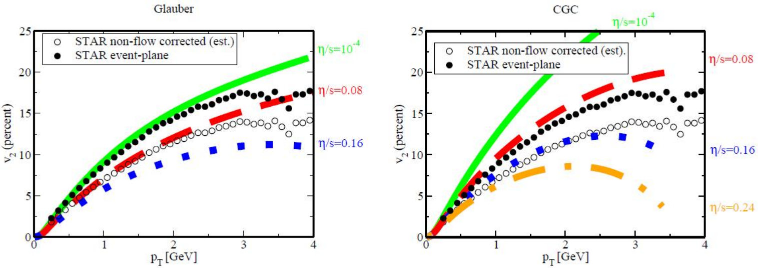

“Experimental” Successes 2 – Condensed Matter Phenomena

Holographic Superconductors

Perhaps a descriptions of high T_c superconductors?

Systems with non-relativistic scaling

Low temperature phase of some condensed matter systems exhibit a non-relativistic scaling symmetry, this has been realized in holographic examples as well.

Non-Fermi Metals

Progress towards understanding so-called non-Fermi liquids (Fermi liquid theory of electrons/holes breaks down) in terms of a dual system

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Consistent Truncations of IIB Supergravity on Sasaki-Einstein Manifolds

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Supersymmetric Generalizations of $AdS_5 \times S^5$

Will consider reductions of IIB supergravity on five-dimensional Sasaki-Einstein manifolds

- Truncations presented will be generic for any SE_5 which includes S^5, $T^{1,1}$, $Y^{p,q}$, etc...
- $AdS_5 \times SE_5$ is a solution of IIB – convenient to consider an effective five-dimensional theory which has AdS_5 solutions
- From AdS/CFT perspective these reductions generically have less supersymmetry than S^5 reduction
- Nice to have a consistent five-dimensional theory containing matter fields for AdS/CFT applications

Motivation – Applied AdS/CFT

Provide further effective theories to explore holographic techniques

Window into quantum gravity?

Potential applications for phenomenology?

Descriptions of strongly coupled SCFTS?

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AdS/CMT (Condensed Matter Theory)

- Holographic techniques provide insight into strongly coupled regimes of certain condensed-matter like systems
  - Holographic superconductors – charged bulk scalar acquires vacuum expectation value in thermal background
  - Non-relativistic geometries – bulk metric has anisotropic scaling symmetry – $(t, x) \sim (\lambda^z t, \lambda x)$
  - Including fermions provide a potential description of non-Fermi metals

HOWEVER

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- AdS/CMT typically takes phenomenological approach – take mass, charge, etc of matter fields as free parameters but require specific ranges for desired effects.
- How can we be sure these models have well defined holographic duals?
- Useful and instructive to embed these into string theory where:
  1. The dual theory is precisely known and the duality is "under control"
  2. Can systematically include "stringy"/quantum effects
  3. Gain insight as to in what sense gauge/gravity duality persists beyond $AdS_5 \times S^5$

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IIB Supergravity

Bosonic Sector of IIB supergravity

Described by the following (pseudo) action:

$$\mathcal{L}_{\mathrm{IIB}} = \frac{1}{16\pi\kappa^2}\int d^{10}x\Big(R * 1 - \frac{1}{2\tau_2^2}d\tau \wedge * d\bar{\tau} - \frac{1}{2}\mathcal{M}_{ij}F_3^i \wedge * F_3^j \\ - \frac{1}{4}\widetilde{F}_5 \wedge * \widetilde{F}_5 - \frac{1}{4}\epsilon_{ij}C_4 \wedge F_3^i \wedge F_3^j\Big),$$

supplemented with the self-duality constraint:

$$* \widetilde{F}_5 = \widetilde{F}_5.$$

$\tau = C_0 + ie^{-\phi}$ – axi-dilaton (complex scalar),
F_3^i – $SL(2,\mathbb{R})$ doublet of three-forms.

Fermionic Sector of IIB Supergravity

Supersymmetry variations:

$$
\begin{aligned}
\delta\lambda &= \frac{i}{2\tau_2}\Gamma^A\partial_A\tau\epsilon^c - \frac{i}{24}\Gamma^{ABC}v_iF^i_{ABC}\epsilon, \\
\delta\Psi_M &= \mathcal{D}_M\epsilon \equiv \left(\nabla_M + \frac{i}{4\tau_2}\partial_M\tau_1 + \frac{i}{16\cdot 5!}\Gamma^{ABCDE}\widetilde{F}_{ABCDE}\Gamma_M\right)\epsilon \\
&\quad + \frac{i}{96}\left(\Gamma_M{}^{ABC} - 9\delta^A_M\Gamma^{BC}\right)v_iF^i_{ABC}\epsilon^c
\end{aligned}
$$

Equations of motion:

$$
\begin{aligned}
0 &= \Gamma^M\mathcal{D}_M\lambda - \frac{i}{8\cdot 5!}\Gamma^{MNPQR}F_{MNPQR}\lambda, \\
0 &= \Gamma^{MNP}\mathcal{D}_N\Psi_P + \frac{i}{48}\Gamma^{NPQ}\Gamma^M v_i^* F^{i*}_{NPQ}\lambda - \frac{i}{4\tau_2}\Gamma^N\Gamma^M\partial_N\tau\lambda^c
\end{aligned}
$$

λ – dilatino, Ψ_M – gravitino
All Weyl spinors – $\Gamma_{11}\epsilon = \epsilon$

$AdS_5 \times S^5$ Solution of IIB supergravity

$AdS_5 \times S^5$ is a solution to the equations of motion of IIB supergravity:

$$ds_{10}^2 = \frac{r^2}{L^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{r^2}dr^2 + L^2 d\Omega_5^2$$

$$\widetilde{F}_5 = \frac{4}{L}(1+*)vol(S^5)$$

- Recall – this shows up as near horizon region of the black 3-brane.
- Can replace S^5 with *any* Einstein space – in particular SE_5.
- SE_5 nice – well defined killing spinors – preserves 1/4 SUSY in AdS_5 vacuum

Sasaki-Einstein Manifolds

Defined such that the cone metric over $ds^2(SE_5)$ is that of a Calabi-Yau-cone

$$ds^2(CY_6) = dr^2 + r^2 ds^2(SE_5)$$

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Just as for S^5, SE_5 can be realized as a $U(1)$ fibration over a Kahler-Einstein base B:

$$ds^2(SE_5) = ds^2(B) + (d\psi + \mathcal{A})^2, \qquad d\mathcal{A} = 2J$$

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Also $SU(2)$ structure exists on SE_5 defined by holomorphic (2,0)-form Ω and (1,1)-Kahler form J on B which satisfy:

$$\begin{gathered} J \wedge \Omega = 0, \qquad \Omega \wedge \bar{\Omega} = 2J \wedge J = 4 *_4 1, \\ *_4 J = J, \qquad *_4 \Omega = \Omega, \\ dJ = 0, \qquad d\Omega = 3i(d\psi + \mathcal{A}) \wedge \Omega. \end{gathered}$$

Dimensional Reduction

Philosophy

- Instead of simply analyzing the $AdS_5 \times SE_5$ solution, dimensionally reduce theory on SE_5 to produce an effective five-dimensional theory – i.e. an effective Lagrangian.
- Replace $ds^2(AdS_5) \to ds_5^2$ and reduce field content on SE_5.
- Obvious method – Kaluza-Klein reduction.

Kaluza-Klein Reduction

Expand 10-dimensional fields along complete set of harmonics on internal space.

$$\Phi(x, y) = \sum_{n}^{\infty} \phi_n(x) Y_n(y), \qquad Y_n(y) = \text{internal harmonic}$$

- Reduced theory is equivalent to the higher dimensional theory
- Effective theory contains infinite tower of massive states
- In simple cases (circle or torus reductions) can completely decouple massive modes by taking compact dimension to be small:

$$m \sim 1/L \to \infty$$

how general is this? – relies on existence of mass-gap between zero modes and KK-modes

Note: theory after decoupling is no longer equivalent to original higher dimensional theory.

Consistent truncation

A non-linear reduction of original theory such that solutions of lower dimensional equations of motion necessarily solve the original equations of motion

- This can be difficult – want to retain only a finite set of fields in Kaluza-Klein tower
- One can always truncate to singlets on internal space – $\Phi(x, y) = \phi_0(x)$ – e.g. massless modes of circle reductions
- Furthermore, it is consistent to truncate to singlets under a *transitively* acting subgroup of the isometry group of the internal manifold

  e.g. for S^5, take singlets under $SU(3) \times U(1) \subset SU(4) \cong SO(6)$

KK reduction of IIB on S^5

[Kim, Romans, van Nieuwenhuizen]

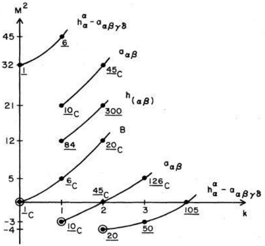
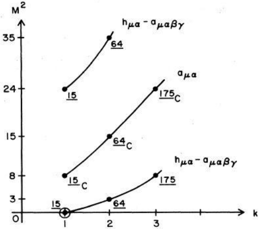


FIG. 2. Mass spectrum of scalars.



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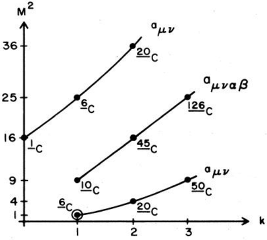

FIG. 3. Mass spectrum of antisymmetric tensors.

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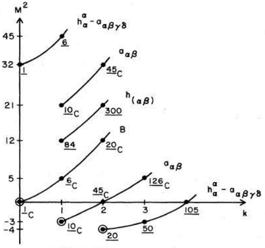

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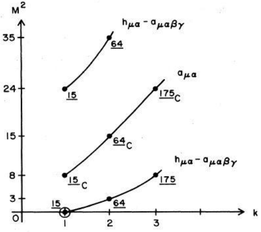

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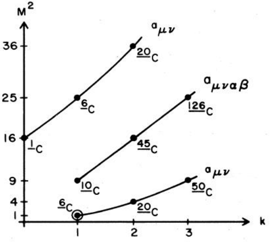

FIG. 3. Mass spectrum of antisymmetric tensors.

- Lowest modes *not* massless – mass due to non-zero curvature of S^5

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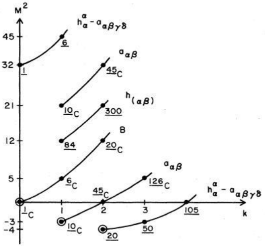

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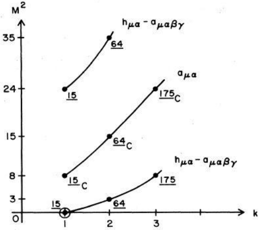

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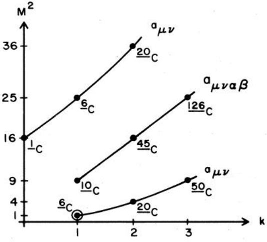

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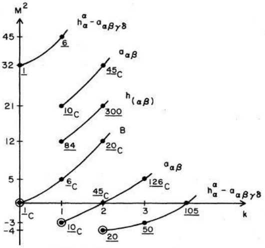

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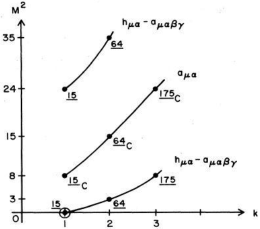

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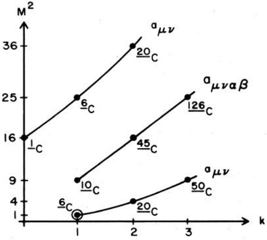

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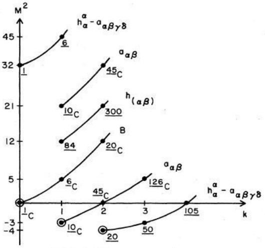

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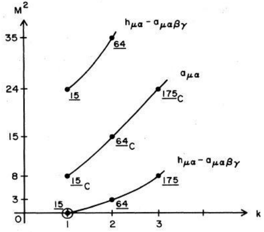

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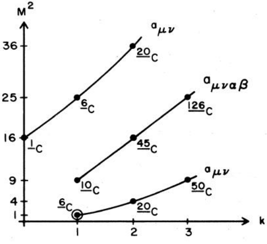

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  [Cvetic,Lu,Pope,Sadrzadeh,Tran]
- Truncation presented will correspond to keeping a subset of the lowest modes of KK tower

Reduction of Bosonic fields on Sasaki-Einstein Manifolds

Metric: Gauge $U(1)$-fiber add A_1 – graviphoton [Buchel,Liu]

$$ds_{10}^2 = ds_5^2 + ds^2(B) + (\underbrace{d\psi + \mathcal{A}}_{\eta} + A_1)^2$$

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also – add "breathing" and "squashing" modes, ρ, σ [Bremer,Duff,Lu,Pope,Stelle]

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Form Fields – expand using $SU(2)$-structure

$$B_2^i = b_2^i + b_1^i \wedge (\eta + A_1) + b_0^i \Omega + \bar{b}_0^i \bar{\Omega}, \qquad F_3^i = dB_2^i$$

$$\widetilde{F}_5 = (1 + *)[(4 + \phi_0) *_4 1 \wedge (\eta + A_1) + \mathbb{A}_1 \wedge *_4 1$$
$$+ p_2 \wedge J \wedge (\eta + A_1) + q_2 \wedge \Omega \wedge (\eta + A_1) + h.c.]$$

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Expanding only along $SU(2)$-structure guarantees consistency, recall:

$$J \wedge \Omega = 0, \qquad \Omega \wedge \bar{\Omega} = 2J \wedge J = 4 *_4 1,$$
$$*_4 J = J, \qquad *_4 \Omega = \Omega,$$
$$dJ = 0, \qquad d\Omega = 3i\eta \wedge \Omega.$$

Reduction of IIB Fermions

Decompose IIB spinors along killing spinor on Sasaki-Einstein, η, and its charge conjugate, η^c:

$$
\begin{aligned}
\Psi_\alpha &= e^{-A/2}\psi_\alpha \otimes \eta \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} + e^{-A/2}\psi'_\alpha \otimes \eta^c \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
\Psi_a &= e^{-A/2}\psi \otimes \tau_a \eta \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} + e^{-A/2}\psi' \otimes \tau_a \eta^c \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
\lambda &= e^{-A/2}\lambda \otimes \eta \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} + e^{-A/2}\lambda' \otimes \eta^c \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix},
\end{aligned}
$$

Reduction of IIB Fermions

Decompose IIB spinors along killing spinor on Sasaki-Einstein, η, and its charge conjugate, η^c:

$$\begin{aligned}
\Psi_\alpha &= e^{-A/2}\psi_\alpha \otimes \eta \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} + e^{-A/2}\psi'_\alpha \otimes \eta^c \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
\Psi_a &= e^{-A/2}\psi \otimes \tau_a\eta \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} + e^{-A/2}\psi' \otimes \tau_a\eta^c \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
\lambda &= e^{-A/2}\lambda \otimes \eta \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} + e^{-A/2}\lambda' \otimes \eta^c \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix},
\end{aligned}$$

IIB susy parameter – appropriate for $\mathcal{N} = 2$ supersymmetry:

$$\epsilon = e^{A/2}\varepsilon \otimes \eta \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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IIB susy parameter – appropriate for $\mathcal{N}=2$ supersymmetry:

$$\epsilon = e^{A/2}\varepsilon \otimes \eta \otimes \begin{bmatrix}1\\0\end{bmatrix}$$

Again $SU(2)$ structure helps:

$$\partial_\psi \eta = \frac{3i}{2}\eta, \qquad \tau^9\eta = -\eta, \qquad \tau^b J_{ab}\eta = i\tau_a\eta,$$
$$\tau^b\Omega_{ab}\eta = 0, \qquad \tau^b\bar{\Omega}_{ab}\eta = 2\tau_a\eta^c.$$

Five-dimensional multiplet structure

n	Multiplet	State	Field
0	supergraviton	$D(4,1,1)_0$	$g_{\mu\nu}$
		$D(3\frac{1}{2},1,\frac{1}{2})_{-1} + D(3\frac{1}{2},\frac{1}{2},1)_1$	$\hat{\psi}_\mu$
		$D(3,\frac{1}{2},\frac{1}{2})_0$	$A_1 + \frac{1}{6}\mathbb{A}_1$
0	LH+RH chiral	$D(3,0,0)_{\pm 2}$	$b^{m^2=-3}$
		$D(3\frac{1}{2},\frac{1}{2},0)_1 + D(3\frac{1}{2},0,\frac{1}{2})_{-1}$	λ'
		$D(4,0,0)_0 + D(4,0,0)_0$	τ
1	LH+RH massive gravitino	$D(5\frac{1}{2},\frac{1}{2},1)_1 + D(5\frac{1}{2},1,\frac{1}{2})_{-1}$	$\hat{\psi}'_\mu$
		$D(5,\frac{1}{2},\frac{1}{2})_0 + D(5,\frac{1}{2},\frac{1}{2})_0$	b_1^i
		$D(5,0,1)_2 + D(5,1,0)_{-2}$	q_2
		$D(6,0,1)_0 + D(6,1,0)_0$	b_2^i
		$D(4\frac{1}{2},0,\frac{1}{2})_1 + D(4\frac{1}{2},\frac{1}{2},0)_{-1}$	$\psi'^{m=5/2}$
		$D(5\frac{1}{2},0,\frac{1}{2})_{-1} + D(5\frac{1}{2},\frac{1}{2},0)_1$	λ
2	massive vector	$D(7,\frac{1}{2},\frac{1}{2})_0$	$\mathbb{A}_1$
		$D(6\frac{1}{2},\frac{1}{2},0)_{-1} + D(6\frac{1}{2},0,\frac{1}{2})_1$	$\psi^{m=-9/2}$
		$D(7\frac{1}{2},0,\frac{1}{2})_{-1} + D(7\frac{1}{2},\frac{1}{2},0)_1$	$\psi^{m=11/2}$
		$D(6,0,0)_0$	σ
		$D(7,0,0)_{\pm 2}$	$b^{m^2=21}$
		$D(8,0,0)_0$	ρ

[Cassani, Dall'Agata, Faedo; Gauntlett, Varela; Bah, Faraggi, Jottar, Leigh, Pando Zayas]

Five-dimensional multiplet structure

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		$D(3\frac{1}{2},\frac{1}{2},0)_1 + D(3\frac{1}{2},0,\frac{1}{2})_{-1}$	λ'
		$D(4,0,0)_0 + D(4,0,0)_0$	τ
1	LH+RH massive gravitino	$D(5\frac{1}{2},\frac{1}{2},1)_1 + D(5\frac{1}{2},1,\frac{1}{2})_{-1}$	$\hat{\psi}'_\mu$
		$D(5,\frac{1}{2},\frac{1}{2})_0 + D(5,\frac{1}{2},\frac{1}{2})_0$	b_1^i
		$D(5,0,1)_2 + D(5,1,0)_{-2}$	q_2
		$D(6,0,1)_0 + D(6,1,0)_0$	b_2^i
		$D(4\frac{1}{2},0,\frac{1}{2})_1 + D(4\frac{1}{2},\frac{1}{2},0)_{-1}$	$\psi'^{m=5/2}$
		$D(5\frac{1}{2},0,\frac{1}{2})_{-1} + D(5\frac{1}{2},\frac{1}{2},0)_1$	λ
2	massive vector	$D(7,\frac{1}{2},\frac{1}{2})_0$	$\mathbb{A}_1$
		$D(6\frac{1}{2},\frac{1}{2},0)_{-1} + D(6\frac{1}{2},0,\frac{1}{2})_1$	$\psi^{m=-9/2}$
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- Reducing equations of motion yields consistent truncation

[Cassani, Dall'Agata, Faedo; Gauntlett, Varela; Bah, Faraggi, Jottar, Leigh, Pando Zayas]

Five-dimensional multiplet structure

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0	LH+RH chiral	$D(3,0,0)_{\pm 2}$	$b^{m^2=-3}$
		$D(3\frac{1}{2},\frac{1}{2},0)_1+D(3\frac{1}{2},0,\frac{1}{2})_{-1}$	λ'
		$D(4,0,0)_0+D(4,0,0)_0$	τ
1	LH+RH massive gravitino	$D(5\frac{1}{2},\frac{1}{2},1)_1+D(5\frac{1}{2},1,\frac{1}{2})_{-1}$	$\hat{\psi}'_\mu$
		$D(5,\frac{1}{2},\frac{1}{2})_0+D(5,\frac{1}{2},\frac{1}{2})_0$	b_1^i
		$D(5,0,1)_2+D(5,1,0)_{-2}$	q_2
		$D(6,0,1)_0+D(6,1,0)_0$	b_2^i
		$D(4\frac{1}{2},0,\frac{1}{2})_1+D(4\frac{1}{2},\frac{1}{2},0)_{-1}$	$\psi'^{m=5/2}$
		$D(5\frac{1}{2},0,\frac{1}{2})_{-1}+D(5\frac{1}{2},\frac{1}{2},0)_1$	λ
2	massive vector	$D(7,\frac{1}{2},\frac{1}{2})_0$	$\mathbb{A}_1$
		$D(6\frac{1}{2},\frac{1}{2},0)_{-1}+D(6\frac{1}{2},0,\frac{1}{2})_1$	$\psi^{m=-9/2}$
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[Cassani, Dall'Agata, Faedo; Gauntlett, Varela; Bah, Faraggi, Jottar, Leigh, Pando Zayas]

Five-dimensional multiplet structure

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0	LH+RH chiral	$D(3,0,0)_{\pm 2}$	$b^{m^2=-3}$
		$D(3\frac{1}{2},\frac{1}{2},0)_1+D(3\frac{1}{2},0,\frac{1}{2})_{-1}$	λ'
		$D(4,0,0)_0+D(4,0,0)_0$	τ
1	LH+RH massive gravitino	$D(5\frac{1}{2},\frac{1}{2},1)_1+D(5\frac{1}{2},1,\frac{1}{2})_{-1}$	$\hat{\psi}'_\mu$
		$D(5,\frac{1}{2},\frac{1}{2})_0+D(5,\frac{1}{2},\frac{1}{2})_0$	b_1^i
		$D(5,0,1)_2+D(5,1,0)_{-2}$	q_2
		$D(6,0,1)_0+D(6,1,0)_0$	b_2^i
		$D(4\frac{1}{2},0,\frac{1}{2})_1+D(4\frac{1}{2},\frac{1}{2},0)_{-1}$	$\psi'^{m=5/2}$
		$D(5\frac{1}{2},0,\frac{1}{2})_{-1}+D(5\frac{1}{2},\frac{1}{2},0)_1$	λ
2	massive vector	$D(7,\frac{1}{2},\frac{1}{2})_0$	$\mathbb{A}_1$
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- Reducing equations of motion yields consistent truncation
- Lagrangian too long to show explicitly
- Five-dimensional $\mathcal{N}=2$ gauged supergravity coupled to various multiplets

[Cassani, Dall'Agata, Faedo; Gauntlett, Varela; Bah, Faraggi, Jottar, Leigh, Pando Zayas]

Five-dimensional multiplet structure

n	Multiplet	State	Field
0	supergraviton	$D(4,1,1)_0$	$g_{\mu\nu}$
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		$D(4,0,0)_0 + D(4,0,0)_0$	τ
1	LH+RH massive gravitino	$D(5\frac{1}{2},\frac{1}{2},1)_1 + D(5\frac{1}{2},1,\frac{1}{2})_{-1}$	$\hat{\psi}'_\mu$
		$D(5,\frac{1}{2},\frac{1}{2})_0 + D(5,\frac{1}{2},\frac{1}{2})_0$	b_1^i
		$D(5,0,1)_2 + D(5,1,0)_{-2}$	q_2
		$D(6,0,1)_0 + D(6,1,0)_0$	b_2^i
		$D(4\frac{1}{2},0,\frac{1}{2})_1 + D(4\frac{1}{2},\frac{1}{2},0)_{-1}$	$\psi'^{m=5/2}$
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- Reducing equations of motion yields consistent truncation
- Lagrangian too long to show explicitly
- Five-dimensional $\mathcal{N} = 2$ gauged supergravity coupled to various multiplets
- Various further truncations exist – in particular can truncate out massive gravitino multiplet

[Cassani, Dall'Agata, Faedo; Gauntlett, Varela; Bah, Faraggi, Jottar, Leigh, Pando Zayas]

Recall KK reduction of IIB on S^5

[Kim, Romans, van Nieuwenhuizen]

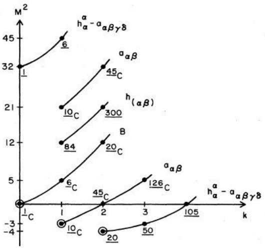


FIG. 2. Mass spectrum of scalars.

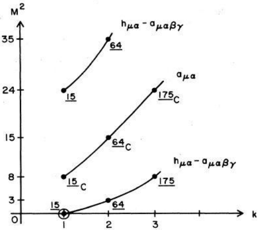


FIG. 1. Mass spectrum of vectors.

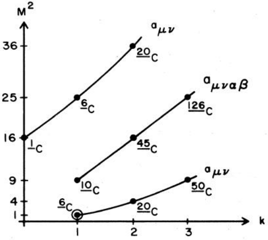
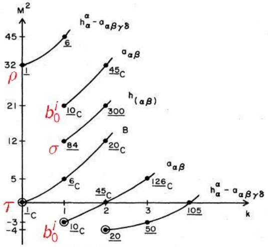


FIG. 3. Mass spectrum of antisymmetric tensors.

Recall KK reduction of IIB on S^5

[Kim, Romans, van Nieuwenhuizen]



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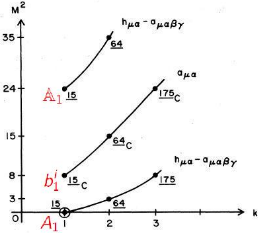

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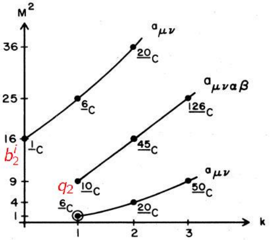
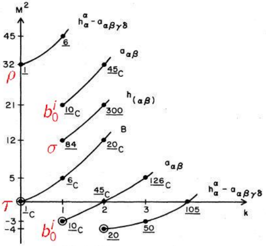


FIG. 3. Mass spectrum of antisymmetric tensors.

- Perform linearized analysis to determine masses and identify spectrum

Recall KK reduction of IIB on S^5

[Kim, Romans, van Nieuwenhuizen]



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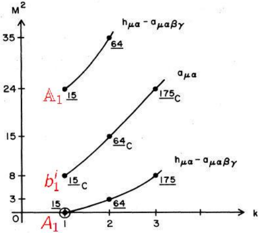

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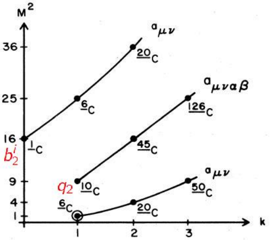

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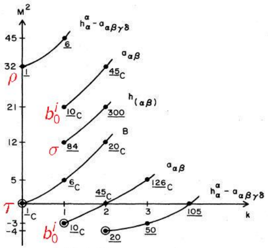
Recall KK reduction of IIB on S^5

[Kim, Romans, van Nieuwenhuizen]



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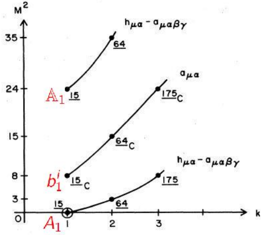

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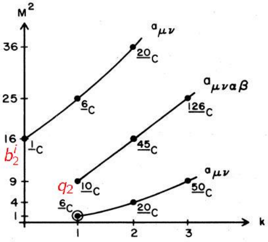

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- Perform linearized analysis to determine masses and identify spectrum
- All modes in consistent truncation lie at bottom of KK-towers
- All belong to $SU(4)$ reps containing singlets under $SU(3) \subset SU(4)$

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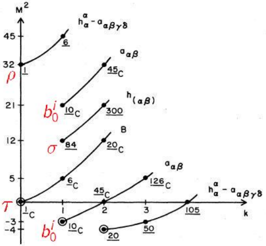
[Kim, Romans, van Nieuwenhuizen]



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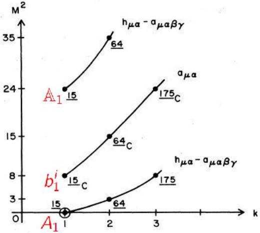

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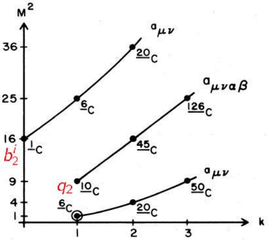

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- Perform linearized analysis to determine masses and identify spectrum
- All modes in consistent truncation lie at bottom of KK-towers
- All belong to $SU(4)$ reps containing singlets under $SU(3) \subset SU(4)$
- Note that much of the massless $\mathcal{N} = 8$ multiplet does not include an $SU(3)$ singlet.

Applications

A holographic supersymmetric superconductor?

$$\mathcal{L} = \mathcal{L}_b + \mathcal{L}_f$$

$$\begin{aligned}\mathcal{L}_b \quad = \quad & R * 1 + \frac{6(2-3\chi)}{(1-\chi)^2} - \frac{d\chi \wedge *d\chi}{2(1-\chi)^2} * 1 - \frac{(1+\chi)d\tau \wedge *d\bar{\tau}}{2(1-\chi)\tau_2^2} - \frac{3}{2}F_2 \wedge *F_2 - \frac{\mathbb{A}_1 \wedge *\mathbb{A}_1}{2(1-\chi)^2} \\ & - \frac{8\tau_2 Db \wedge *D\bar{b}}{1-\chi} - \frac{2i}{1-\chi}(\bar{b}Db \wedge *d\bar{\tau} - bD\bar{b} \wedge *d\tau) - A_1 \wedge F_2 \wedge F_2,\end{aligned}$$

$$\begin{aligned}e^{-1}\mathcal{L}_f \quad = \quad & \bar{\psi}_\alpha \gamma^{\alpha\beta\sigma} D_\beta \psi_\sigma + \tfrac{3i}{8}\bar{\psi}_\alpha \left(\gamma^{\alpha\beta\rho\sigma} + 2g^{\alpha\beta}g^{\rho\sigma}\right) F_{\beta\rho}\psi_\sigma + \tfrac{1}{2}\bar{\lambda}\gamma^\alpha D_\alpha \lambda + \tfrac{3i}{16}\bar{\lambda}\gamma^{\mu\nu}F_{\mu\nu}\lambda \\ & + \tfrac{1}{2}e^{-4B}\left(3\tau_2(bD_\mu\bar{b} - \bar{b}D_\mu b)\bar{\lambda}\gamma^\mu\lambda + \tfrac{3}{2}(1+8\tau_2|b|^2)\bar{\lambda}\lambda\right) \\ & + e^{-4B}\left(-\tfrac{3}{2}\bar{\psi}_\alpha\gamma^{\alpha\sigma}\psi_\sigma + \tau_2(\bar{b}D_\beta b - bD_\beta\bar{b})\bar{\psi}_\alpha\gamma^{\alpha\beta\sigma}\psi_\sigma\right) \\ & + \tau_2^{1/2}e^{-4B}\left(D_\mu b\bar{\psi}_\alpha\gamma^\mu\gamma^\alpha\lambda + 3b\bar{\psi}_\alpha\gamma^\alpha\lambda + h.c.\right) \\ & + \frac{e^{-2B}}{\tau_2^{1/2}}\left(-b\bar{\psi}_\alpha\gamma^{\alpha\beta\sigma}\partial_\beta\tau\psi_\sigma^c + \tau_2^{1/2}\bar{\psi}_\alpha\gamma^\mu\partial_\mu\tau\gamma^\alpha\lambda^c + h.c.\right), \qquad e^{4B} = 1 - 4\tau_2|b|^2\end{aligned}$$

Embedded a holographic superconductor model into $\mathcal{N} = 2$ supergravity

[Gubser, Herzog, Pufu, Tesileanu]

Outline

"Real World" Motivations – gauge/gravity duality

Theoretical Motivations – (for this work)

Consistent Truncations of IIB Supergravity on Sasaki-Einstein Manifolds

Final Comments

Useful for other types of compactifications?

Can "pull-in" radial coordinate and relate these truncations to cone compactifications:

$$ds^2_{10} = \underbrace{e^{2Y(r)} h_{\mu\nu}(x) dx^\mu dx^\nu + e^{2X(r)}[dr^2}_{ds^2_5} + e^{2Z(r)} ds^2(SE_5^{squashed})]$$

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Allows for relation to some classes of flux compactifications

- For $SE_5 = T^{1,1}$ can reproduce the Klebanov-Strassler solution

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Allows for relation to some classes of flux compactifications

- For $SE_5 = T^{1,1}$ can reproduce the Klebanov-Strassler solution
- Perhaps these truncations can be utilized to find other such solutions?

Useful for other types of compactifications?

Can "pull-in" radial coordinate and relate these truncations to cone compactifications:

$$ds_{10}^2 = \underbrace{e^{2Y(r)} h_{\mu\nu}(x) dx^\mu dx^\nu + e^{2X(r)}[dr^2}_{ds_5^2} + e^{2Z(r)} ds^2(SE_5^{squashed})]$$

Allows for relation to some classes of flux compactifications

- For $SE_5 = T^{1,1}$ can reproduce the Klebanov-Strassler solution
- Perhaps these truncations can be utilized to find other such solutions?
- Application is somewhat limited – can only describe dependence on radial "cone" coordinate - r.

Some loose ends and future work...

What about stability of these truncations?

- Should analyze spectrum of fluctuations of these truncations about vacuum solutions – $AdS_5 \times S^5$ vacuum is stable – what about other solutions?

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Constructing non-relativistic solutions

- There has been work relating these truncations and similar constructions to non-relativistic geometries [Narayan, Balasubramanian; Gauntlett, Donos; Kraus, Perlmutter; Cassani, Faedo; Halmagyi, Petrini, Zaffaroni]

Some loose ends and future work...

Truncations on the five-sphere?

Given the relative ease of these constructions can one consider going back to S^5 and working out consistent truncation to the full massless sector of the $\mathcal{N} = 8$ theory?

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- Truncation to $\mathbf{20}'$ scalars and $\mathbf{15}$ vectors known

  [Cvetič,Lü,Pope,Sadrzadeh,Tran]

- $\mathbf{1} + \bar{\mathbf{1}}$ scalars from axi-dilaton
- $\mathbf{10} + \bar{\mathbf{10}}$ scalars and $\mathbf{6} + \bar{\mathbf{6}}$ tensors come from $3-$forms
- Keeping entire massless sector of $\mathcal{N} = 8$ perhaps not as bad as anticipated?

Thank you!