Modal analysis of Casimir interactions

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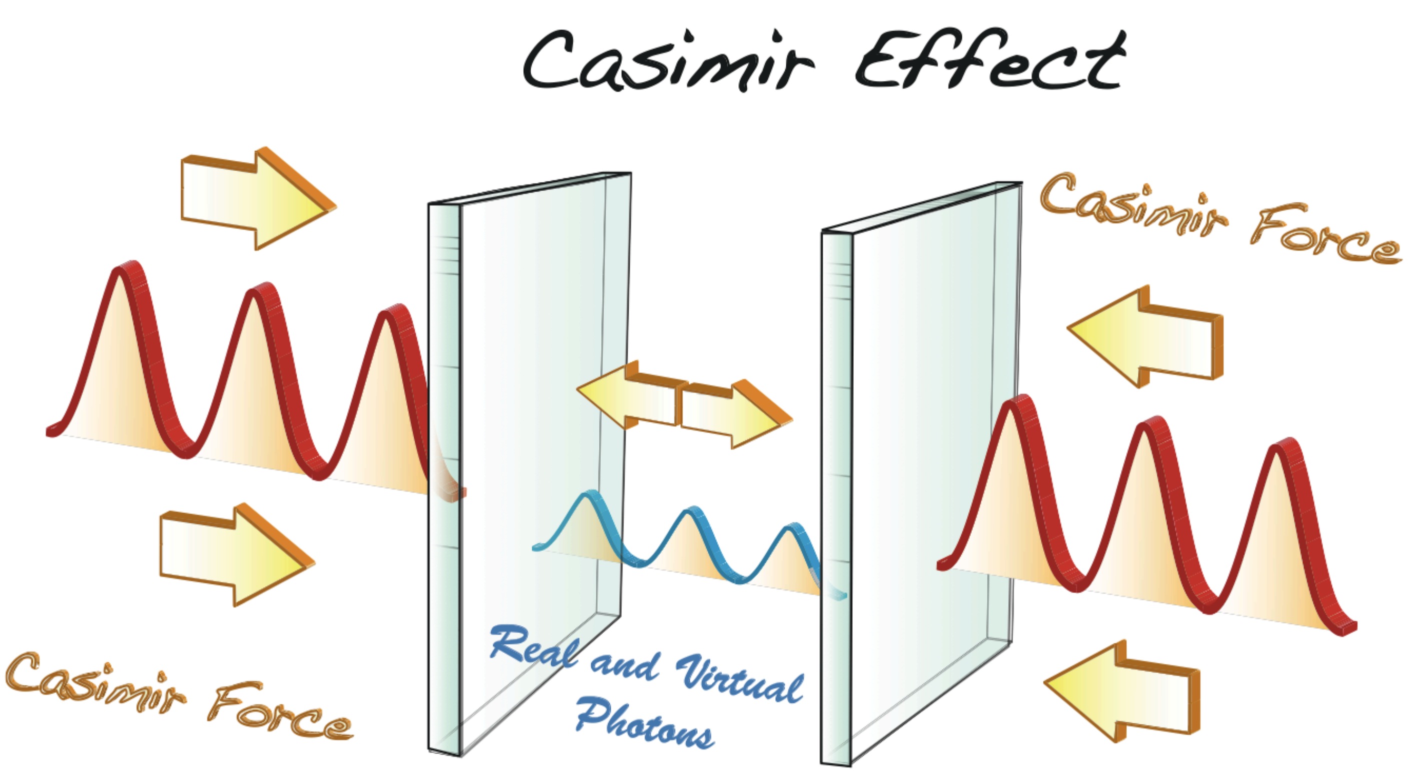
P. Milonni, LANL

P. Davids, SNL
R. Decca, IUPUI
V. Aksyuk, NIST
D. Lopez, ANL
....

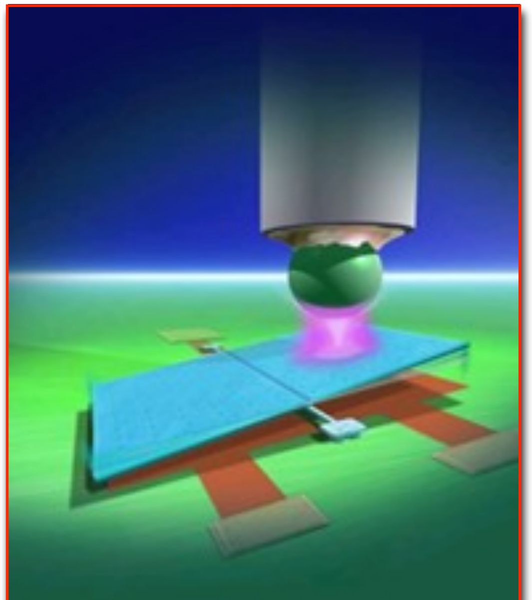
The Casimir effect (1948) is:

- a mesoscopic **"quantum"** force between neutral non magnetic object
- Of the order of 0.1 μN for a surface of 1cm^2 and a distance of 1μm (**equivalent to the weight of 10 μg**)
- A dispersion force (van der Waals). It goes as a powerlaw

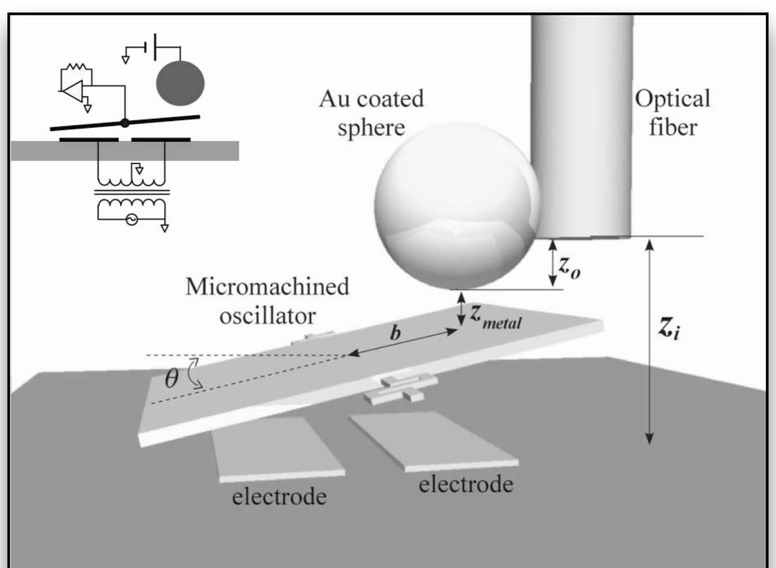
$$F \propto \frac{\hbar c}{d^\alpha}$$



A force relevant for nano- and micro-machines



H. Chan et al.

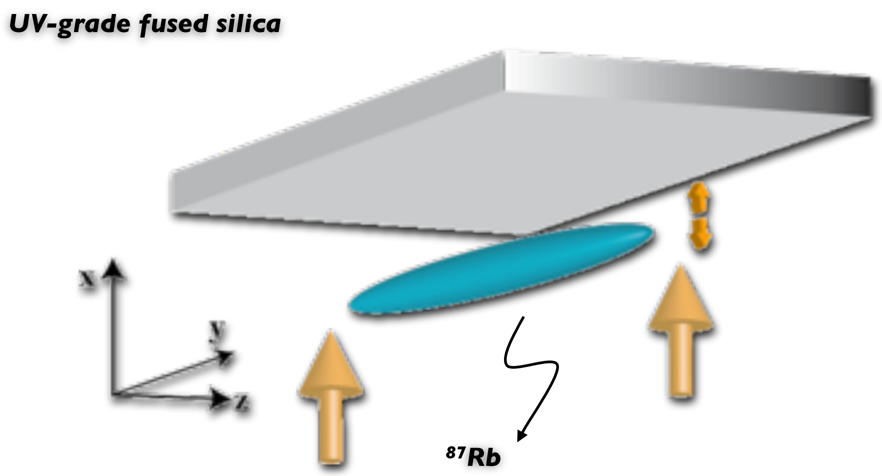


Decca et al.

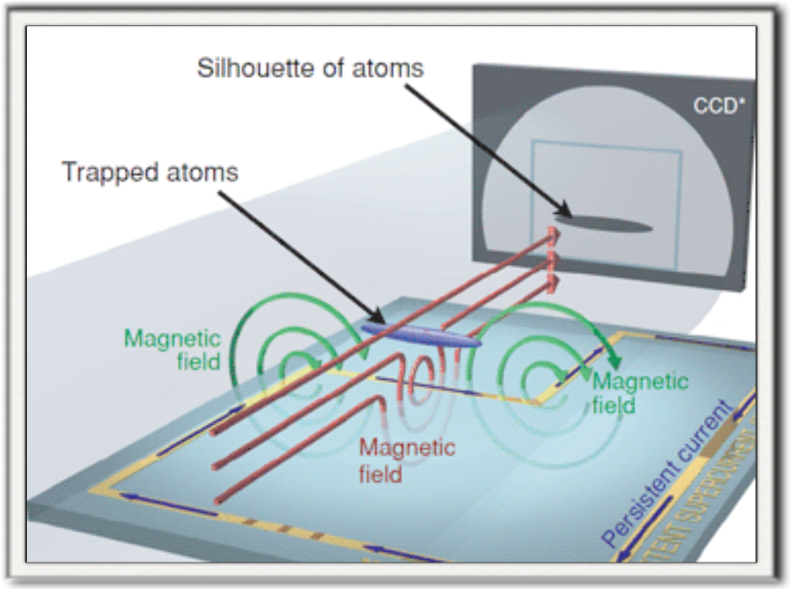
Sticking/ Contactless Force

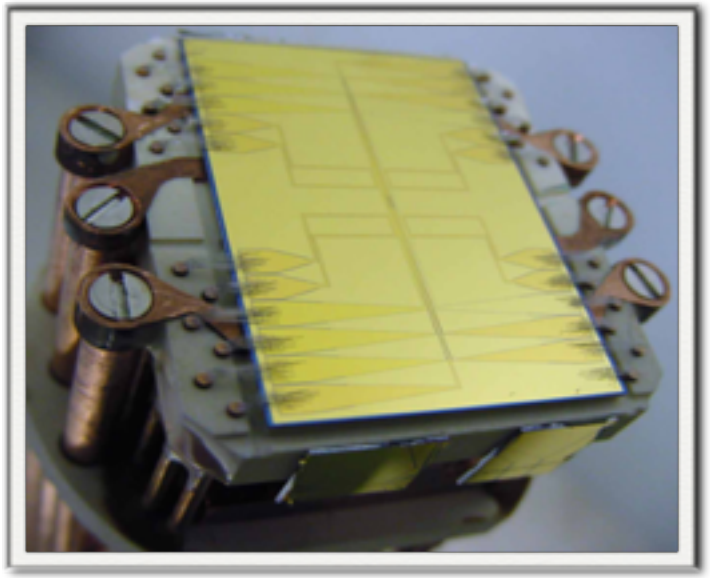
Recent experiments with atoms

Bose-Einstein Condensates



D. M. Harber et al. Phys. Rev. A **72**, 033610 (2005)





Courtesy of R. Folman

Quantum Computation

Test of Gravity

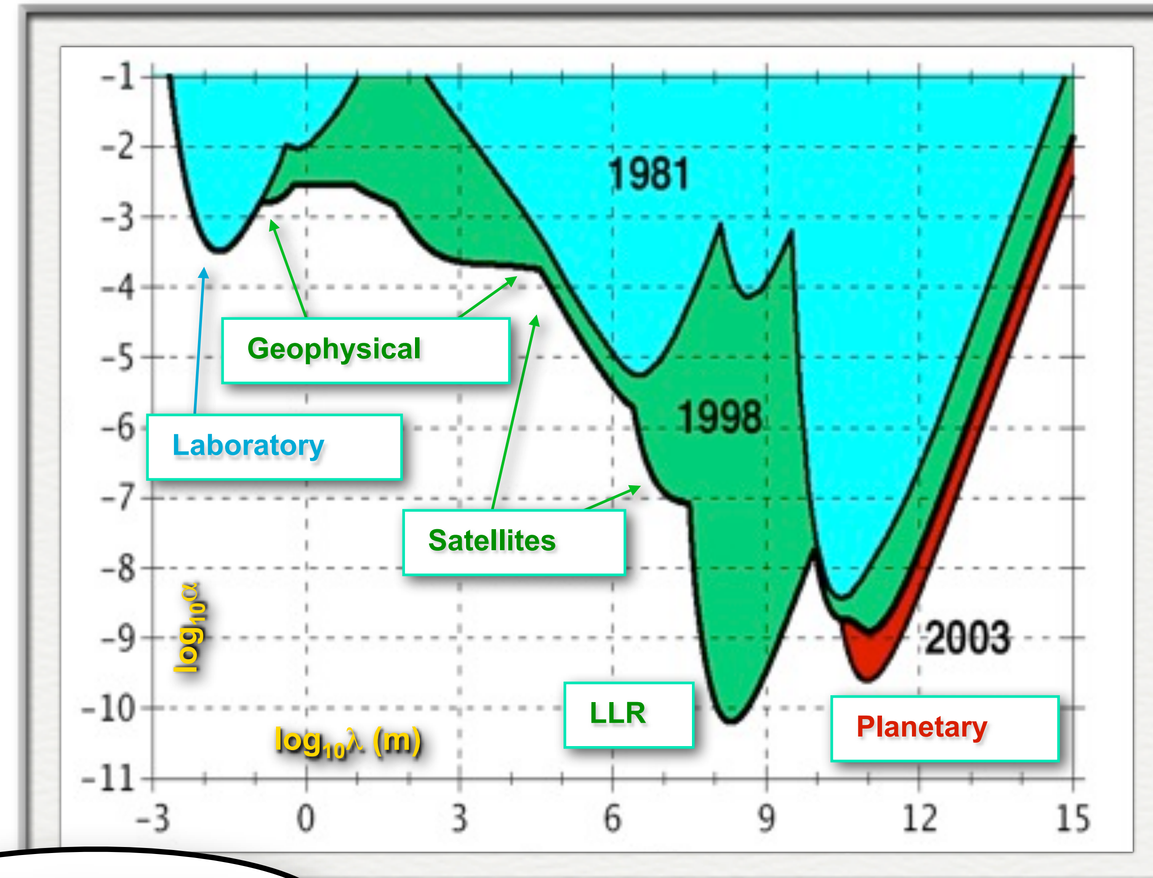
Large distances [Mass energy equivalence]:

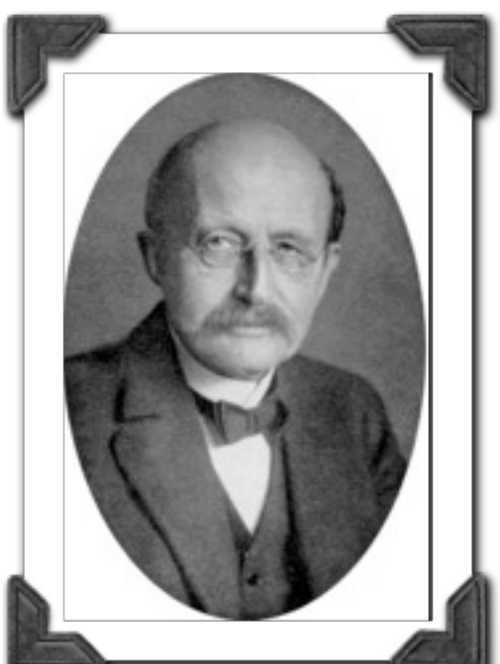
- Cosmological constant
- Dark Matter/Energy

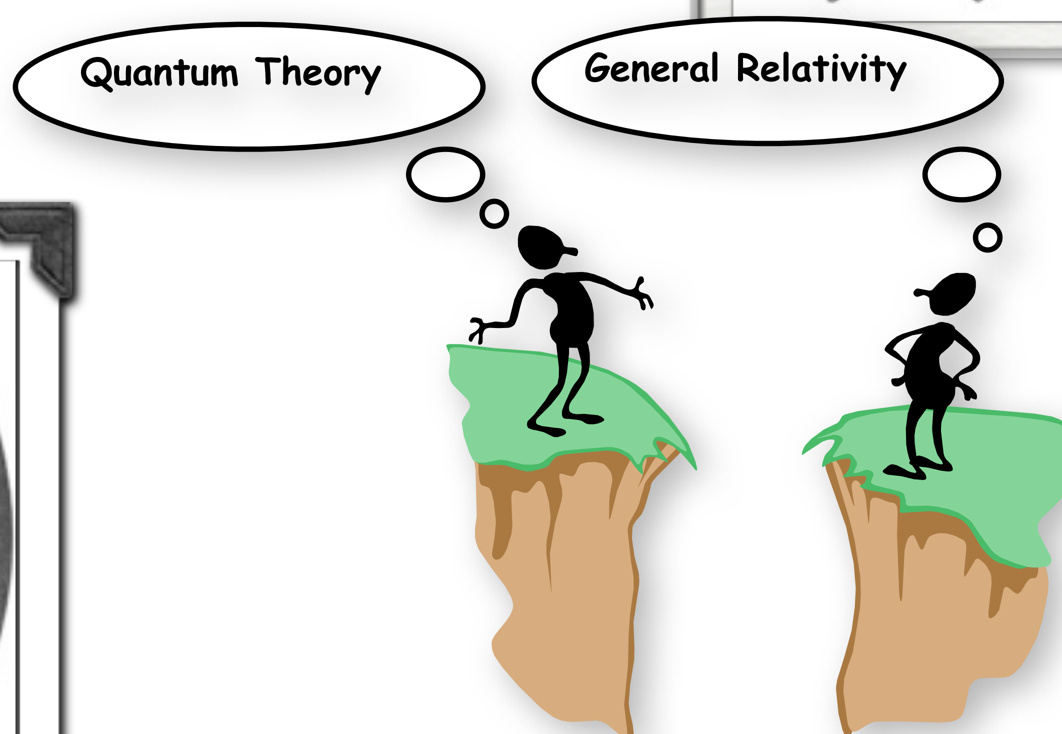
Short distances [Fifth force]

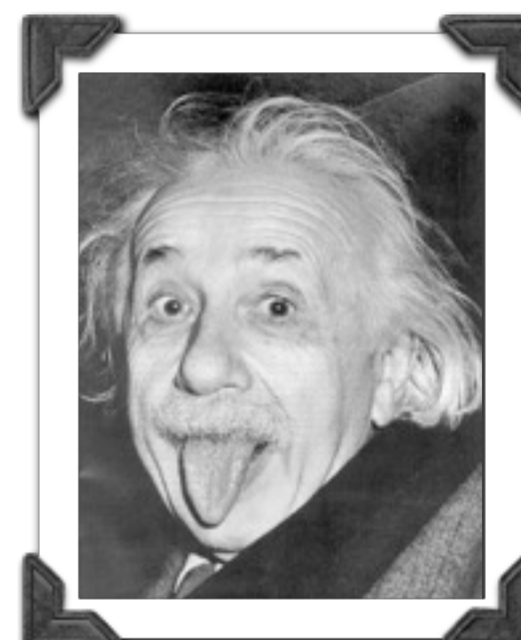
Casimir force is the dominant effect between two neutral objects at distances between the nanometer and the millimeter

$$E = -\frac{GM_1M_2}{r}(1 + \alpha e^{-r/\lambda})$$



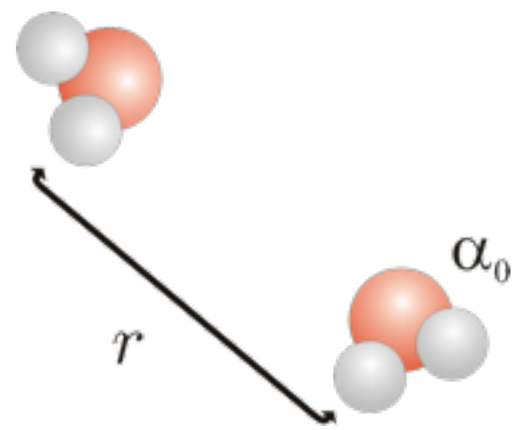






Fluctuation-induced interactions: The Casimir Effect

Non additivity !



Retarded van der Waals Interaction

$$F \approx -\frac{\lambda_a C_6}{r^6(r+\lambda_a)}$$

Difference of pressure

$$\mathcal{F}(L) = k_B T \int_0^\infty d\omega \ln\left(2\sinh\left[\frac{\hbar\omega}{2k_BT}\right]\right)\Delta\rho(\omega, L)$$

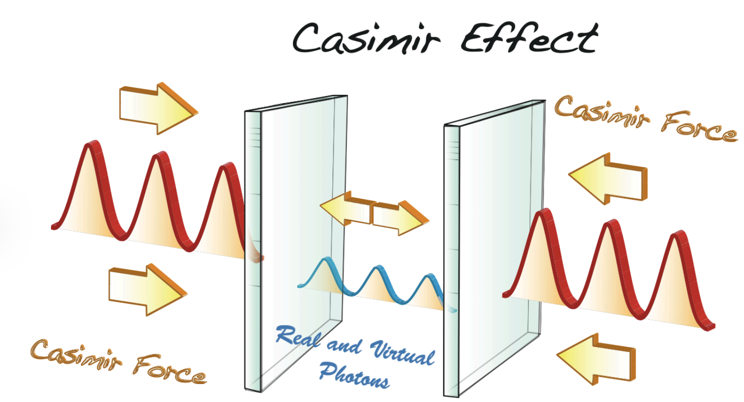
Free energy per mode

DOS Change

$$\Delta\rho(\omega, L) = -\frac{1}{\pi}\partial_\omega \mathrm{Im}\sum_{p,\mathbf{k}} \ln\left(1 - r^p_{\mathbf{k}}[\omega]^2 e^{2ik_zL}\right)$$

Magdeburg hemispheres

$$F \approx -\frac{\lambda_p C_3}{L^3(L+\lambda_p)}$$

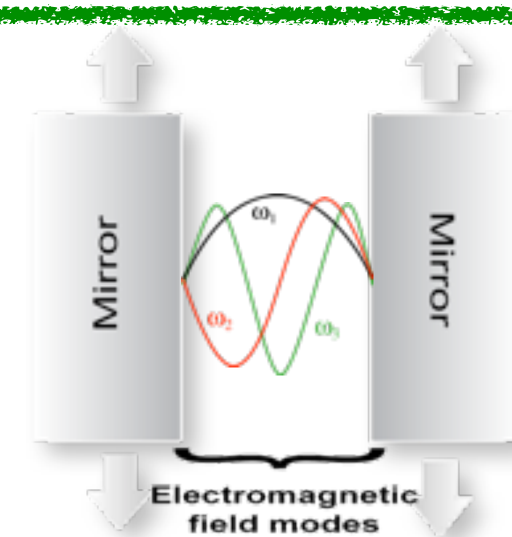


Sum over the zero point energies

H. Casimir

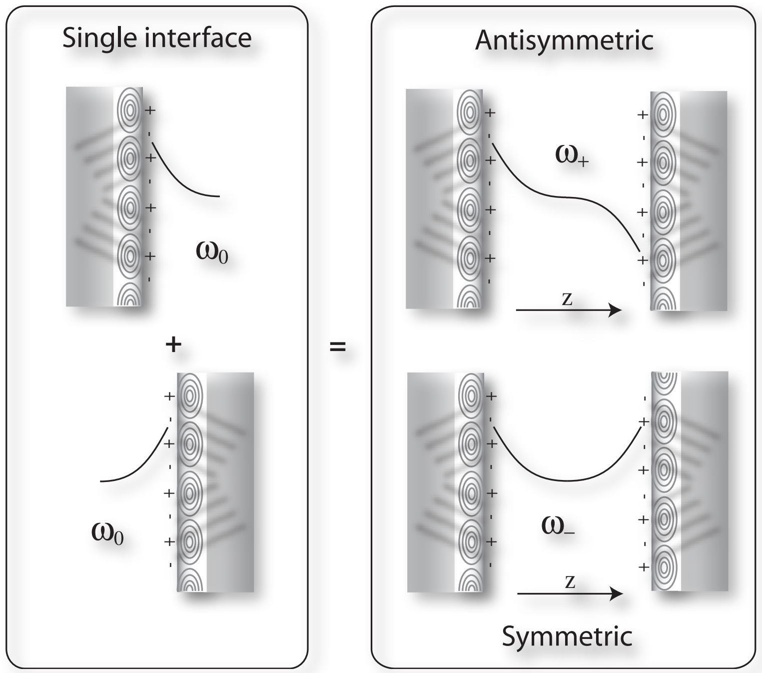
H. Casimir, Proc. kon. Ned. Ak. Wet. **51**, 793 (1948)

$$E = \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2}\left[\sum_n \omega^p_n\right]_L}_{\text{Infinite zero point energy}} - \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2}\left[\sum_n \omega^p_n\right]_{L\to\infty}}_{\text{Setting the zero}}$$

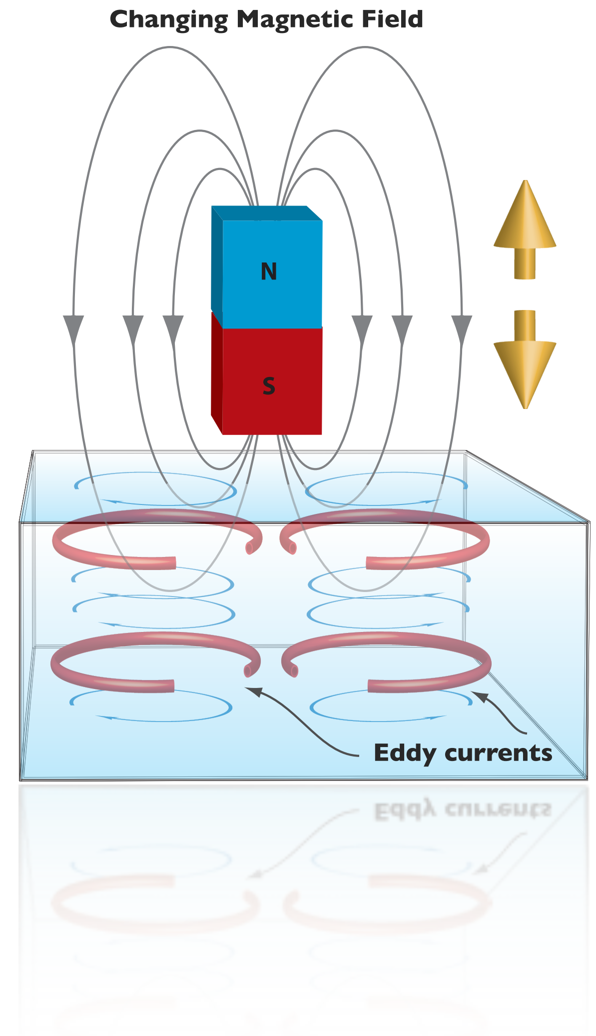


Today's topics

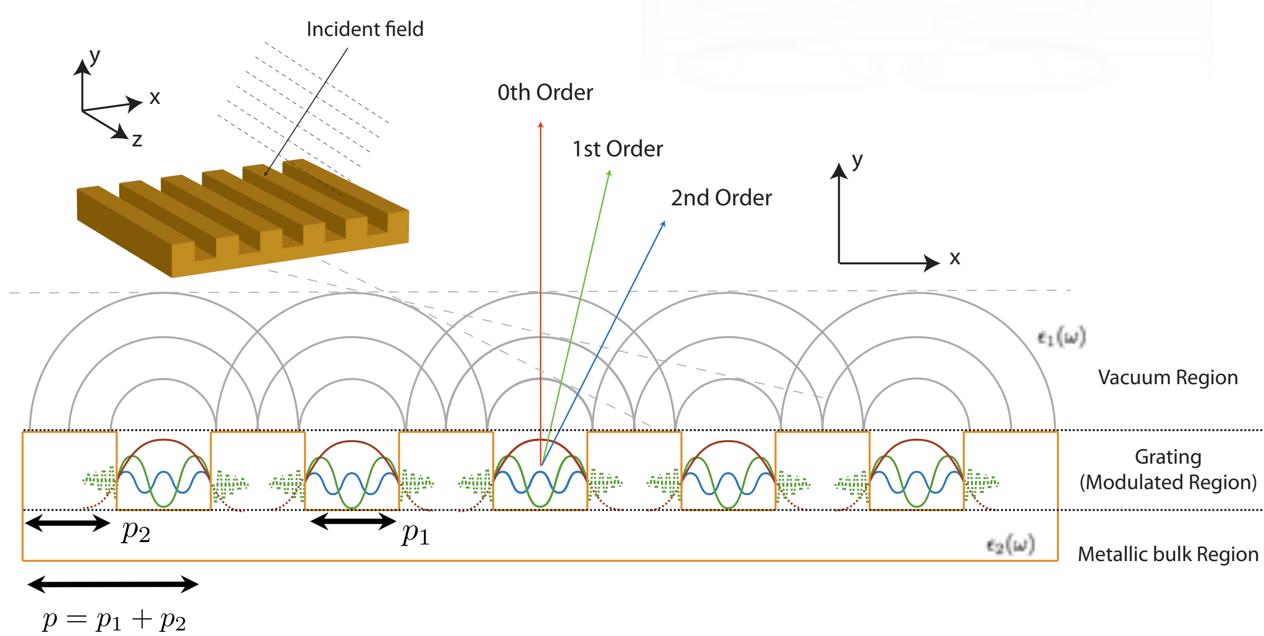
Plasmonics



Diffusive Electrodynamics



Geometrical effects



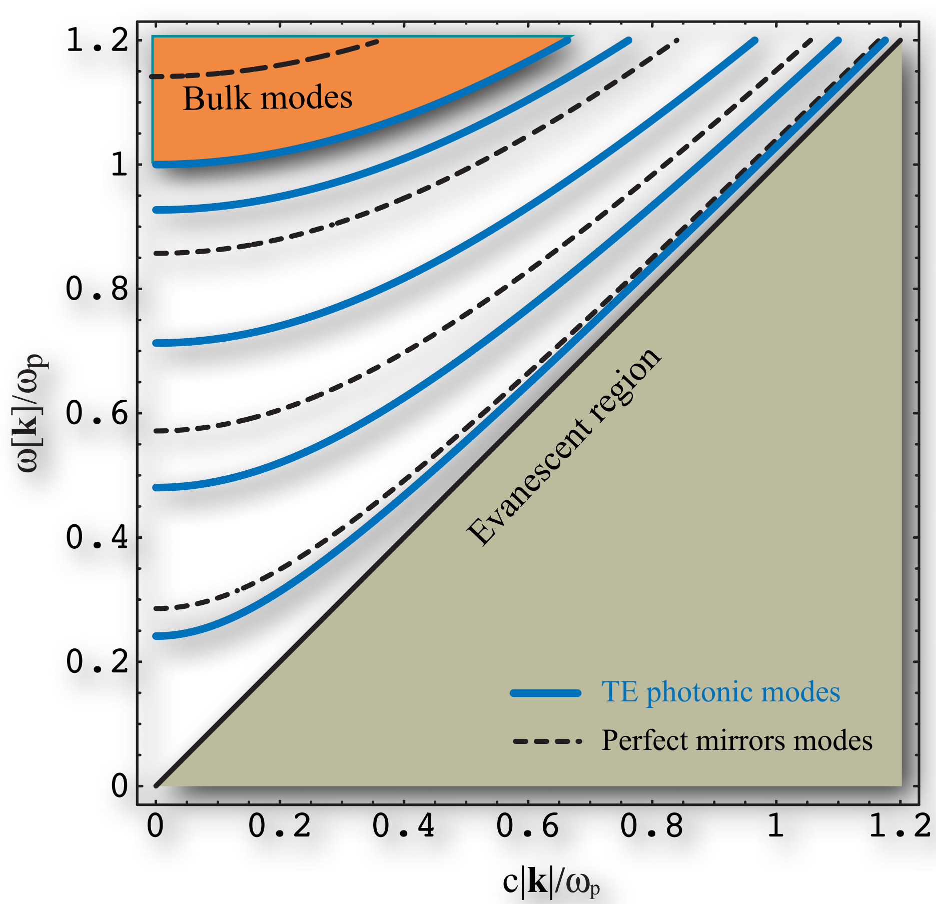
Casimir effect and surface plasmons

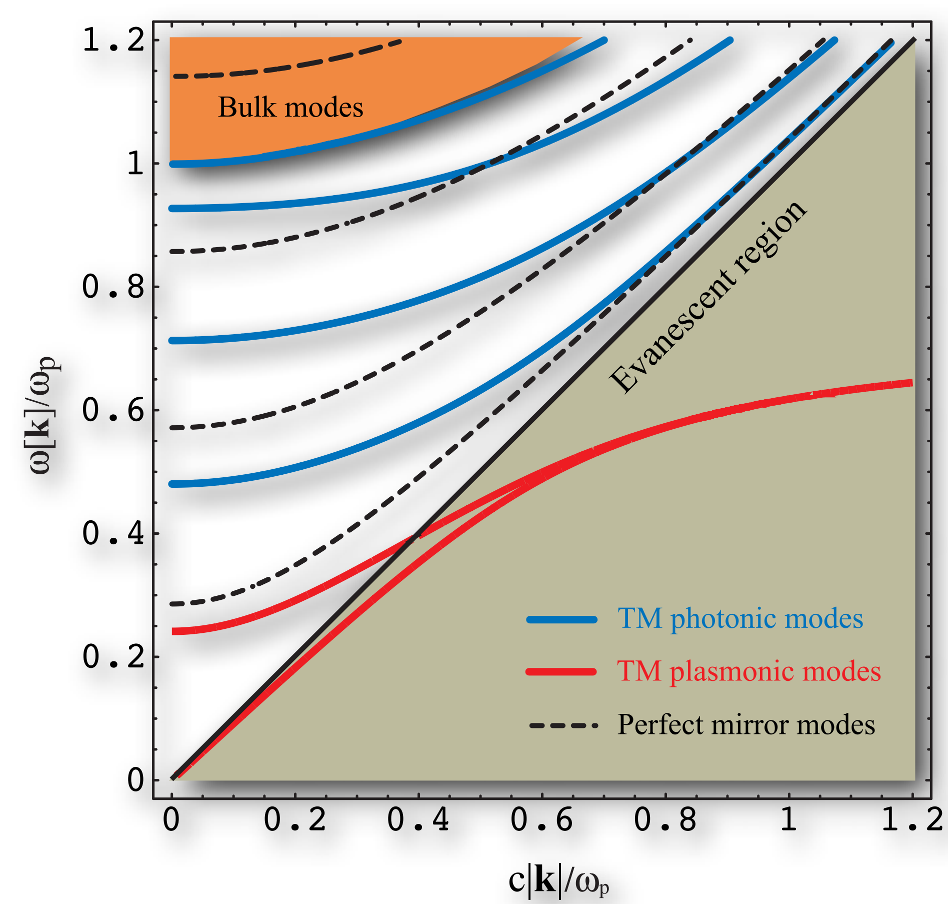
Mode Contributions: non-dissipative case

F. I. and A. Lambrecht, *Phys. Rev. Lett.* **94**, 110404 (2005).
F. I., C. Henkel and A. Lambrecht. *Phys. Rev. A* **76**, 033820 (2007).

Plasma model

$$\left.\begin{array}{r}\mu[\omega]=1\\ \epsilon[\omega]=1-\frac{\omega_p^2}{\omega^2}\end{array}\right\}\rightarrow \qquad E=\underbrace{\sum_{\mathbf{k}}\frac{\hbar}{2}\left[\omega_{+}+\omega_{-}\right]^{L}_{L\to\infty}}_{\text{Plasmonic contribution }(E_{pl})}+\underbrace{\sum_{p,\mathbf{k}}\frac{\hbar}{2}\left[\sum_{m}\omega_m^p\right]^{L}_{L\to\infty}}_{\text{Photonic contribution }(E_{ph})}$$





All the TE-modes belong to the propagative sector

They differ from the perfect mirrors modes because of the dephasing due to the non perfect reflection coefficient.

TM-modes propagative modes look qulitatively like TE modes.

There are only two evanescent modes. They are the generalization to al distances of the coupled plasmon modes.

Plasmonic contribution in metallic plates

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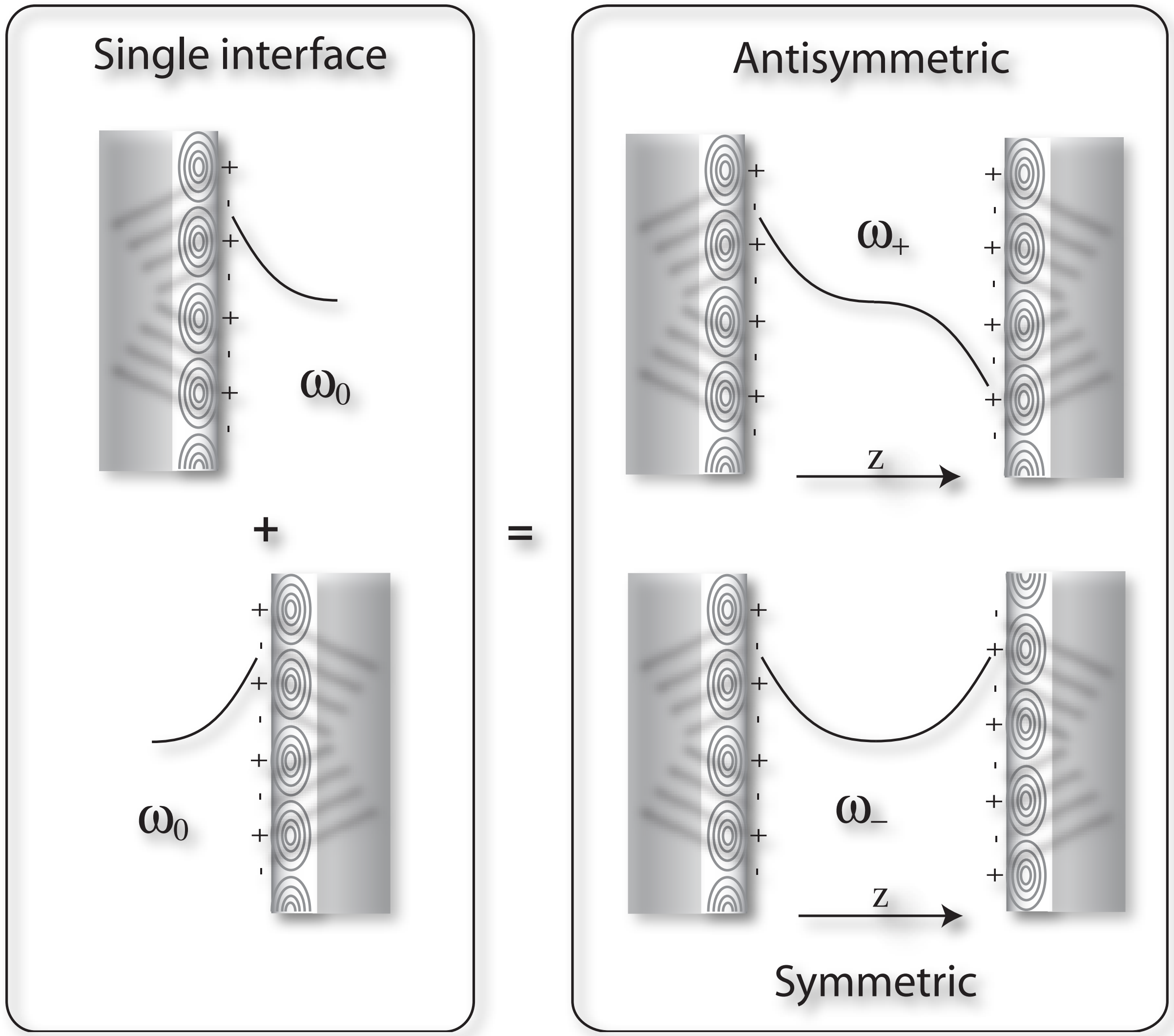
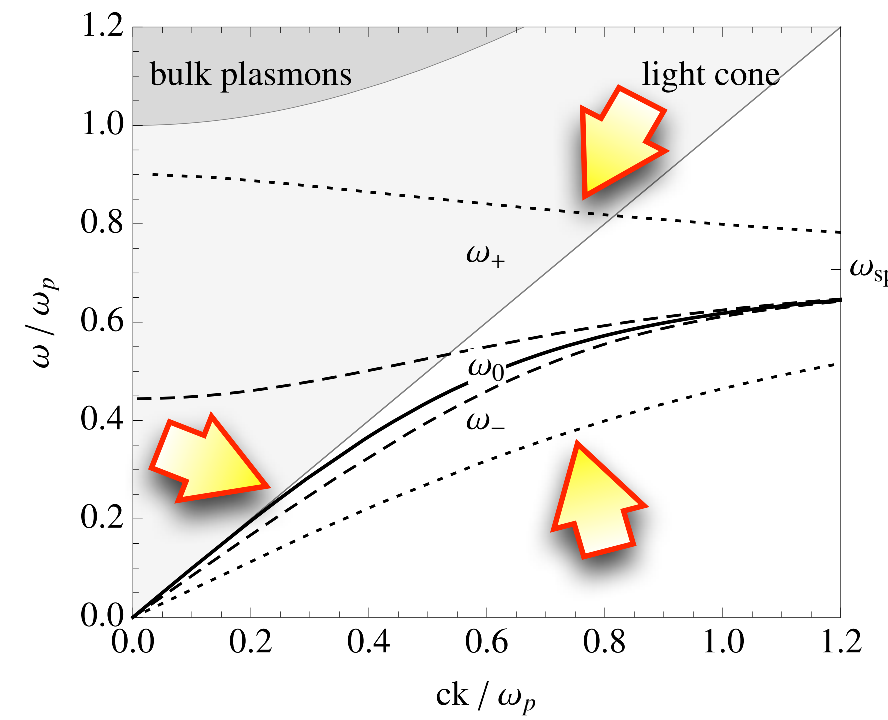
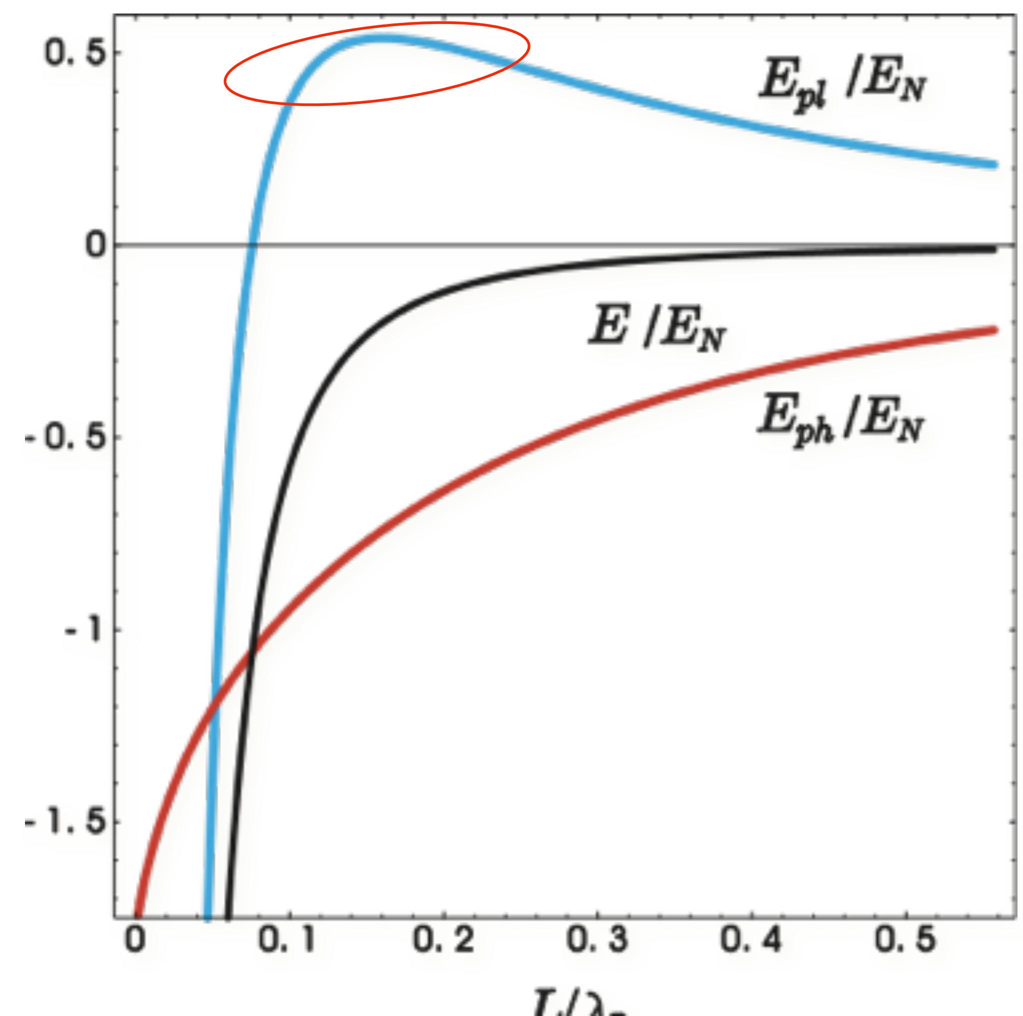
F. I. and A. Lambrecht, *Phys. Rev. Lett.* **94**, 110404 (2005).

F. I., C. Henkel and A. Lambrecht. *Phys. Rev. A* **76**, 033820 (2007).







- Attractive and dominant at short separations
- Large and repulsive at large separations
- Unusual power law ($E \sim L^{-5/2}$) at large separations

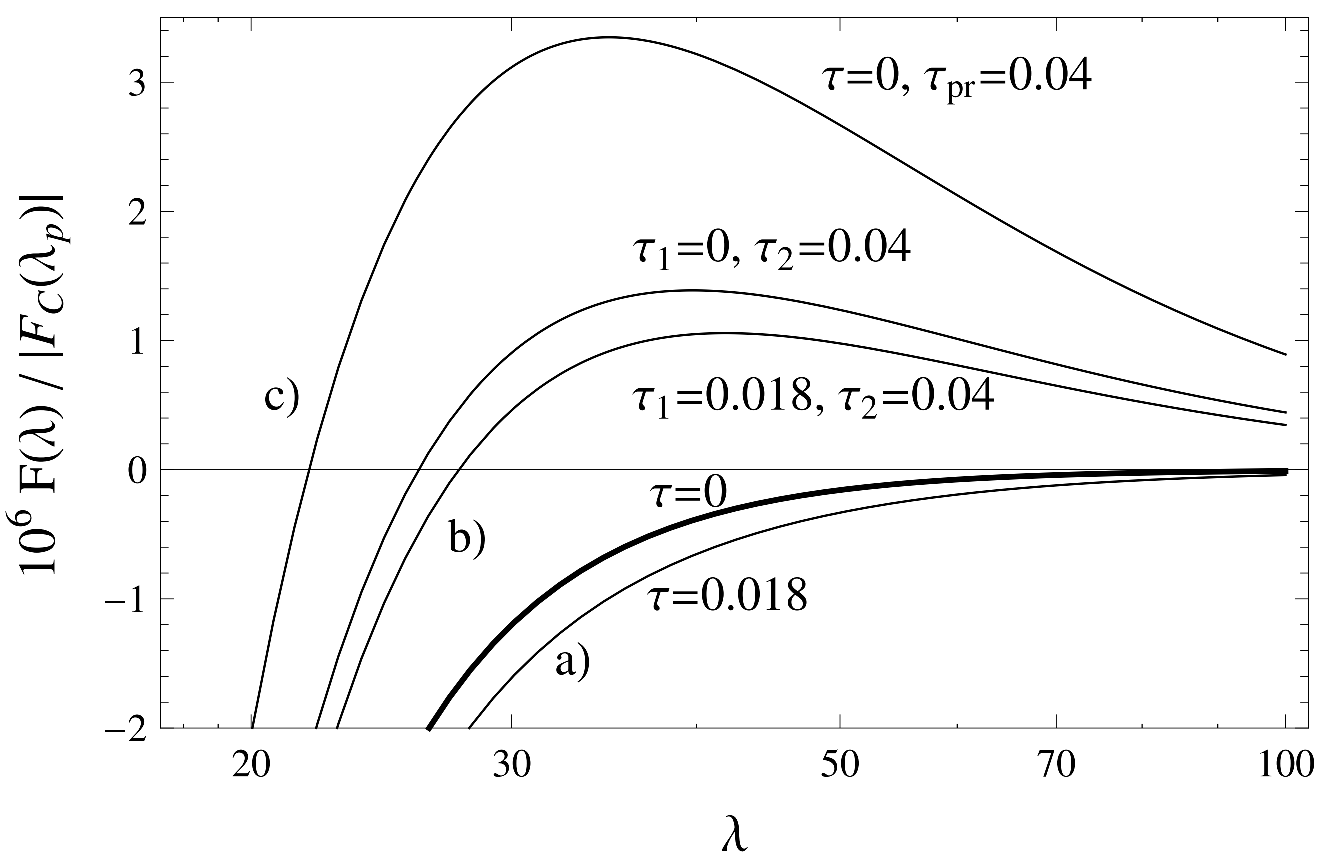
Can one reduce the force or get repulsion changing the balance of the two contributions?

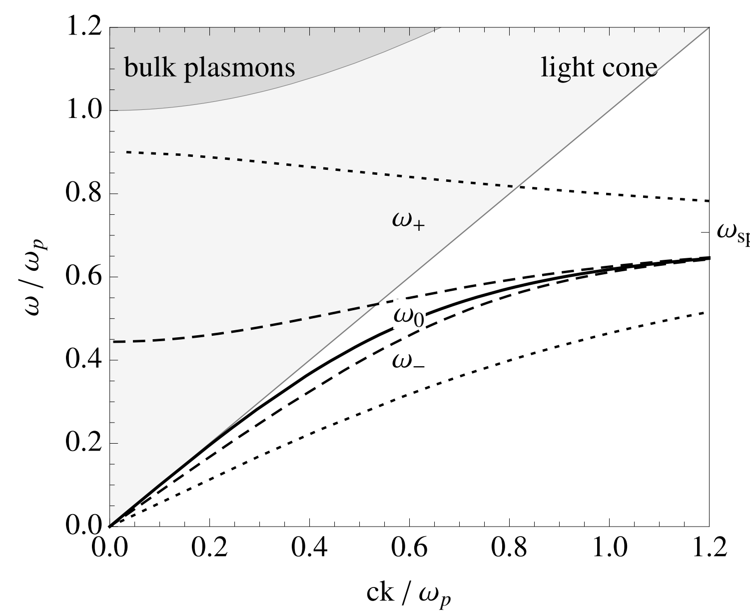
Out of equilibrium plasmons

H. Haakh, F.I. and C. Henkel, Phys. Rev. A **82,** 012507 (2010)

Total Casimir force (per unit area) in thermal equilibrium (thick line) and in different non-equilibrium scenarios.

$$T = 300\,\mathrm{K}(665\,\mathrm{K}) \Rightarrow \tau \approx 0.018(0.04) \quad 10^{-6}|F_\mathrm{C}(\lambda_\mathrm{p})| \approx 3.65\,\mu\mathrm{Pa} \quad \text{(for gold)}$$





a) Total equilibrium

b) Surface plasmon modes of one plate out of equilibrium

c) All modes at equilibrium, except for the propagating branch of the plasmonic mode.

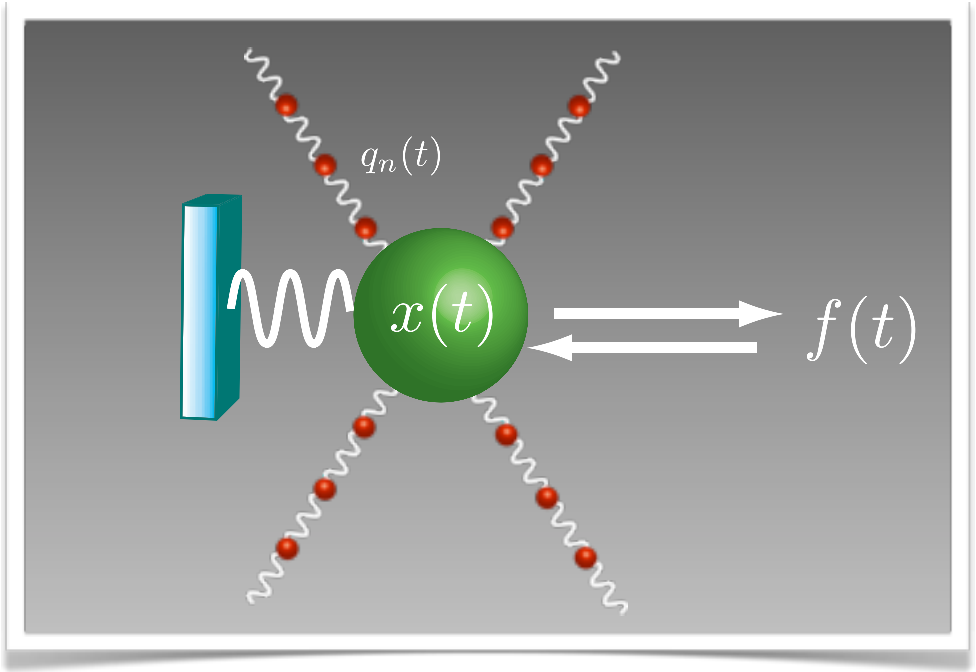
The sign change to repulsion (positive pressure) would occur for gold at distances between ~ 2.7 μm and ~3.7 μm

Role of dissipation: diffusive electrodynamics

Dissipative case: Open system

The "system+bath" paradigm

G. W. Ford, M. Kac, and P. Mazur., J. Math. Phys. **6**, 504 (1965).
K. E. Nagaev and M. Buttiker, Europhys. Lett. **58**, 475 (2002)
F. I. et al. Phys. Rev. A **67**, 042108 (2003)
and a lot more...



In this picture, the observables relevant to a mode are damped in time because they are strongly coupled to a system with infinitely many degrees of freedom that are not directly accessible.

Sum over complex frequencies

$$E = \sum_{p,\mathbf{k}} \frac{\hbar}{2} \mathrm{Re}\left[\sum_m{}' \omega_m - \frac{2\mathrm{i}\omega_\mathrm{m}}{\pi} \ln \frac{\omega_m}{\Lambda}\right]_{L\to\infty}^{L}$$

"Entropic" contribution

Sum Rule!

$$\left[\sum_m \mathrm{Im}\ \omega_m^p\right]_{L\to\infty}^{L} = 0$$

F.I. and C. Henkel, J. Phys. A: Math. Gen. **41**, 164018 (2008)
F. I. and C. Henkel, Phys. Rev. Lett. **103**, 130405 (2009)
F. Intravaia and R. Behunin, arXiv:1209.6072 (2012).

In the complex frequency plane

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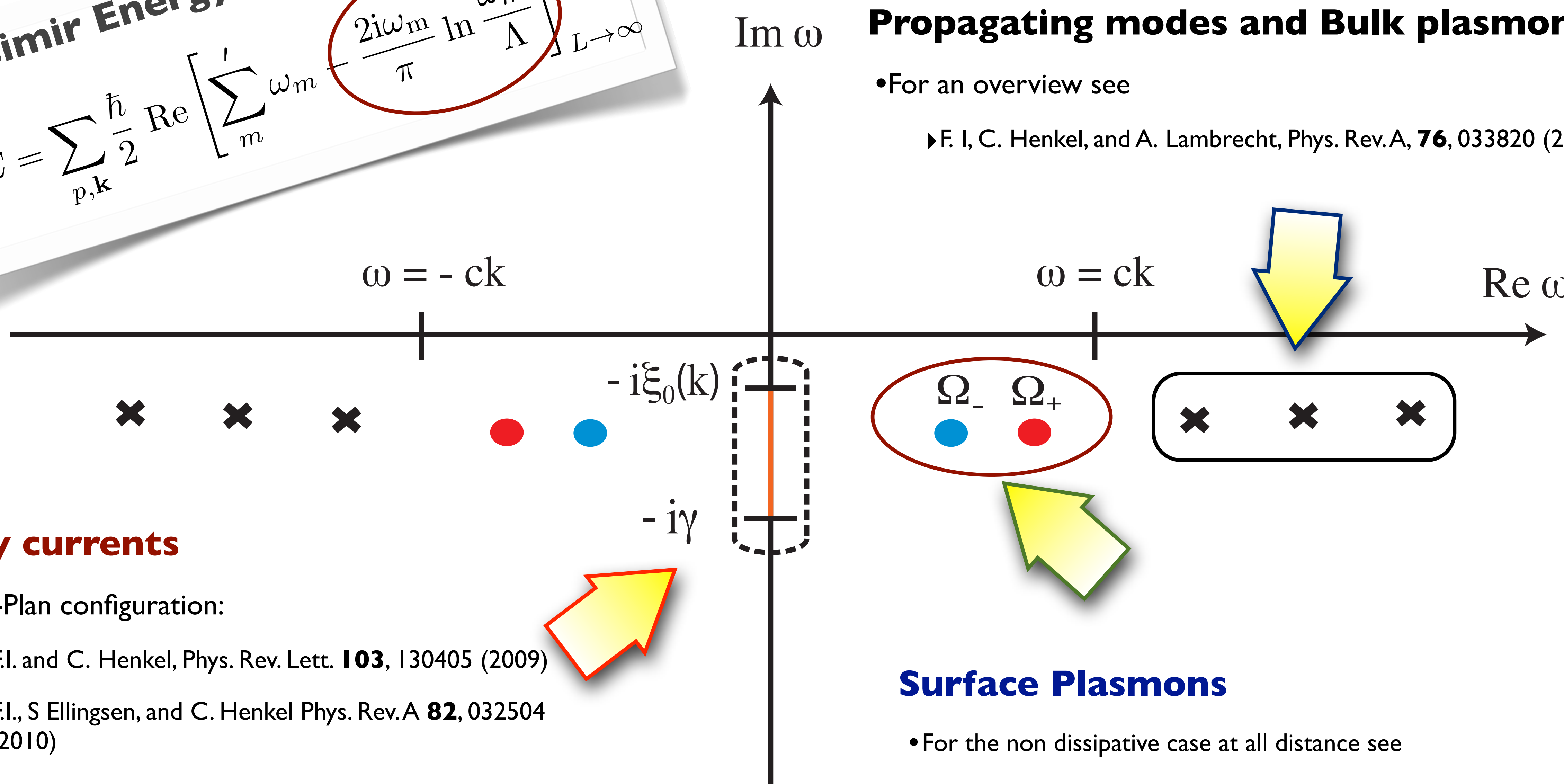
Casimir Energy

$$E = \sum_{p,\mathbf{k}} \frac{\hbar}{2} \mathrm{Re}\left[\sum_m{}' \omega_m - \frac{2i\omega_m}{\pi}\ln\frac{\omega_m}{\Lambda}\right]^L_{L\to\infty}$$



Propagating modes and Bulk plasmons

- For an overview see
  - F. I, C. Henkel, and A. Lambrecht, Phys. Rev. A, **76**, 033820 (2007)

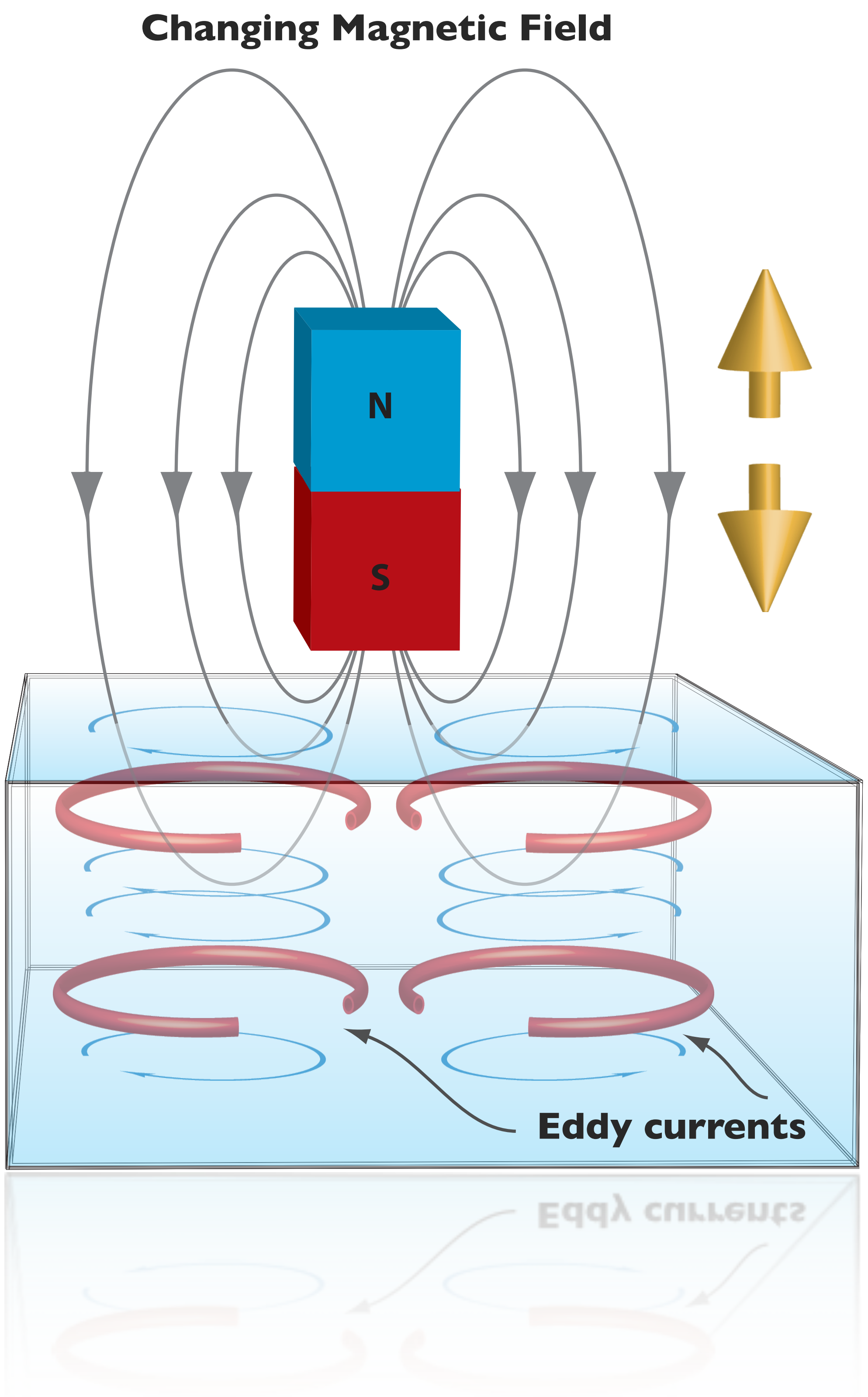
Eddy currents

- Plan-Plan configuration:
  - F.I. and C. Henkel, Phys. Rev. Lett. **103**, 130405 (2009)
  - F.I., S Ellingsen, and C. Henkel Phys. Rev. A **82**, 032504 (2010)
  - F. I and C. Henkel, arXiv:0911.3490v1
  - C. Henke and F.I., arXiv:0911.3489
- Effects on atom-surface interaction:
  - H. Haakh, F.I., C. Henkel et al., Phys. Rev. A **80**, 062905 (2009)

Surface Plasmons

- For the non dissipative case at all distance see
  - F. I and A. Lambrecht, Phys. Rev. Lett., **94**, 110404 (2005).
  - F. I, C. Henkel, and A. Lambrecht, Phys. Rev. A, **76**, 033820 (2007)
- Dissipative case at short distance
  - F. I and C. Henkel, arXiv:0911.3490

Foucault-Eddy currents



Principle (Classical View)

- A changing magnetic field generates swirling current (Lorentz force).
- These circulating eddies of current create electromagnets with magnetic fields that opposes the change of the magnetic field (Lenz rule).

Applications

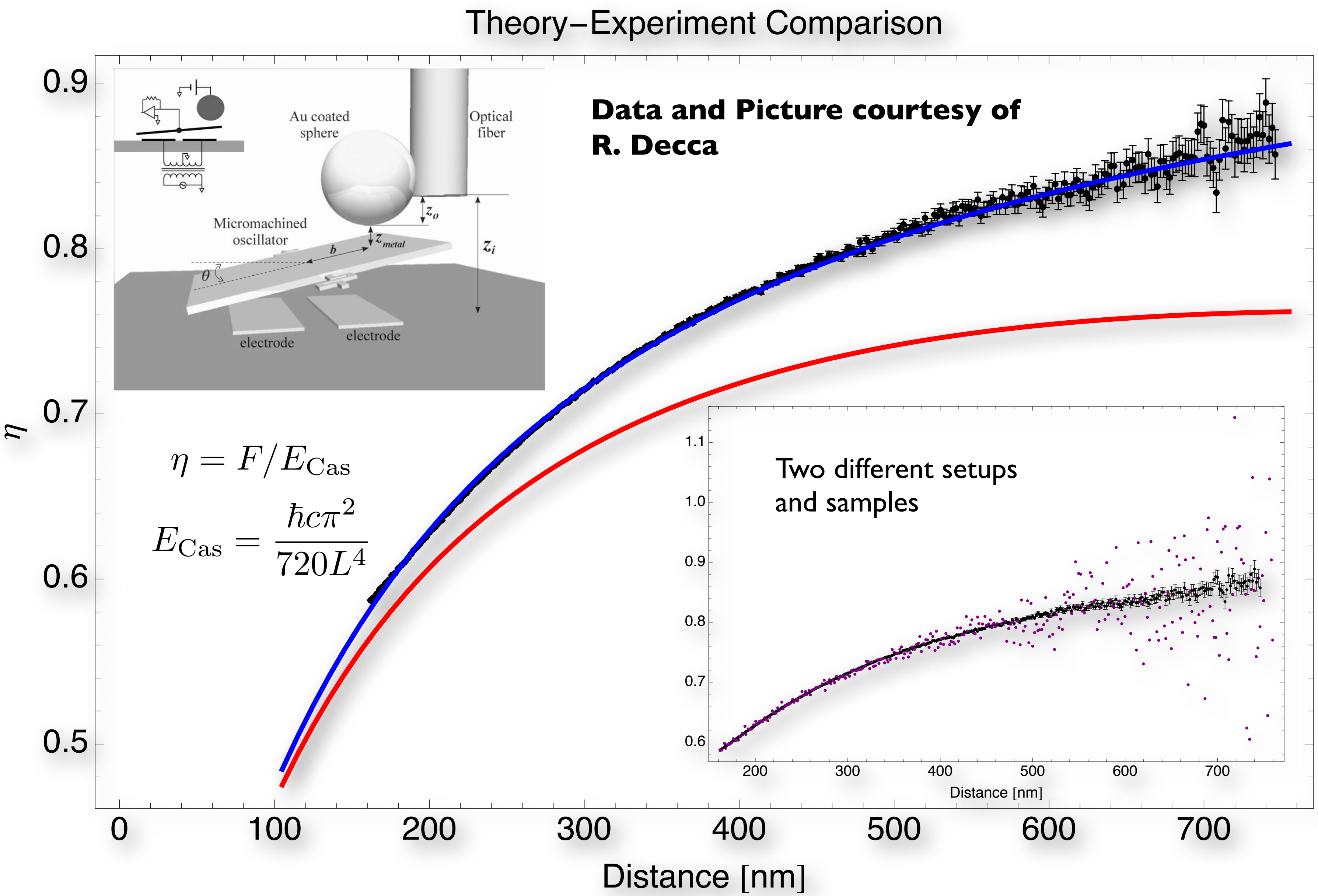
- Magnetic brakes
- Induction oven
- Structural Testing of metallic structures

Dynamycs: Diffusion

- Low frequency diffusive phenomenon
- Heat-conduction-like equation for their vector potential

$$D\ \nabla^2 A = \partial_t A, \quad D = \gamma\lambda^2$$

Theory-Experiment Comparison



A matter of models

$$\epsilon_{\mathrm{Pl}}(\omega) = 1 + A(\omega) - \frac{\omega_p^2}{\omega^2}$$

$$\epsilon_{\mathrm{D}}(\omega) = 1 + A(\omega) - \frac{\omega_p^2}{\omega(\omega + \mathrm{i}\gamma)}$$

(Interband contribution)

Ellipsometric measurements

$$\omega_p = 8.9\,\mathrm{eV} \qquad \gamma = 0.0375\,\mathrm{eV}$$

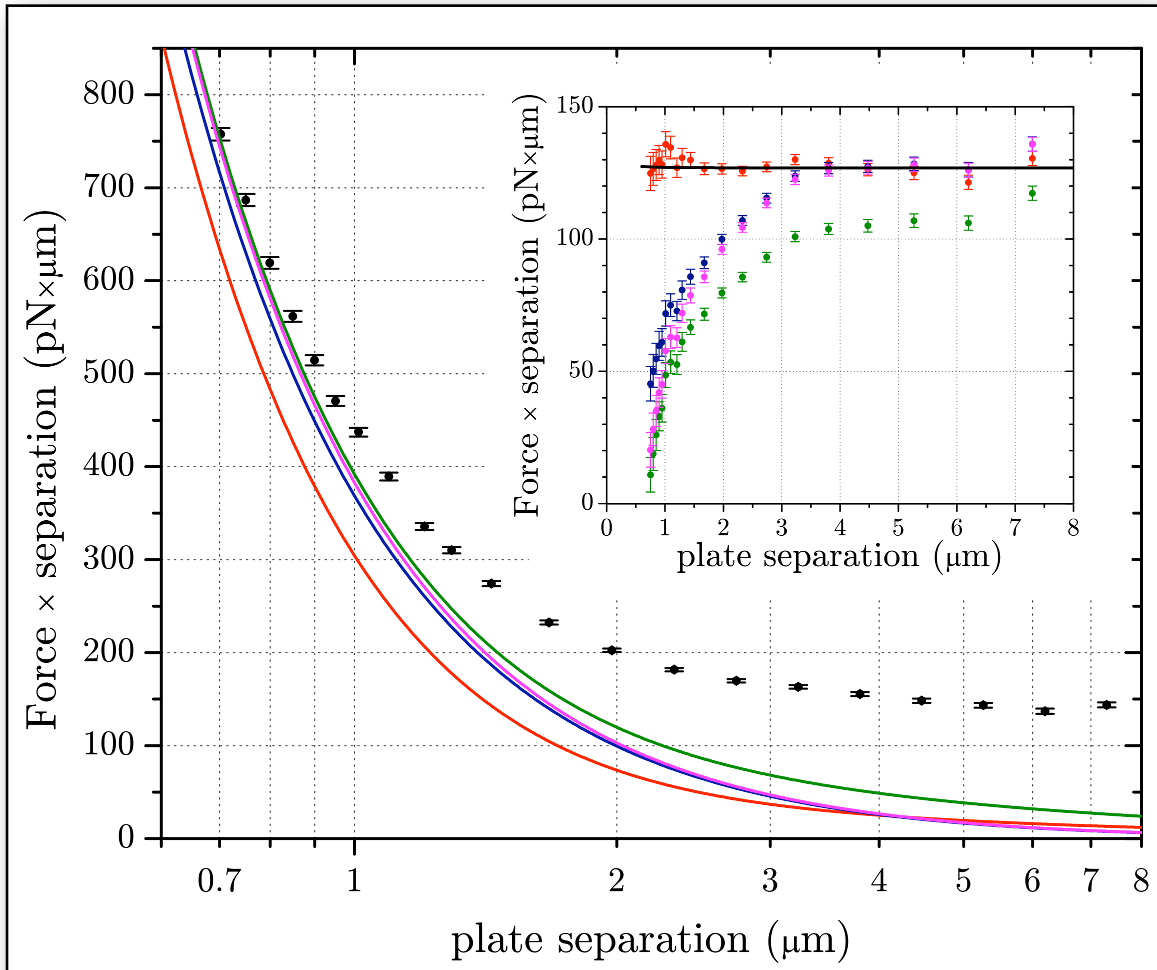
Not very sensitive to the temperature

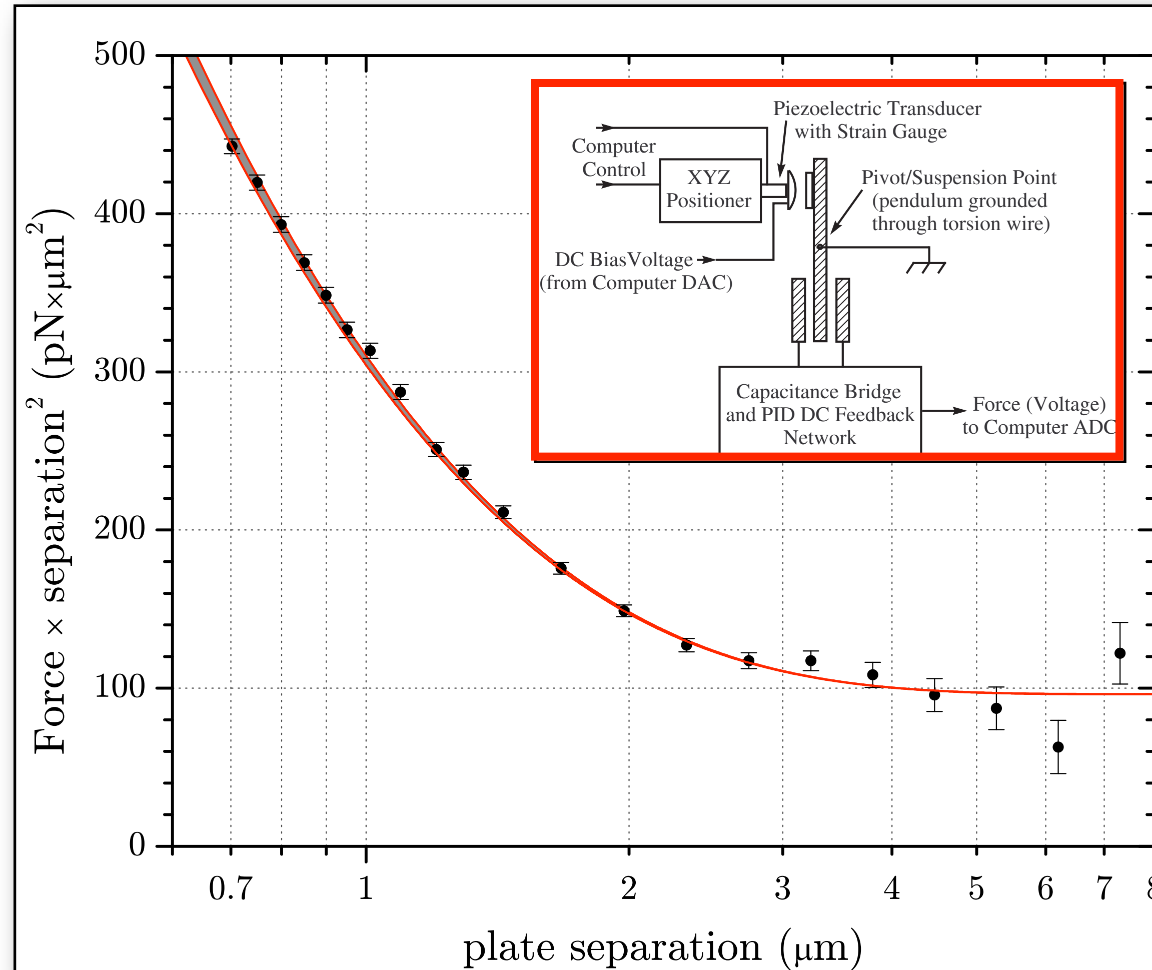
"Maybe..."

- Electrostatic (Patches potentials?) R. O. Behunin, F. I. D. A. R. Dalvit, P. A. M. Neto, and S. Reynaud. Phys. Rev. A **85**, 012504 (2012).
- PFA: seems to be a very good approximation for the experimental distances / overestimate at larger distance
- ...

Recent experiments: Torsional balance

A very recent experiment at Yale measured the Casimir force between 0.7 and 7.5 μm and shows good **agreement with the Drude model** is one takes into account **patch-potentials**

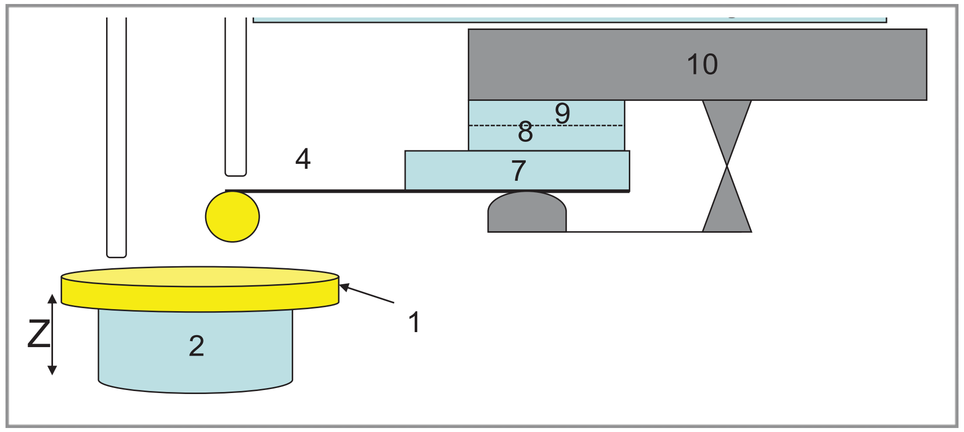




A. O. Sushkov, W. J. Kim, D. A. R. Dalvit, and S. K. Lamoreaux. Nat. Phys. **7**, 230 (2011)

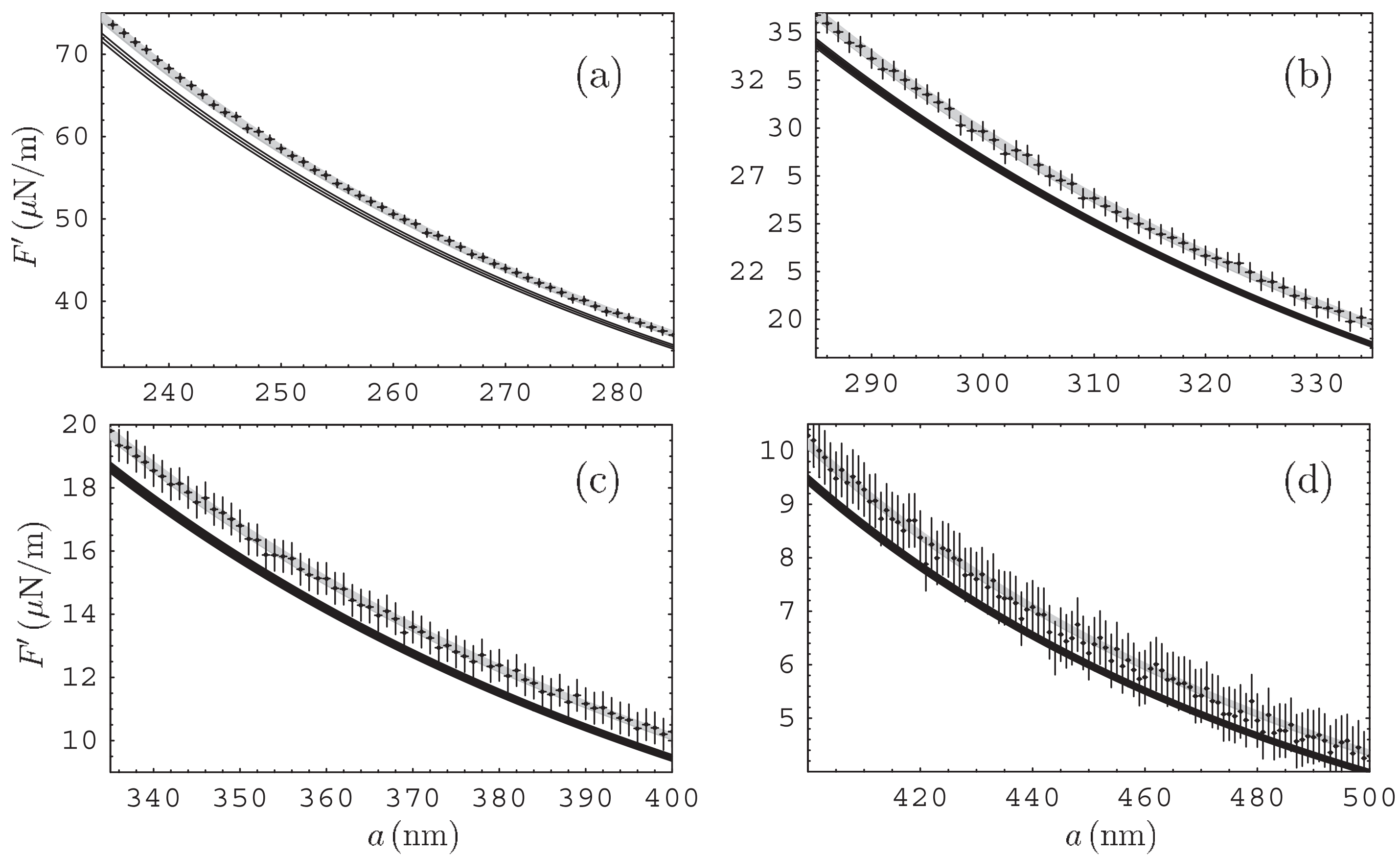
R. O. Behunin, F. I. D. A. R. Dalvit, P. A. M. Neto, and S. Reynaud. Phys. Rev. A **85**, 012504 (2012).

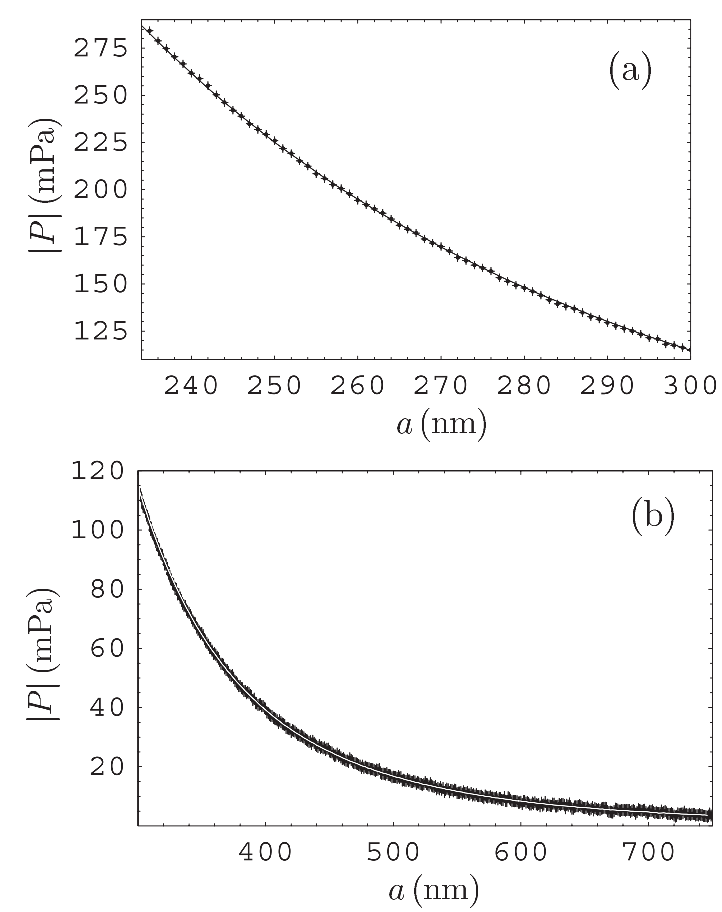
Recent experiments: Dynamic AFM



A even more recent experiment at Riverside confirmed the **agreement with the Plasma model. It shows a very good agreement with the results of the previous experiment.**

C.-C. Chang et al, Phys. Rev. B **85**, 165443 (2012).





Turning the Eddy currents off

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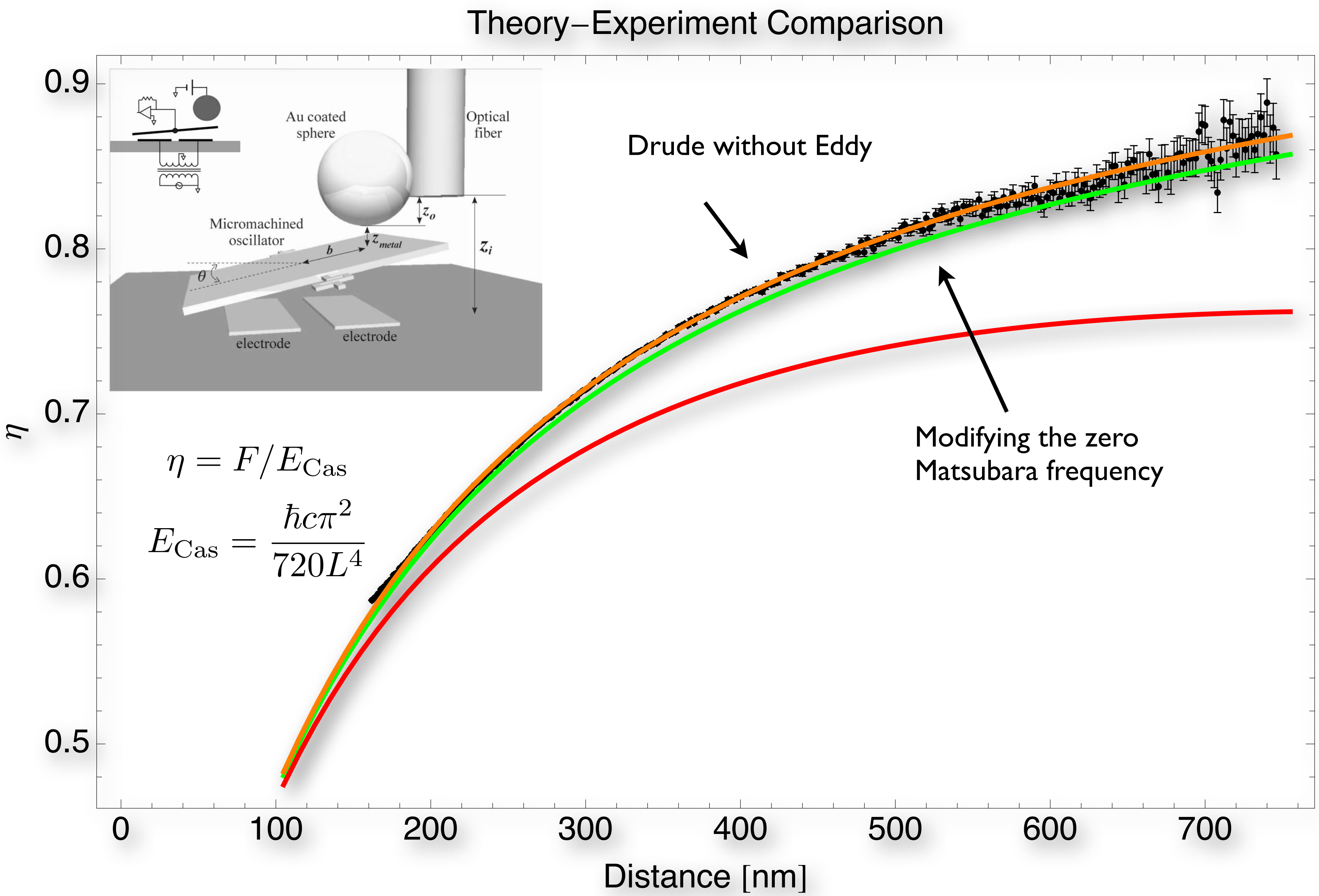


A matter of models

$$\epsilon_{\mathrm{Pl}}(\omega) = 1 + A(\omega) - \frac{\omega_p^2}{\omega^2}$$

$$\epsilon_{\mathrm{D}}(\omega) = 1 + A(\omega) - \frac{\omega_p^2}{\omega(\omega + \mathrm{i}\gamma)}$$

(Interband contribution)

Ellipsometric measurements

$$\omega_p = 8.9\,\mathrm{eV} \qquad \gamma = 0.0375\,\mathrm{eV}$$

Matsubara Summation

T. Matsubara, Prog. Theor. Phys. **14**, 351 (1955)

$$\mathcal{F} = k_B T \sum_{p,n}{}' \Gamma_p(\mathrm{i}\xi_n, L)$$

$$\xi_n = 2\pi n k_B T/\hbar$$

$$\Gamma_{TM}(L,\omega) \leftrightarrow r^{TM}(\omega) \xrightarrow{\omega \to 0} 1$$

$$\Gamma_{TE}(L,\omega) \leftrightarrow r^{TE}(\omega) \xrightarrow{\omega \to 0} \begin{cases} 0 & \text{Drude (Bohr-von Leeuwen)} \\ \neq 0 & \text{Plasma, BCS, ...} \end{cases}$$

H.-J.V. Leeuwen, J. Phys. Radium 2, **361** (1921).

G. Bimonte, H. Haakh, C. Henkel, and F. I., J. Phys. A **43**, 145304 (2010)

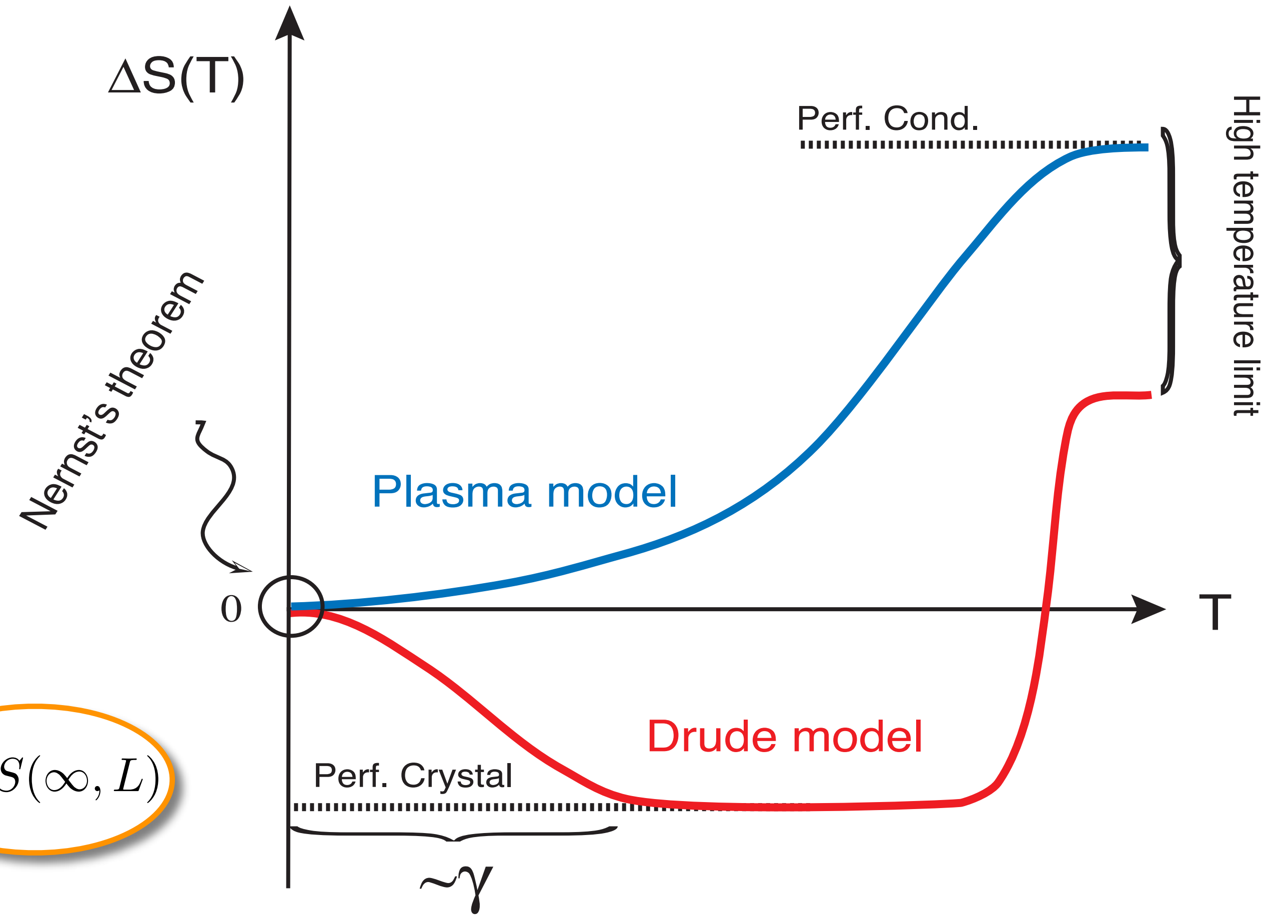
Entropy “Pathologies”

“Perfect crystal” limit:

$$\epsilon_{\mathrm{D}}(\omega) = 1 + A(\omega) - \frac{\omega_p^2}{\omega(\omega + \mathrm{i}\gamma(T))}$$

$$\gamma(T) \xrightarrow{T \to 0} T^{\alpha} \quad , \alpha > 1 \longrightarrow -\Delta S(\infty, L)$$

$$\Delta S(T, L) = S(T, L) - S(T, L \to \infty)$$



$\Delta S(\infty, L)$

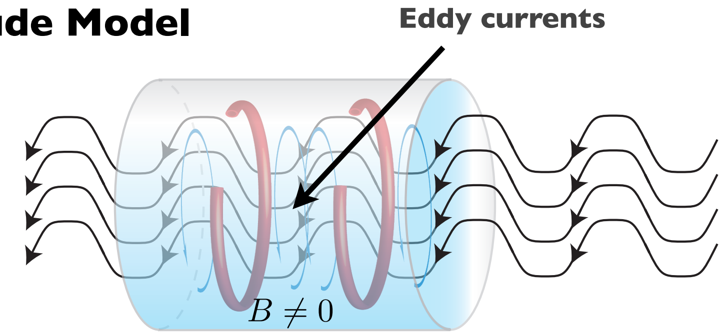
The sign of the entropy has no physical meaning since we are looking at difference of entropies

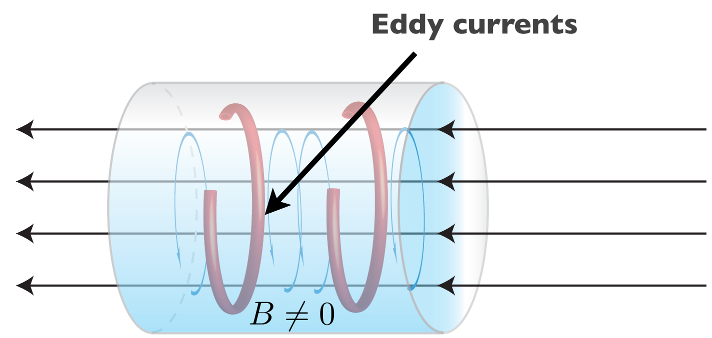
The third law of thermodynamics states that the entropy of a system at absolute zero is a well-defined constant. For a quantum system at zero temperature the entropy is determined only by the degeneracy of the ground state.

Reason for this "Pathology"

F.I. and C. Henkel, Phys. Rev. Lett. **103**, 130405 (2009)

Drude Model





A Drude metal is transparent for low frequency magnetic fields (Bohr - van Leeuwen).

H.-J. van Leeuwen, J. Phys. Radium **2**, 361 (1921)

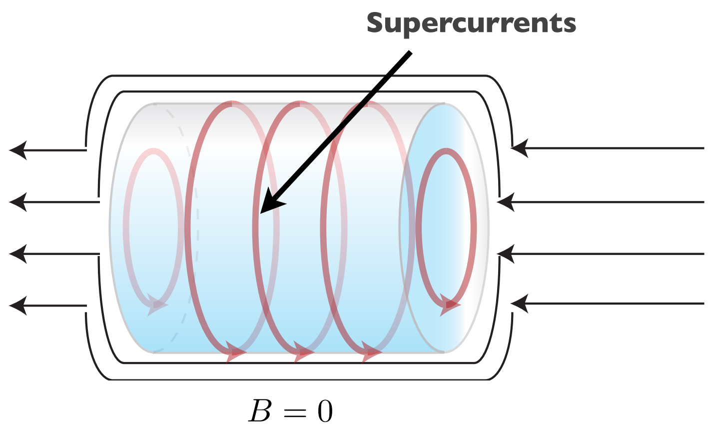
P. R. Buenzli and P. A. Martin, Europhys. Lett., **72**, 42 (2005).

G. Bimonte, Phys. Rev. A, **79**, 042107 (2009).

BCS and Plasma Model

Remarkably, a superconductor will actively exclude any magnetic field present when it makes the phase change to the superconducting state (Meissner Effect). In agreement with London's theory the plasma model is a simple description of a superconductor (initial condition *B=0 inside the metal)*.

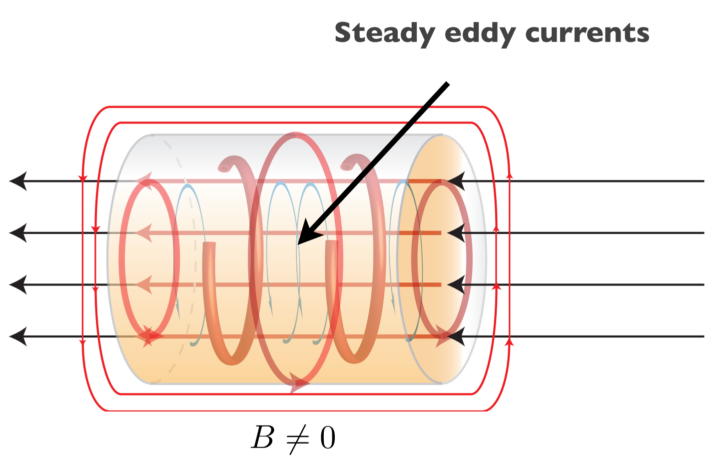
F. London and H. London, Proc. Royal Soc. A **149,** 71 (1935)



"Perfect cristal - Perfect cunductor"

If a perfect metal already had a steady magnetic field through it and was then cooled through the transition to a zero resistance state the magnetic field is be expected to stay the same (solution discarded by London's theory).

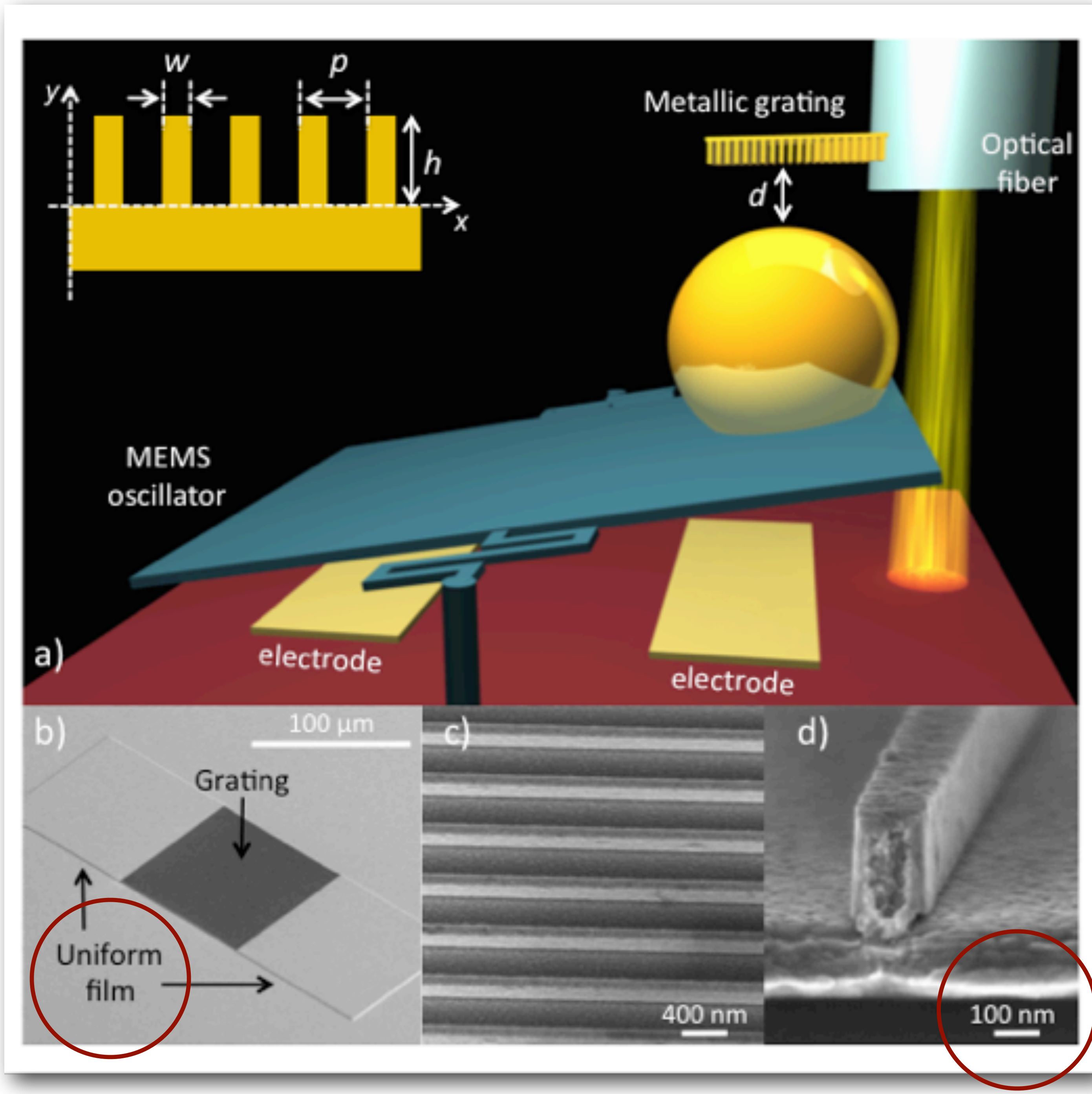
In the "perfect crystal" the disorder is frozen down to zero temperature Situation very similar to a glass (Faucault-Glass)! The Nernst's theorem does not give zero in this case



Casimir effect and geometry effects

Casimir interaction with metallic gratings

R. Decca (IUPUI). World most precise set up!



Motivations:

- Control the Casimir force
- Strong signal
- Plasmonic effects
  (Intravaia & Lambrecht PRL 2005)
  (Intravaia, Henkel & Lambrecht PRA 2007)
  (Haakh, Intravaia & Henkel PRA 2010)

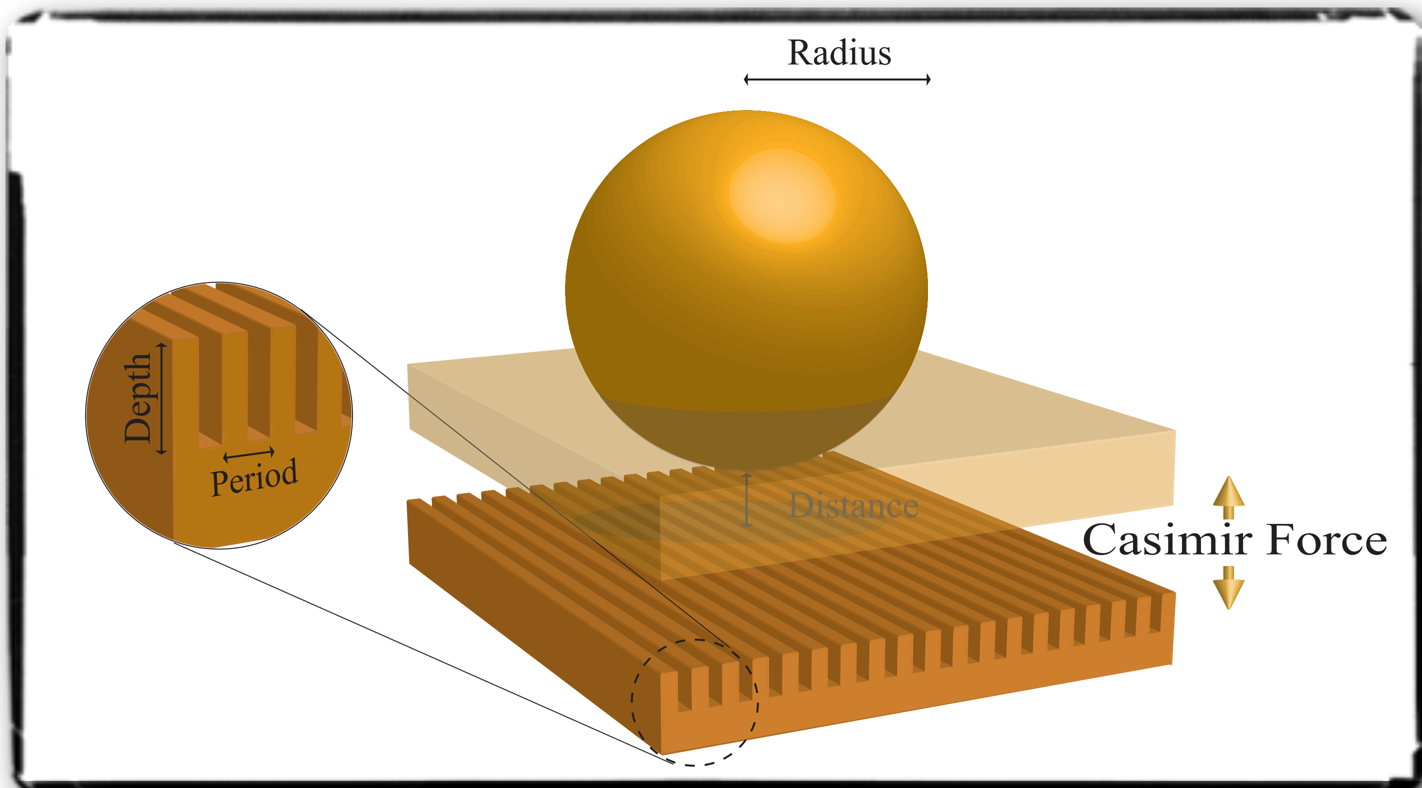
Difficulties:

- Fabrication challenges
- Calibration
- Measurement

} **Experiment**

- High conductivity } **Theory**

Replacing a sphere with a plane



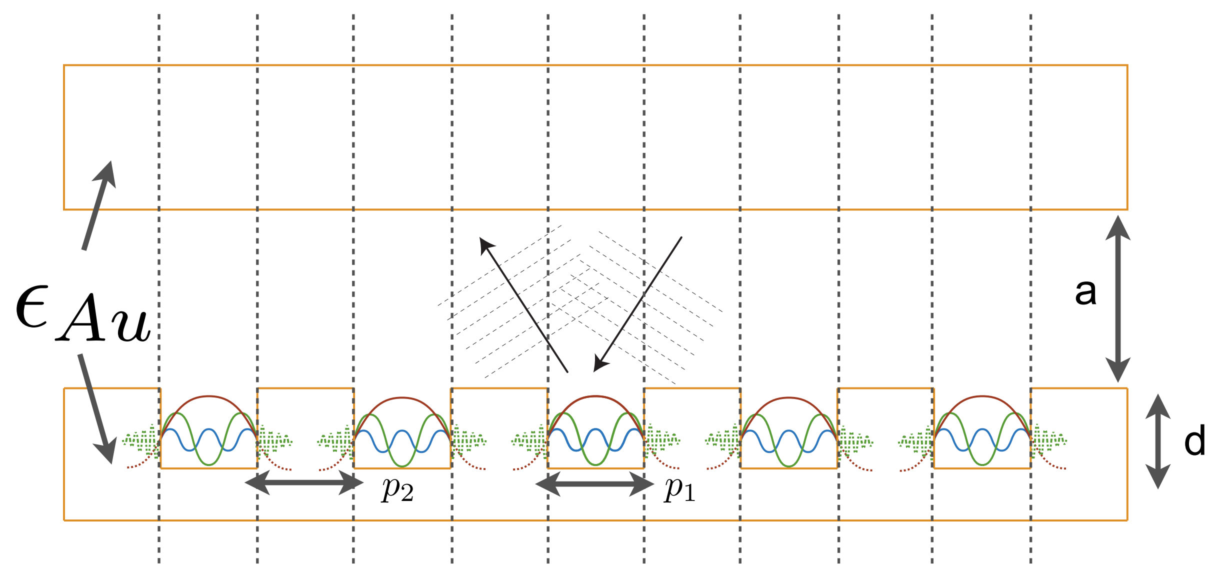
If one of the objects is a sphere with very large radius we can use the Proximity Force Approximation (PFA) and evaluate the interaction as if the sphere was a plane

$$P_{\mathrm{sph-g}}(a) = 2\pi R\, \mathcal{F}_{\mathrm{pl-g}}(a)$$

The reflection coefficients of the plane are given by the Fresnel coefficients.

What to expect? A weaker force

Proximity force approximation

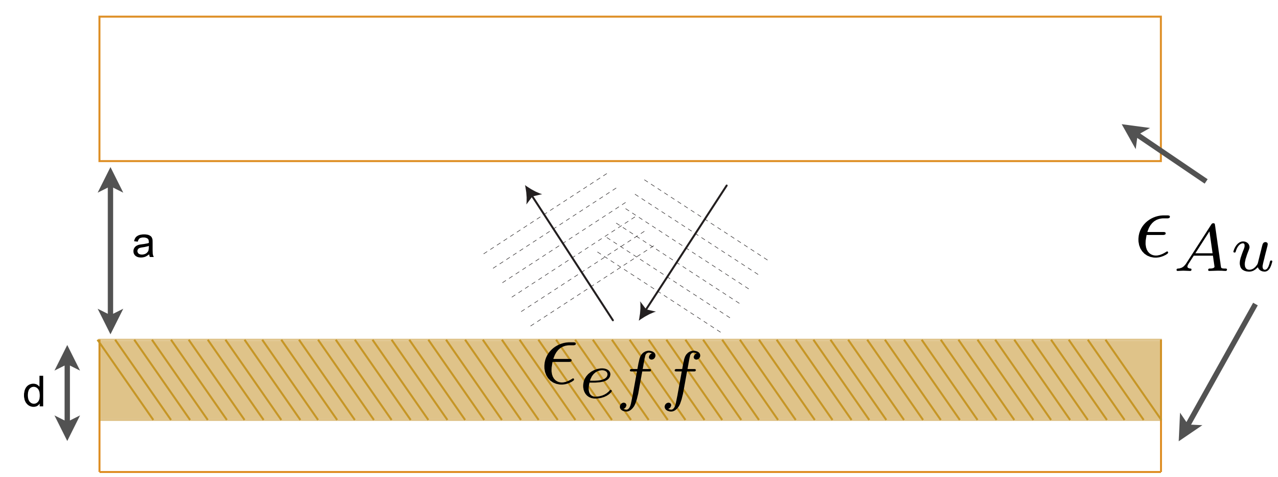


$$\epsilon(\omega, x) = \begin{cases} 1 & |x| \le \frac{p_1}{2} \\ \epsilon(\omega) & \frac{p_1}{2} \le |x| \le \frac{p_1+p_2}{2} = \frac{p}{2} \end{cases}$$

$$P_{\mathrm{PFA}}(a) = fP(a) + (1-f)P(a+d) < P(a)$$

Good approximation for short distance

Effective medium approach



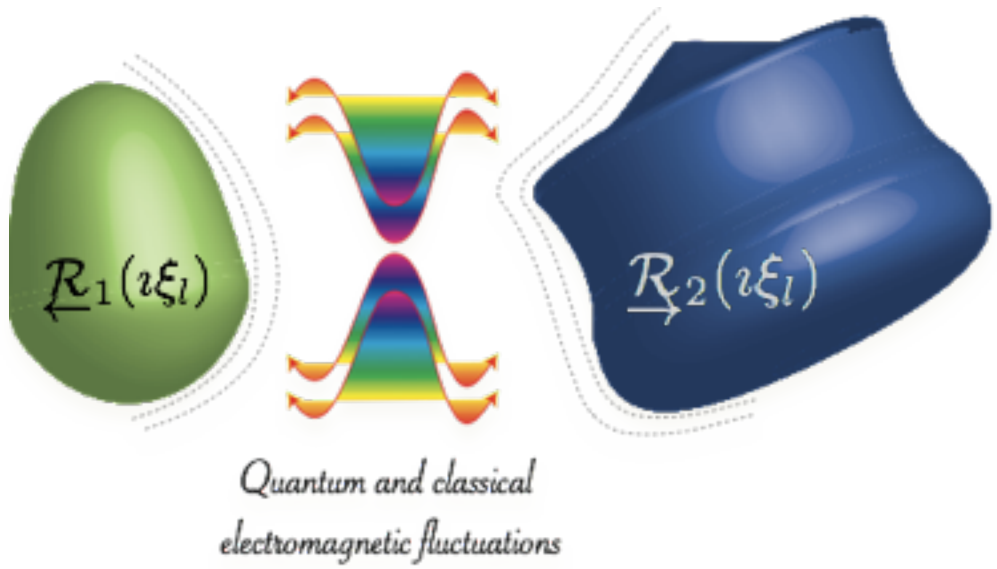
$$\overleftrightarrow{\epsilon} = \begin{pmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix}$$

$$\epsilon_i \le \epsilon_{Au} \Rightarrow P_{\mathrm{EMA}}(a) \le P(a)$$

Good approximation for large distance

what about non trivial effects?

Casimir interaction with general geometries



Scattering approach

The scattering formula allows to calculate the free energy between two bodies the Casimir if we know the reflection operator of each isolated object.

Reviews
Kenneth & Klich, PRL 2006
Lambrecht et al, NJP 2006
Rahi et al, PRD 2009

Previous work
Büscher & Emig, PRA 2004
Lambrecht & Marachevsky, PRL 2008
Rodriguez, Joannopoulos, & Johnson, PRA 2008
Davids et al, PRA 2010
....

Sum over Matsubara frequencies

Reflection operators

$$\mathcal{F}(a) = k_B T \sum_{l=0}^{\infty}{}' \log \mathrm{Det}\left[1 - \mathcal{R}_1(\mathrm{i}\xi_l) \cdot \mathcal{X}(\mathrm{i}\xi_l, a) \cdot \mathcal{R}_2(\mathrm{i}\xi_l) \cdot \mathcal{X}(\mathrm{i}\xi_l, a)\right]$$

Translation operators

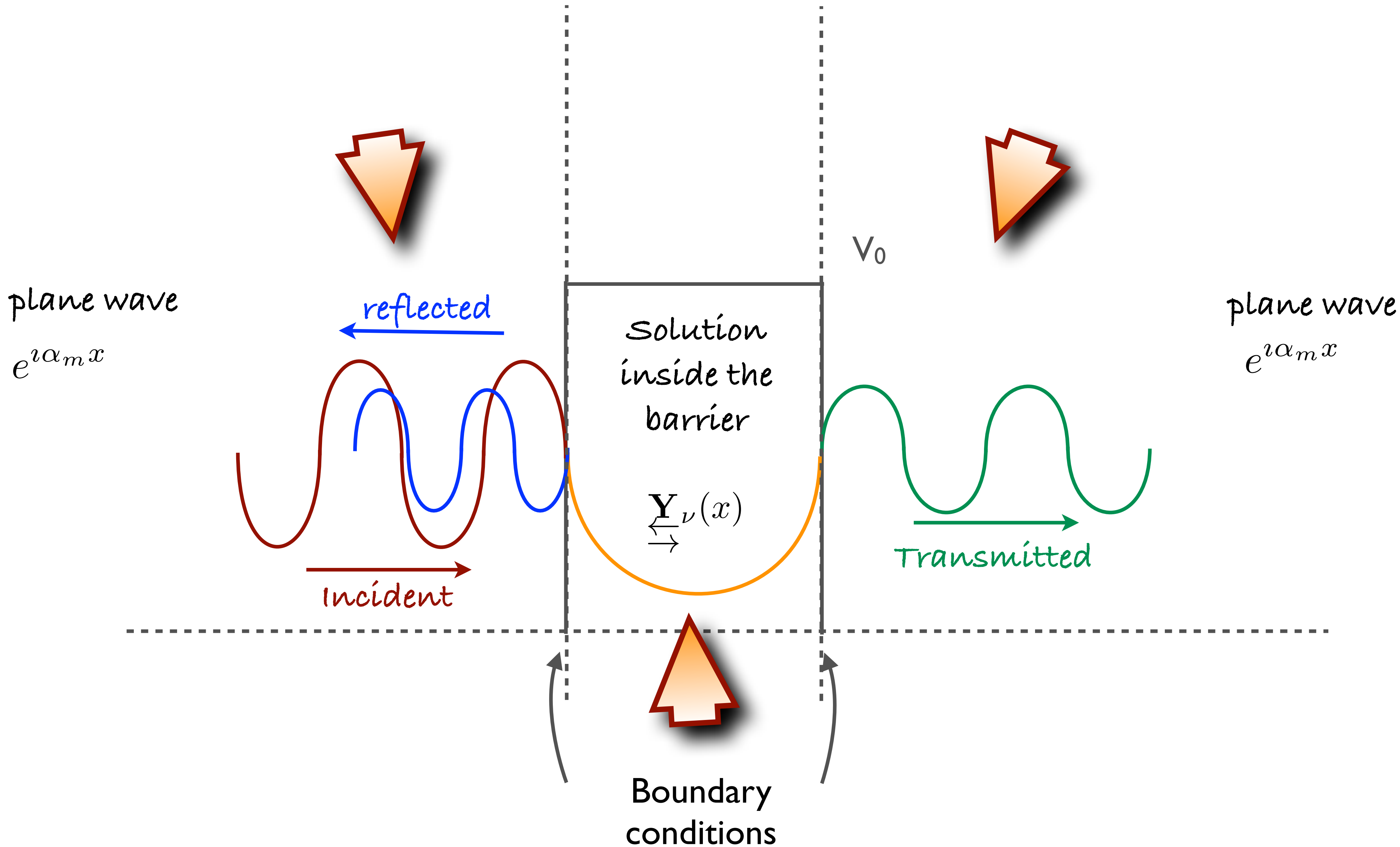
The sum starts with zero. Therefore the reflection operators must be correctly evaluated at zero frequency

$$\xi_l = \frac{2\pi k_B T}{\hbar} l$$

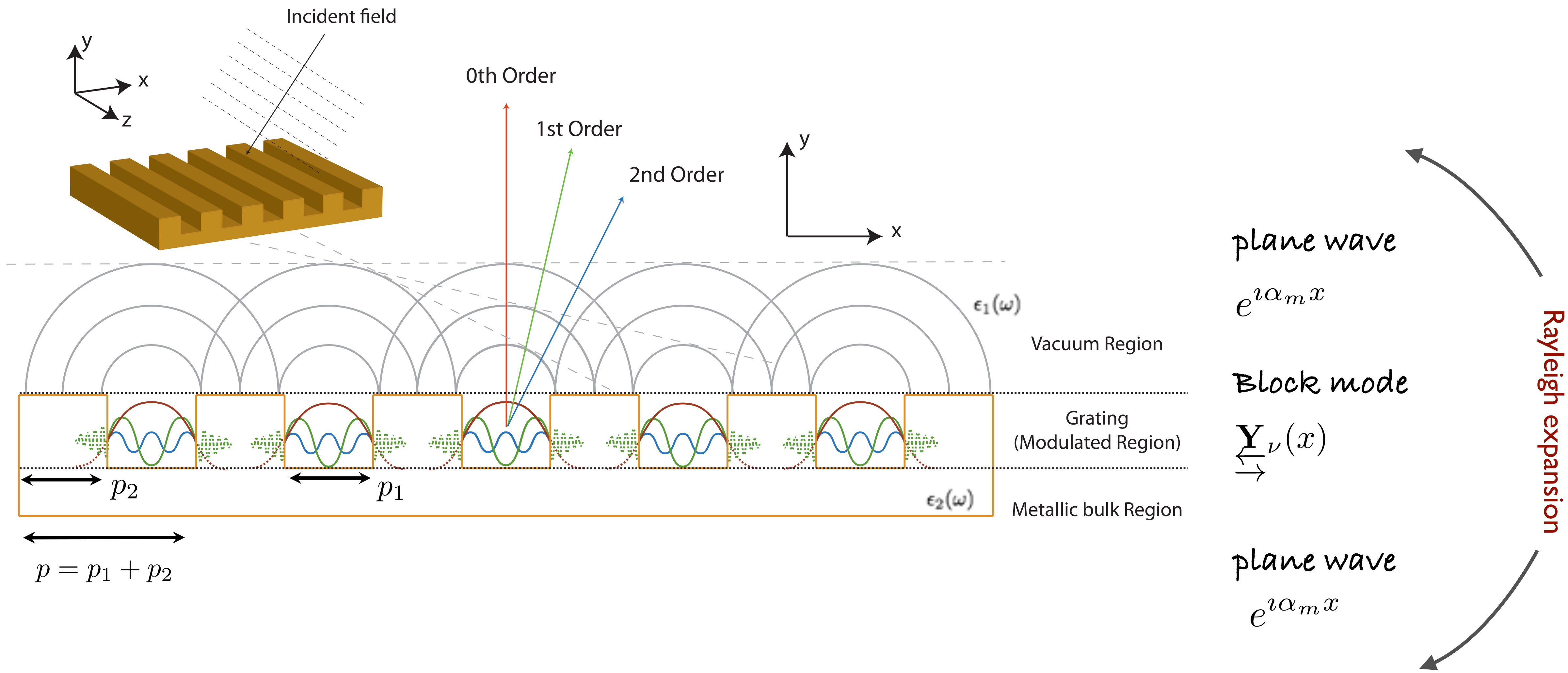
The translation operators carry no information about the objects but their position in space

How do you get the reflection properties

Simple example: Particle against a potential barrier



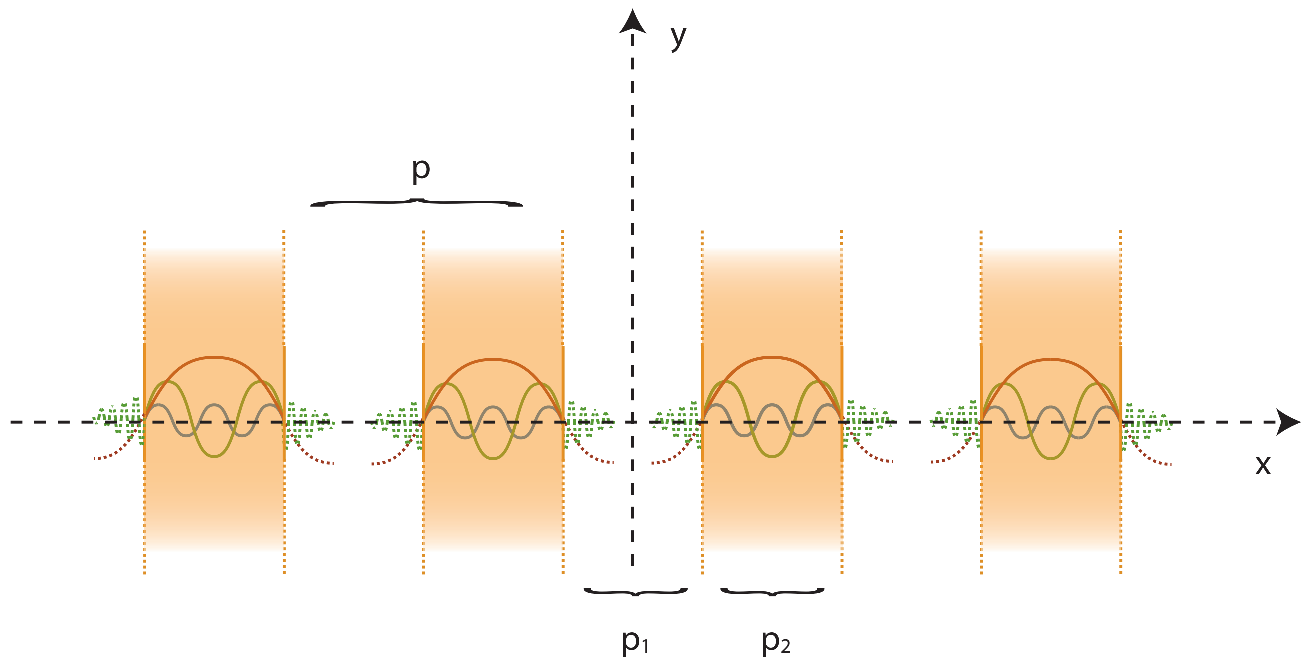
Reflection from a lamellar grating



In the simple case of a lamellar grating this can be done almost completely in an analytical way

An almost full analytical approach

Li, J. Mod. Optics 1993



Recasting Maxwell equations

$$\frac{\partial_y}{\imath}\begin{pmatrix} E_z(x,y) \\ E_x(x,y) \\ H_z(x,y) \\ H_x(x,y) \end{pmatrix} = \boxed{\mathcal{H}[x,\partial_x]} \cdot \begin{pmatrix} E_z(x,y) \\ E_x(x,y) \\ H_z(x,y) \\ H_x(x,y) \end{pmatrix}$$

! 4x4 differential non self-adjoint operator!

Two polarizations:

$$s = e \rightarrow E_x = 0$$
$$s = h \rightarrow H_x = 0$$

Analytical expressions

Transcendental equation

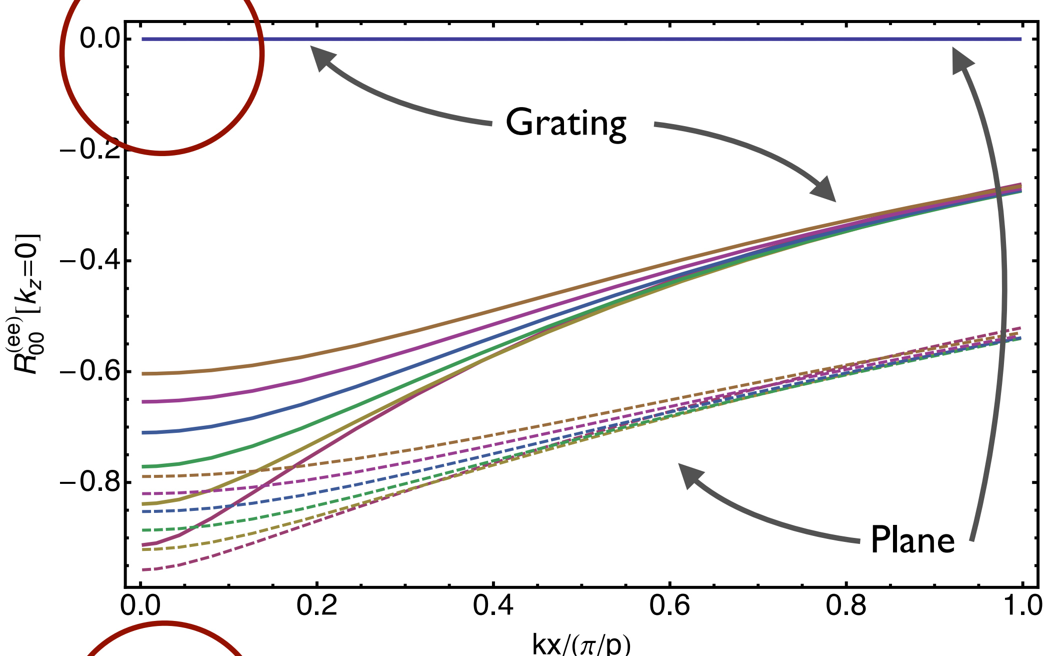
$$\begin{pmatrix} E_z(x,y) \\ E_x(x,y) \\ H_z(x,y) \\ H_x(x,y) \end{pmatrix}_i = \sum_{\nu,s} A_\nu^{(s,i)} \mathbf{Y}^{(s,i)}[x, \underline{\eta_\nu^{(s,i)}}] e^{\imath\lambda[\eta_\nu^{(s,i)}]y}$$

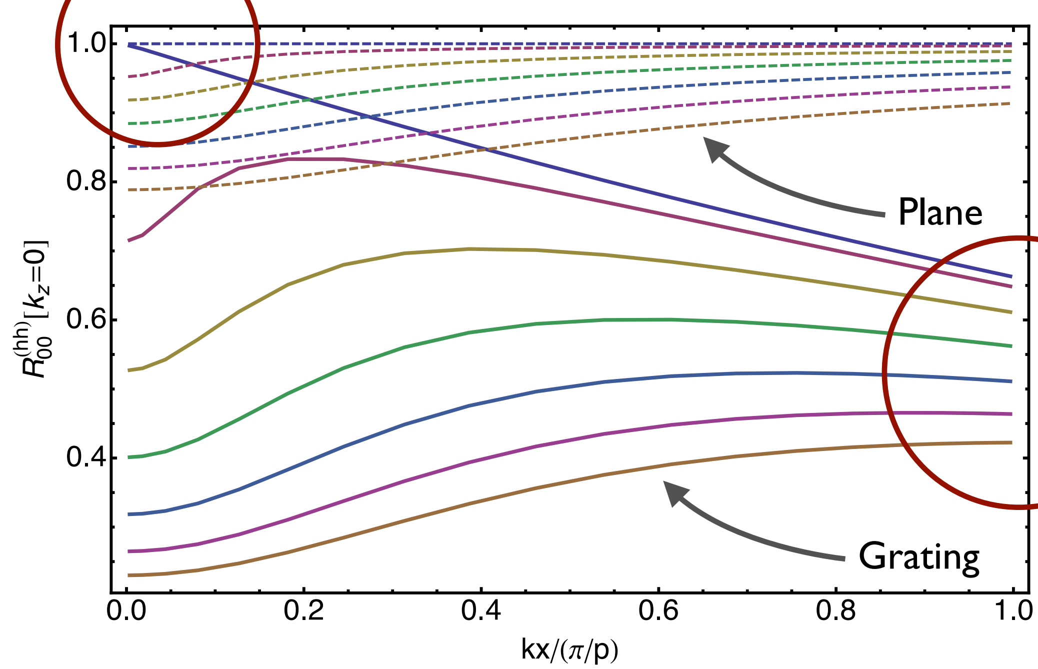
$$D^{(s)}(\underline{\eta}) = 0$$

Eigenvalues

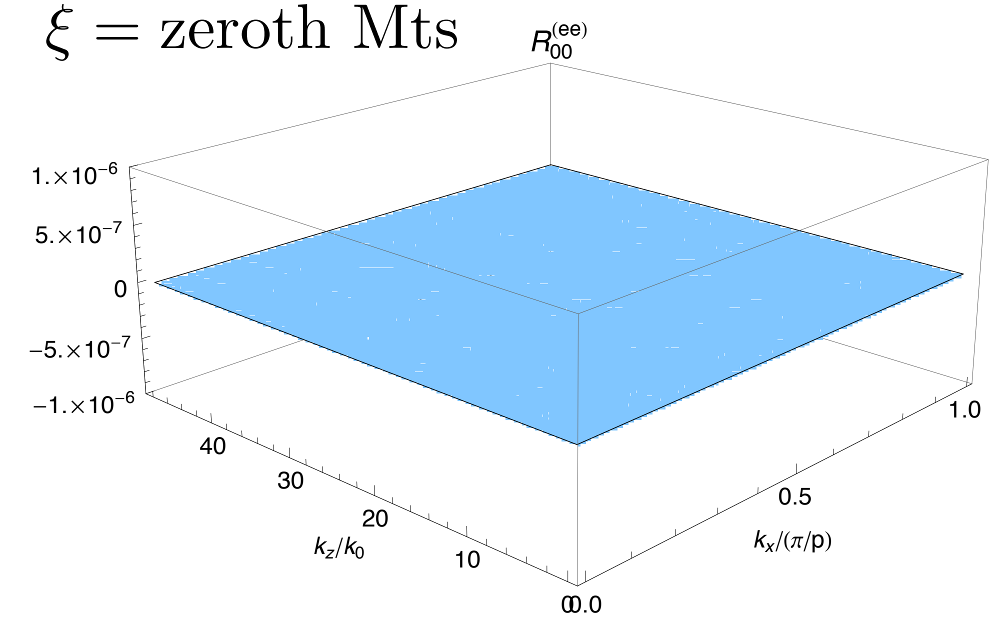
Reflection Matrices

Reflection coefficient within the first Brillouin zone and comparison with plane: first seven Matsubara frequencies



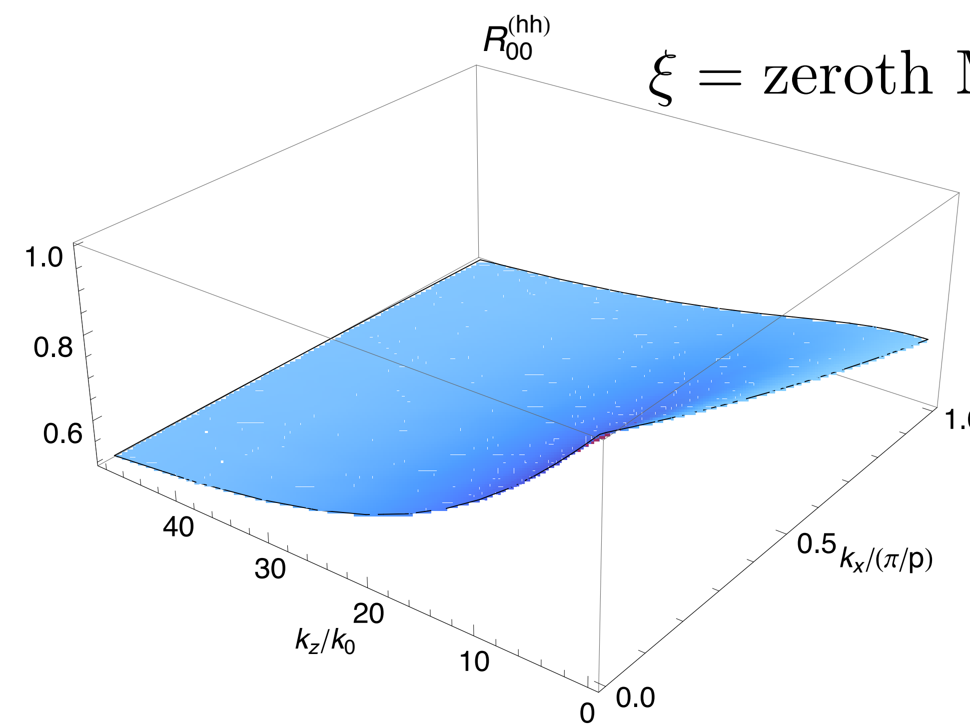


$\xi = \text{zeroth Mts}$

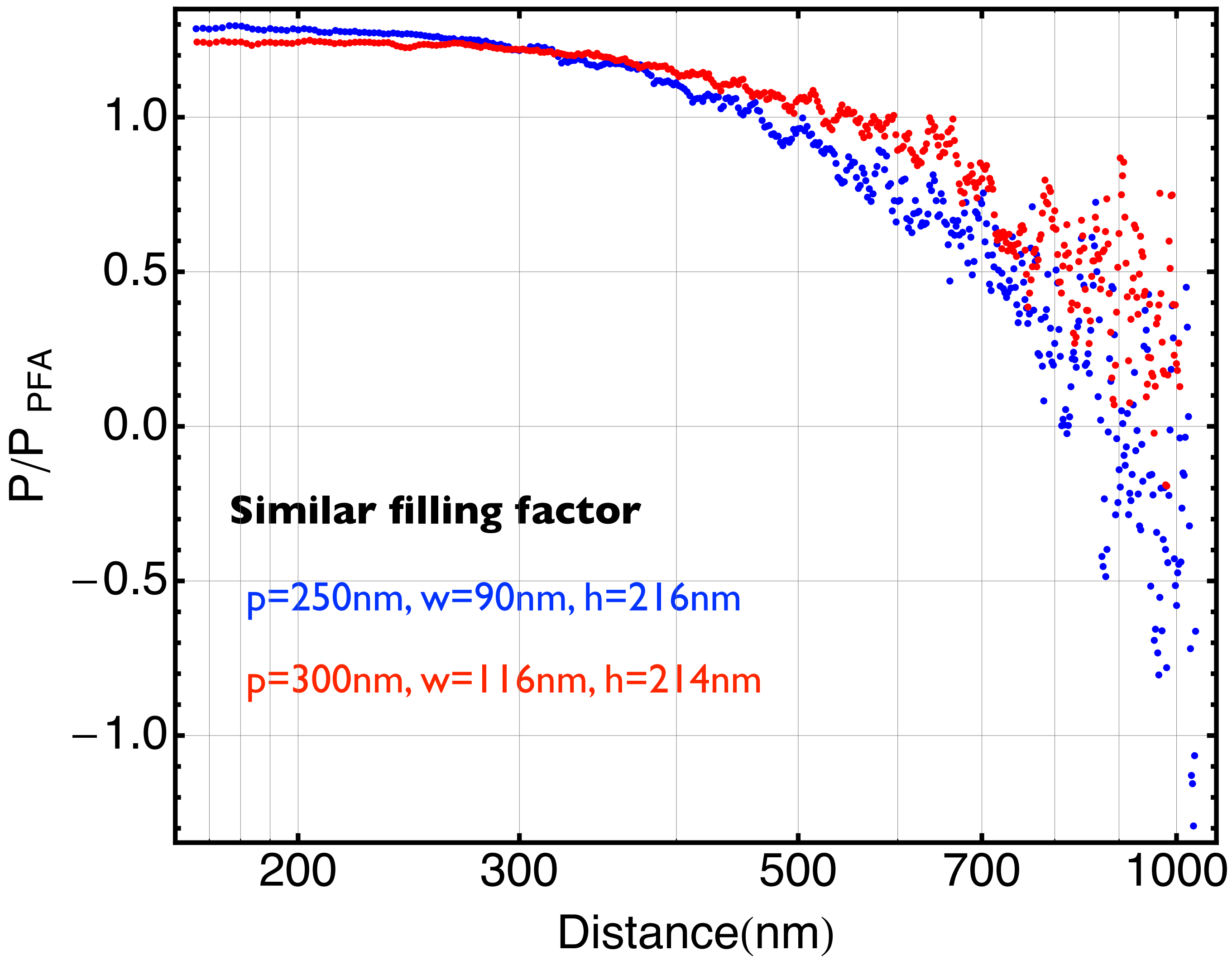


This result can be shown analytically!

$\xi = \text{zeroth Mts}$



Experimental results vs PFA



Small separations: *PFA underestimates the total pressure*.

Large separations: *PFA overestimates the exact pressure*.

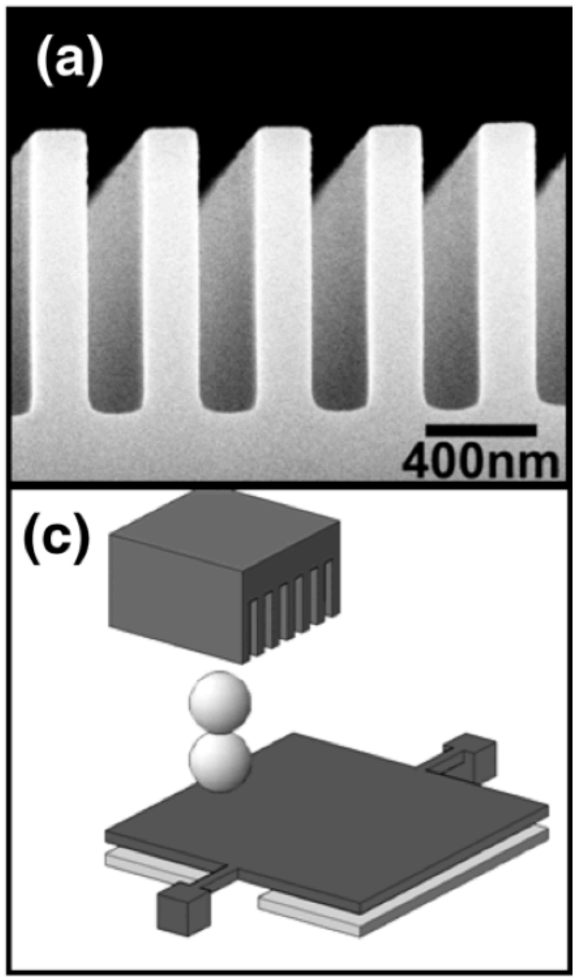
Pressure is going to zero faster than d^{-4}

Since for large separations distances $P_{PFA}(d) \propto d^{-4}$, the Casimir pressure is decreasing as a power law with an exponent larger than four.

Trends (similar filling factor but with a different period):
it can be understood with a scaling argument.

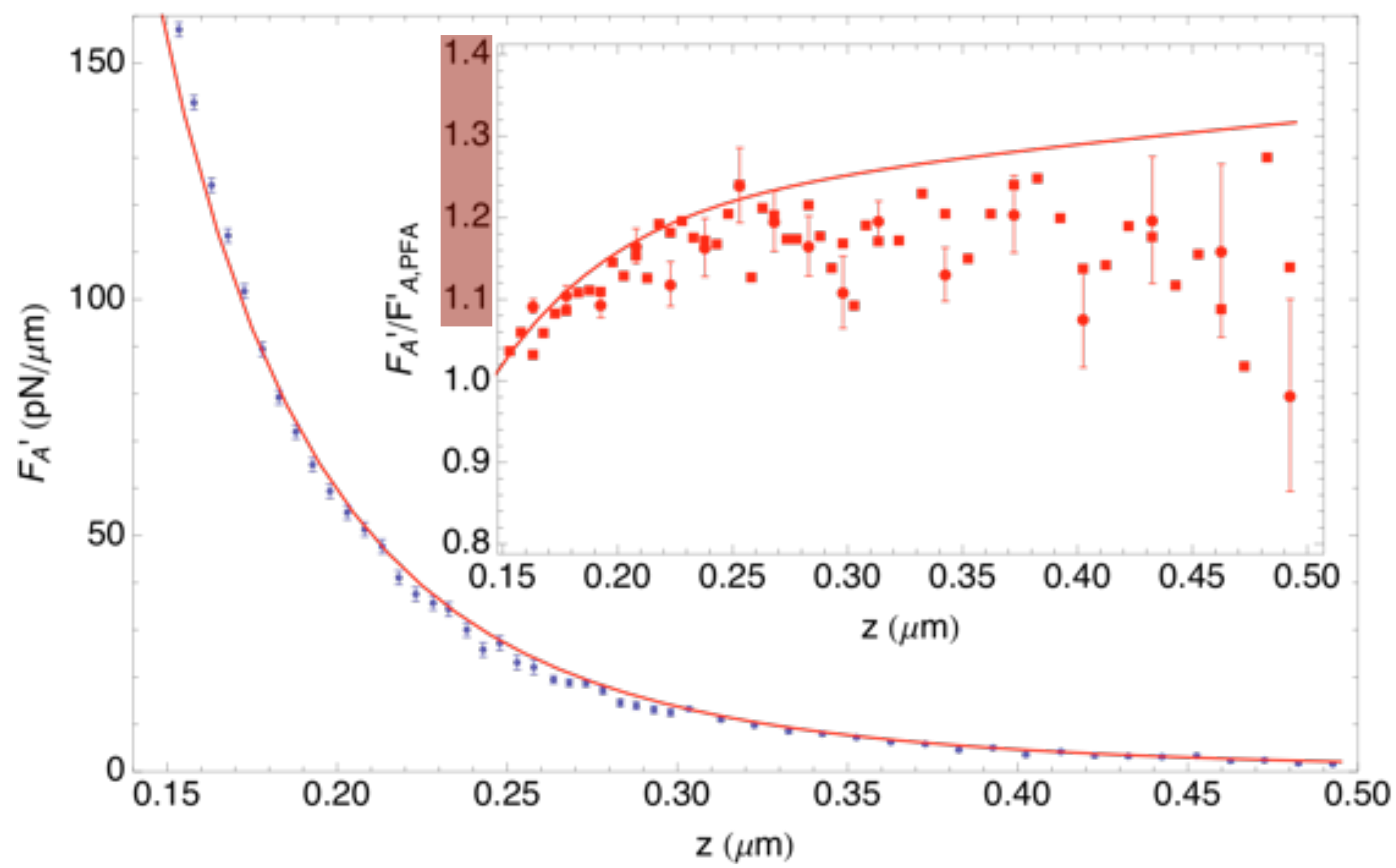
- ➡ At short distance the shorter the grating period, the larger the enhancement of the Casimir pressure with respect to PFA,
- ➡ at large distances the opposite happens - shorter period leads to a stronger reduction of the Casimir force.

Previous work with Si gratings



H. B. Chan et al. PRL 2008
Lambrecht & Marachevsky, PRL 2008
Davids et al, PRA 2010

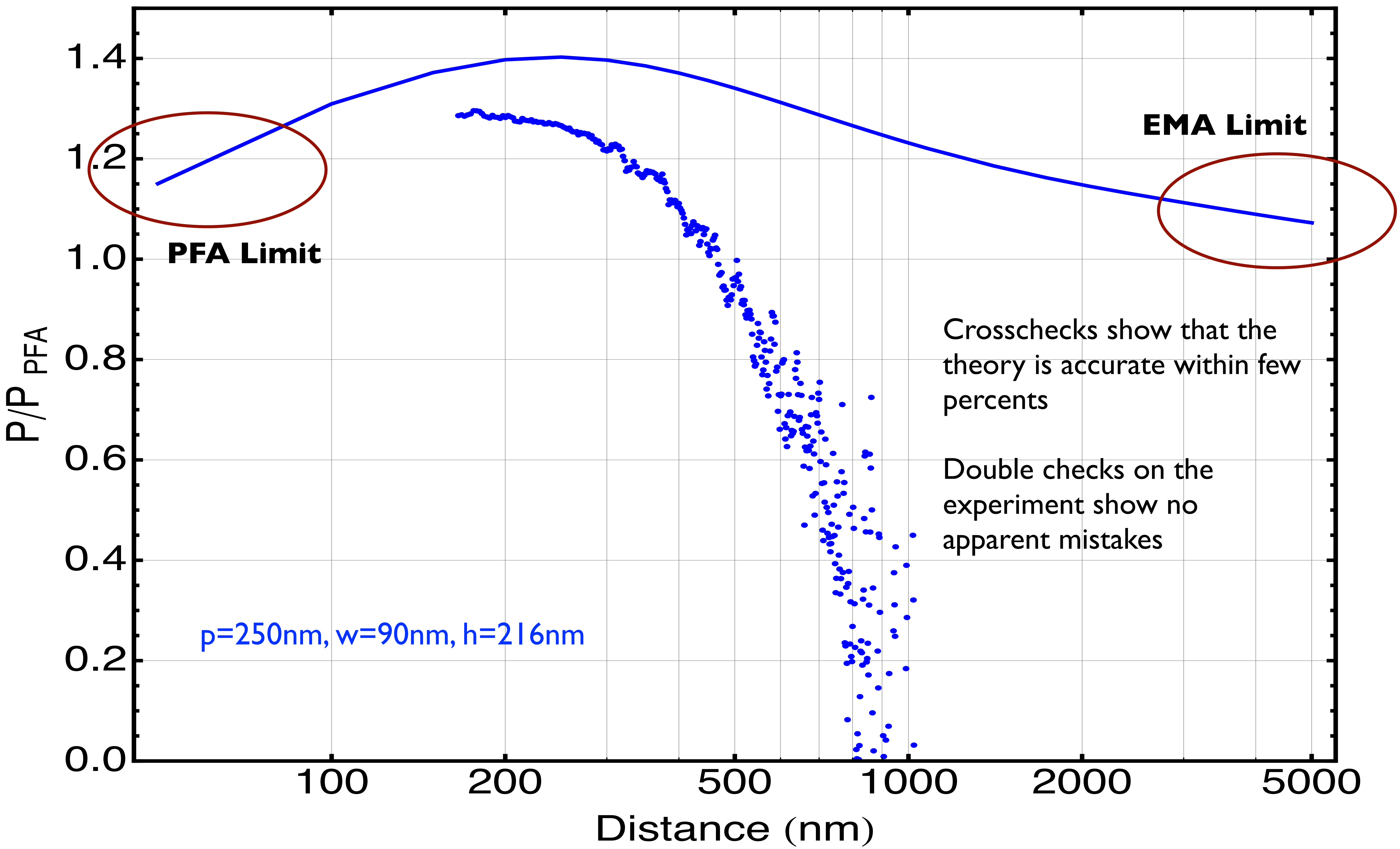
Sample A: period=1μm, filling a=0.510



PFA underestimates the real force

Comparison: Experiment/Theory

Disagreement between theory and experiment !



Crosschecks show that the theory is accurate within few percents

Double checks on the experiment show no apparent mistakes

Conclusions

Modes analysis an very useful tool

- ➡Connection with plamsonics
- ➡Connection with diffusive electrodynamics (and superconductors and glass physics)
- ➡Study of complex geometries

Comparison with experiments

- ➡ Puzzling agreements/disagreements between theory and experiment
- ➡ Dissipation or not?
- ➡ Geometry effects: Power law faster than d^{-4}; Stronger decay for shorter periods

So what is going on?

- ➡Are we correctly describing the experiment?
- ➡Is the experiment correct?
- ➡Is something wrong with the theory?
- ➡Further analysis are on going