Emergent Phenomena And Universality In Quantum Systems Far From Thermal Equilibrium

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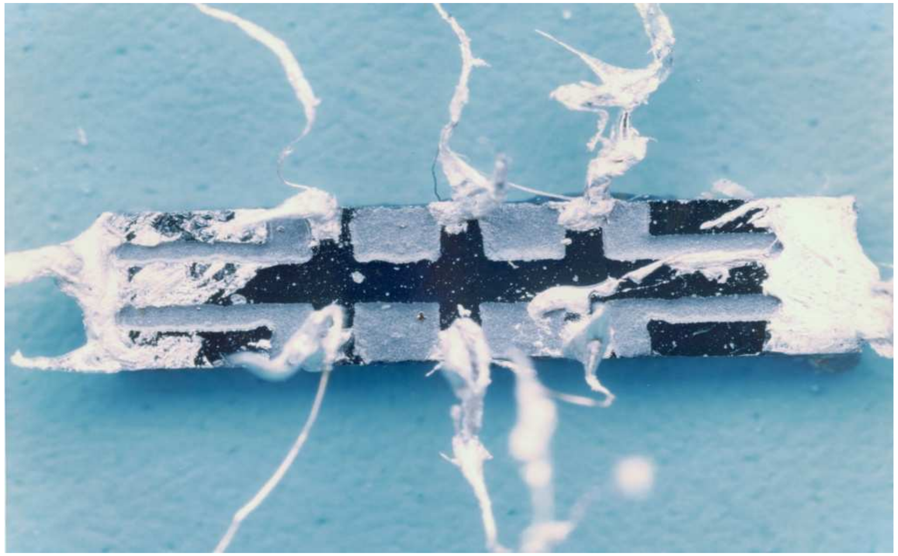
A typical experiment in traditional Condensed Matter physics (equillibrium)

1. Take a piece of junk:



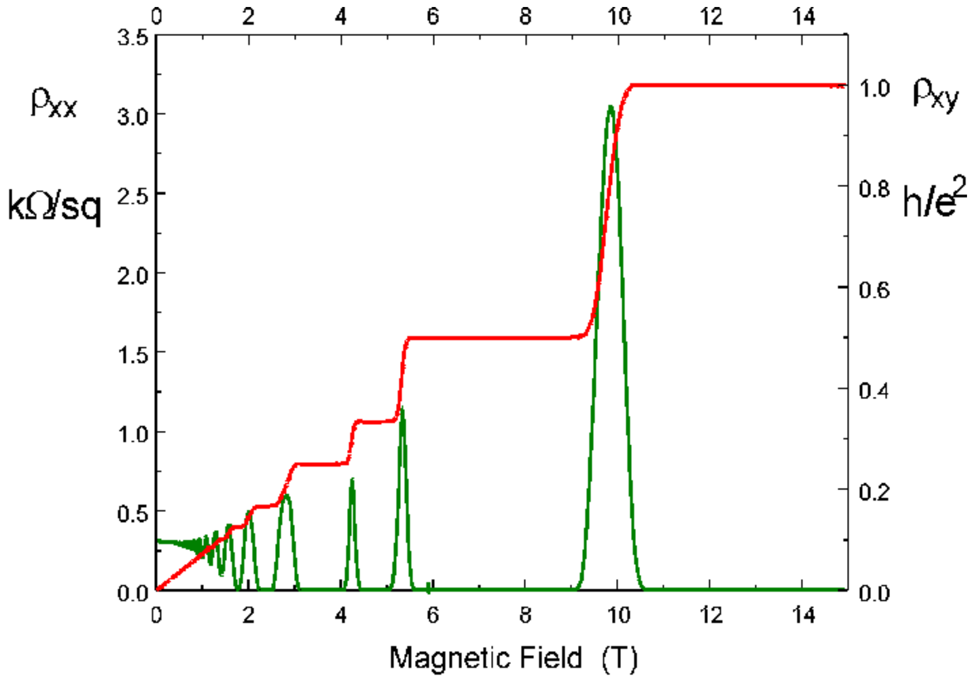
2. Cool it down

3. Measure linear response to a small perturbation:
   e.g transport σ_{ij} (T,ω) , $\kappa(T)$ or scattering intensity S($\mathbf{q}$,ω)

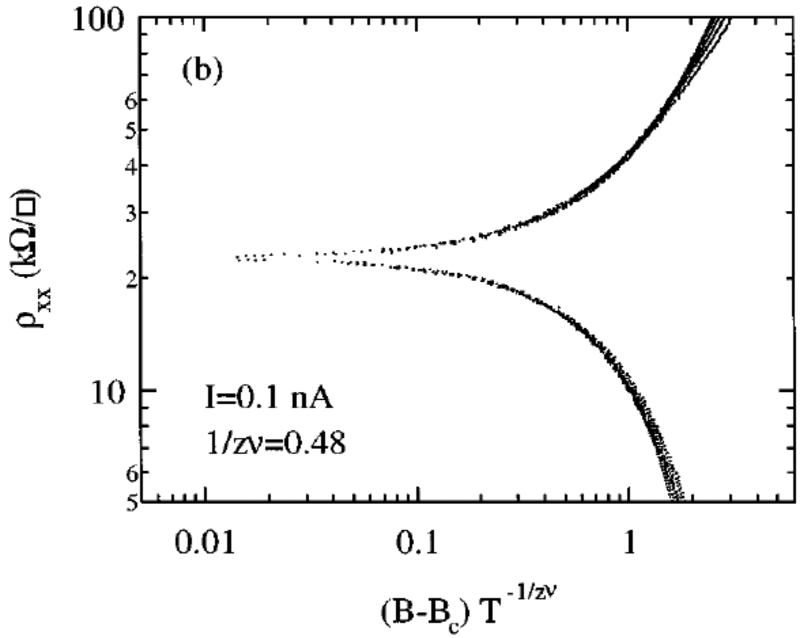

And (sometimes) a miracle occurs !

Observe beautiful universal behavior insensitive to sample details

Example: precise quantization of the Hall resistivity in the Quantum Hall effect



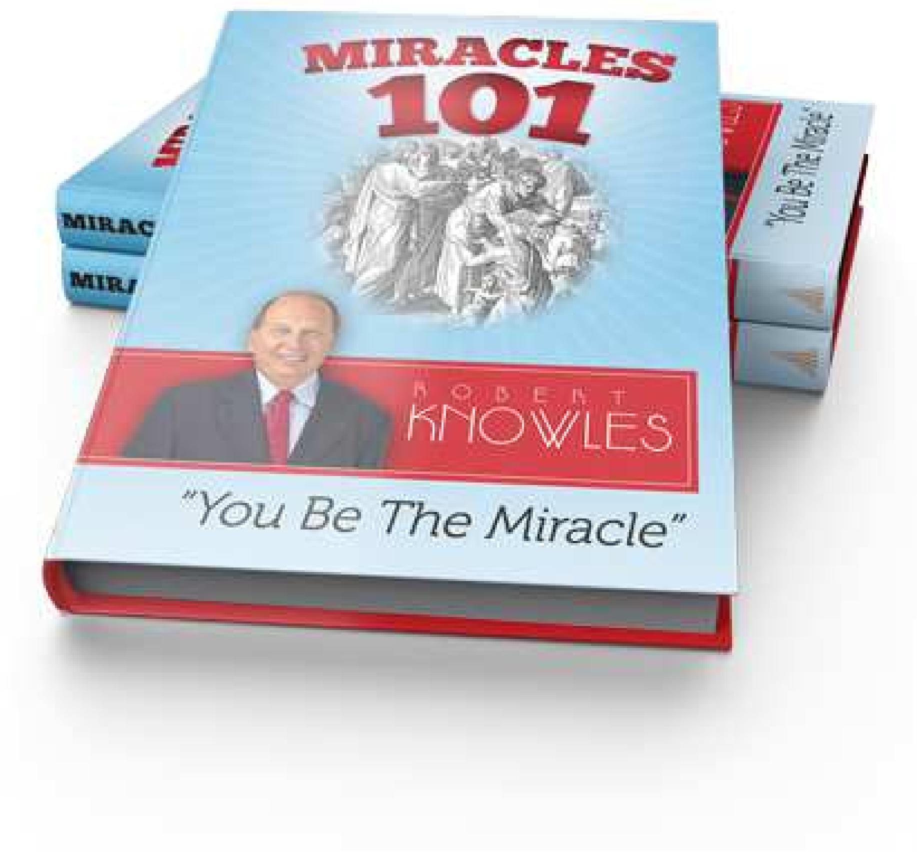
Transition of ρ_{xx} on going from plateau to step:

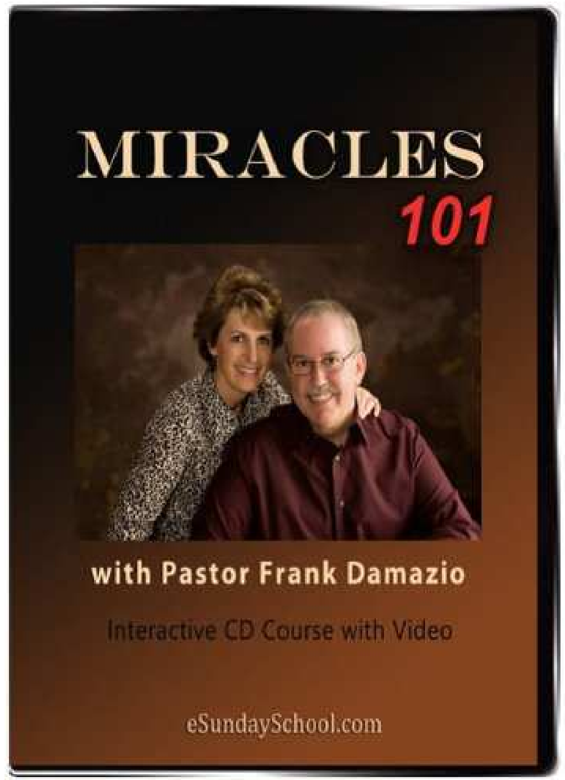


Emergent universal behavior is what allows predictive power in spite of the underlying complexity of materials

How do such miracles arise?

Can we expect to find them in quantum systems out of thermal equilibrium (e.g. cold atoms)?





Outline

- Universality in low temperature equilibrium physics: Renormalization and quantum phases.

- Ultracold atomic systems as a non equilibrium laboratory

- Focus questions:

  - Are there generic systems that do not thermalize?

  - Phase transitions from a non thermalizing to a thermalizing state?

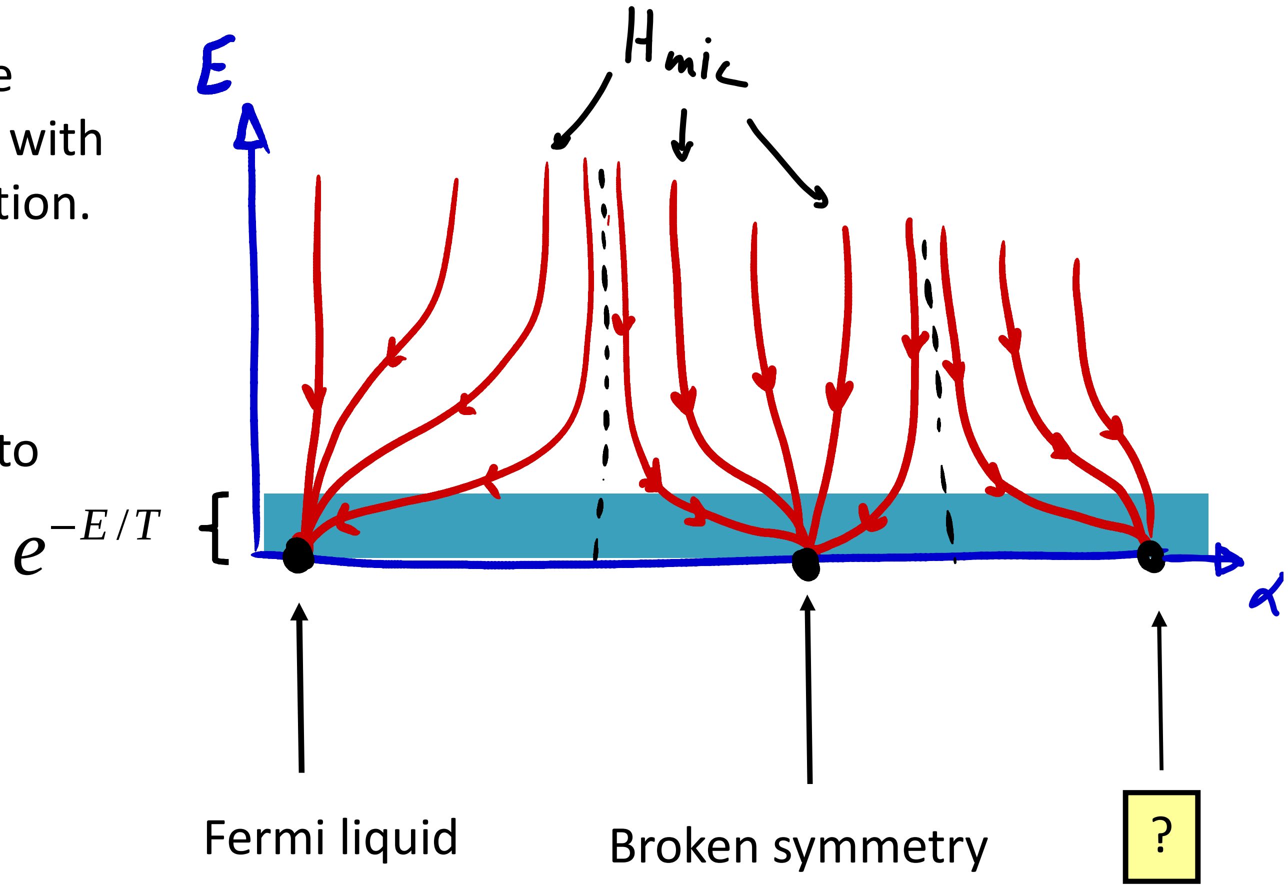
- From Anderson localization to many-body localization:
  - Renormalization group perspective on quantum dynamics
  - Emergent integrals of motion

Low temperature equilibrium physics

$$H_{mic} = \sum_i \frac{P_i^2}{2M_i} + \sum_e \frac{p_e^2}{2m_e} + \sum_{ie} V_{ie}(r_e - R_i) + \sum_{ij} V_{ii}(R_i - R_j) + \sum_{ee'} V_{ee}(r_e - r_{e'})$$

Renormalization = How the system appears to a probe with low spatiotemporal resolution.

The system itself samples only low energies thanks to the Boltzmann factor

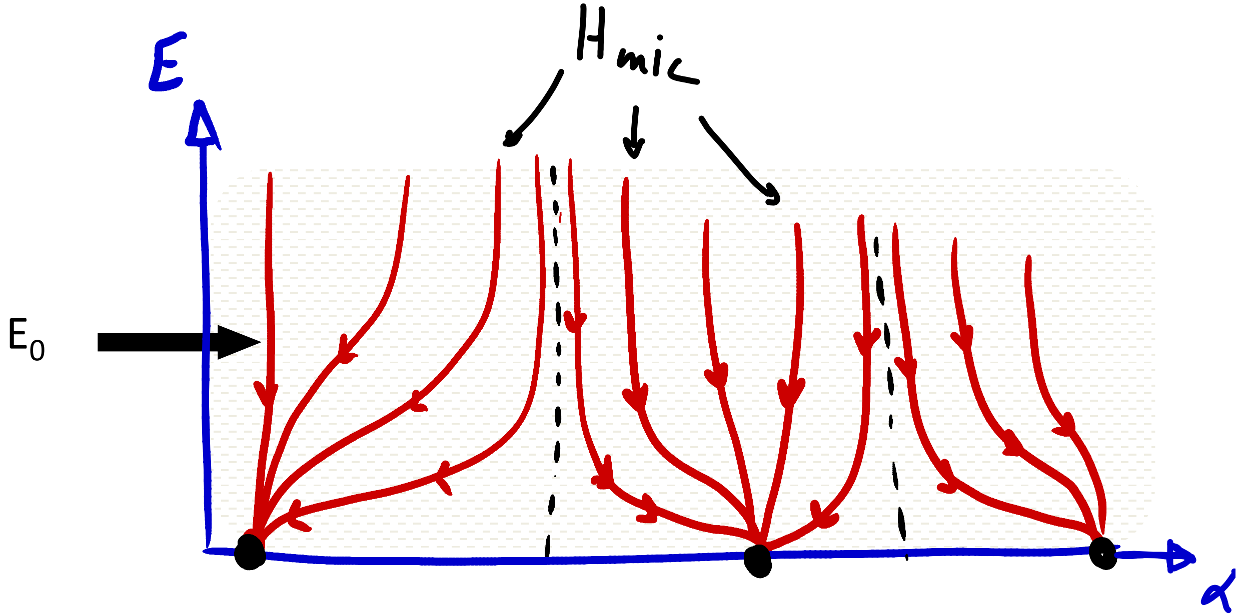


Universality classes

= quantum phases

Far from equilibrium

Inject the system with high energy.
e.g by rapid change of system parameters or by continuous drive



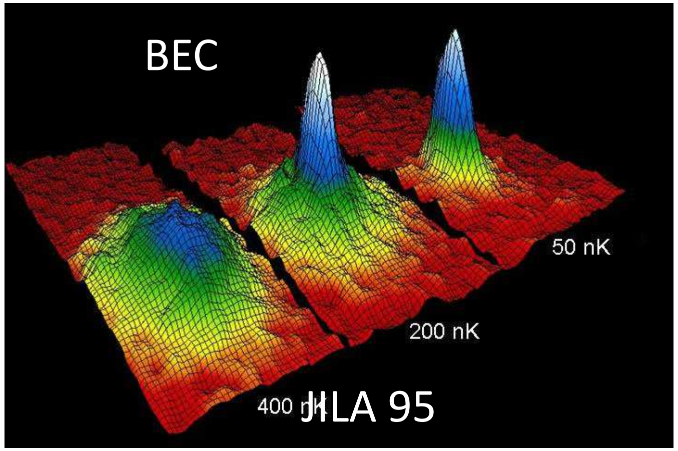
Dynamics involves all energy scales

Can the complexity of quantum dynamics generate emergent universal phenomena far from equilibrium?

Can one still define quantum phase transitions?

Ultracold atoms – a new class of CM system

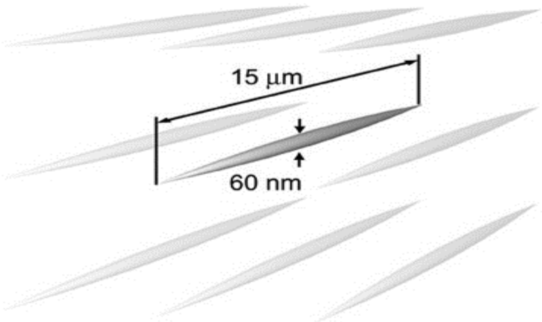
$n \sim 10^{14}\,\mathrm{cm}^{-3}$ $\qquad T_{\mathrm{BEC}} \sim 1\mu\mathrm{K}$



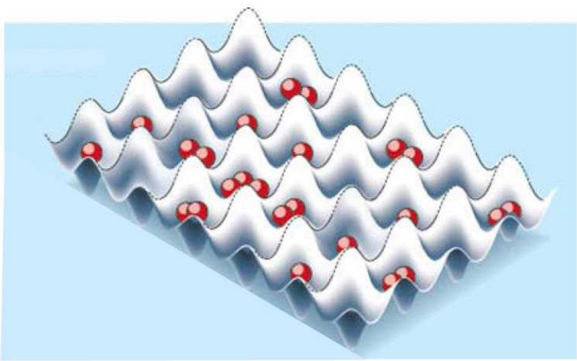
Extremely dilute,
interaction naturally weak

Various ways devised to enhance quantum correlations:

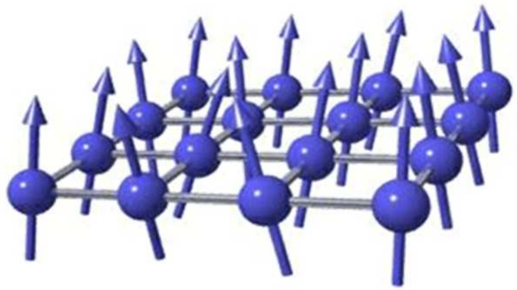
Low dimensions



Optical lattice

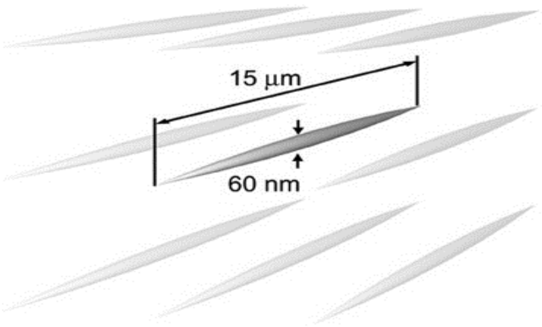


Dipolar molecules

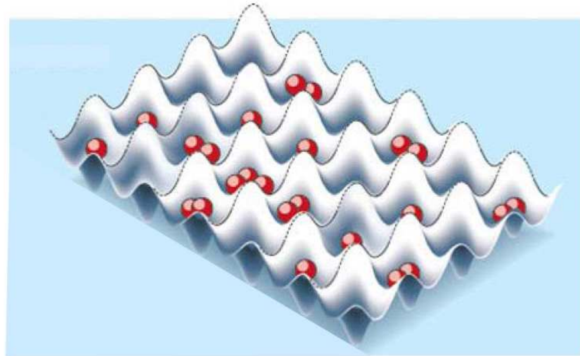


Ultracold atoms – a new class of CM system

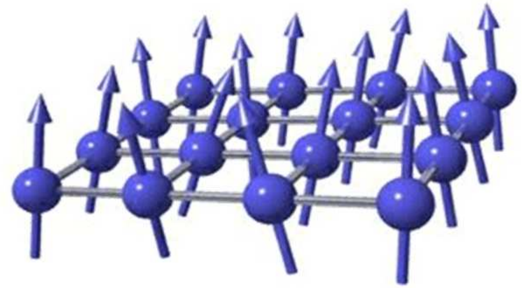
Low dimensions



Optical lattice



Dipolar molecules



- Highly tunable (Hamiltonian and state engineering)
- Almost toally isolated (Closed systems)
- Long natural timescales (KHz compared to GHz -THz in solids)

➡ Ideal laboratory for studying non-equilibrium quantum dynamics

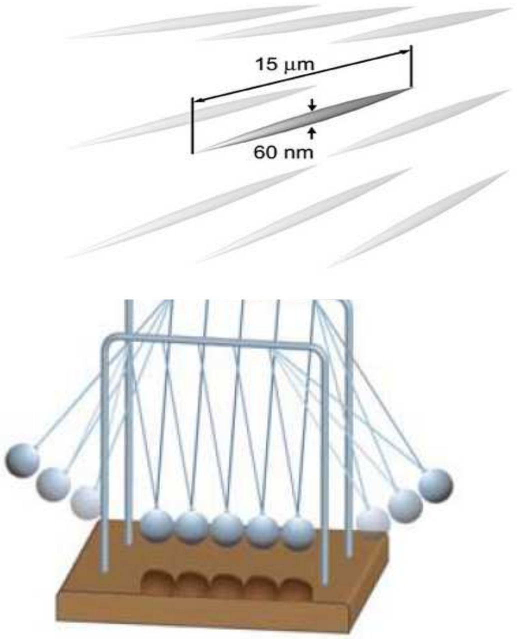
<u>A typical experiment:</u>

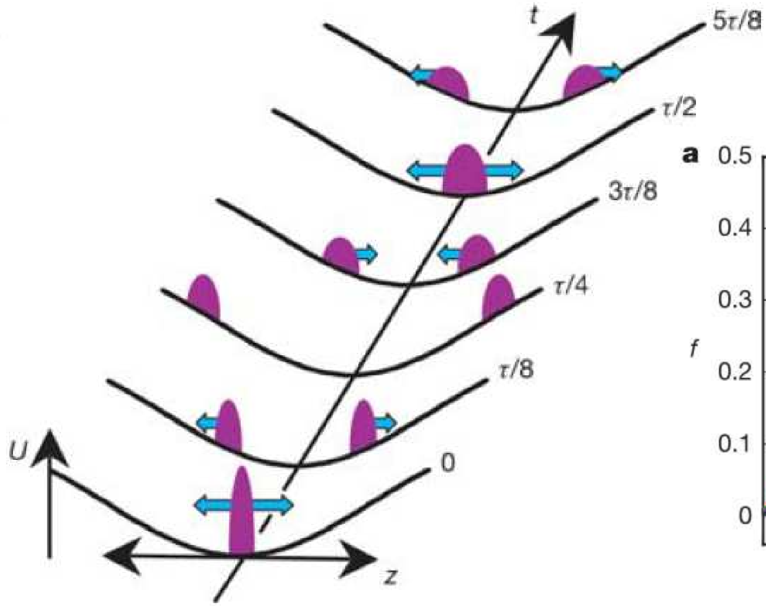
1. Prepare a well defined initial state
2. Unitary evolution with a known Hamiltonian. $$|\,\psi(t)\,\rangle = e^{-iHt}\,|\,\psi_0\,\rangle$$
3. Observe at varying times

Example 1: **A quantum Newton's cradle**

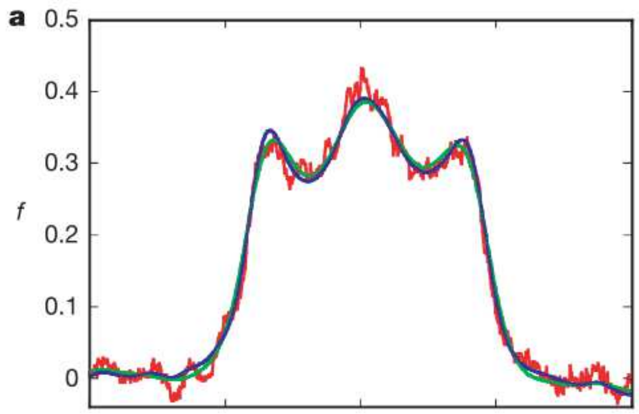
Toshiya Kinoshita[1], Trevor Wenger[1] & David S. Weiss[1]   Nature (2006)

(Take a 1d gas and kick it in the balls)





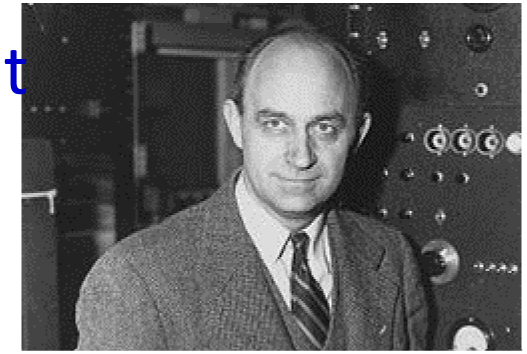
n(p)



Constant non-thermal momentum distribution seen at long times

- Is there a simple description of the time evolution?
- What is the nature of the steady state?
- Does the system eventually thermalize?

The “quantum Newton’s cradle” experiment
is a quantum analogue
of the famous FPU problem



STUDIES OF NON LINEAR PROBLEMS

E. FERMI, J. PASTA, and S. ULAM
Document LA-1940 (May 1955).

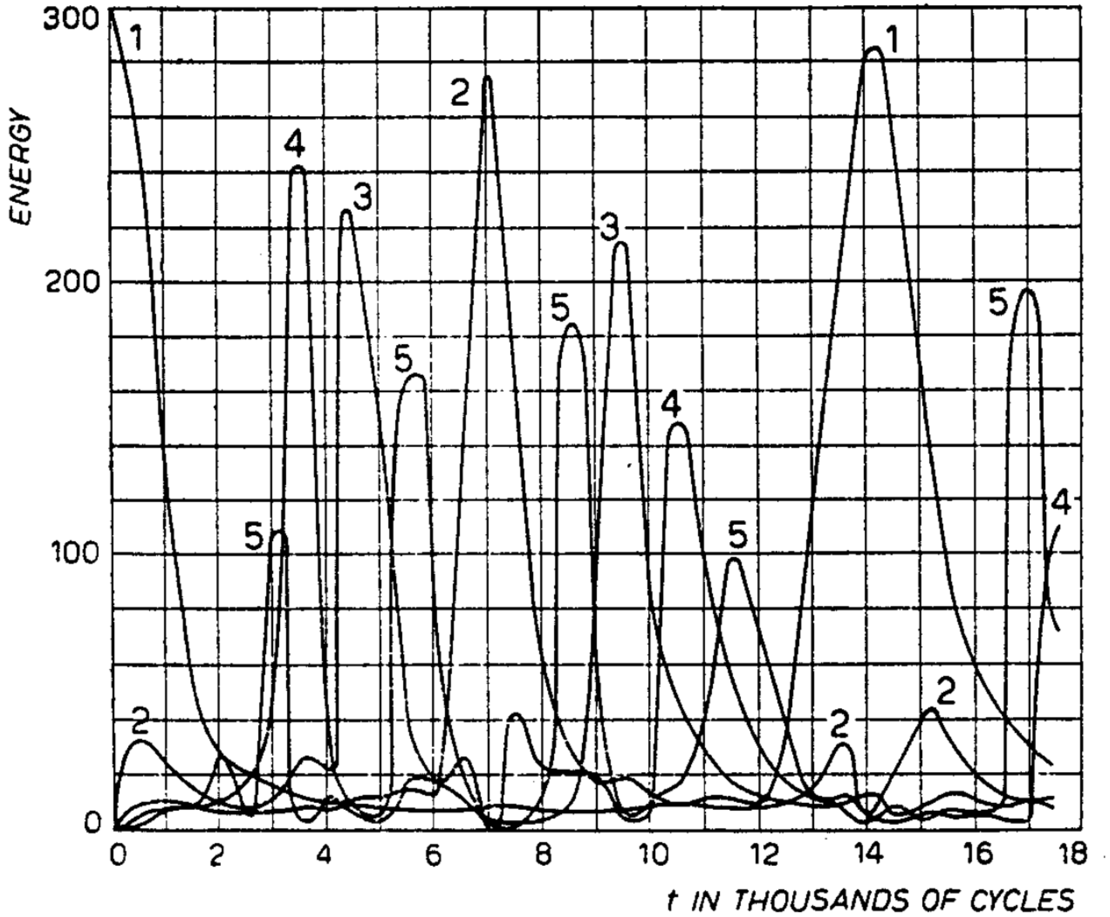
A one-dimensional dynamical system of 64 particles with forces between neighbors containing nonlinear terms has been studied on the Los Alamos computer MANIAC I. The nonlinear terms considered are quadratic, cubic, and broken linear types. The results are analyzed into Fourier components and plotted as a function of time.

The results show very little, if any, tendency toward equipartition of energy among the degrees of freedom.

$$\ddot{x}_i = (x_{i+1} + x_{i-1} - 2x_i) + \alpha\left[(x_{i+1} - x_i)^2 - (x_i - x_{i-1})^2\right]$$

$$(i = 1, 2, \cdots, 64),$$

No equipartition of energy:



The system is close to an integrable KdV equation.

Long lived soliton solutions delay thermalization

Kruskal and Zabusky (1965).

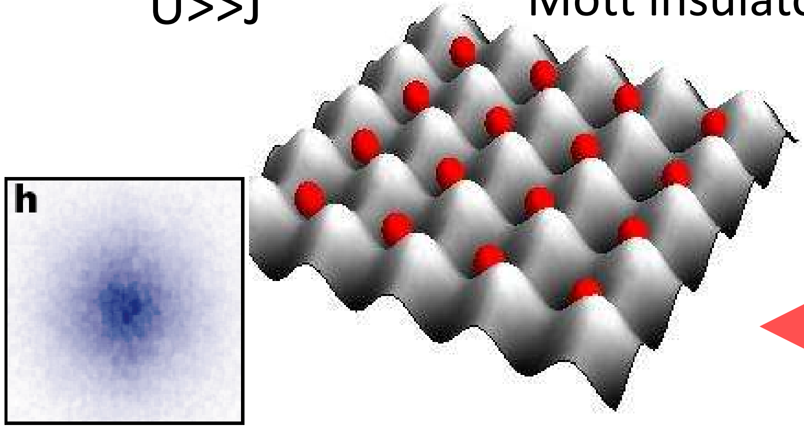
Example 2: Sudden quench from weak to strong lattice

$$H = -J\sum_{\langle ij\rangle}(b_i^\dagger b_j + \mathrm{H.c.}) + U\sum_i n_i(n_i - 1)$$

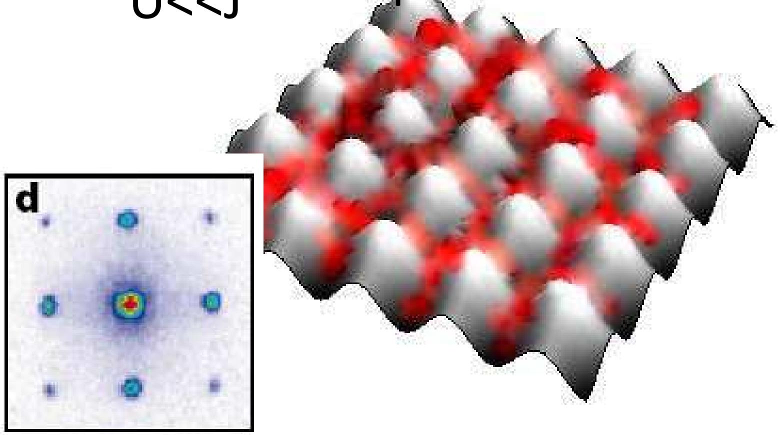
Greiner et. al. Nature 415, 2002;
ibid 419, 2002

Equilibrium phases:

U>>J Mott insulator



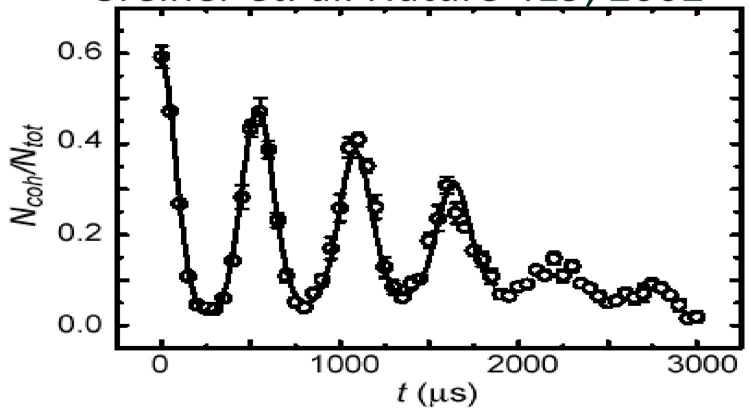
U<<J Superfluid



Quench: Decaying oscillations of phase coherence

- What determines decay time?
- Nature of steady state seen in exp.?

Greiner et. al. Nature 419, 2002



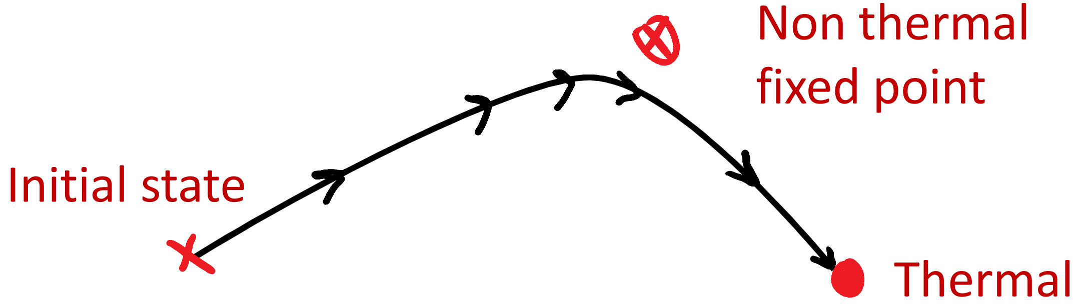
Non-thermal! (Kollath, Lauchli & EA PRL 07)

Lessons from the classical world

1. Things tend to thermalize
(Approach maximal entropy)

Simple → messy → Thermal

2. The “mess” can have interesting structure amenable to theory



The experiments in the previous slides probably represent a similar trajectory interrupted before the advent of true equilibration.

Can thermalization be avoided altogether?

At least it can be avoided in somewhat pathologic systems:

Integrable system = Infinite number of "local" conserved quantities

$$[H, I_n] = 0$$

Example: free fermions $$H = \sum_k \epsilon_k n_k$$

Conjectured Equilibration to Generalized Gibbs ensemble:

Jaynes, Phys. Rev. (57), Rigol et. al. PRL (07)

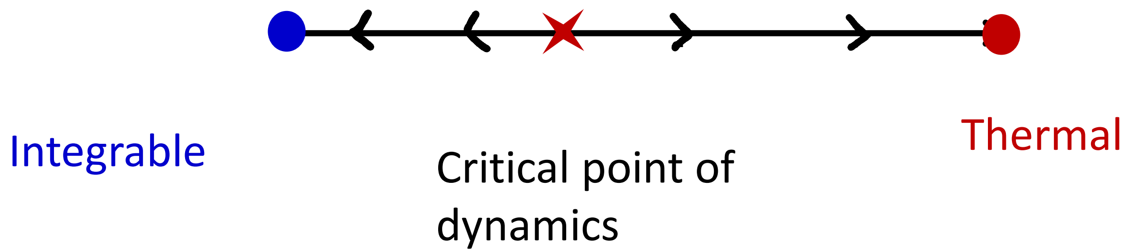
Maximum entropy subject to infinite set of constraints:

$$\rho[\beta] = \frac{1}{Z} e^{-\beta H} \longrightarrow \rho[\{\beta_n\}] = \frac{1}{Z} e^{-\sum_n \beta_n I_n}$$

Lagrange multipliers β_n fixed by initial values $\langle I_n(t=0) \rangle$

Can thermalization be avoided more generically?

Situation where weak breaking of integrability is ***irrelevant*** for the long time evolution

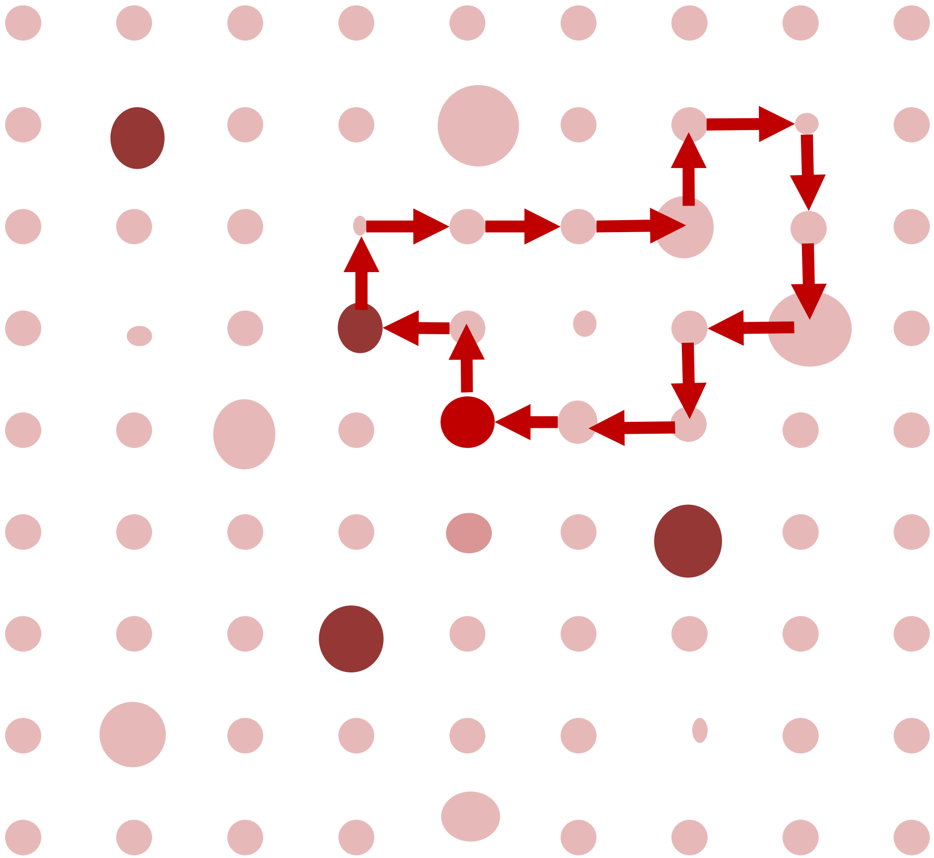


Remainder of this talk: a concrete example

From Anderson localization to many-body localization

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)



Single particle localization

Absence of Diffusion in Certain Random Lattices

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

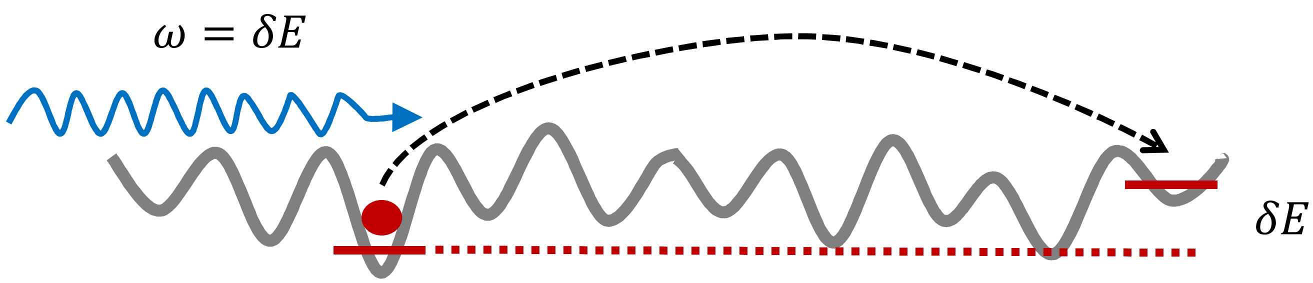
Such a theorem is of interest for a number of reasons: first, because it may apply directly to spin diffusion among donor electrons in Si, a situation in which Feher[3] has shown experimentally that spin diffusion is negligible; second, and probably more important, as an example of a real physical system with an infinite number of degrees of freedom, having no obvious oversimplification, in which the approach to equilibrium is simply impossible; and third, as the irreducible minimum from which a theory of this kind of transport, if it exists, must start. In particular, it re-emphasizes the caution with which we must treat ideas such as "the thermodynamic system of spin interactions" when there is no obvious contact with a real external heat bath.

Anderson was actually interested in many-body localization (problem of quantum spin diffusion).
Used a single particle model as a (over) simplification.

Conductivity in Anderson “insulators” (T>0)

Phonon assisted hopping (Mott):

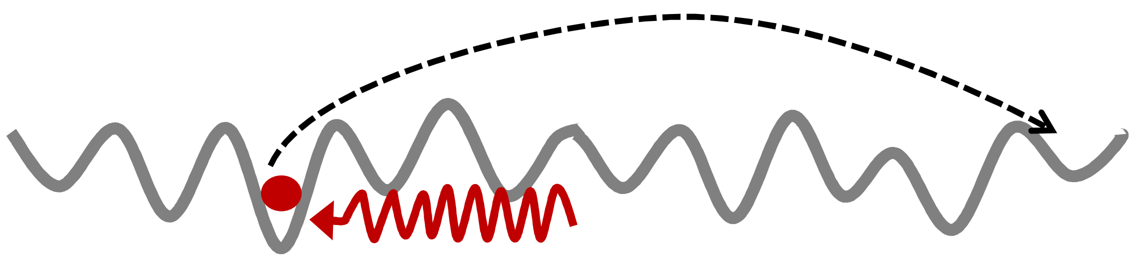
$$\sigma = \sigma_0 e^{-(T_0/T)^{1/(d+1)}}$$



Closed system with interactions (no phonon bath) :

Q: Can intrinsic collective modes (plasmons) replace phonons as the bath?

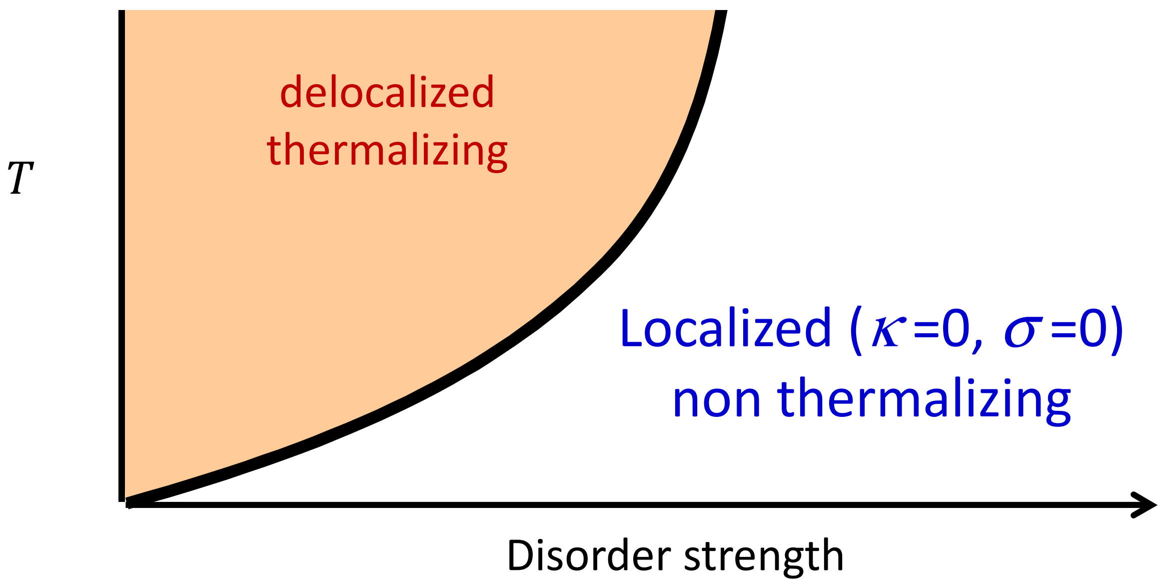
A: These modes can be localized, have a discrete local spectrum,
and thus fail to serve as a bath



Many-Body Localization transition

<u>Basko, Aleiner, Altshuler (2005):</u>
Insulating phase stable below a critical temperature, metal above it.

Disorder tuned transition at $T = \infty$ in a system with ***bounded spectrum***
Oganesyan and Huse *(2007), Pal and Huse (2010)*

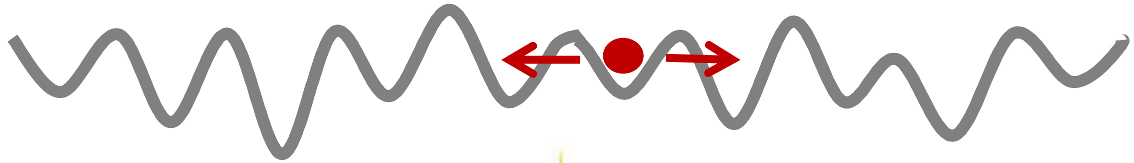


- Arguments not fully controlled
- Nature of transition?
- Of localized phase?

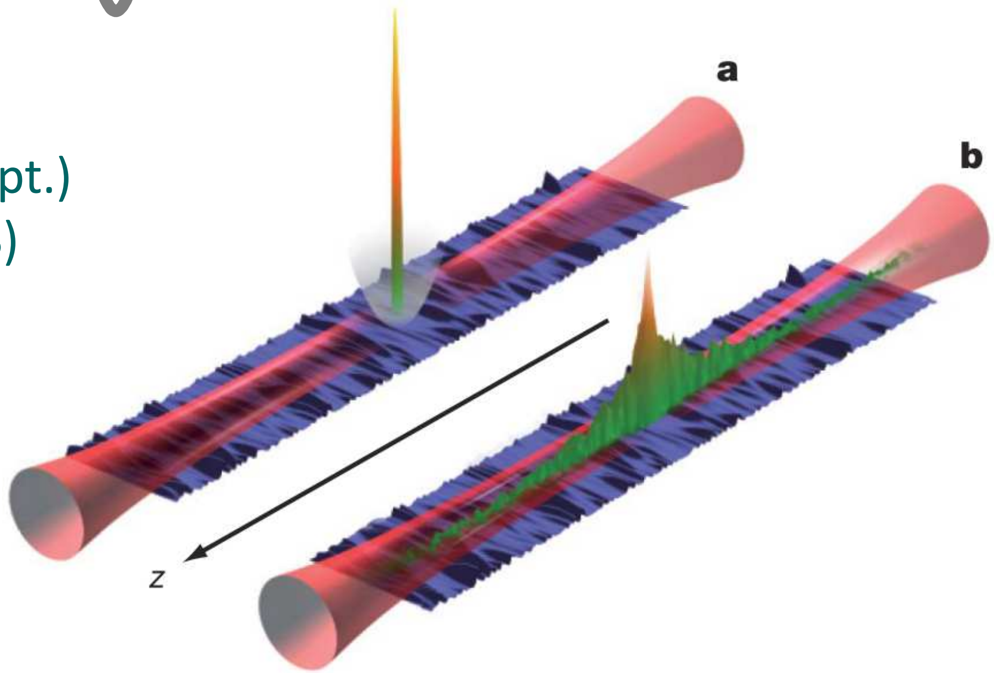
Experiments with ultra-cold atoms

<u>Anderson localization:</u>

Ready…Set…Go!

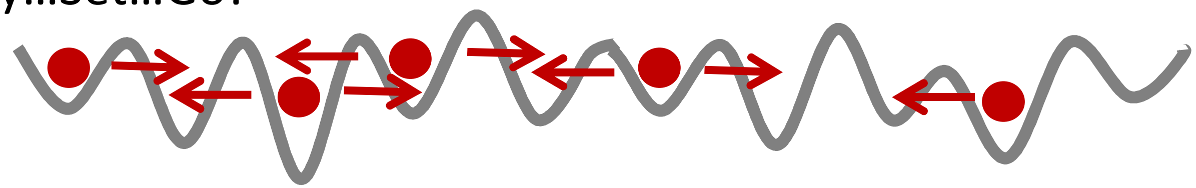


J. Billy et. al. Nature 2008 (Inst. Opt.)
G. Roati et. al. Nature 2008 (LENS)



<u>Many-body localization?</u>

Ready…Set…Go!



Setup of a model calculation

Disordered fermion chain:

$$H = J\sum_i \left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + \Delta n_i n_{i+1}\right) + \sum_i h_i n_i$$

= random spin chain:

$$H = J\sum_i \left(S_i^+ S_{i+1}^- + S_{i+1}^+ S_i^- + \Delta S_i^z S_{i+1}^z\right) + \sum_i h_i S_i^z$$

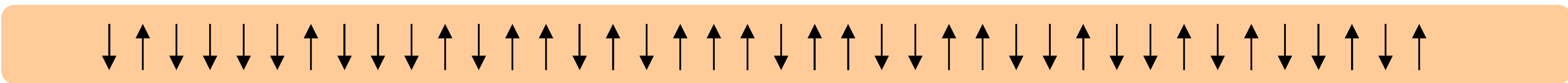
$h_i \in [-h, h]$

hopping

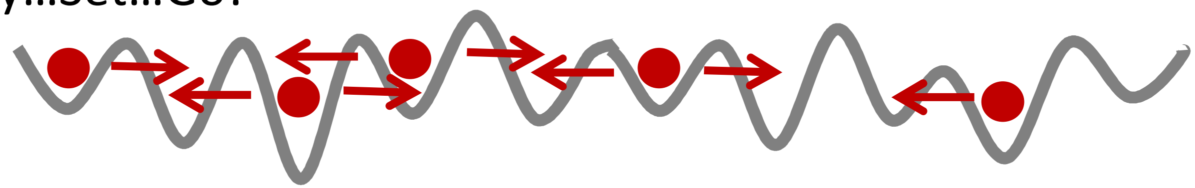
Interaction

On site disorder

Initial state: particles in well defined positions



Ready…Set…Go!



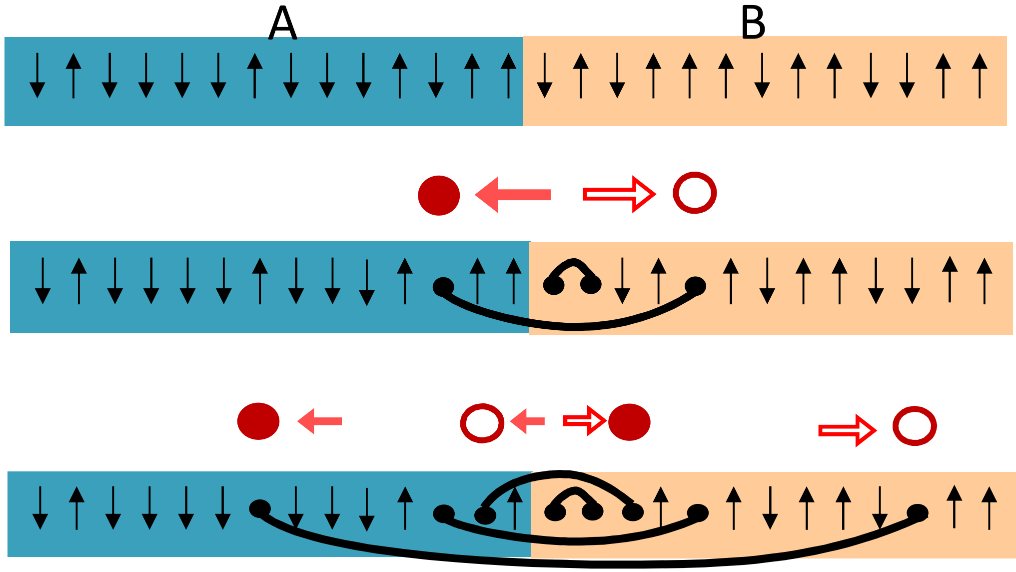
$$e^{-iHt} \mid \Psi_0 \rangle$$

Entropy growth following the quench

$$\rho(0) = \rho_A \otimes \rho_B$$

$$\rho(t_1)$$

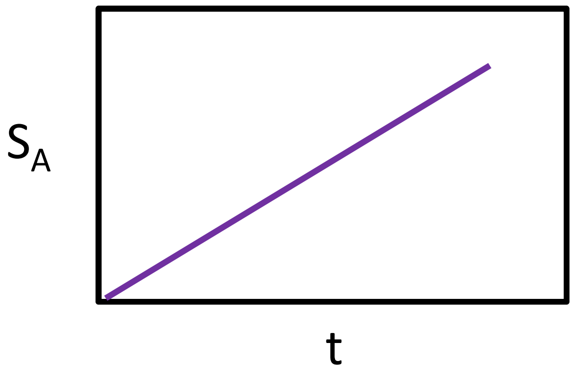
$$\rho(t_2)$$



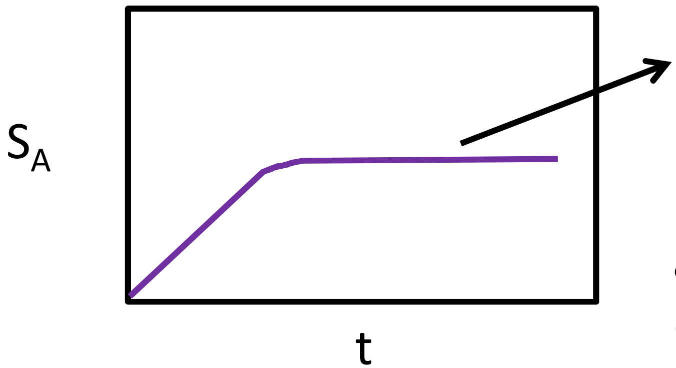
Growing entanglement between the two halves is measured by the Von-Neuman entropy:

$$S_A(t) = -Tr\left[\rho_A(t) \ln \rho_A(t)\right]$$

clean system



localized system (?)



$S_{saturation} \sim \xi_{localization}$

Bounded entanglement allows efficient numerics (using DMRG).

Numerical simulation – Entropy growth

Bardarson, Pollmann & Moore. PRL (2012)

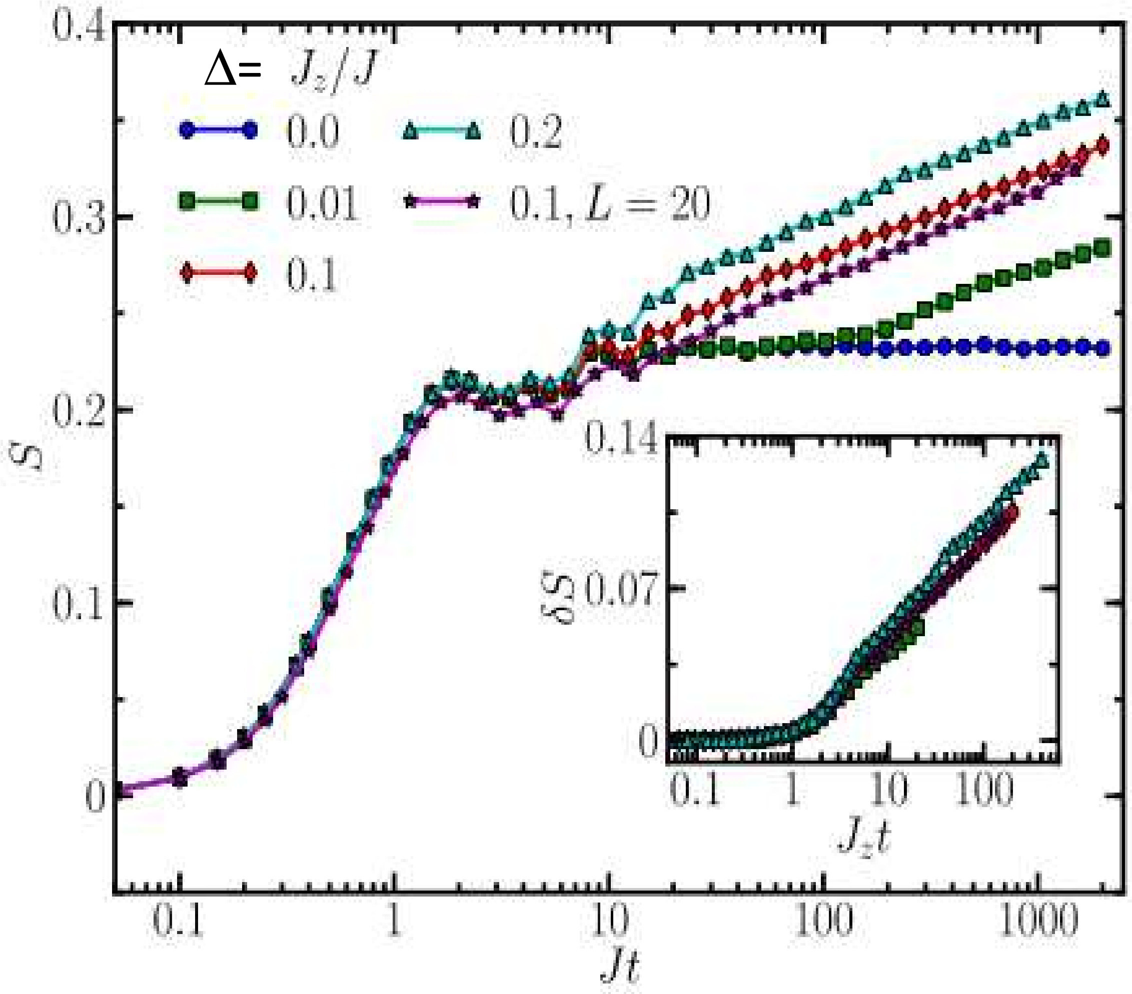
Surprise!

Unbounded log growth
(saturation only for finite L
or without interaction)

With interaction:

- Log(t) increase.
- Delay time ~ $1/\Delta J \sim 1/J_z$

$S_A(t)$



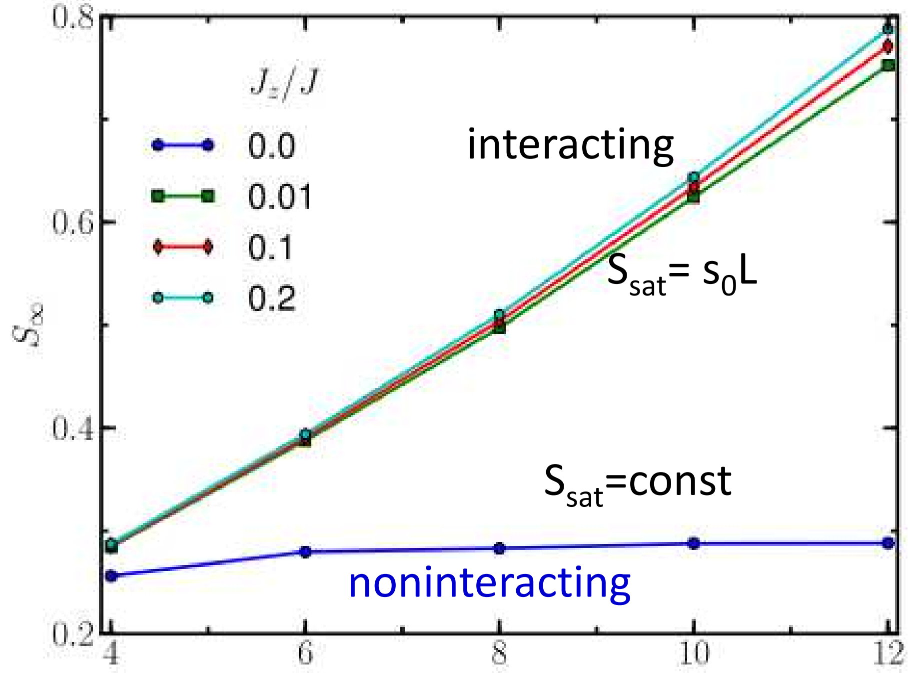
Earlier numerical studies: De Chiara et. al. (2006); Znidaric et. al. (2008)

Numerical simulation – Check thermalization

Bardarson, Pollmann & Moore. PRL (2012)

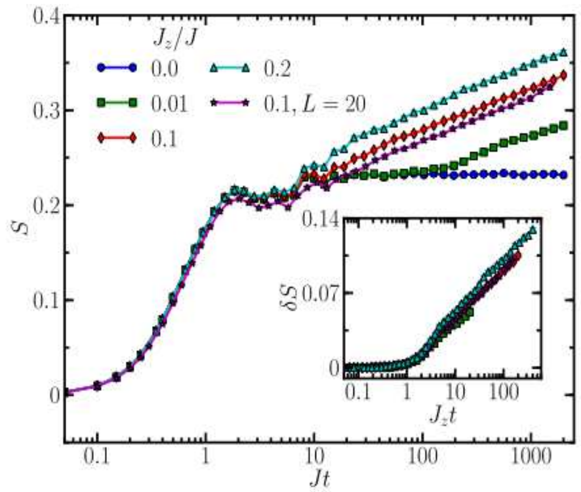
Computed the saturation value of the entropy in a finite system (L).

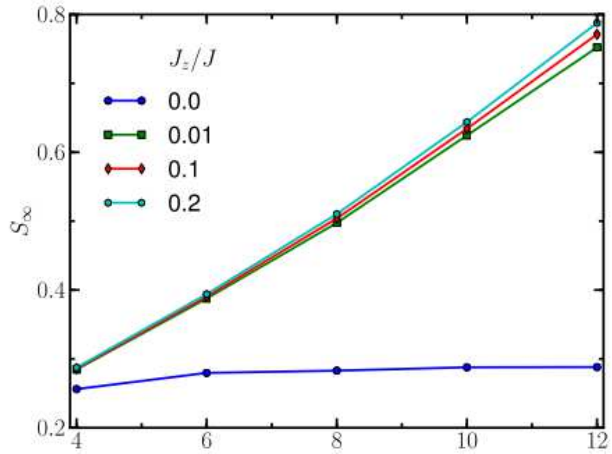
Saturation entropy is extensive, but much smaller than thermal entropy



Even the interacting system does not thermalize!

Goals for theory



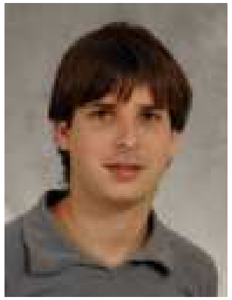


- Explain the universal evolution of the entanglement entropy in this “localized” state as seen in numerics.

- Description of the non thermal state at long times

Renormalization group perspective

Ronen Vosk and EA, arXiv:1205.0026



Model:

$$H = \frac{1}{2}\sum_i J_i \left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta_i S_i^z S_{i+1}^z\right)$$

Random exchange

$$J_i \in [-\Omega, \Omega]$$

Anisotropy (=interaction)

$$|\Delta_i| \ll 1$$

Initial state: $\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow$

We want to compute:

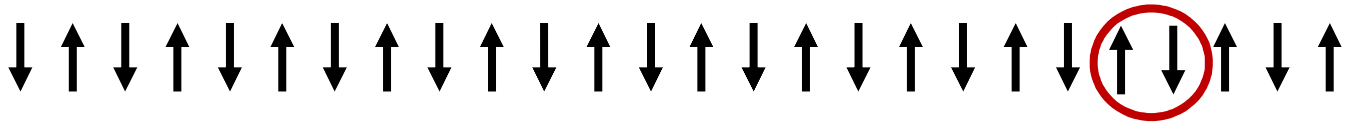
$$\rho(t) = e^{iHt}\rho(0)e^{-iHt} = \;?$$

Renormalization group perspective

Ronen Vosk and EA, arXiv:1205.0026

$$H = \frac{1}{2}\sum_i J_i \left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2\Delta_i S_i^z S_{i+1}^z\right)$$

$H(\Omega)$



<u>Short times ($t \approx 1/\Omega$) :</u>

Pairs on strong bonds J=Ω perform rapid oscillations

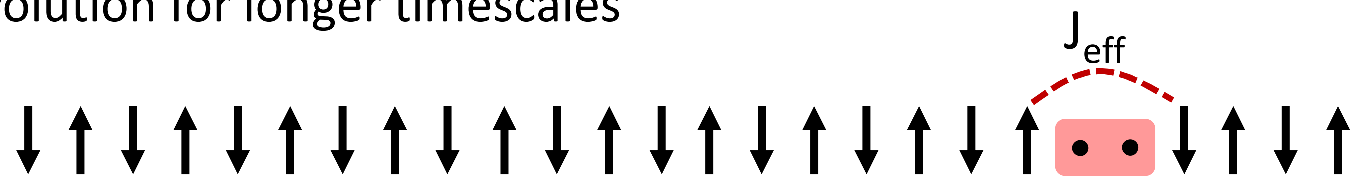
Other spins essentially frozen on this timescale.

<u>Longer times ($t \gg 1/\Omega$) :</u>

Eliminate rapid oscillations perturbatively.

Obtain effective evolution for longer timescales

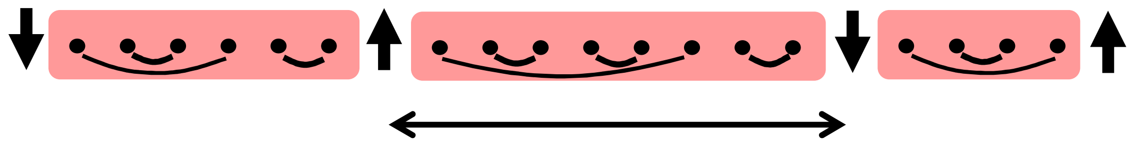
$H(\Omega - \delta\Omega)$



Similar idea for ground states: Dasgupta & Ma 1980, D. Fisher 1994

RG results I – Entropy growth

Renormalized chain
at time t (scale Ω=1/t):



$L(t) \sim [\,\log(t)\,]^2$

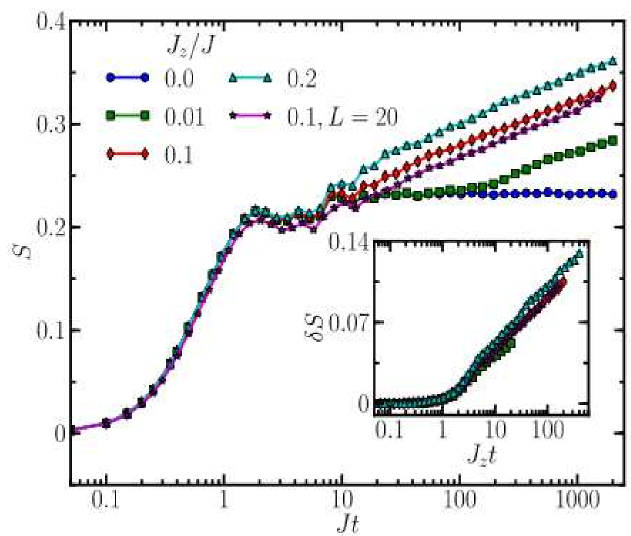
Very slow growth of decimated (dynamic) clusters

$$S(t) \sim [\log(t)]^{2/\phi}\Theta(t - t_{delay}) + \log(\log t)$$

$$\phi = (1+\sqrt{5})/2$$

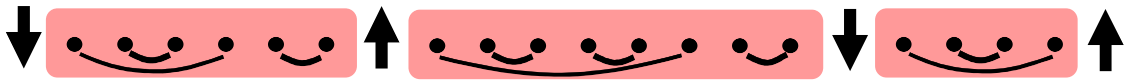
$$t_{\rm delay} = 2\Omega_0/(J_0^2\Delta_0) \quad \sim 1/J_z$$

Explains universal features in numerical result

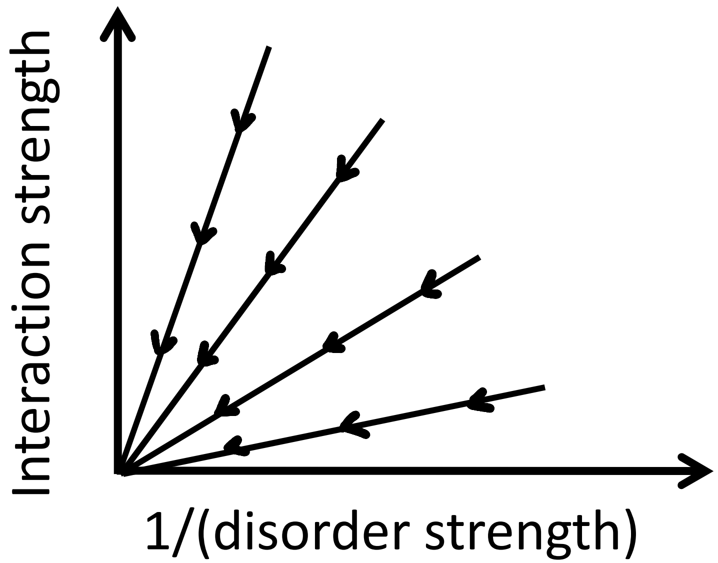


RG results II – flow to infinite randomness

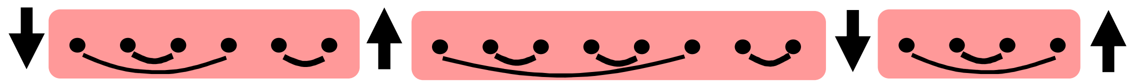
Renormalized chain
at time t (scale Ω=1/t):

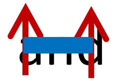
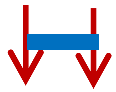


Flows to infinite-randomness
fixed point. RG is asymptotically exact
at long times.



RG results III – Emergent conservation laws



In every decimated pair of spins the states  and  are never populated therefore S(L)<(L/2)ln2

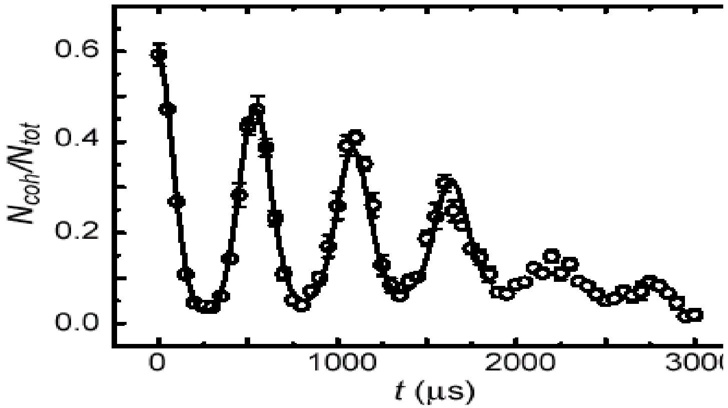
Approximate integrals of motion: $I_p = [S_1^z S_2^z]_{pair}$

Approach exact conservation rules at long times

Non-thermalization - asymptotic generalized Gibbs ensemble

Summary

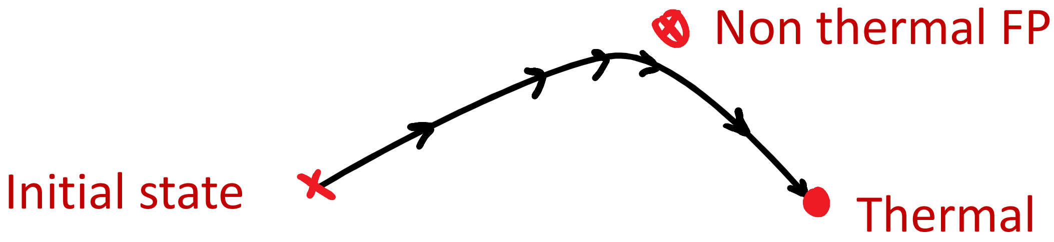
Greiner et. al. Nature 419, 2002



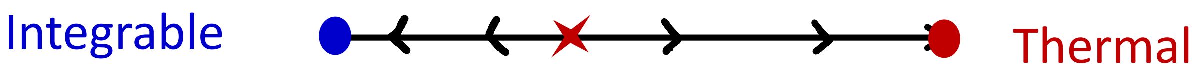
- Ultracold gases: novel laboratory for quantum dynamics.
  Closed systems.

- Prospects for universal behavior

  – <u>Prethermalization</u>:



  – <u>Non-thermalization in many body localization:</u>



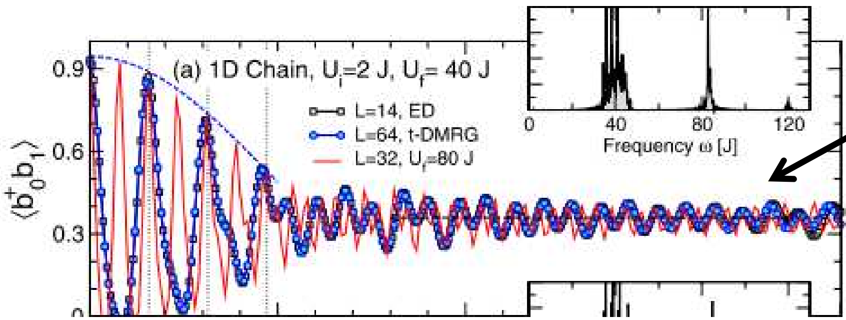
Q: nature of the Critical point ?

PRL 98, 180601 (2007) PHYSICAL REVIEW LETTERS week ending 4 MAY 2007

Quench Dynamics and Nonequilibrium Phase Diagram of the Bose-Hubbard Model

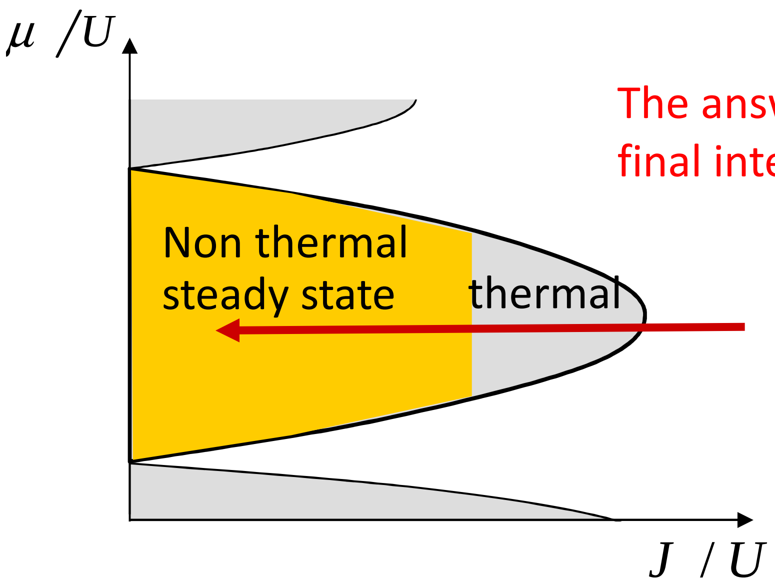
Corinna Kollath,[1] Andreas M. Läuchli,[2] and Ehud Altman[3]

DMRG calculation of: $$H = -t\sum_{\langle i,j\rangle}\left(b_i^\dagger b_j + h.c.\right) + \frac{U}{2}\sum_i n_i(n_i-1)$$

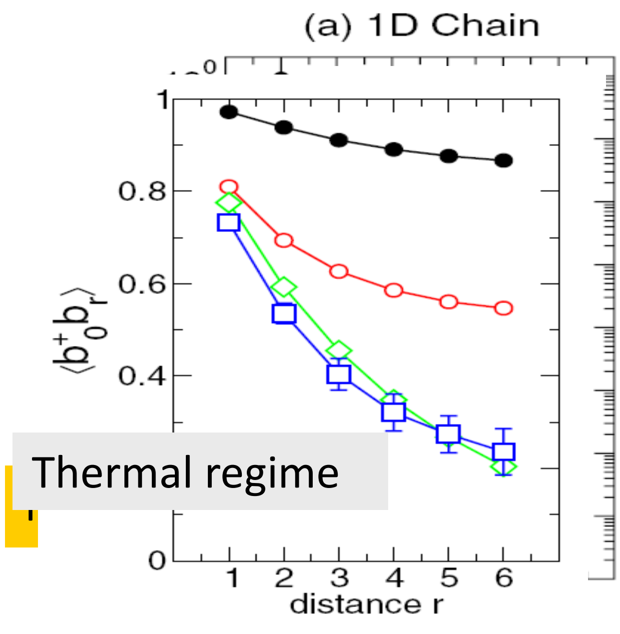


Is this a thermal steady state ?

Compare correlations in steady state to expected equilibrium correlations



The answer depends on final interaction strength



Thermalization is an “emergent scale in this regime