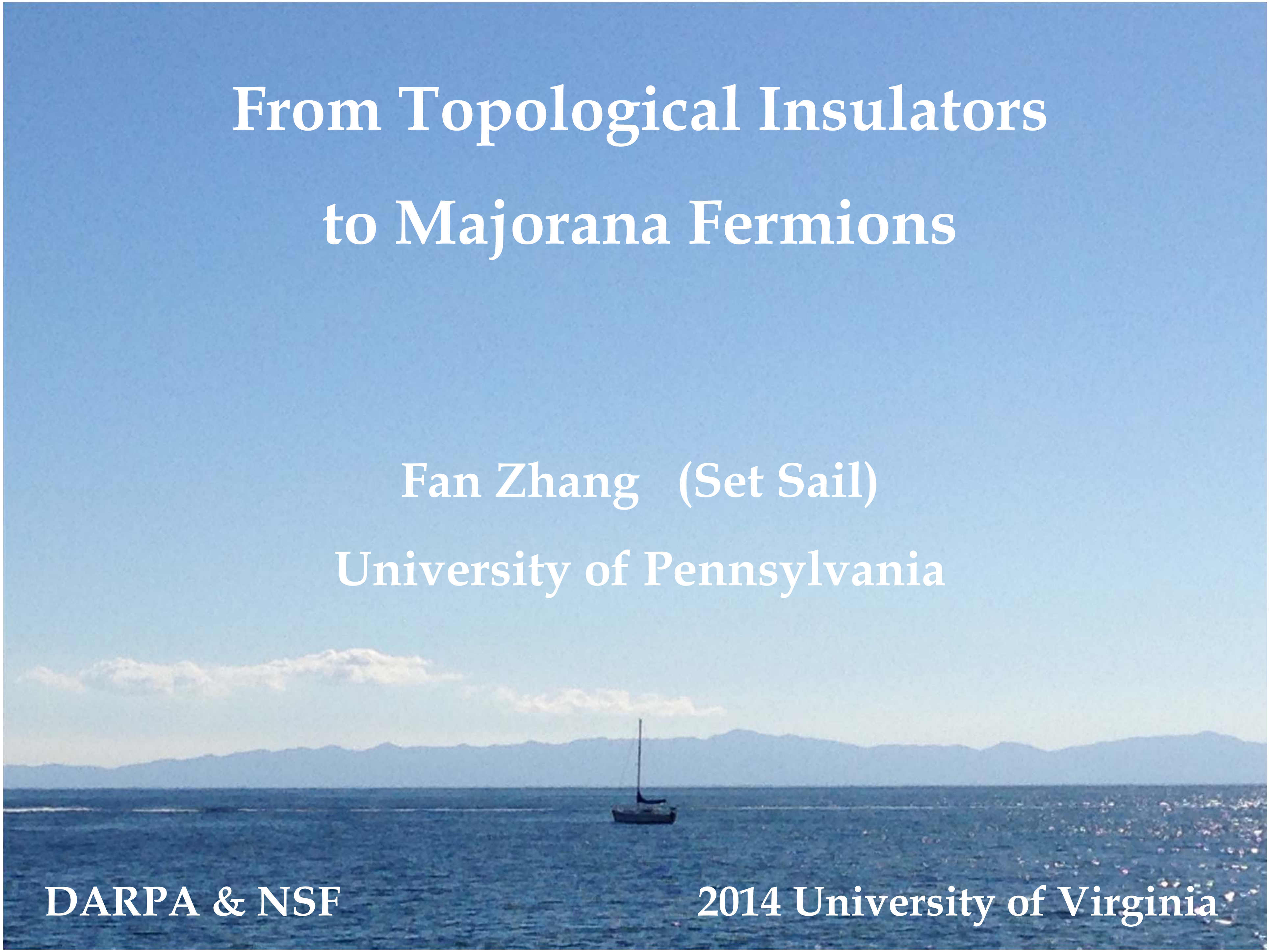

From Topological Insulators to Majorana Fermions

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DARPA & NSF

2014 University of Virginia

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Abel Yang, Jeil Jung (Singapore)
Hongki Min (Seoul)
Bhagawan Sahu (AMD)
Q. Niu, G. Fiete, Xiao Li (Austin)

My Captains

Charles Kane

Eugene Mele

(Since Nov. 2011)

Allan MacDonald

(Since Jan. 2008)

Topological Insulators and Superconductors

- A recurring theme in CMP has been the **discovery and classification** of different states of matter.

- In conventional Landau picture, states can be classified **by symmetry breaking**.

crystals	translational symmetry
ferromagnets	rotational symmetry
superconductors	gauge symmetry

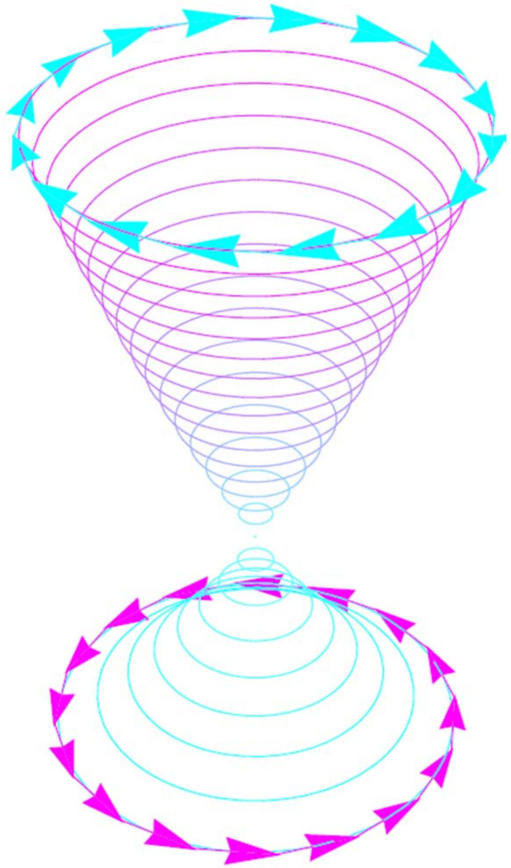
Topological Insulators and Superconductors

What about states in the same symmetry class?

(beyond the Landau picture)

- Are all electronic states with energy gaps topologically equivalent to the vacuum (e.g. atomic insulator)?

- The answer is no, and the counterexamples are fascinating states of matter.



Outline

Part I: Introductions (three messages)

- Topological insulator = band inversion + symmetry protection
- Examples of band inversion mechanisms
- There are a variety of topological insulators/superconductors

  [fit into an elegant "periodic table"]

Part II: Most Recent Progress (in 2013)

- Time-reversal-invariant topological superconductivity
- Majorana Kramers pairs
- $Z_2 \times Z_2$ fractional Josephson effects

  ["periodic building" with the "table" being its ground floor]

Notes: (a) at free-fermion level

(b) experimentally observed/realizable

From Bulk to Boundary

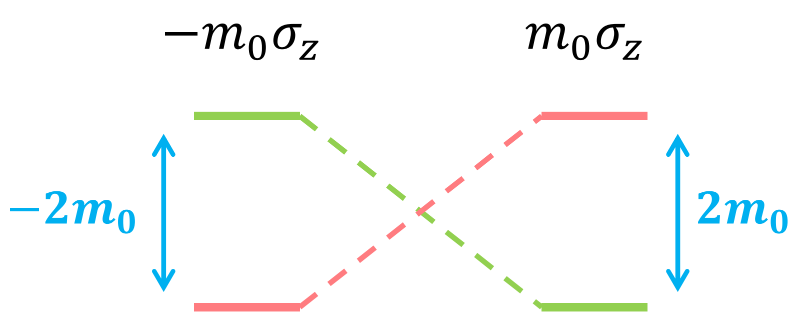
1D Massive Dirac Fermion:

$$H = vk_z\sigma_y + m_0\mathrm{sgn}(z)\sigma_z$$

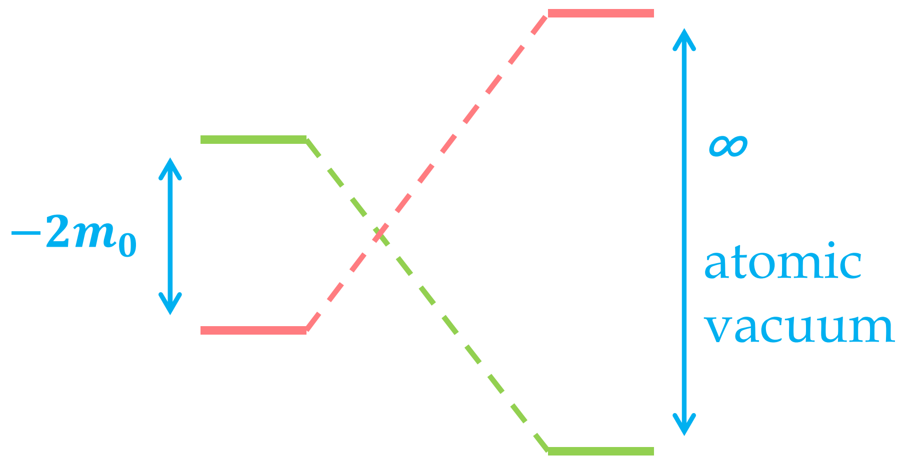
1D boundary problem in Q.M. with a special solution:

(i) E = 0   (ii) $\sigma_x = 1$ (iii) z = 0

gapless   fractionalized   localized



band crossing near z=0



Jackiw-Rebbi, PRD 13, 3398 (1976)

Passivation on the Boundary

1D Massive Dirac Fermion:

$$H = vk_z\sigma_y + m_0\mathrm{sgn}(z)\sigma_z$$

a special solution: (i) E = 0 (ii) $\sigma_x = 1$ (iii) z = 0

gapless fractionalized localized

The zero mode
is not protected !!!

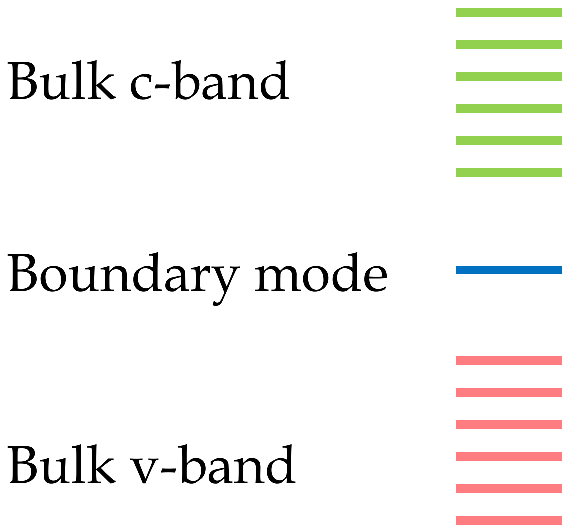
Symmetry Protected Topological Insulator

1D Massive Dirac Fermion:

$$H = vk_z\sigma_y + m_0\text{sgn}(z)\sigma_z$$

a special solution: (i) $E = 0$ (ii) $\sigma_x = 1$ (iii) $z = 0$

gapless fractionalized localized



Chiral (particle-hole) symmetry:

$$\sigma_x H \sigma_x = -H$$

- The mode is protected!
- Integer invariant ($H_{boundary}$=0)!

Su-Schrieffer-Heeger (1978 polyacetylene)

Symmetry Protected Topological Insulators

- Mass switching sign (or band inversion) may lead to gapless boundary states,

- however, they are not robust against perturbations, unless there is a symmetry to protect them,

- Protected gapless boundary states are exotic.

Topological Equivalence

Sphere

$$|UVA\rangle = |110\rangle \quad \longleftrightarrow \quad |FAN\rangle = |101\rangle$$

Rules of topological deformation:

- Do not allow any energy band crossing (involving a valence band)
- Do not break any essential symmetry

“Mass” in Solid State Physics

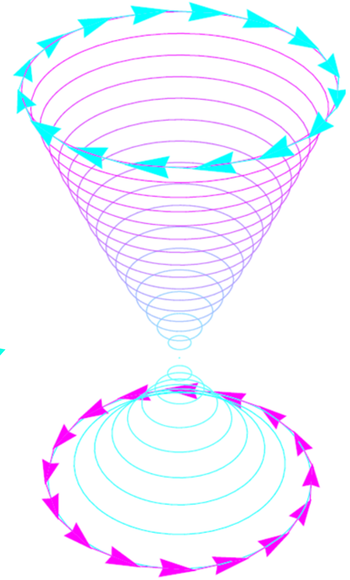
- Insulators: energy gap in the band structure
- Superconductors: energy gap for quasi-particles

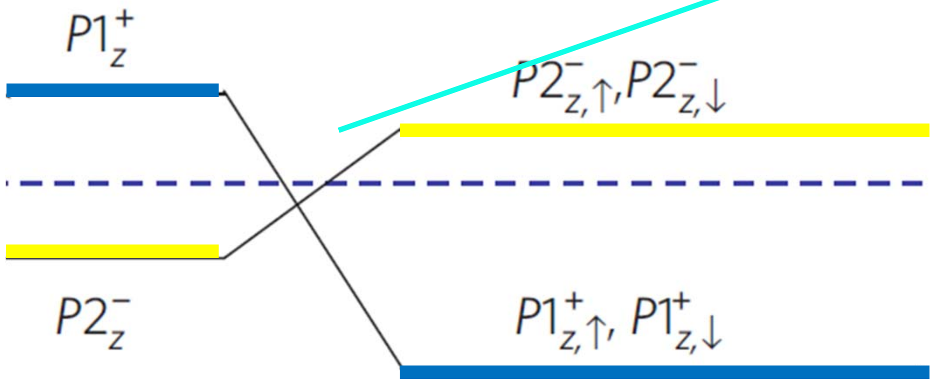
Why Mass Switches Sign
(Band Inversion)

Band Inversion Mechanism

(I) spin-orbit couplings

e.g. Bi_2Se_3, Bi_2Te_3, …
(Z_2 topological insulators
protected by time-reversal symmetry)





Helical surface states
(spin-momentum locking)

Hasan-Kane, RMP2010

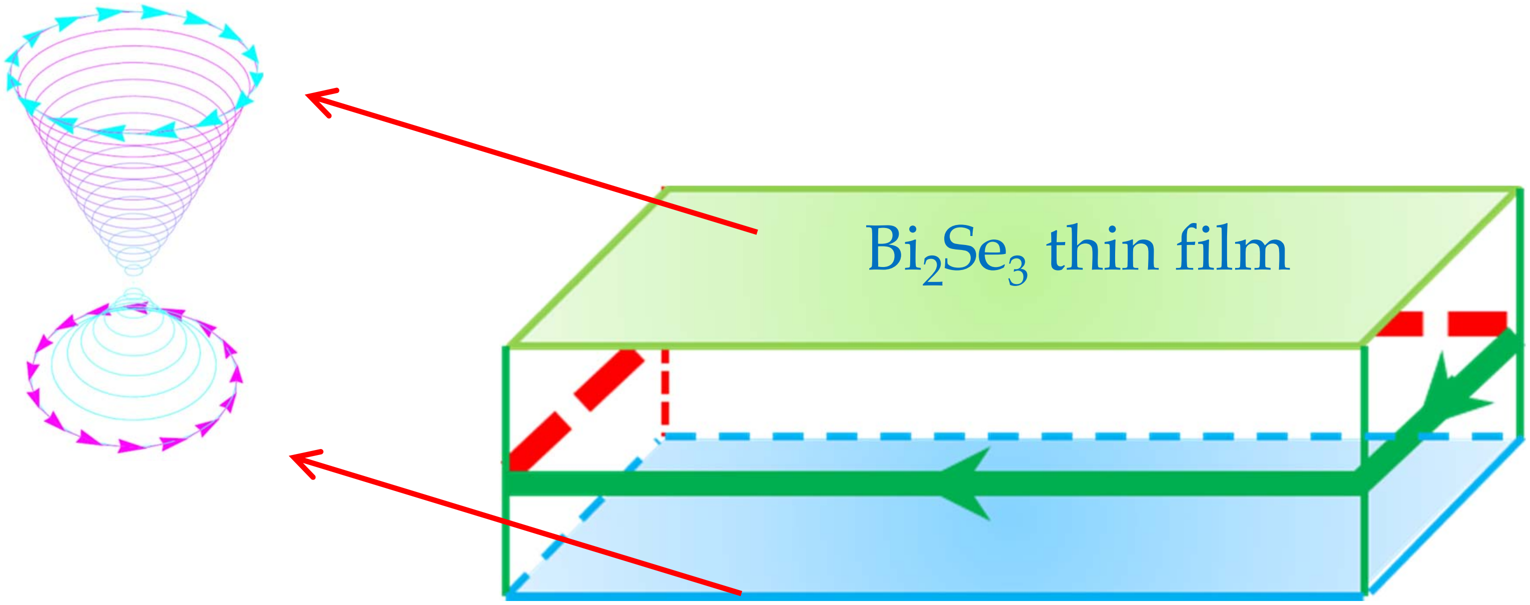
Band Inversion Mechanism

(II) “strong” magnetic fields

e.g. integer quantum Hall states (orbital effect)

quantum anomalous Hall states (Zeeman effect)

Quantum Anomalous Hall Insulators Driven by Zeeman fields

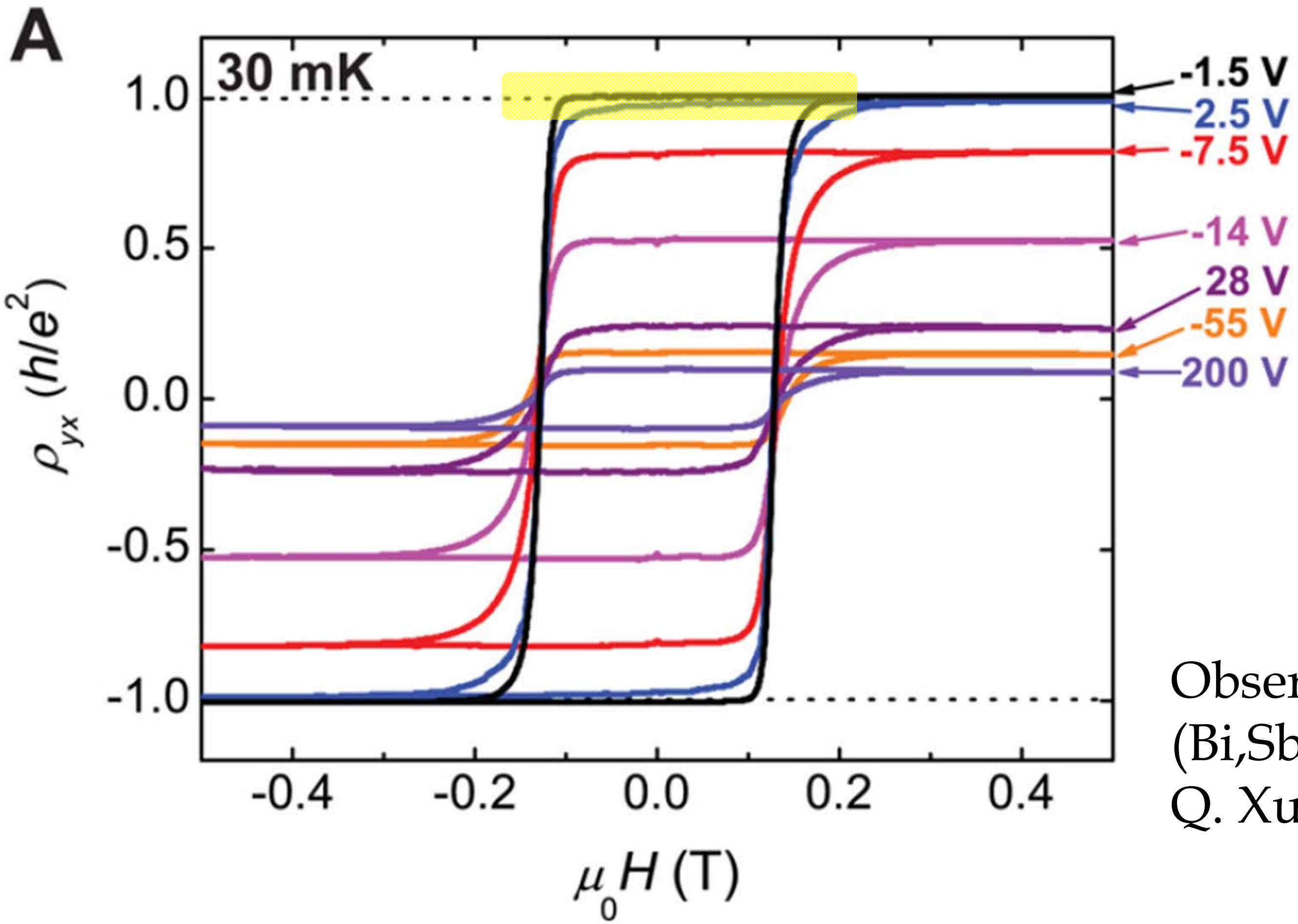


Hall conductivity induced by Zeeman field:

$$\pm\frac{1}{2}\pm\frac{1}{2}=\mathbf{0},\pm\mathbf{1} \quad [e^2/h]$$

- General criterion: **FZ**-Kane-Mele
- DFT of thin film geometry: Dai-Fang group

Quantum Anomalous Hall Insulators Driven by Zeeman fields



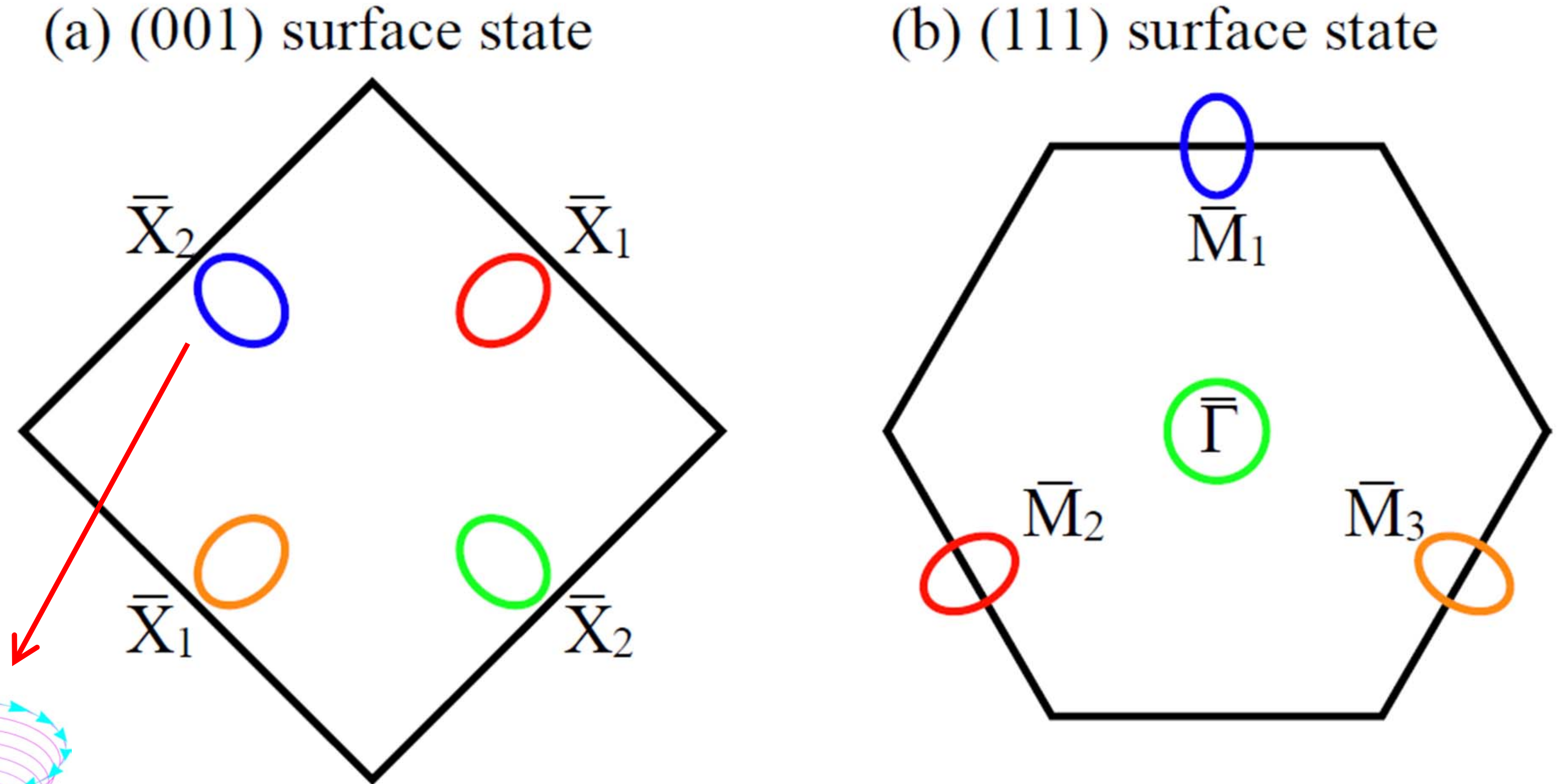
Observation in Cr-doped $(Bi,Sb)_2Te_3$ thin film by Q. Xue group, Science2013

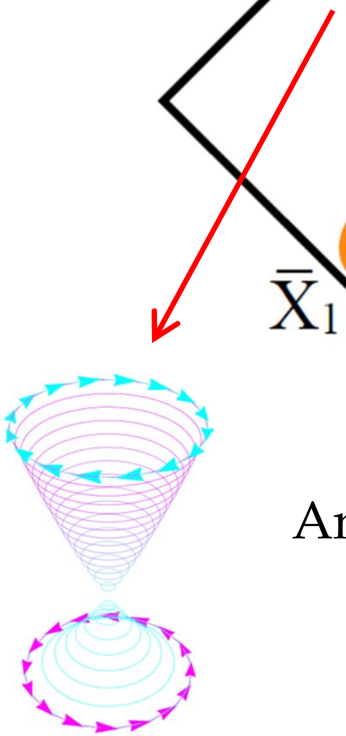
Theory:

- General criterion: **FZ**-Kane-Mele
- Thin film geometry: Dai-Fang group

N. Samarth group
L. Molenkamp group
…

Surface States of SnTe





Anomalous Hall conductivity induced by Zeeman field:

$$\pm\frac{1}{2}\pm\frac{1}{2}\pm\frac{1}{2}\pm\frac{1}{2} \quad [e^2/h]$$

FZ et al, arXiv:1309.7682

How to Tune the Hall Conductance

σ_H = -4, -3, -2, -1, 0, 1, 2, 3, 4 $[e^2/h]$

- Combining perpendicular Zeeman and electric fields, strain effects, and interlayer couplings

  --- A. Bernevig Princeton group 2013

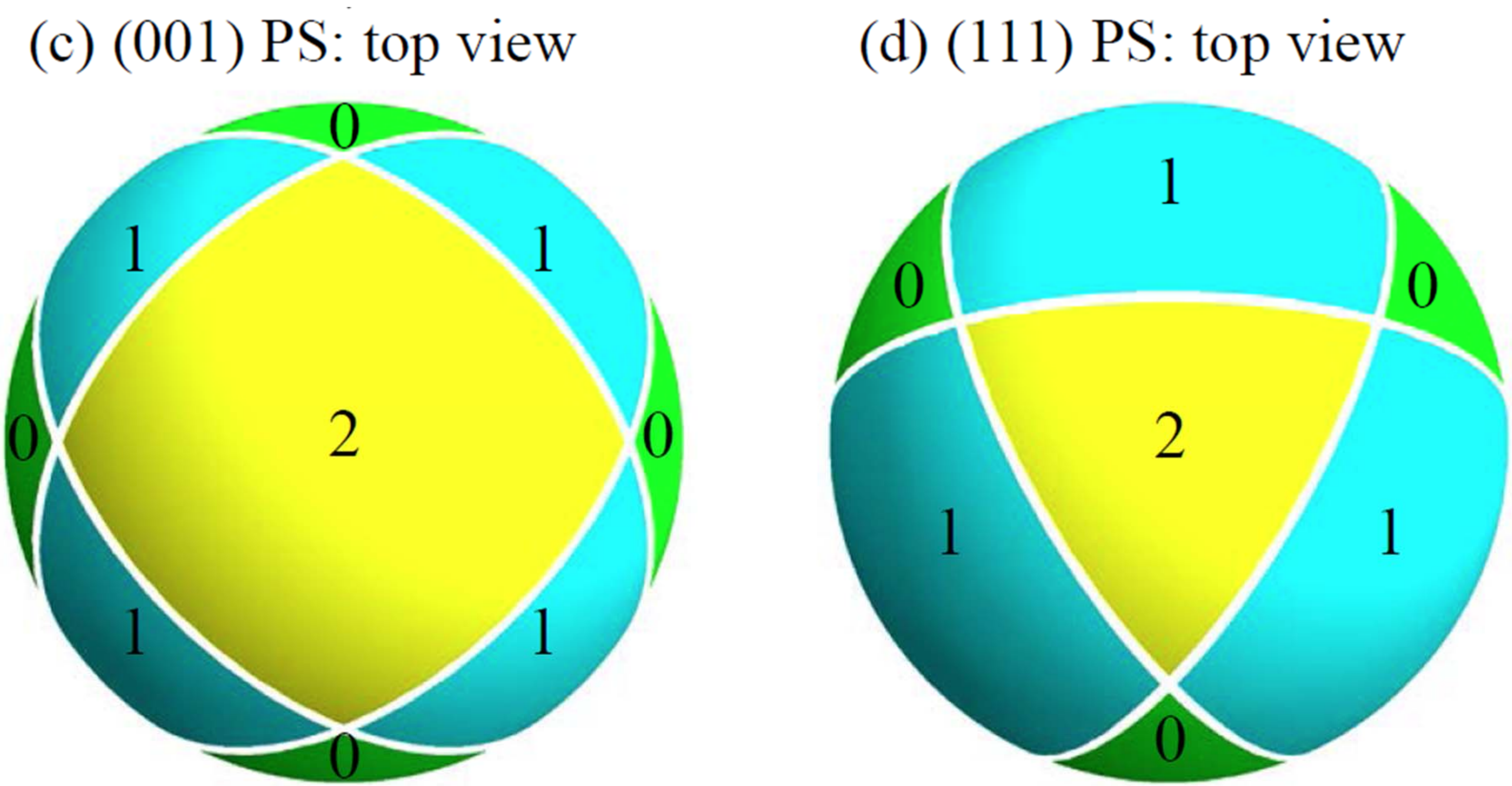
- Varying the direction of Zeeman fields, making use of the crystal symmetries and anisotropies.

  --- **FZ** et al. 2013

Rotating the Zeeman Field : tuning the Hall conductance

σ_H = -4, -3, -2, -1, 0, 1, 2, 3, 4 $[e^2/h]$

Phase diagram for one surface:

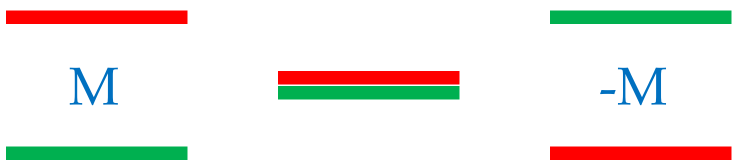


FZ et al, arXiv:1309.7682

Band Inversion Mechanism

(III) electron-electron interactions

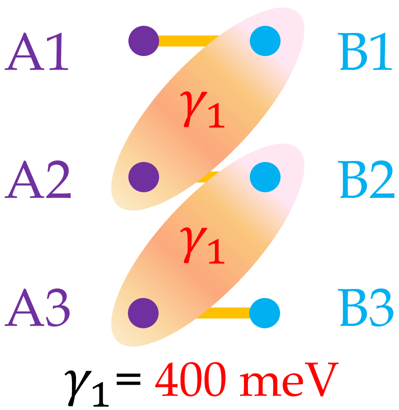
e.g. topological superconductors
topological Kondo insulators (SmB_6)
chiral (ABC) graphene layers
…

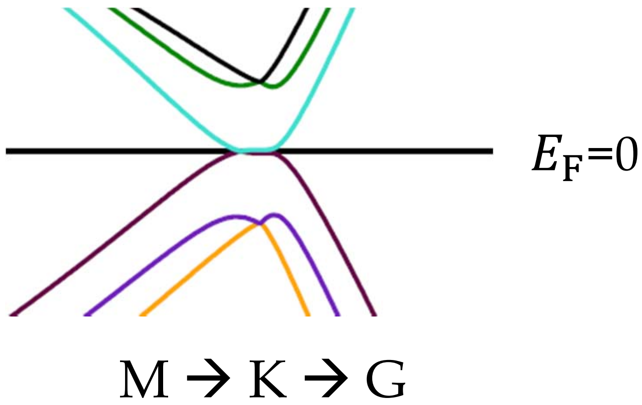


ABC Graphene: 2D Chiral Electron Liquids

$$\mathcal{H}_\mathrm{N} = \frac{(v_0 p)^N}{(-\gamma_1)^{\mathrm{N}-1}} [\cos(N\phi_{\boldsymbol{p}})\sigma_\mathrm{x} + \sin(N\phi_{\boldsymbol{p}})\sigma_\mathrm{y}]$$

Gapless: chiral (A_1-B_N sublattice) symmetry
i.e. $\sigma_z H \sigma_z = -H$

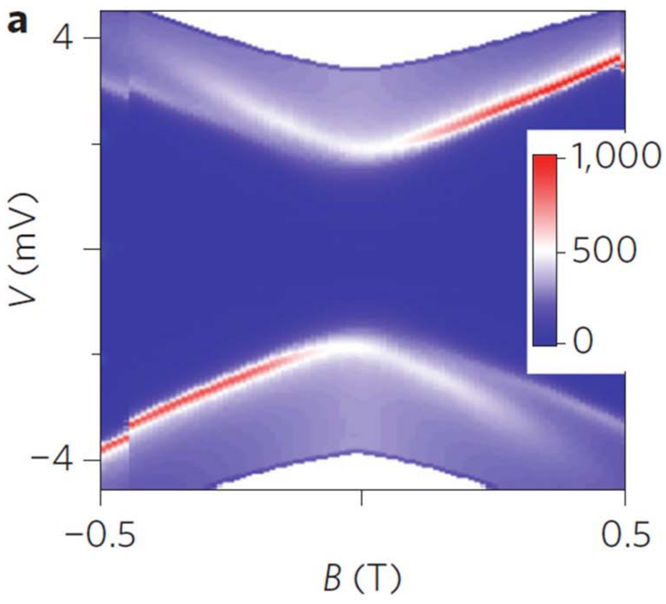


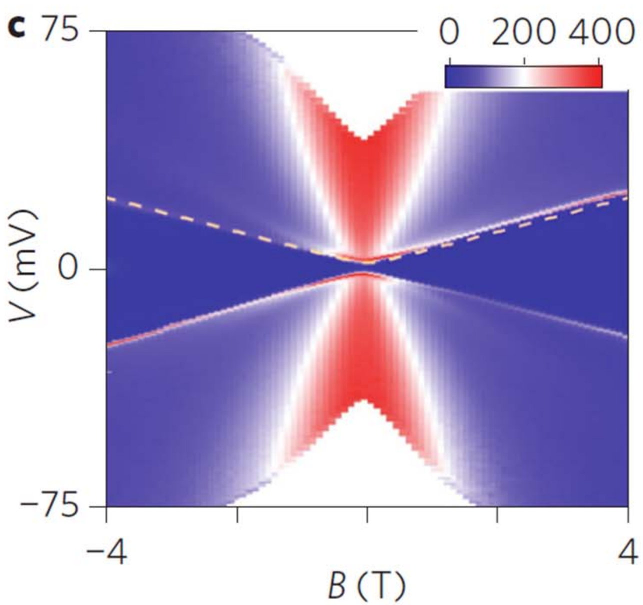


Spontaneous chiral symmetry breaking & metal-insulator transition

Theories: **FZ**-MacDonald

Spontaneous Chiral Symmetry Breaking





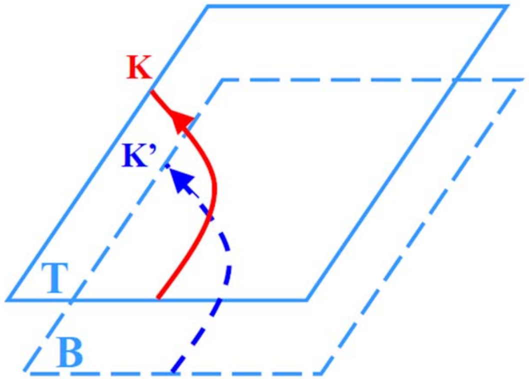
- Layer-antiferromagnetic ground state at n=B=E=0
- In dual gated, suspended, high mobility samples
- AB bilayers: 2-3 meV
  ABC trilayers: 45 meV

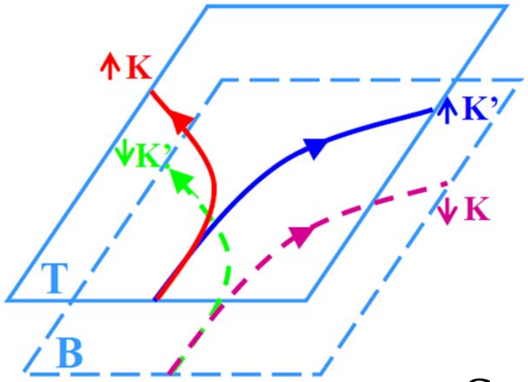
Predictions: **FZ** et al, PRL (2011)

Observation: Lau-**FZ**, Nature Nano (2012)
and many other groups

Spontaneous Quantum Hall (SQH) States

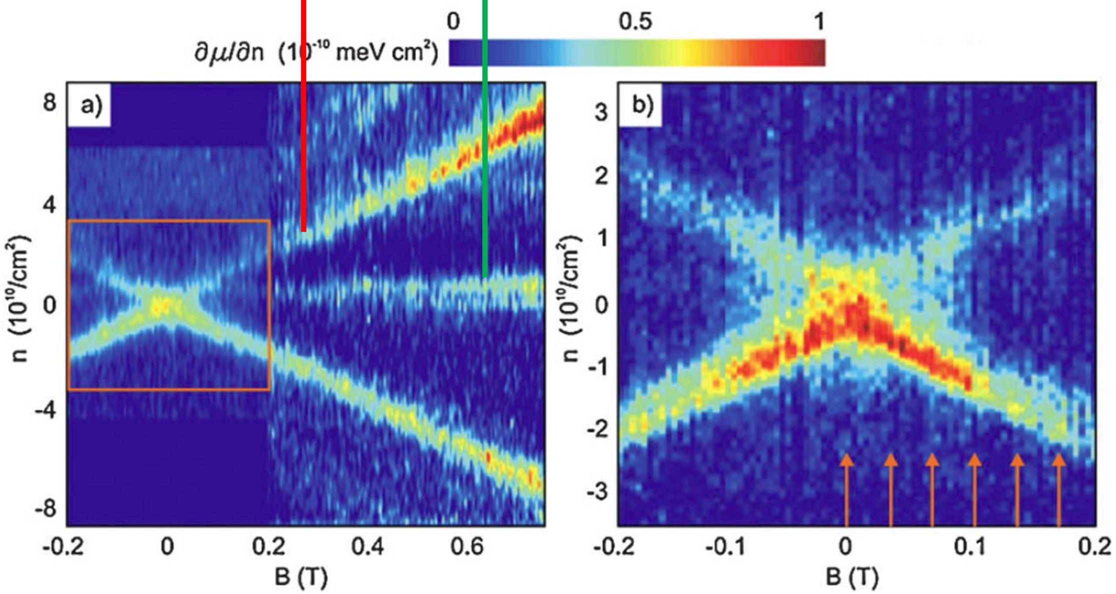
Valley K and K′
have opposite
gaps !
$\sigma_H = 4$





Spin up and
down have
opposite gaps !
$\sigma_H = 0$

σ_H = 4 0



Theory: **FZ**-MacDonald

Observation in bilayers:
A. Yacoby group at Harvard

Band Inversion Mechanism

(a) Spin-orbital couplings

(b) Magnetic (Zeeman or orbital) fields

(c) Electron-electron interactions

(d) Disorders, electron-phonon interactions, ……

Band Inversion in Real Life

Crazy proposal for China-Hong Kong connection

Pearl River Necklace bridge comes with a twist !

The "Omnipotent" Periodic Table

(free fermion systems)

Band inversions + Adding dimensions

+ Imposing symmetries (protection)

(**Kitaev table**; Schnyder-Ryu-Furusaki-Ludwig)

s	AZ	Symmetry Θ^2	Ξ^2	Π^2	Dimension ($\boldsymbol{k}$) 0	1	2	3	4	5	6	7
0	A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
1	AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
0	AI	1	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
1	BDI	1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
2	D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
3	DIII	−1	1	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
4	AII	−1	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
5	CII	−1	−1	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
6	C	0	−1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
7	CI	1	−1	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

FZ-Kane "periodic build" (2013-)

Outline

Part I: Introductions

- Topological insulator = band inversion + symmetry protected
- Examples of band inversion mechanisms
- There are a variety of topological insulators/superconductors

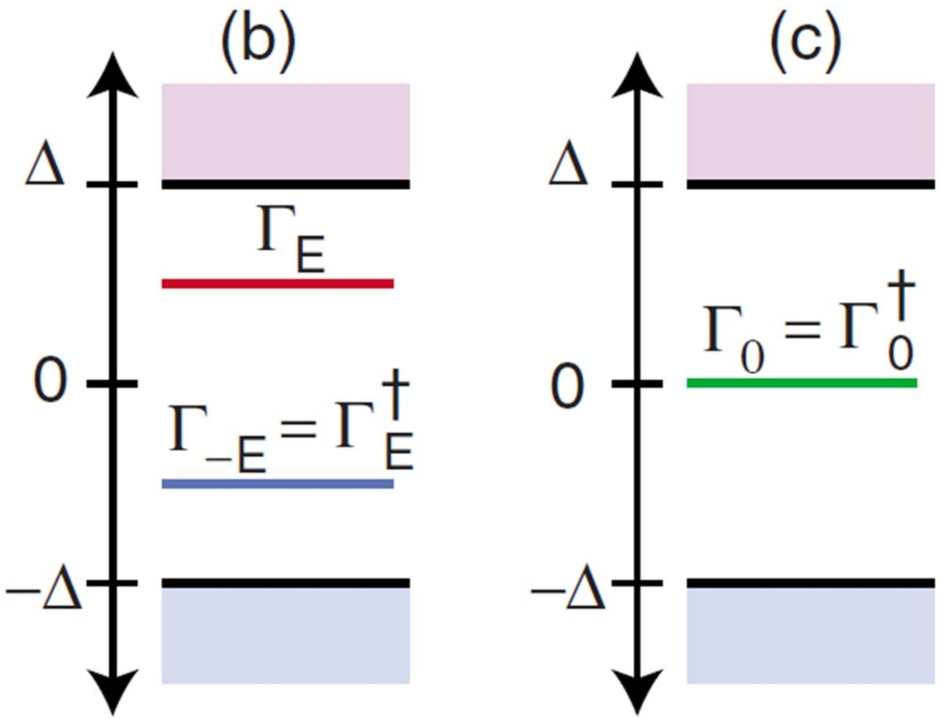
  [fit into an elegant "periodic table"]

Part II: Most Recent Progress (in 2013)

- Time-reversal-invariant topological superconductivity
- Majorana Kramers pairs
- $Z_2 \times Z_2$ fractional Josephson effects

  ["periodic building" with the "table" being its ground floor]

1D Topological Superconductors



- Energy gap for quasi-particles (not for Cooper pairs)
- Intrinsic anti-unitary part-hole symmetry for BdG Hamiltonians
- E=0 mode (at k=0) = Majorana fermion

From Hasan-Kane, RMP2010

Topological Superconductor without Time-Reversal Symmetry

Dimensions	0	1	**2**
Class D	Z_2	Z_2	**Z**

- **In 2D: Z = Chern number = TRS must be broken**

$$H_{BDG} = H_N + H_\Delta$$

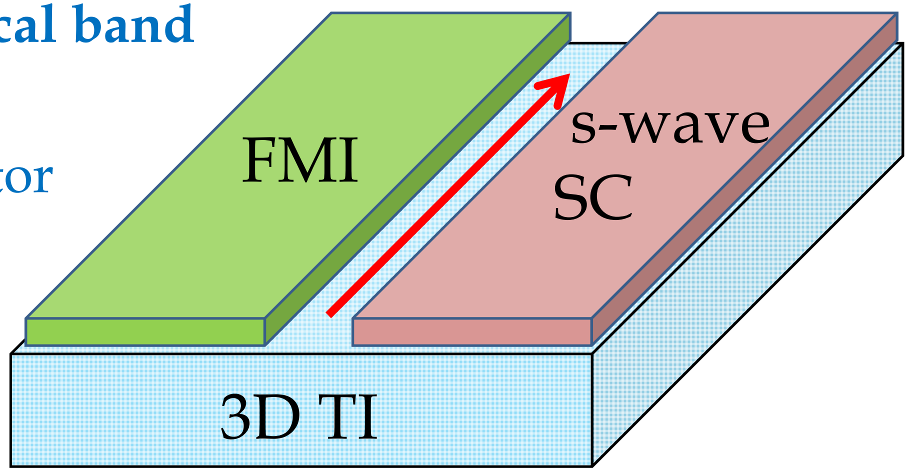
Idea i: H_N is normal whereas H_Δ has a winding number,

[e.g. p wave SC, Read&Green2000, Kitaev2001, Ivanov2001]

Topological Superconductor without Time-Reversal Symmetry

Idea ii: H_Δ is normal whereas H_N has a winding number

normal state = a single helical band

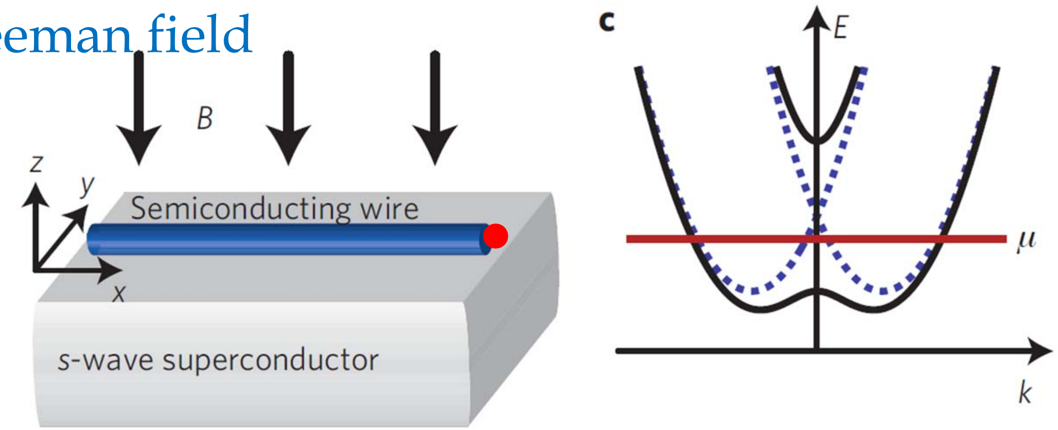


(a) the surface of a topological insulator

+ breaking time-reversal symmetry

Fu-Kane 2008

(b) Rashba semiconductors + Zeeman field



Das Sarma group 2010,
Oreg-Refael-von Oppen 2010,
Alicea 2010
…

(2000-, 2010-) Topological Superconductor without Time-Reversal Symmetry

Ambitious Questions

(i) Is there anything new?
[conceptually novel & experimentally realizable]

(ii) Is it possible to build a topological superconductor that **respects** time-reversal symmetry **without** using any exotic interactions?

Time-Reversal-Invariant Topological Superconductors
(Class DIII)

Dimensions	0	**1**	2	3
Class DIII	0	$\mathbf{Z_2}$	Z_2	Z

Criterion I: [by Qi-Hughes-Zhang]

# of Fermi surfaces:
(enclosing k=0 or π points)

Even = Odd + Odd

The sign of SC pairing: + –

(mass switches signs)

Time-Reversal-Invariant Topological Superconductors
(Class DIII)

Criterion II: [by **FZ**-Kane-Mele]

Without using any e-e interaction or Josephson effect,

pure s-wave pairing is impossible to induce TRI TSC.

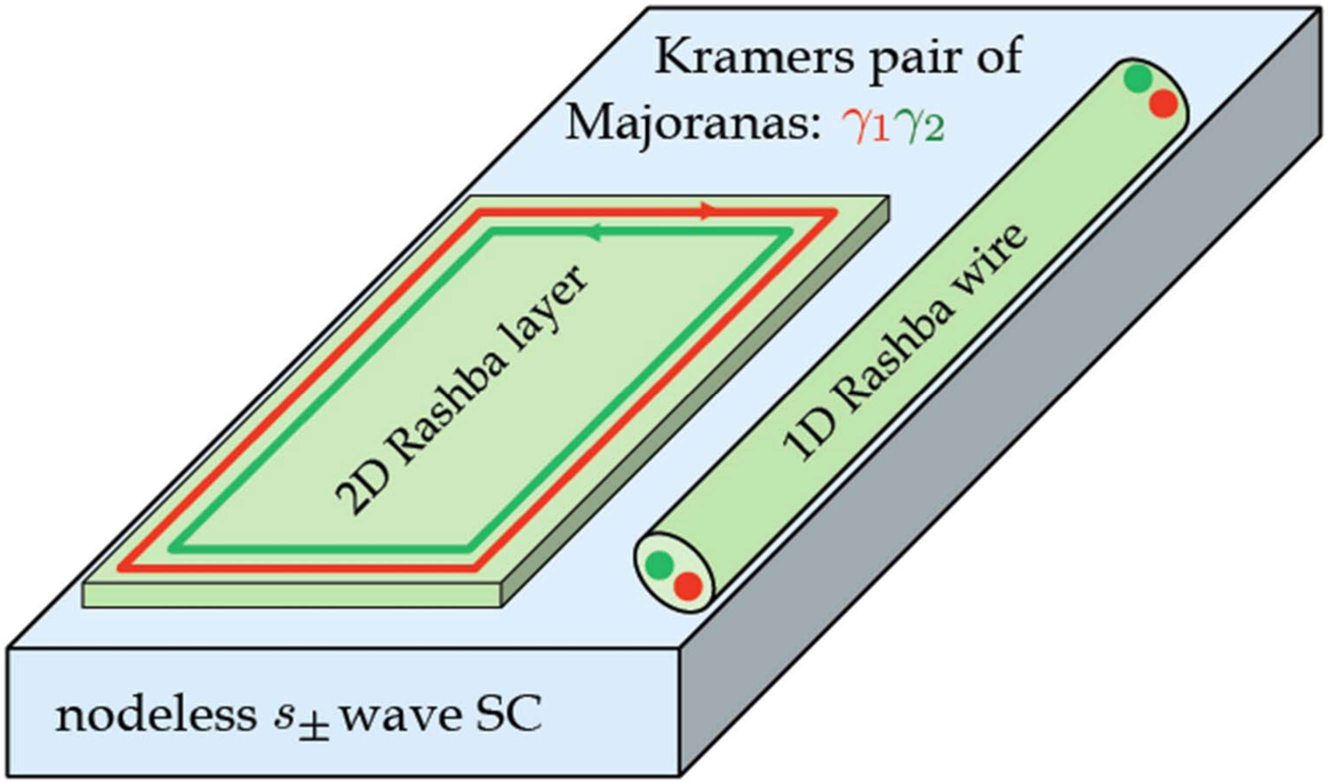
The simplest solution: [probably the best, too]

extended s-wave + nodeless

e.g. Iron pnictides [I. I. Mazin, Nature (2010)]

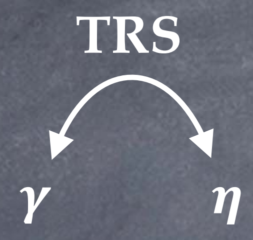
Proximity Coupling an Extended S-wave SC and a Rashba semiconductor

- Time-reversal symmetry: no magnetic perturbation
- No interactions: using proximity effect



FZ-Kane-Mele, PRL 111, 056402 (2013)

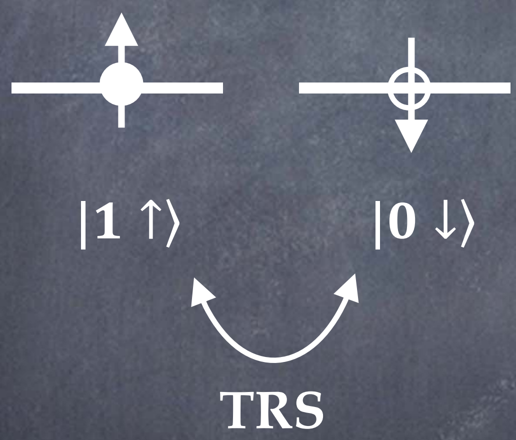
Majorana Kramers Pair (MKP)



MF's: $\gamma^\dagger = \gamma,\ \eta^\dagger = \eta$

TRS: $\gamma \to \eta \to -\gamma$

MKP forms a fermion level



Define: $c_\uparrow = \gamma + i\eta$

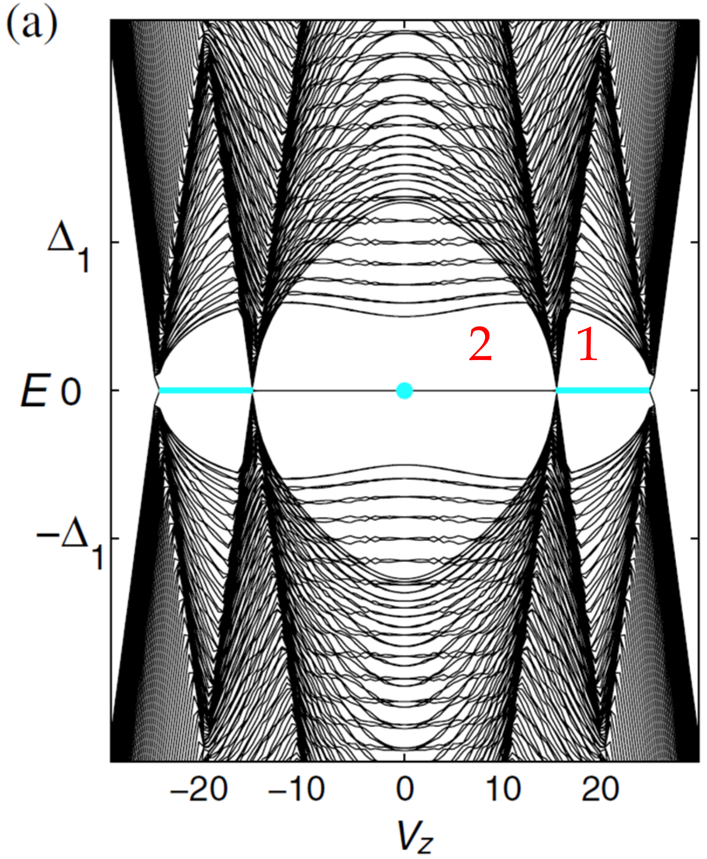
TRS:
$$c_\downarrow = \eta + i\gamma = i(\gamma - i\eta) = ic_\uparrow^\dagger$$

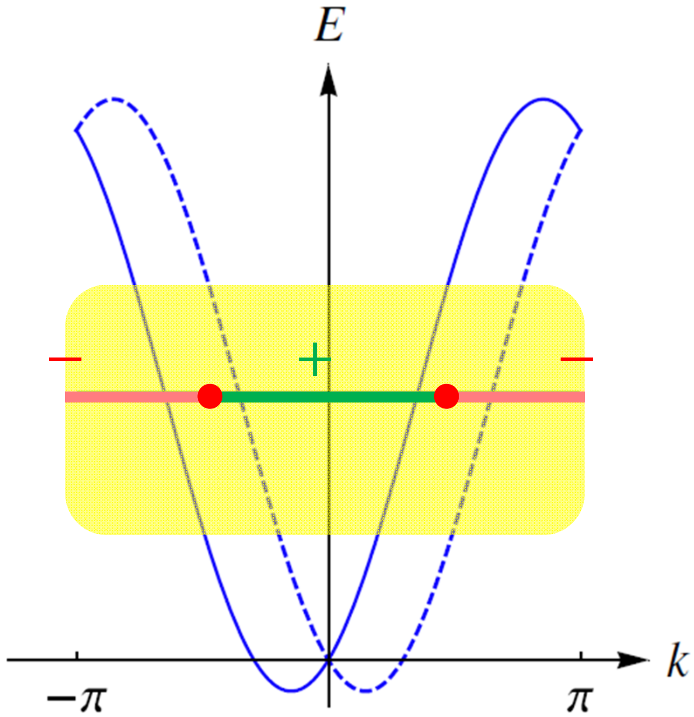
TRS = Super Symmetry

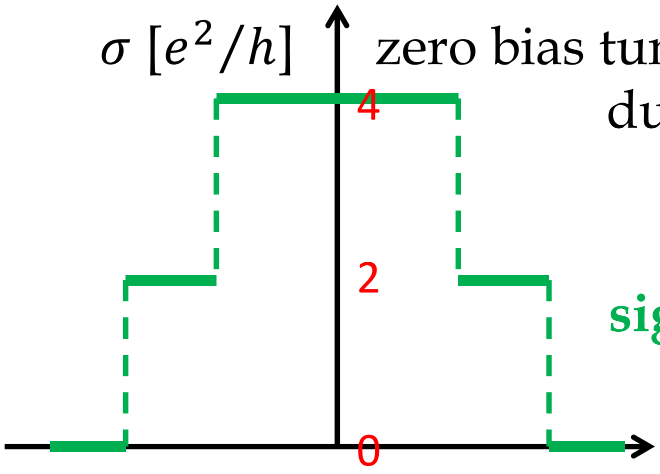
$$c_\sigma = ic_{\bar{\sigma}}^\dagger$$

FZ-Kane-Mele, PRL 111, 056402 (2013)
FZ-Kane-Mele, PRL 111, 056403 (2013)

Evolution of a Majorana Kramers Pair in Zeeman Fields







zero bias tunneling conductance

due to **resonant** Andreev reflections

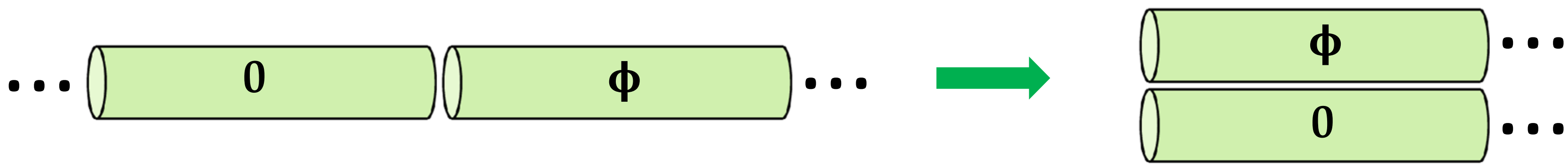
signatures in tunneling spectroscopy !

FZ-Kane-Mele, PRL 111, 056402 (2013)

Non-Abelian Statistics? 4π Josephson Effects?

- Tunneling Cooper pairs or electrons? [Yes, 4π]
- One minus sign for each Majorana? [No, 2π]

Periodicity: $2\pi \frac{2e}{Q} \sim \frac{h}{Q}$



- the Josephson effects can thus be interpreted as the boundary consequences of the bulk invariant of $H(k,\phi)$;
- topological classification of $H(k,\phi)$?

Anomalous Pumps $H(k, \phi)$

Symm.	*PHS*	*TRS*	*PHS × TRS*
k	$-$	$-$	$+$
r	$+$	$+$	$+$
ϕ	$+$	$-$	$-$
θ	$-$	$+$	$-$

- The cases with TRS cannot be understood by the original table
- We formulated a new class of problems
  ["periodic building" for topological phases: 10×8**×8**
  with the aforementioned table being its ground floor]

FZ-Kane (2013)

Anomalous Pumps $H(k, r, \phi, \theta)$

- 0, Z, and Z_2 classes on the ground floor
- New classes Z × Z and $Z_2 \times Z_2$ upstairs

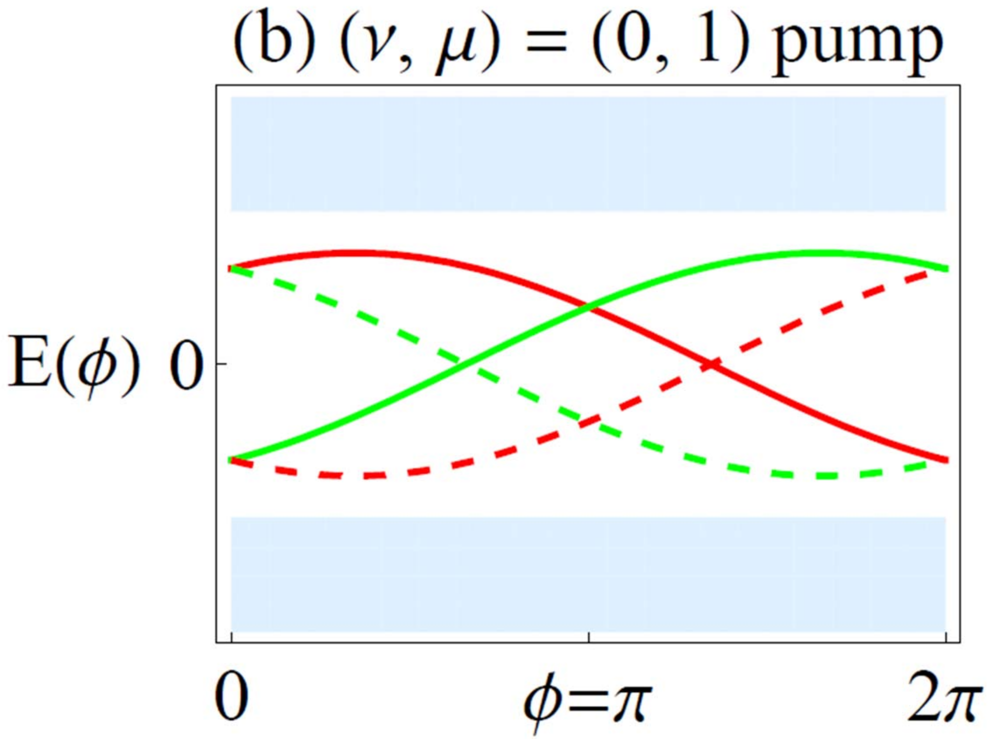
Class DIII:

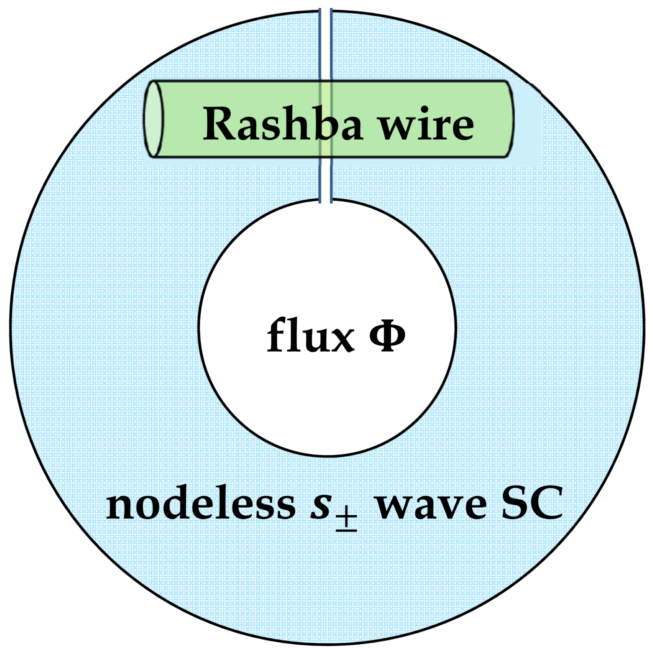
$(d_k - d_r) \bmod 8$		$0, 4, 5, 6$	1	2	3	7
$d_\phi - d_\theta$	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$2\mathbb{Z}$
	1	0	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}$	$2\mathbb{Z} \times 2\mathbb{Z}$

Fractional Josephson effects related to Majorana Kramers pairs

Homotopy Argument: **FZ**-Kane arXiv:1310.5281.

The 1st Z_2 Invariant (answering the question)

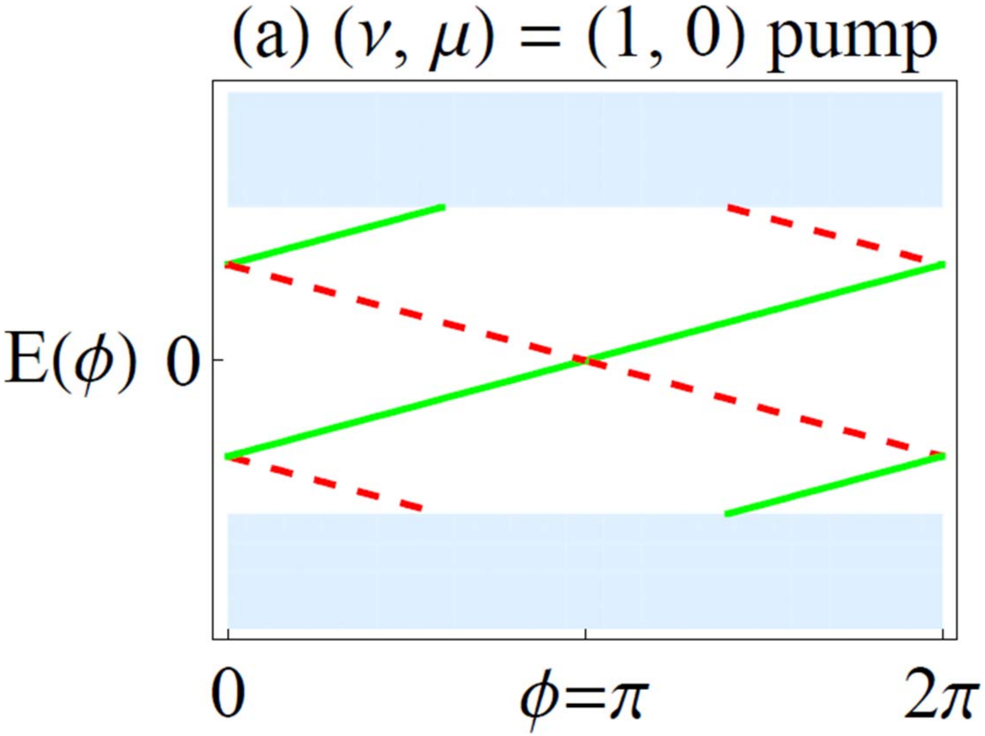


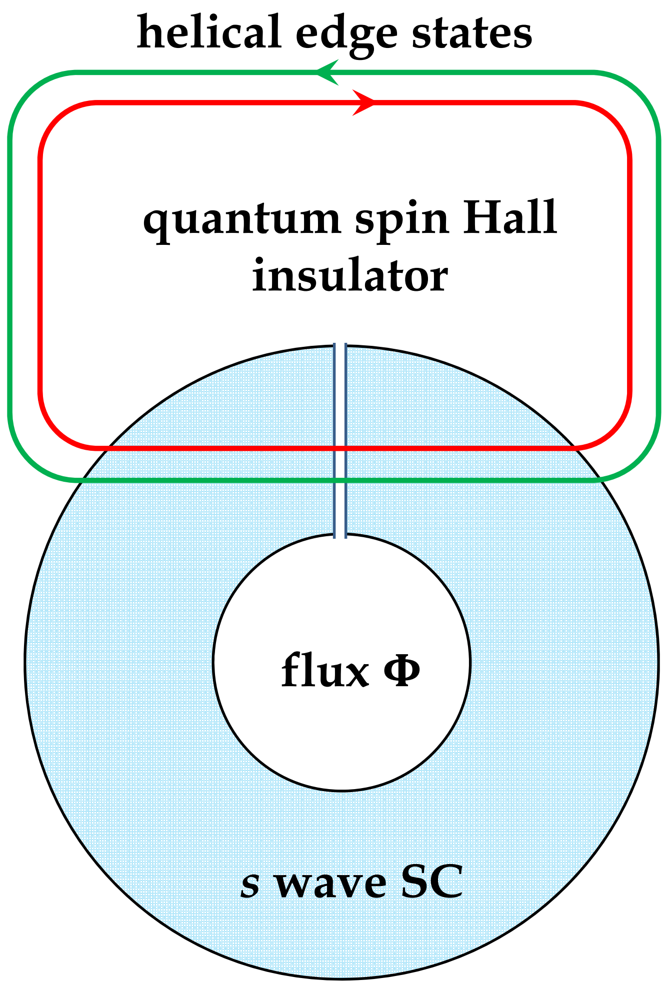


- 4π periodic Josephson effect (non-Abelian statistics of Majorana Kramers pairs)
- The adiabatic pumping of FP and "spin" between two superconductors

FZ-Kane-Mele, PRL 111, 056402 (2013); **FZ**-Kane arXiv:1310.5281.

The 2nd Z_2 Invariant



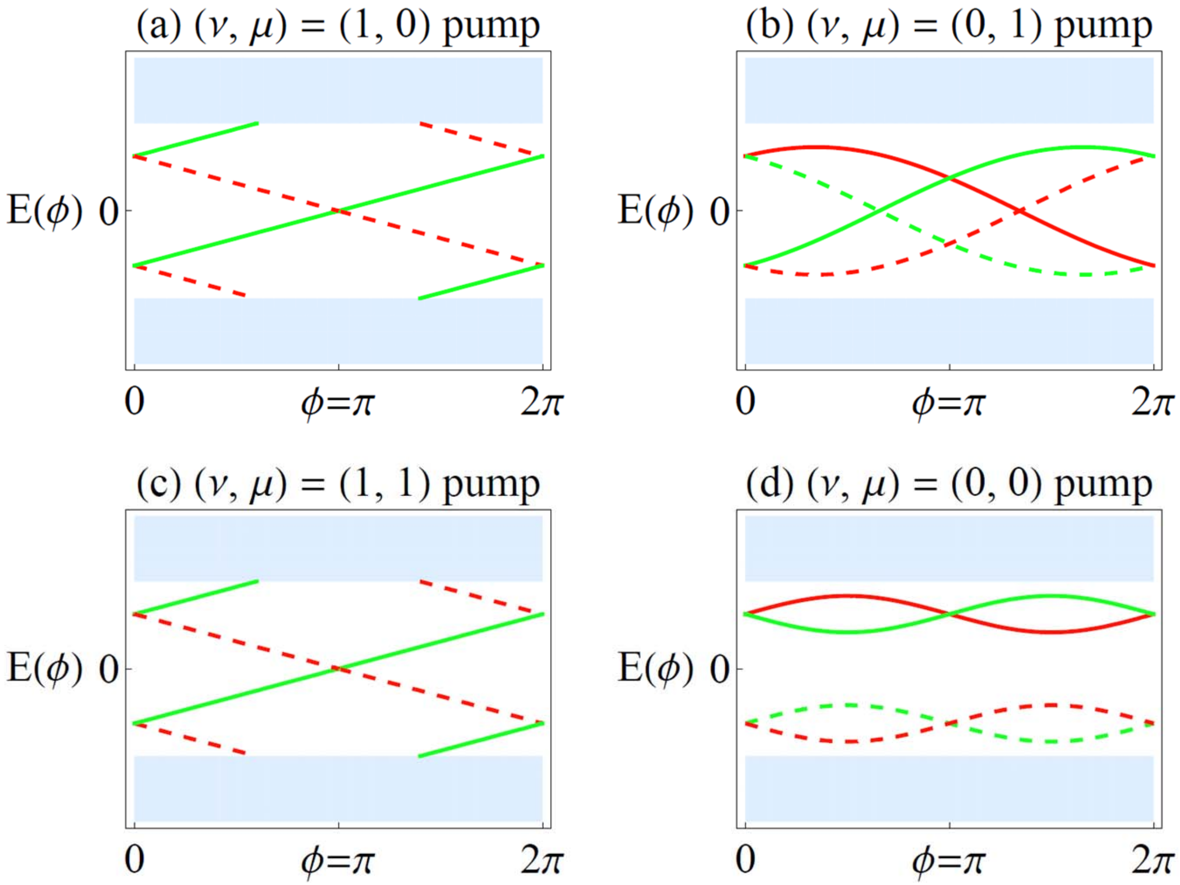


- **Robust to TRS breaking**
- **Fractional Josephson effect**
  (non-Abelian statistics of MF's)
- Ongoing experiments:
  L. Molenkamp, A. Yacoby, R. Du, …

  (HgTe/CdTe)   (InAs/GaSb)

$Z_2 \times Z_2$ Topological Class

- three distinct Z_2 topological pumps, one trivial pump
- two copies of each topological pump = gapped out= trivial
- combine any two distinct Z_2 pumps = the third Z_2 pump



Physics Today

Insulators	Superconductors
• SSH model (1978 polyacetylene)	• Kitaev chain (2000)
• Integer quantum Hall states (orbital 1980', Zeeman 2013)	• p-wave SCs (2000-; 2009-)
• TRI Topological insulators (Kane-Mele 2005…)	• TRI topological SCs (**FZ**-Kane-Mele 2013…)
• Topological crystalline insulators (Fu et al. 2012)	• Topological Mirror SCs (**FZ**-Kane-Mele 2013)
• 2D Weyl (Dirac) semimetal (few-layer graphene)	• 2D Weyl (Dirac) SCs (cuprates)
• 3D Weyl semimetal (Murakami …)	• 3D Weyl SCs (Yang-**FZ** 2014)
• 3D Dirac semimetal: Na_3Bi, Cd_3As_2 (Young et al. Fang et al. 2012)	• 3D Dirac SCs (Yang-**FZ** 2014)

Physics Tomorrow

- Realizations in labs, unique phenomena, and applications (very promising)

- $Z_2 \times Z_2$ and $Z \times Z$ topological classes and the "periodic build" (FZ-Kane 2013-)

- Strong Interactions and fractionalization (ex: FQH states. Parafermions? Bosons?)

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