Whither Quantum Computing?

Barry C. Sanders

Thanks to sponsor:
Optical Society of America

4 April 2014

Computing

Programmable Machine to Perform Logical Operations

Solves computational problems (e.g., Decision or Sampling) by executing an algorithm (input, procedure, output) with available resources (e.g., memory, space, time).

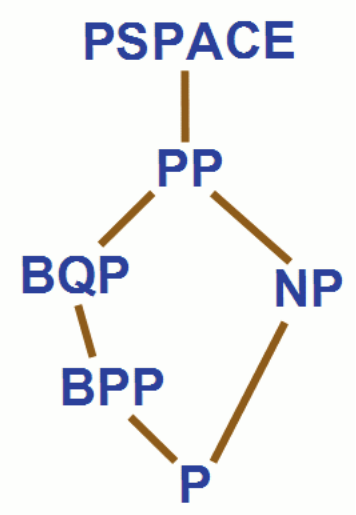
Church-Turing Thesis

Calculable function (efficiently?) computed on a Turing machine.

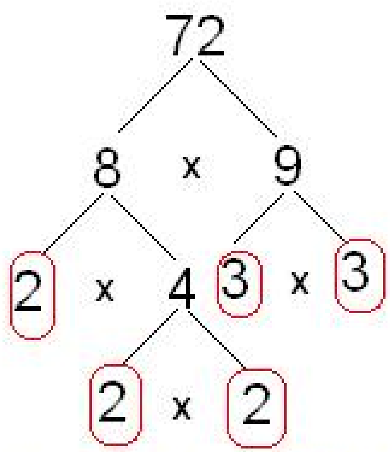
Problem Size and Efficiency

Efficiency is polynomial scaling of resources with problem size (# bits to specify input)

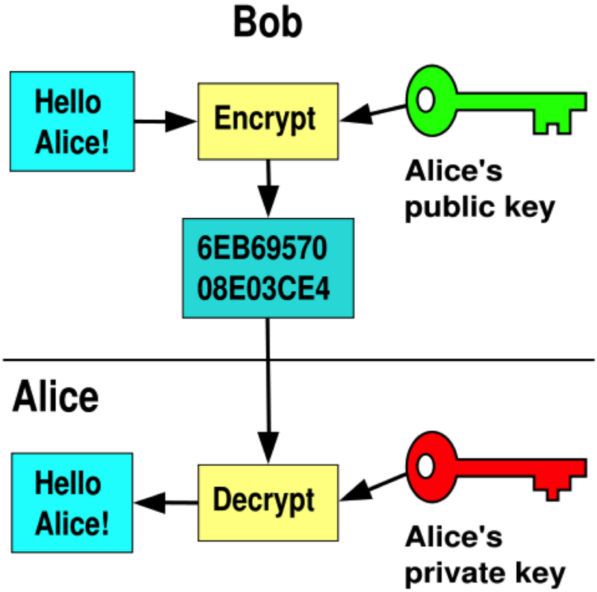
Decisions and Efficiency



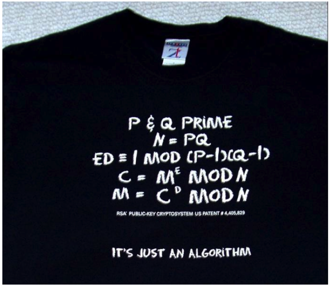
Prime Factorization: exponential speedup

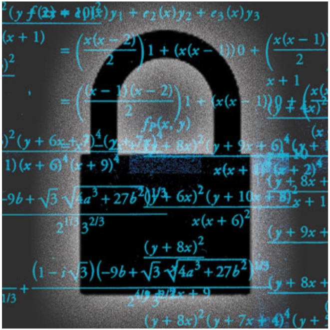


The prime factorization 72 is: $2 \times 2 \times 2 \times 3 \times 3 = 72$

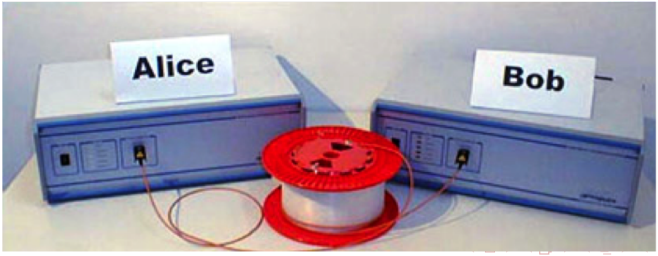

Bob
Hello Alice!
Encrypt
Alice's public key
6EB69570 08E03CE4
Alice
Hello Alice!
Decrypt
Alice's private key

Generating the Key





Security From ℚ Key Distribution

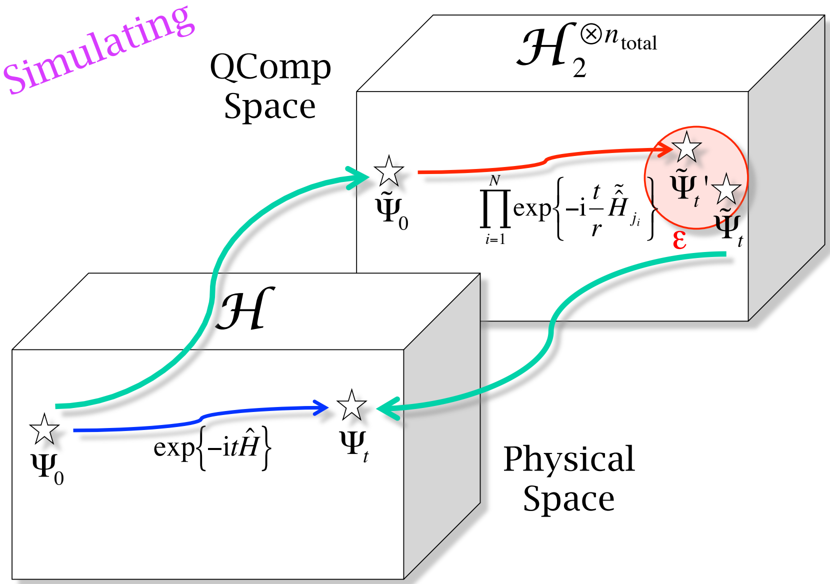



THE
MATRIX

What is the Matrix?

Feynman, Int. J. Th. Phys. 1982 §5

Can a $\mathbb{Q}$ system be probabilistically simulated by a $\mathbb{Q}$ (probabilistic, I'd assume) universal computer? In other words, a computer which will give the same probabilities as the $\mathbb{Q}$ system does. If you take the computer to be the $\mathbb{C}$ kind I've described so far (not the $\mathbb{Q}$ kind described in the last section) and there're no changes in any laws, and there's no hocus-pocus, the answer is certainly, No! This is called the hidden-variable problem: it is impossible to represent the results of $\mathbb{Q}$ mechanics with a $\mathbb{C}$ universal device.


Simulating
QComp Space
$\mathcal{H}_2^{\otimes n_{\text{total}}}$
$\tilde{\Psi}_0$
$\prod_{i=1}^{N} \exp\left\{-\mathrm{i}\frac{t}{r}\tilde{\hat{H}}_{j_i}\right\}$
$\tilde{\Psi}_t{}'$
$\tilde{\Psi}_t$
ε
$\mathcal{H}$
Ψ_0
$\exp\left\{-\mathrm{i}t\hat{H}\right\}$
Ψ_t
Physical Space

ℚ linear equation solver [Harrow Hassidim, Lloyd 2009]

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

Building Blocks for a $\mathbb{Q}$ Computer

$\mathbb{Q}$ bits and $\mathbb{Q}$ gates

- $\mathbb{Q}$bits: Superpositions of $\mathbb{Q}$ logic states $|0\rangle$ and $|1\rangle$.
- Represent states as vectors: $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.
- $\mathbb{Q}$ gates map states to states so, for one $\mathbb{Q}$bit, a gate is a 2×2 unitary matrix.
- Preparation: initial state is 'zero' $|00\ldots0\rangle$.
- Measurement in computational basis, e.g., $|0\rangle\langle 0| \otimes |1\rangle\langle 1| \otimes |1\rangle\langle 1|$.

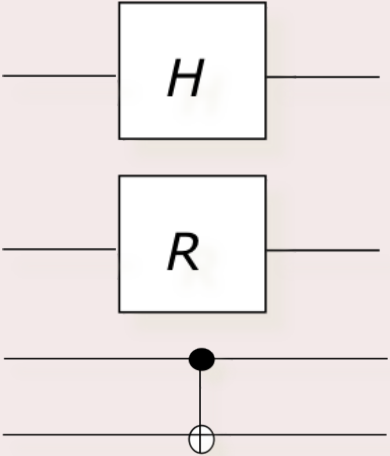
Universal ℚ Gate Set

2 1-ℚbit and 1 entangling 2ℚ gate

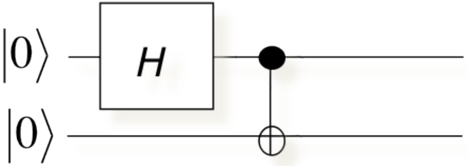
- $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$,
- $R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & \exp(2\pi \mathrm{i} \cos^{-1}(3/5)) \end{pmatrix}$,
- $\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$.

Circuit Representation of Universal ℚ Gate Set

2 1-ℚbit and 1 entangling 2ℚ gate

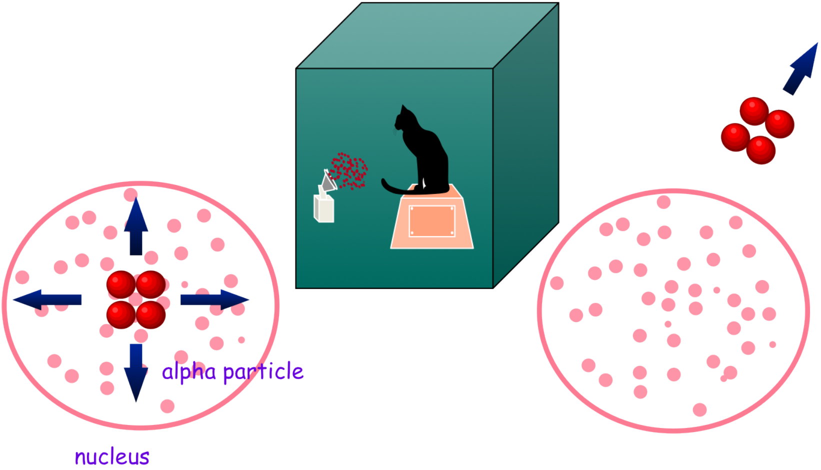


Entangling Gate

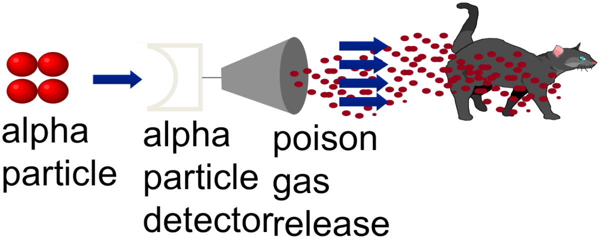


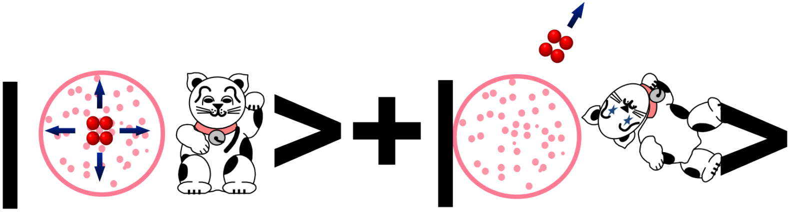
$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Schrödinger's cat schematic



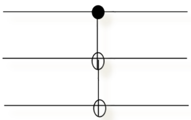
Schrödinger's cat entanglement concept





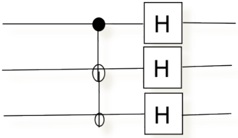
Quantum Error Correction

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |0\rangle, \quad |0\rangle$$



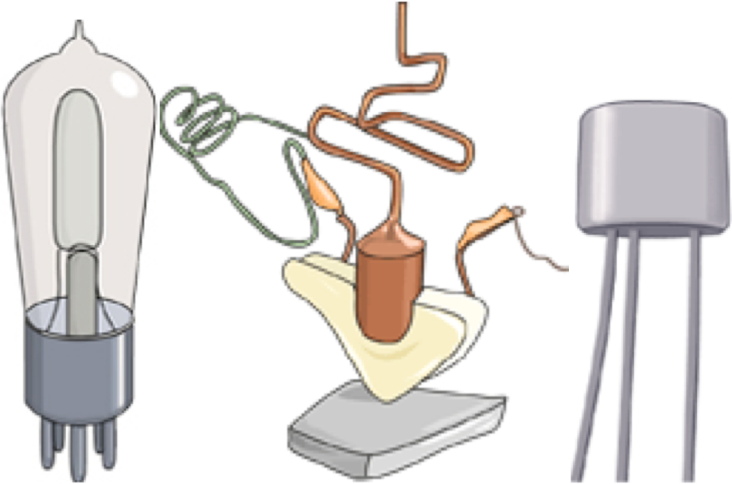
$$\left.\right\} \alpha|000\rangle + \beta|111\rangle = \alpha|0\rangle_L + \beta|1\rangle_L = |\psi\rangle_L$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |0\rangle, \quad |0\rangle$$

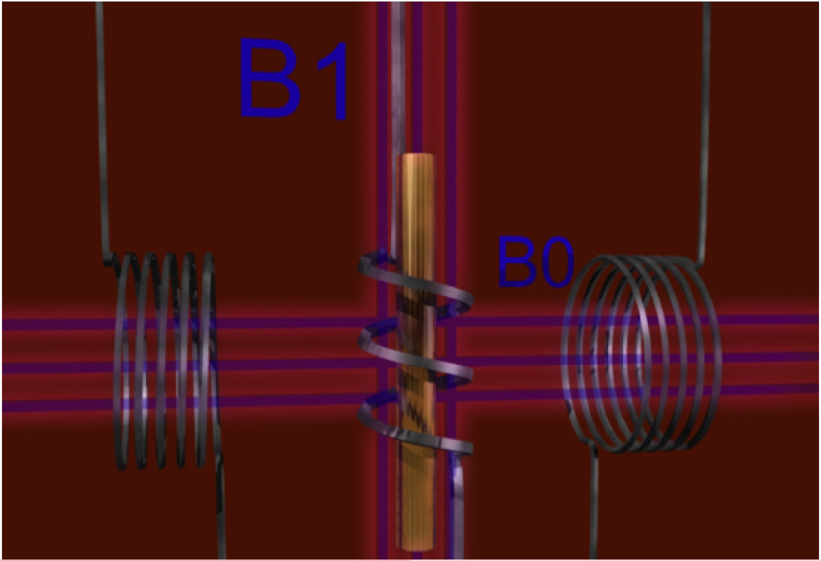


$$\alpha|\bar{0}\bar{0}\bar{0}\rangle + \beta|\bar{1}\bar{1}\bar{1}\rangle = \alpha|\bar{0}\rangle_L + \beta|\bar{1}\rangle_L = |\bar{\psi}\rangle_L$$

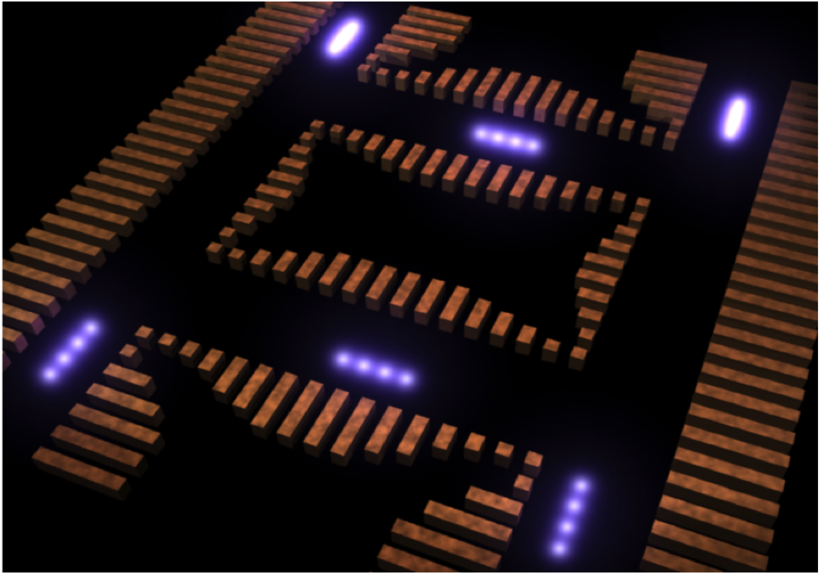
Classical Switches



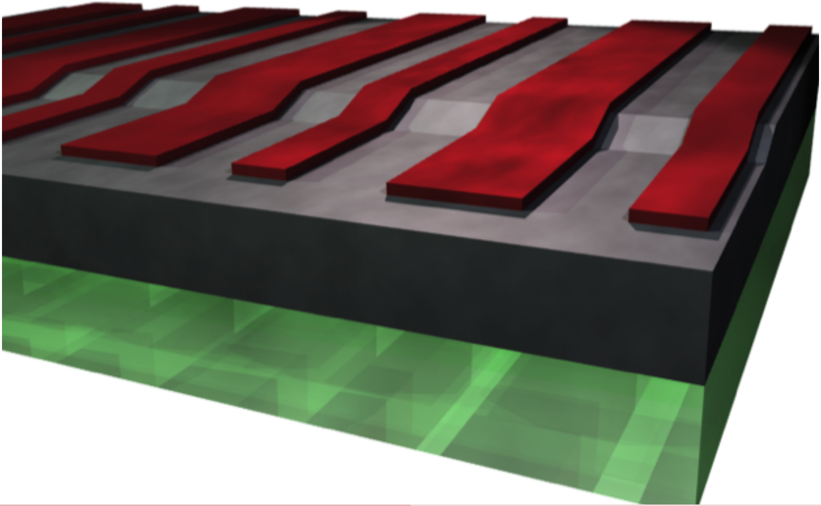
Quantum Computer Technologies: Nuclear Magnetic Resonance



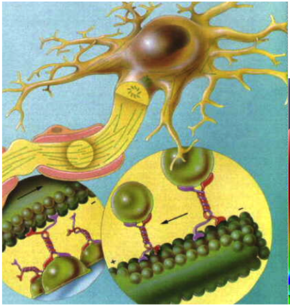
Quantum Computer Technologies: Trapped Ions

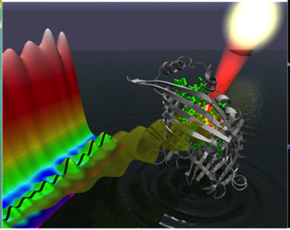


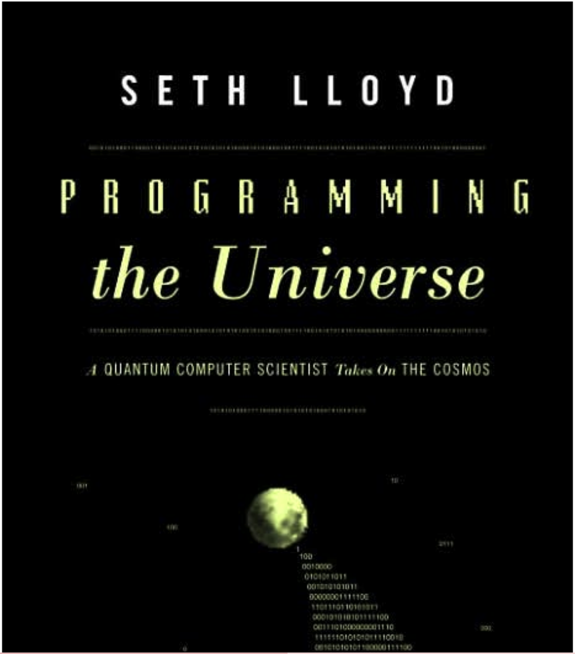
Quantum Computer Technologies: Trapped Ions



Quantum Biology






SETH LLOYD
PROGRAMMING
the Universe
A QUANTUM COMPUTER SCIENTIST Takes On THE COSMOS

Feynman

I [hypothesize] that ultimately physics will not require a mathematical statement, that in the end the machinery will be revealed, and the laws will turn out to be simple, like the checker board with all its apparent complexities.