INSTITUTO DE FÍSICA
Universidade Federal Fluminense

Exploiting the Entanglement in Classical Optics Systems

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Summary

- Optical Vortices
- Spin-Orbit Entanglement

  Optical devices for Quatum Information

  Topological Phase in Spin-Orbit transformations
- Conclusions

Optical Vortices

Light with **Orbital Angular Momentum** (OAM)

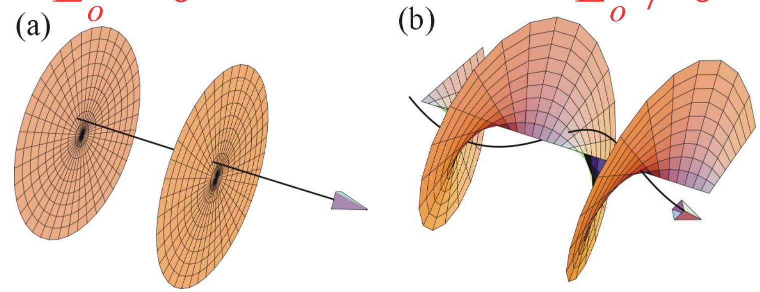
From the Classical Electromagnetism

$$\vec{L} = \varepsilon_0 \int_V (\vec{r} \times \vec{p})\, dV = \vec{L}_s + \vec{L}_o$$

where $\vec{l} = \vec{r} \times \vec{p} = \varepsilon_0 \vec{r} \times \left(\vec{E}(\vec{r},t) \times \vec{B}(\vec{r},t) \right)$ *is the angular momentum density.*

$L_s \rightarrow$ Spin component – polarization
$L_o \rightarrow$ Orbital component - wavefront

$L_o = 0$ $\qquad$ $L_o \neq 0$



Poynting vector parallel to optical axis

Poynting vector rotates around optical axis.

Light with **Orbital Angular Momentum** (OAM)

Paraxial optics – lasers

$$\nabla^2 \vec{E} - \left(\frac{1}{c}\right)^2 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$ *(Wave Equation)*

considering $\vec{E}(\vec{r},t) = \hat{\varepsilon} u(\vec{r}) e^{-i\omega t}$ *with* $u(\vec{r}) = \psi(x,y,z) e^{ikz}$,

$$\nabla^2 \psi(x,y,z) + 2ik\hat{z} \cdot \nabla \psi(x,y,z) = 0$$ *(Helmholtz Equation)*

Considering too $$\left|\frac{\partial^2 \psi}{\partial z^2}\right| \ll \left|\frac{\partial^2 \psi}{\partial x^2}\right|, \left|\frac{\partial^2 \psi}{\partial y^2}\right| \text{ e } 2k\left|\frac{\partial \psi}{\partial z}\right|$$

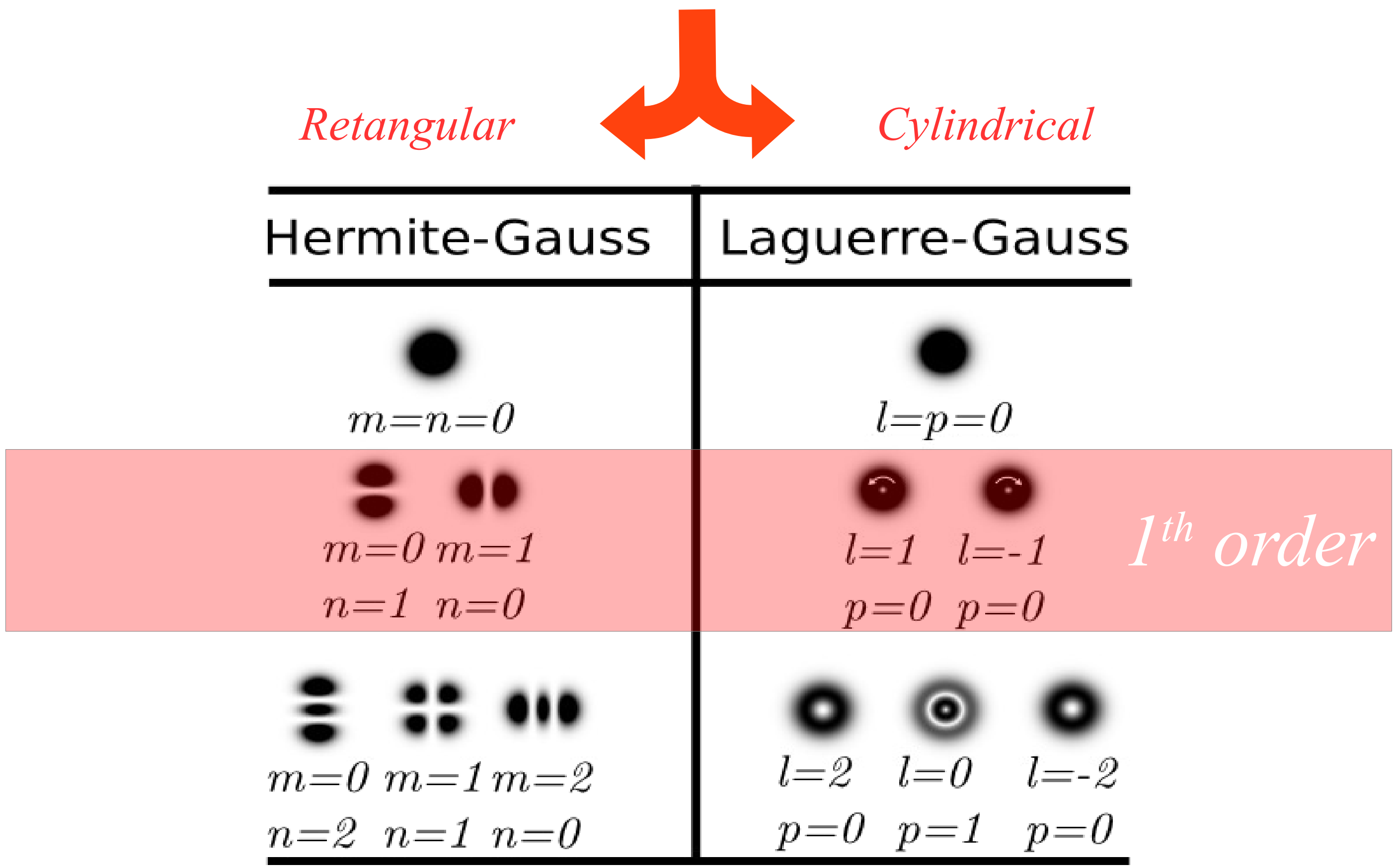
$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + 2ik\frac{\partial \psi}{\partial z} = 0$$ ***(Paraxial Equation)***

Optical Vortices

Light with **Orbital Angular Momentum** (OAM)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + 2ik\frac{\partial \psi}{\partial z} = 0$$

(Paraxial Equation)



Optical Vortices

Light with **Orbital Angular Momentum** (OAM)

Paraxial optics – lasers

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+2ik\frac{\partial\psi}{\partial z}=0$$

(Paraxial Equation)

Solutions of the paraxial equation in ***retangular coordenates****: HG modes*

$$\psi_{n,m}(\vec{r}) = \frac{A_{mn}}{\omega(z)} H_n\left(\sqrt{2}\frac{x}{\omega(z)}\right) H_m\left(\sqrt{2}\frac{y}{\omega(z)}\right) \times$$

GOUY phase

$$\times\, exp\left\{-\frac{x^2+y^2}{\omega(z)^2}\right\} exp\left\{-i(k\frac{x^2+y^2}{2R(z)} - \boxed{\frac{n+m+1}{2}\arctan\frac{z}{z_R}})\right\}$$

N=m+n

Solutions of the paraxial equation in ***cylindrical coordenates****: LG modes*

$$\psi_p^l(\vec{r}) = \frac{A_p^l}{\omega(z)}\left[\frac{\sqrt{2}r}{\omega(z)}\right]^{|l|} L_p^l\left(\frac{2r^2}{\omega^2(z)}\right) exp\left\{-\frac{r^2}{\omega^2(z)}\right\} \times$$

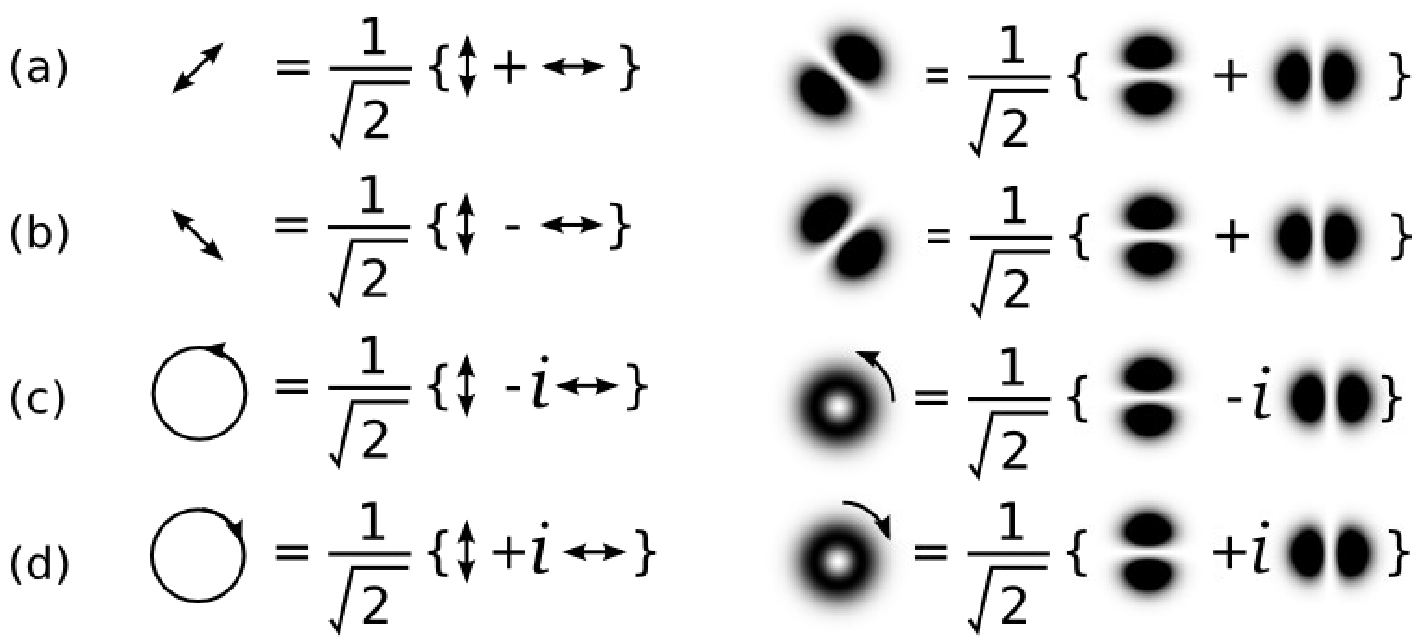
Topological charge

$$\times\, exp\left\{-i(\frac{kr^2}{2R(z)} + (2p+|l|+1)\arctan\frac{z}{z_R} + l\phi)\right\}.$$

*N=2p+|l| - l=n-m - p=*min(*m,n*)

Analogy on the decompositions: **Degrees of Freedom**

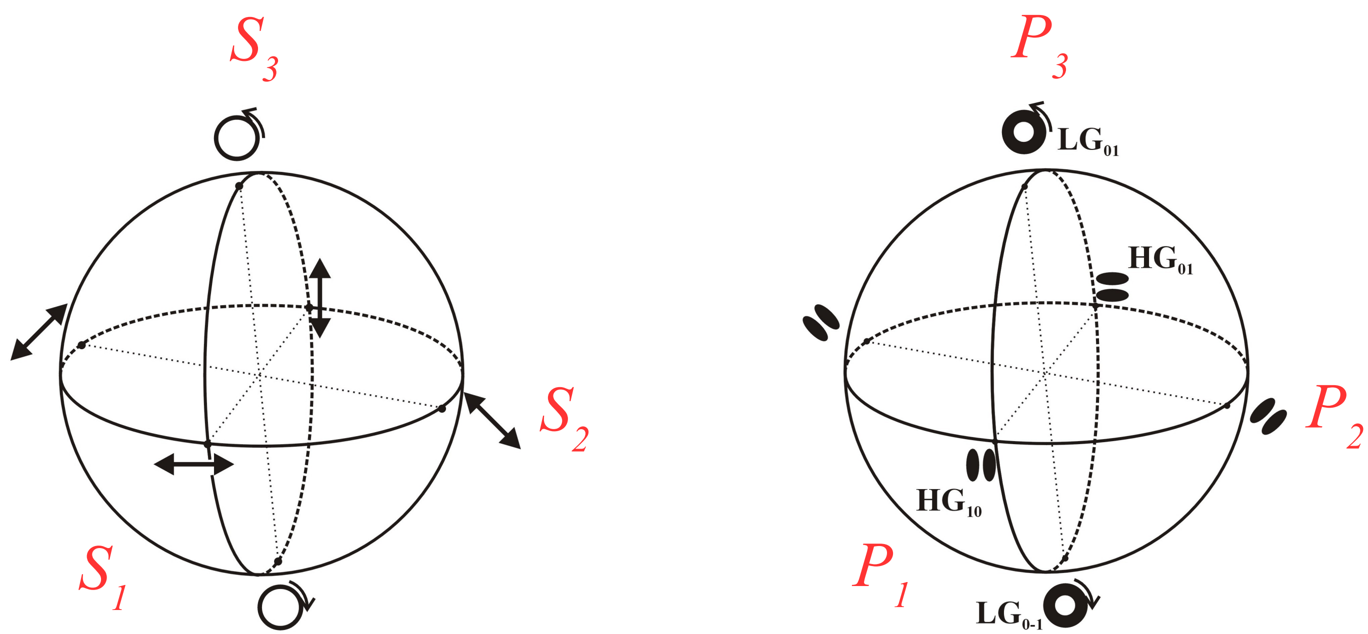
Polarization — *1th order modes*



Optical Vortices

Analogy on the decompositions: Degrees of Freedom

Poincaré Sphere for polarization modes | *Poincaré Sphere for first order modes*

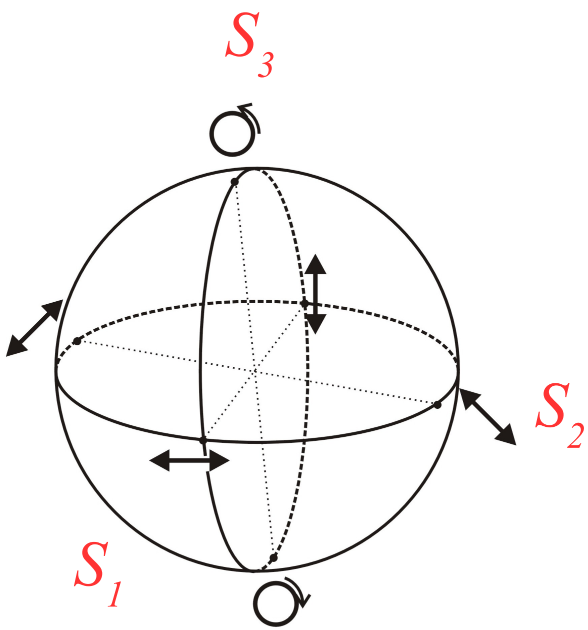


S_i, P_i; i=1,2,3 are Stokes Parameters.

Padgett e Courtial, Opt. Lett. **24** 7 430 (1999)

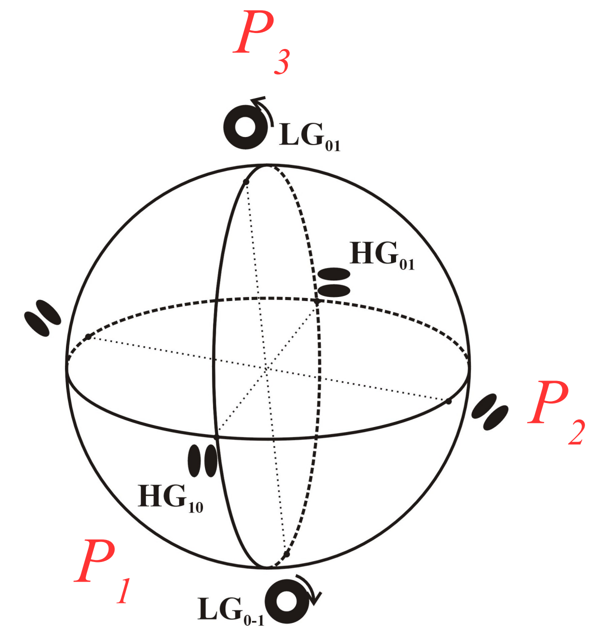
Optical Vortices

Photonic Qubits: **Bloch Sphere** analogy



Polarization qubit

Encoding qubits into the electromagnetic field



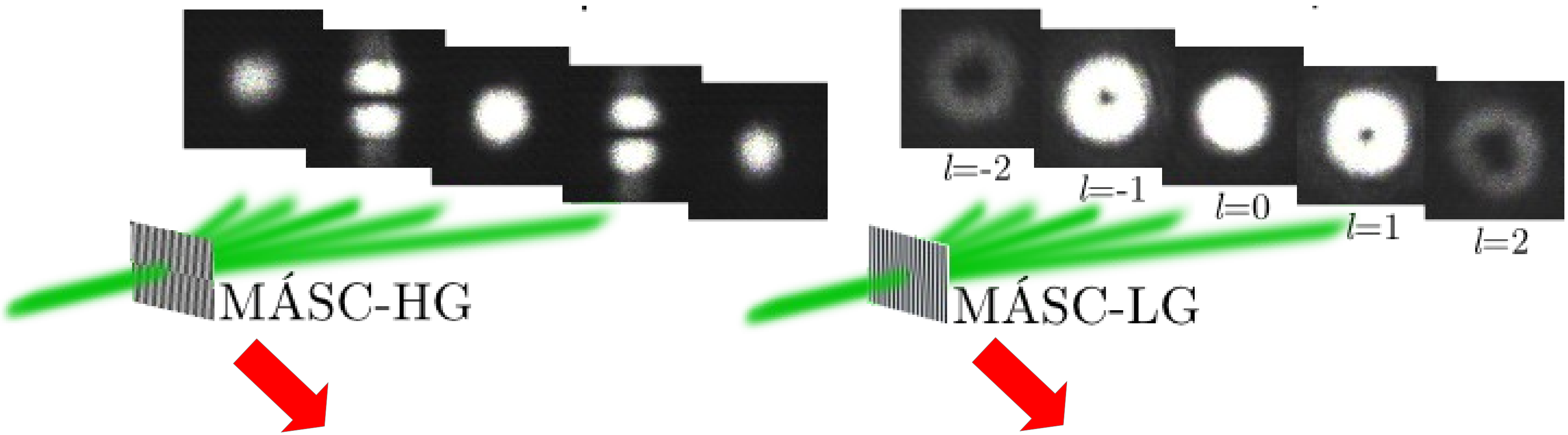
Transverse Spatial degree qubit

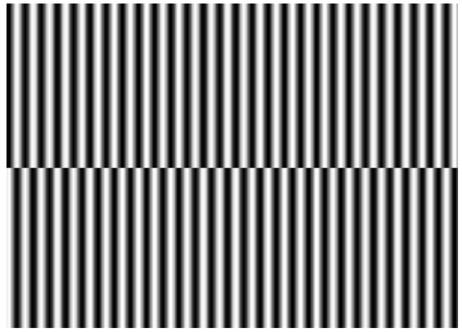
$$|\theta, \phi\rangle = \cos(\theta)\,|H\rangle + e^{\imath\phi}\sin(\theta)\,|V\rangle$$

Optical Vortices

Light with **Orbital Angular Momentum** (OAM)

Experimental optical vortices: LG and HG modes Phase Masks



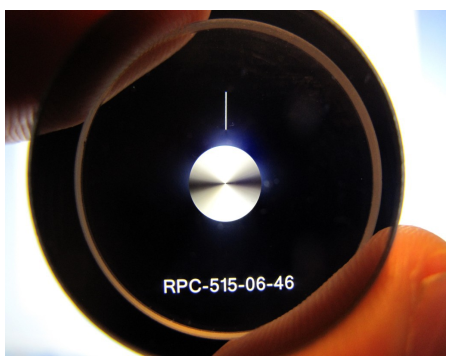


Optical Vortices

Light with **Orbital Angular Momentum** (OAM)

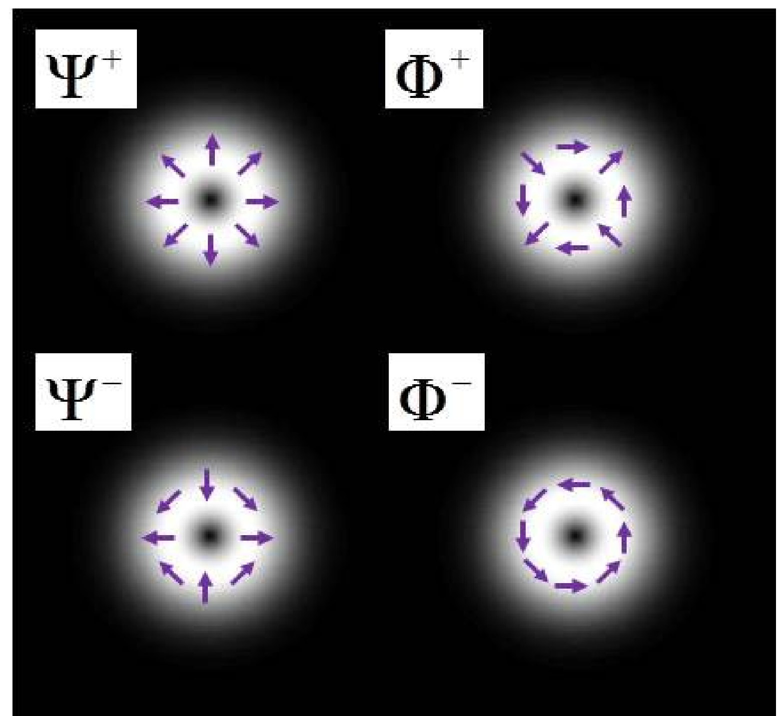
Experimental optical vortices: LG, HG modes and radial polarization

S-WAVEPLATE (Radial Polarization Converter)



Sold by Altechna

Nanostrutured silica glass device



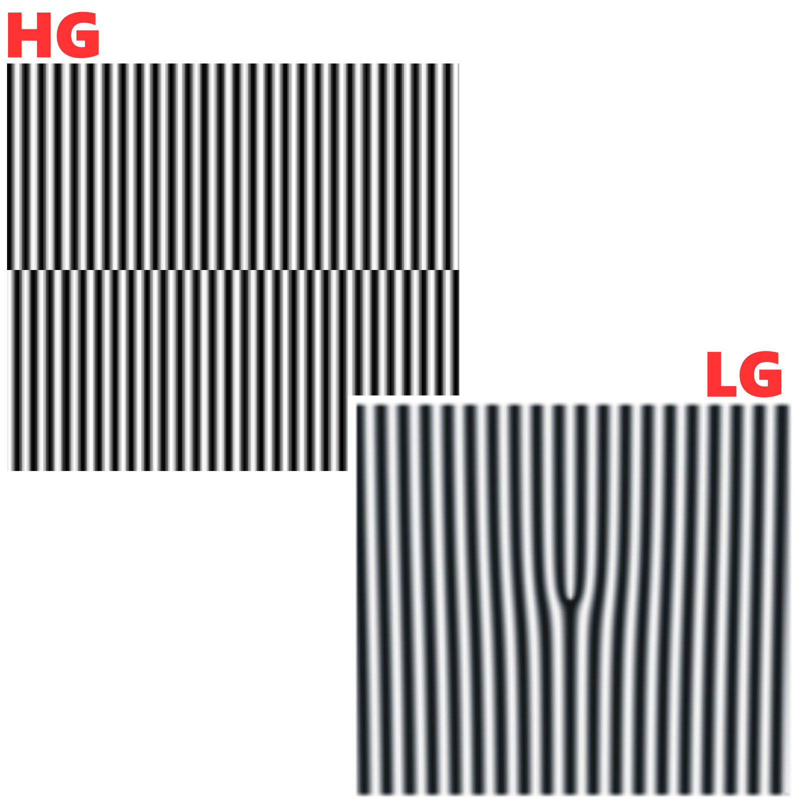
Martynas Beresna et al, APPLIED PHYSICS LETTERS **98**, 201101 (2011)

Optical Vortices

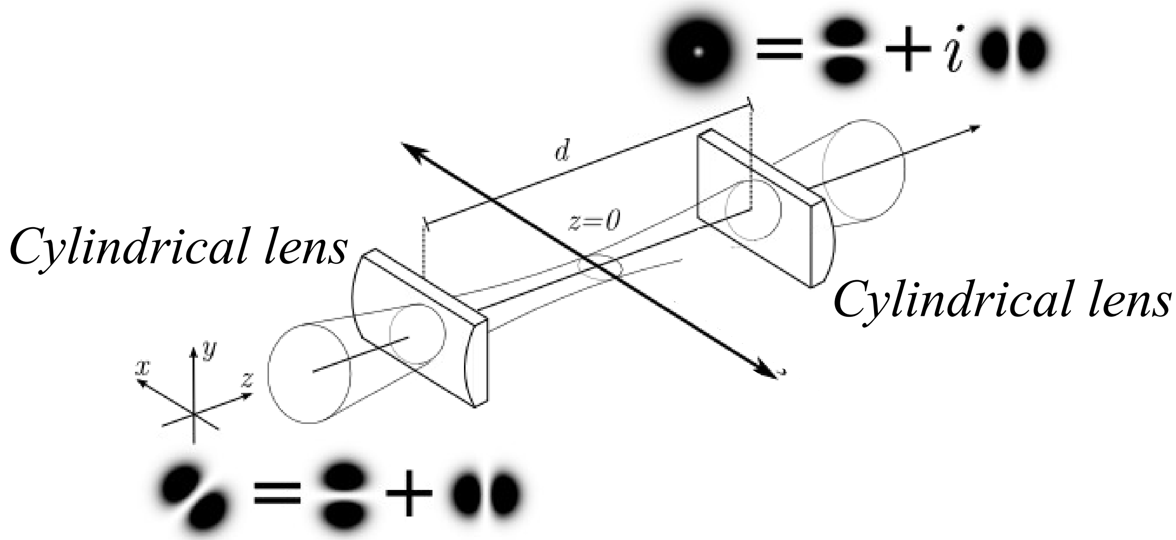
Light with **Orbital Angular Momentum** (OAM)

Experimental optical vortices: LG and HG modes

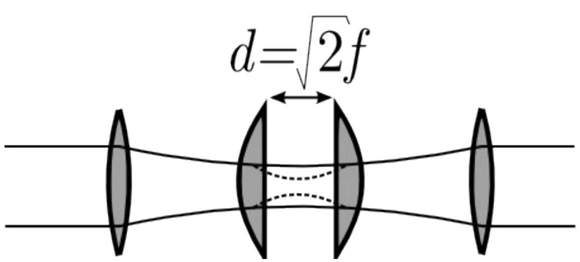
SLM (Spatial Light Modulators)



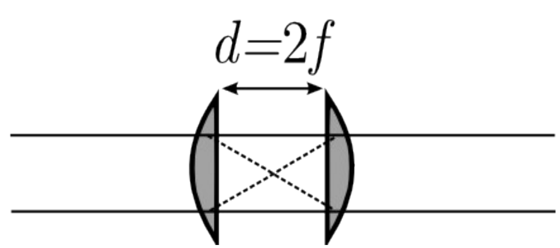
Mode Transformation: Astigmatic Mode Converters (AMC)



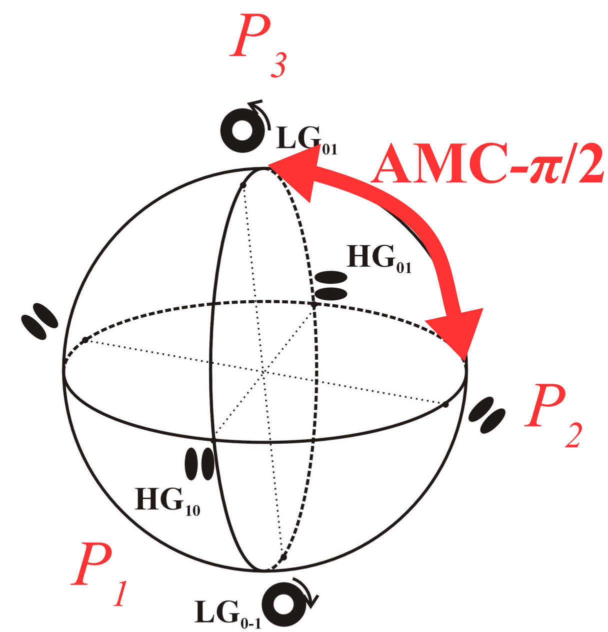
Astigmatic
Mode converter - π/2



Astigmatic
Mode converter - π



Poincaré Sphere for first order modes



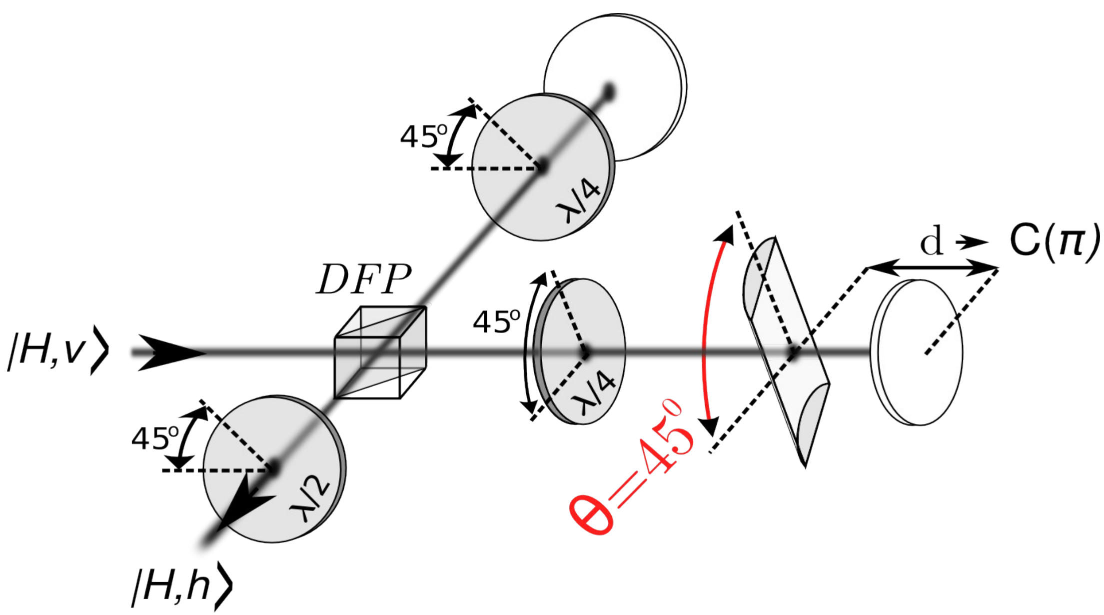
L. Allen et al, Phys. Rev. A **50**. 115 (1992) e M. W. Beijersbergen et al, Opt Commun. **93** 123 (1993).

Spin-Orbit Entanglement

Optical Setups for Quantum Information

C-NOT universal gate → *Spin-Orbit mode*

This gate applies the inversion operation on one qubit (target) depending on the state of a second qubit (control).



Truth table

POL MAO		
$\lvert V,h\rangle$	→ CNOT →	$\lvert V,h\rangle$
$\lvert V,v\rangle$	→ CNOT →	$\lvert V,v\rangle$
$\lvert H,v\rangle$	→ CNOT →	$\lvert H,h\rangle$
$\lvert H,h\rangle$	→ CNOT →	$\lvert H,v\rangle$

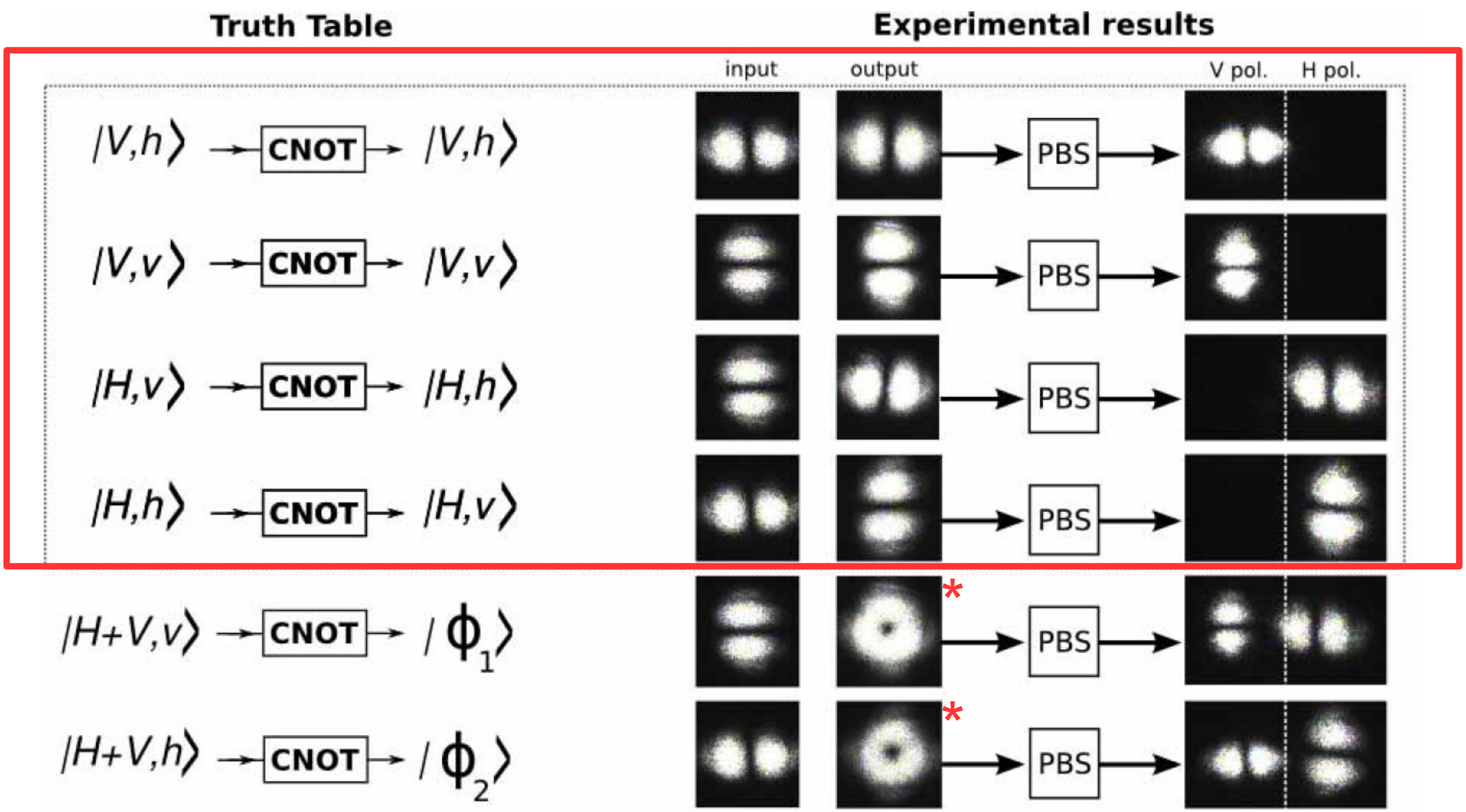
$|\mathrm{H}\rangle \rightarrow \leftrightarrow$; $|\mathrm{V}\rangle \rightarrow \updownarrow$; $|h\rangle \rightarrow$  ; $|v\rangle \rightarrow$  ;

C. E. R. Souza e A. Z. Khoury, Opt. Express. 18 9207-9212 (2010).

Optical Setups for Quantum Information

C-NOT universal gate → *Spin-Orbit mode*

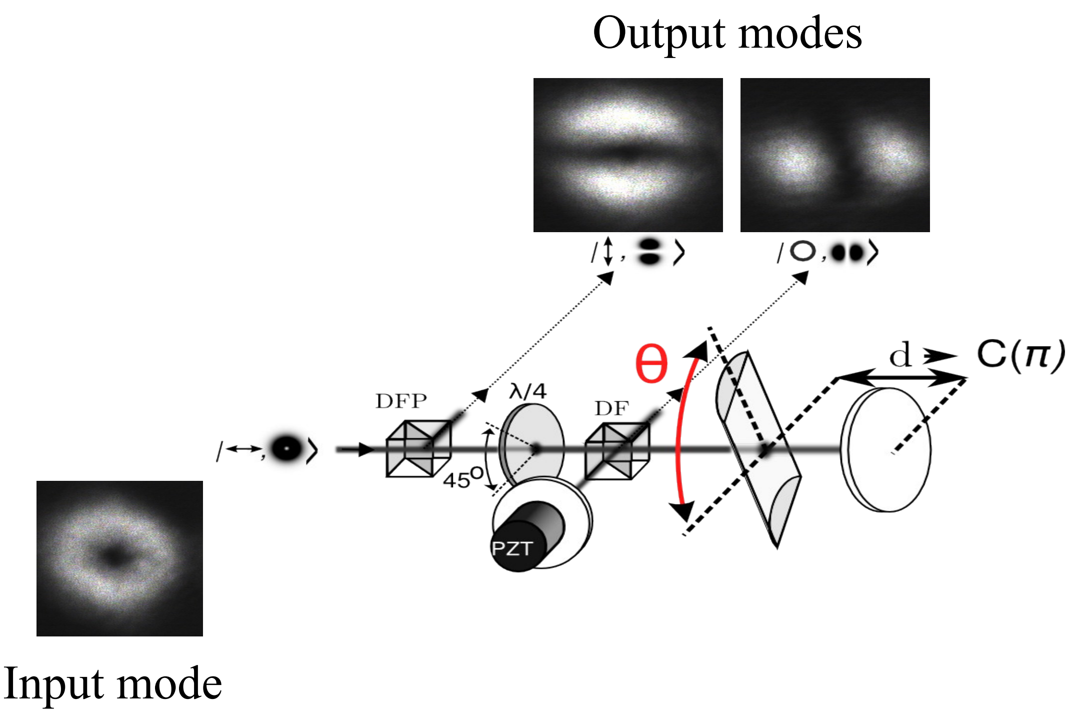
Experimental results: classical beams

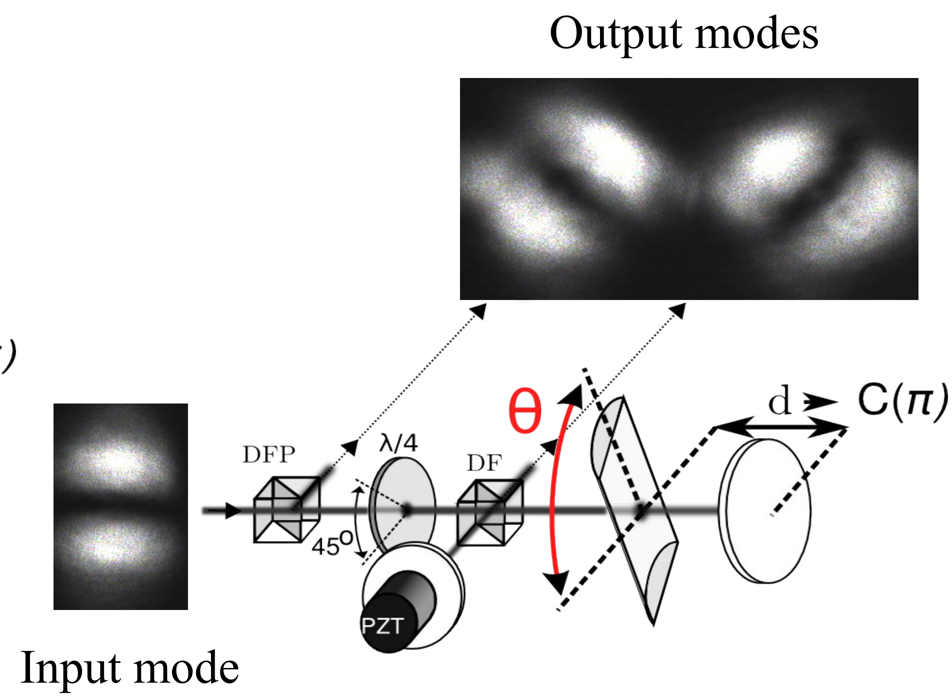


* nonseparable mode → Bell's State when the mode is ocupped by one photon

C. E. R. Souza e A. Z. Khoury, Opt. Express. 18 9207-9212 (2010).

Transverse Mode Selector: Splits 1st order transverse modes in HG components

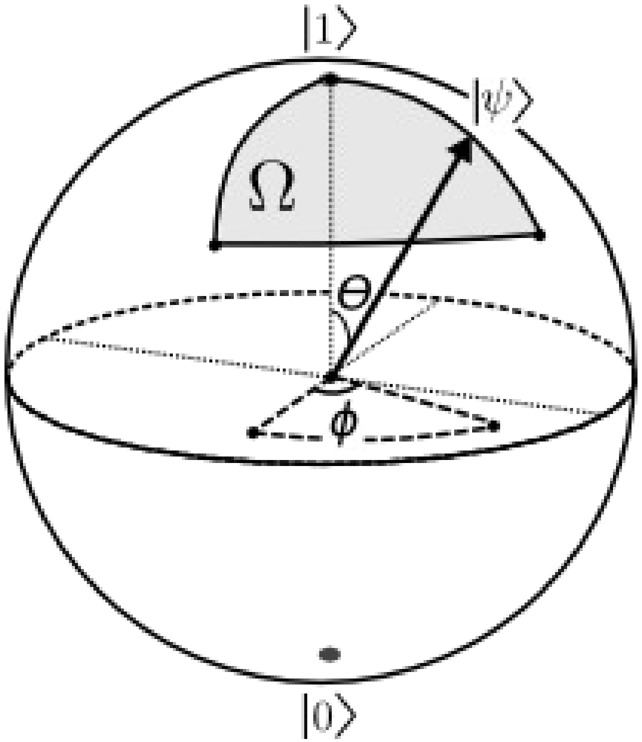




Classical Entanglement:
Topological Phase in Spin-Orbit modes

Topological Phase in Spin-Orbit modes

Geometrical Phase in Spin-Orbit modes: one qubit



In a cyclical evolution where,

$$|\psi(T)\rangle = e^{i\phi}|\psi(0)\rangle,$$

the total phase is given by:

$$\phi = \phi_{din} + \phi_{geo}.$$

$$\boxed{\phi_{geo} = \Omega/2}$$

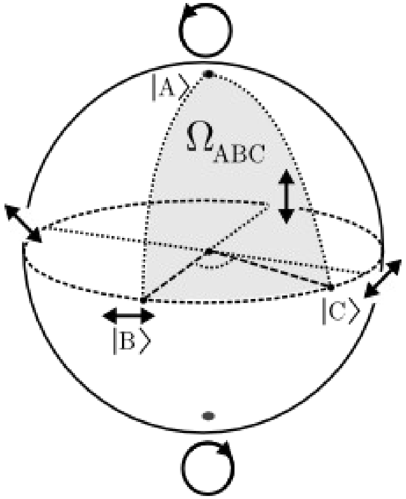
1- M. Berry, Proc. R. Soc. Lond. A Sci. 392, 45 (1984)

Topological Phase in Spin-Orbit modes

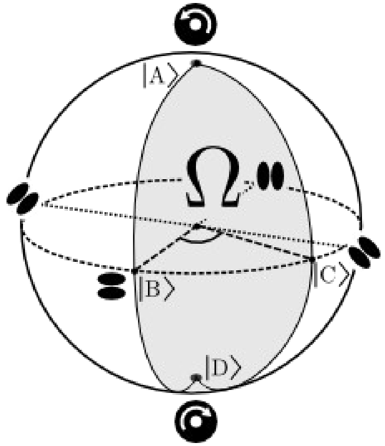
The geometrical phase is observed in classical systems too.

$$\langle A|A'\rangle = \exp\{-\imath\Omega_{ABC}/2\}$$

Pancharatnam Phase in polarization modes[1]



Geometrical phase in first order modes[2]



1- S. Pancharatnam, Proc. Indian Acad. Sci. A 44, 247 (1956). 2- Galvez et al, Phys. Rev. Lett. 90. 203901 (2003).

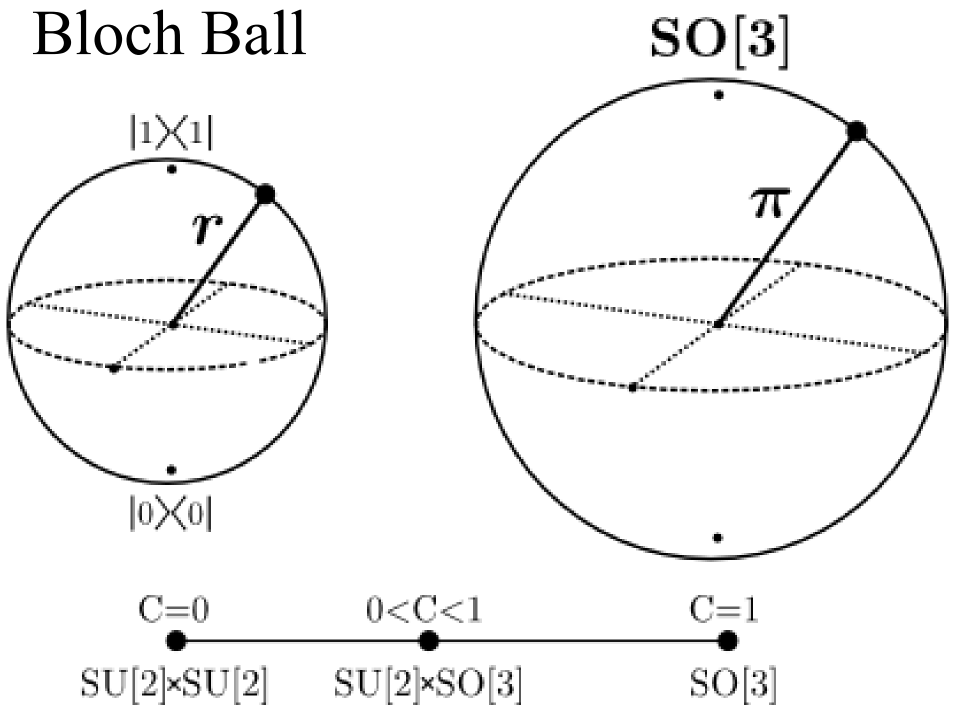
Topological Phase in Spin-Orbit modes

Photonic Qubits:

Geometrical phase in Spin-Orbit modes: two qubit pure state

$$|\psi_0\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

$$r = \sqrt{1 - C_r^2 - C_i^2}$$



Concurrence $\rightarrow$ $C \equiv |C_r + iC_i| = 2|\alpha\delta - \beta\gamma|$

Topological Phase in Spin-Orbit modes

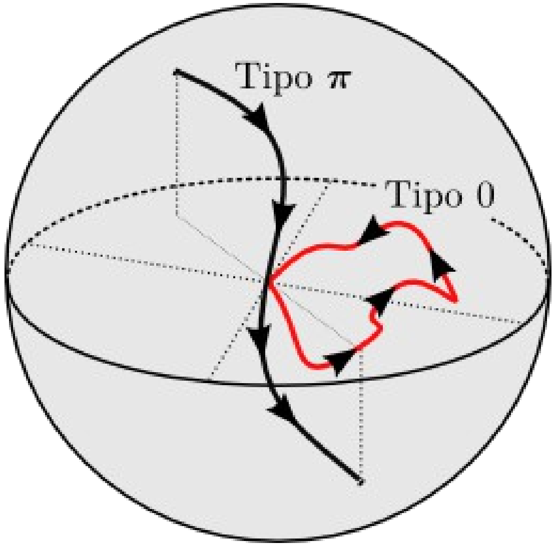
Photonic Qubits:

Geometrical phase in Spin-Orbit modes:

two qubit pure maximally entangled state

$$|\psi_0\rangle = \alpha|00\rangle + \beta|01\rangle - \beta^*|10\rangle + \alpha^*|11\rangle$$

$$|\alpha|^2 + |\beta|^2 = \frac{1}{2}$$



$$C = 2|\alpha\delta - \beta\gamma| = 1.$$

C=0 $\rightarrow$ separable state / C=1 $\rightarrow$ Maximally Entangled state

Topological Phase in Spin-Orbit modes

Photonic Qubits:

Geometrical phase in Spin-Orbit modes:

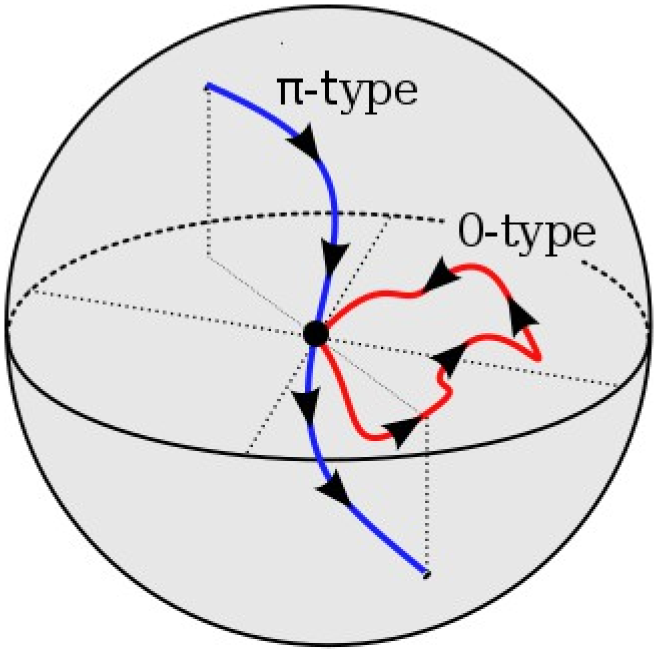
two qubit pure maximally entangled state

$$|\psi\rangle = \alpha\,|00\rangle + \beta\,|01\rangle - \beta^*\,|10\rangle + \alpha^*\,|11\rangle$$

Two Homotopy class

0-type | *π-type*

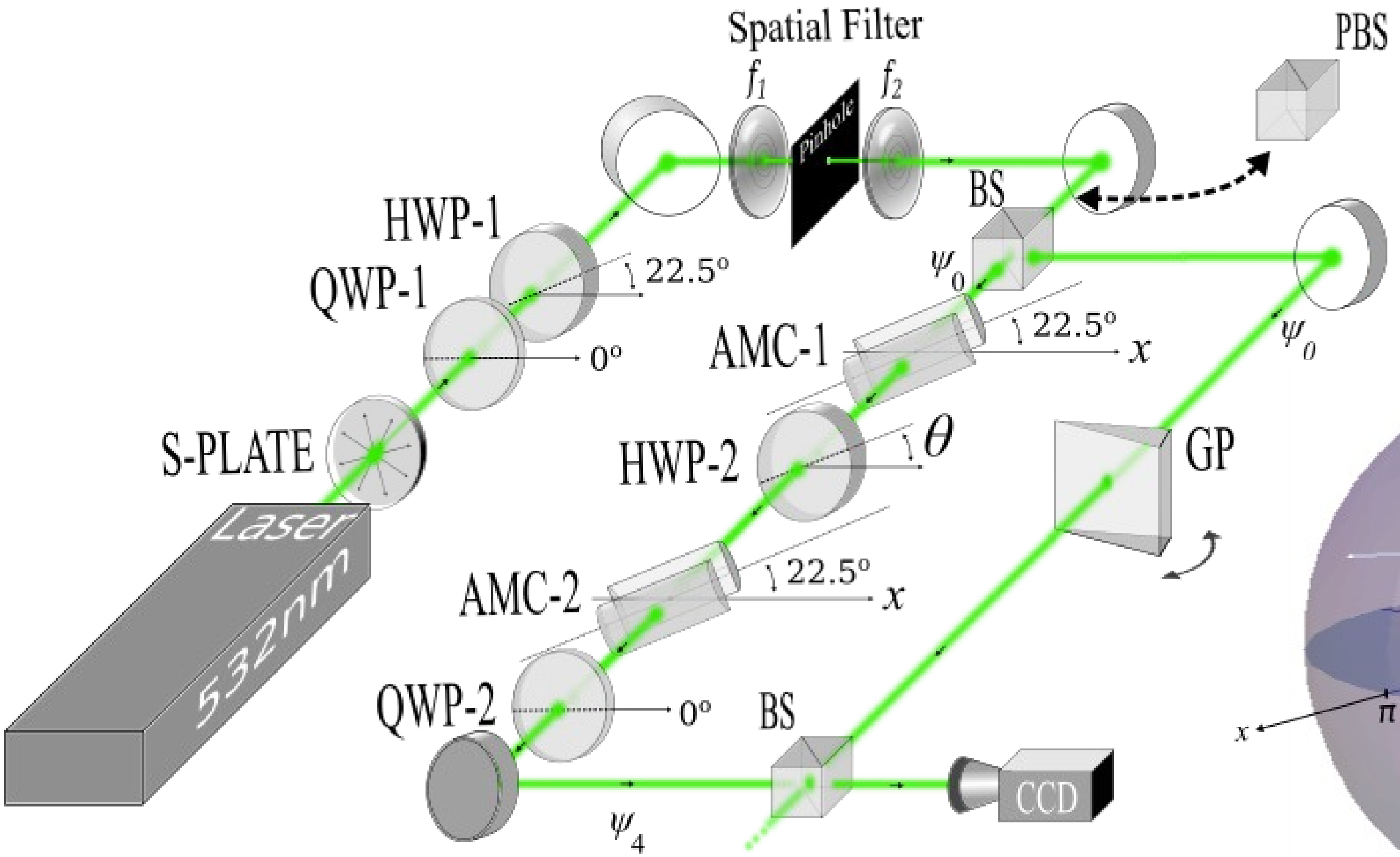
$$|\psi(t)\rangle = |\psi(0)\rangle \qquad |\psi(t)\rangle = -\,|\psi(0)\rangle$$

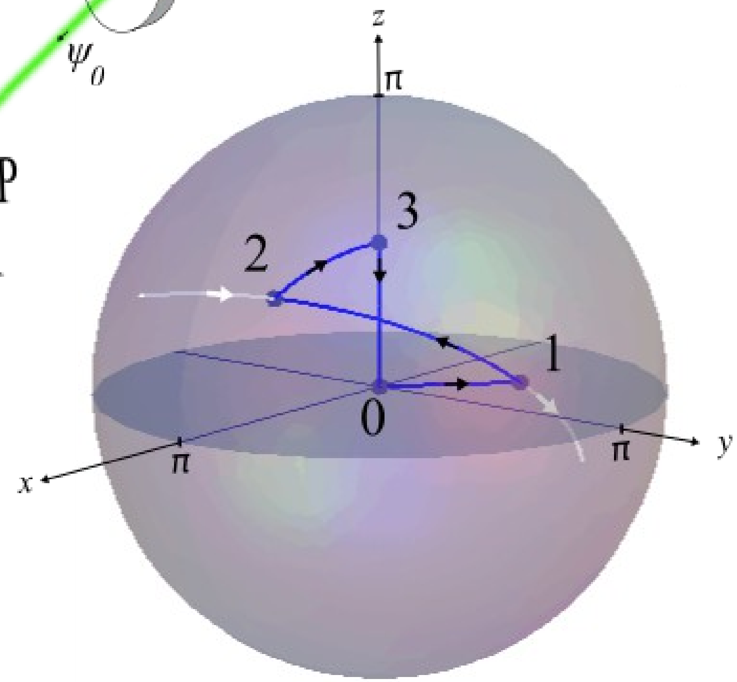


C=1 → Maximally Entangled state

Topological Phase in Spin-Orbit modes

Geometrical phase in Spin-Orbit modes: two qubit pure state maximally entangled





Topological Phase in Spin-Orbit modes

Combining the spin and orbital degree of freedom in the framework of the classical theory

$$\vec{E}(\vec{r}) = \alpha\psi_+(\vec{r})\vec{e}_H + \beta\psi_+(\vec{r})\vec{e}_V + \gamma\psi_-(\vec{r})\vec{e}_H + \delta\psi_-(\vec{r})\vec{e}_V$$

$$\Updownarrow$$

$$|\psi\rangle = \alpha\,|00\rangle + \beta\,|01\rangle + \gamma\,|00\rangle + \delta\,|11\rangle$$

Topological Phase in Spin-Orbit modes

Combining the spin and orbital degree of freedom in the framework of the classical theory

Maximally Entangled State

$$\vec{E}(\vec{r}) = \alpha\psi_+(\vec{r})\vec{e}_H + \beta\psi_+(\vec{r})\vec{e}_V - \beta^*\psi_-(\vec{r})\vec{e}_H + \alpha^*\psi_-(\vec{r})\vec{e}_V$$

$$\Updownarrow$$

$$|\psi\rangle = \alpha\,|00\rangle + \beta\,|01\rangle - \beta^*\,|10\rangle + \alpha^*\,|11\rangle$$

Separable State

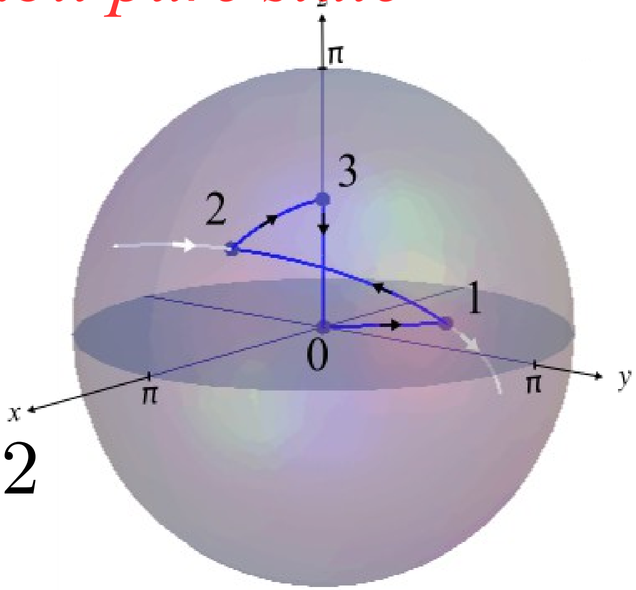
$$\vec{E}(\vec{r}) = \{\alpha_+\psi_+(\vec{r}) + \alpha_-\psi_-(\vec{r})\}\{\beta_H\vec{e}_H + \beta_V\vec{e}_V\}$$

$$\Updownarrow$$

$$|\psi\rangle = \{\alpha_-\,|0\rangle + \alpha_-\,|1\rangle\}\{\beta_+\,|0\rangle + \beta_-\,|1\rangle\}$$

Topological Phase in Spin-Orbit modes

Geometrical phase in Spin-Orbit modes: two qubit pure state maximally entangled



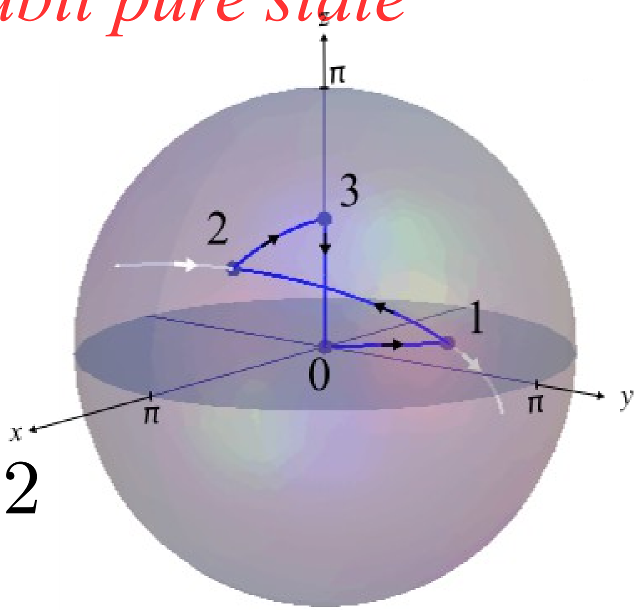
$$\vec{E}_0 = \frac{1}{\sqrt{2}}\{\psi_+ \vec{e}_H + \psi_- \vec{e}_V\} \quad (\alpha = 1; \beta = 0)$$

Parametrization

$$\begin{cases} \alpha = \cos a/2 - \imath k_z \sin a/2 \\ \beta = -(k_x + k_y) \sin a/2. \\ \text{with } \vec{k} = (-k_x, -k_y, -k_z) \text{ and } a \in [0, \pi] \end{cases}$$

Topological Phase in Spin-Orbit modes

Geometrical phase in Spin-Orbit modes: *two qubit pure state maximally entangled*



$$\vec{E}_0 = \frac{1}{\sqrt{2}}\{\psi_+ \vec{e}_H + \psi_- \vec{e}_V\} \quad (\alpha = 1; \beta = 0)$$

Parametrization

$$\begin{cases} \alpha = \cos a/2 - \imath k_z \sin a/2 \\ \beta = -(k_x + k_y) \sin a/2. \\ \textit{with } \vec{k} = (-k_x, -k_y, -k_z) \ \textit{and} \ \ a \in [0, \pi] \end{cases}$$

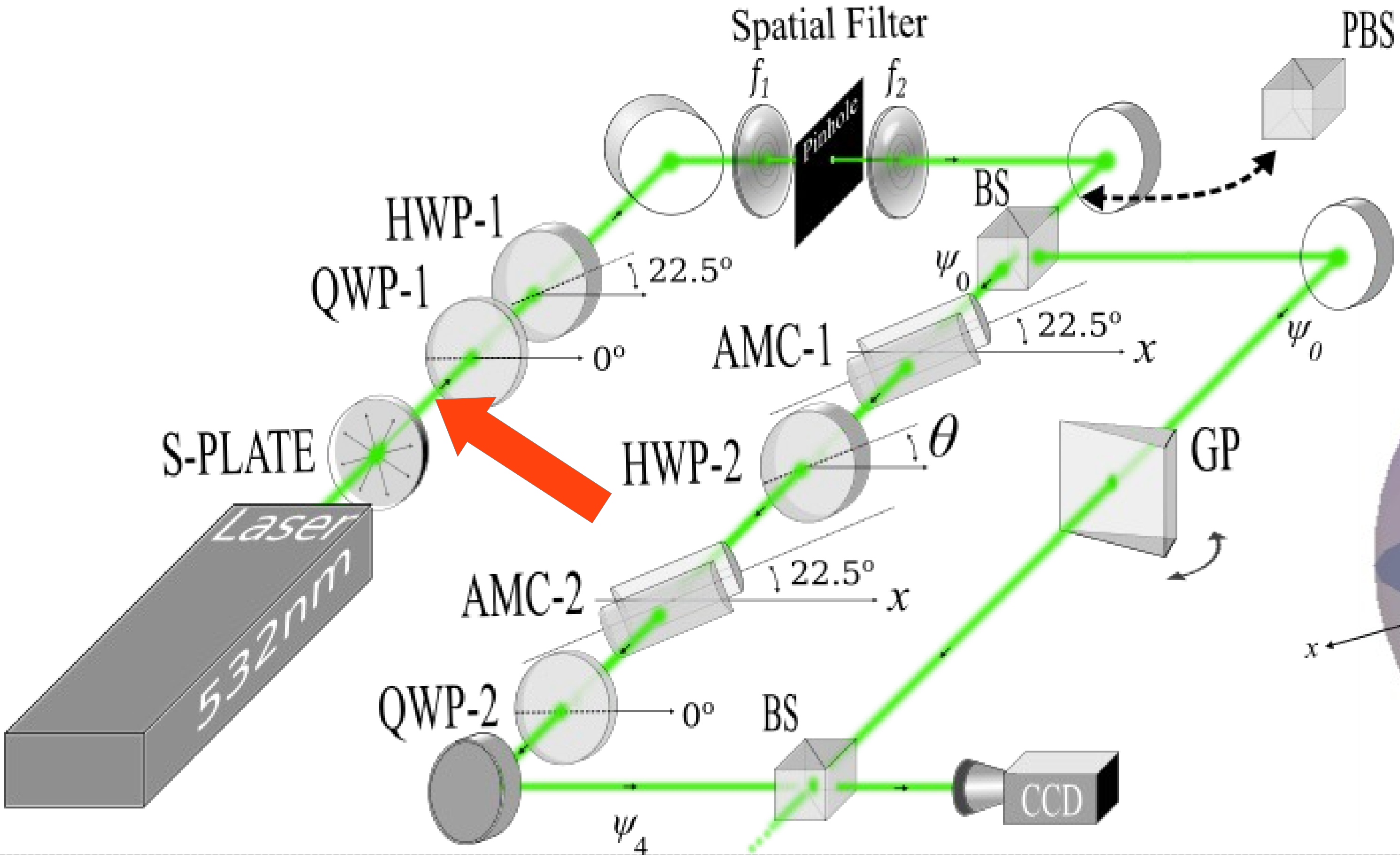
Waveplates $W(\phi,\theta) = \begin{bmatrix} \cos(\phi/2) + \imath\cos(2\theta)\sin(\phi/2) & \imath\sin(2\theta)\sin(\phi/2) \\ \imath\sin(2\theta)\sin(\phi/2) & \cos(\phi/2) - \imath\cos(2\theta)\sin(\phi/2) \end{bmatrix}$

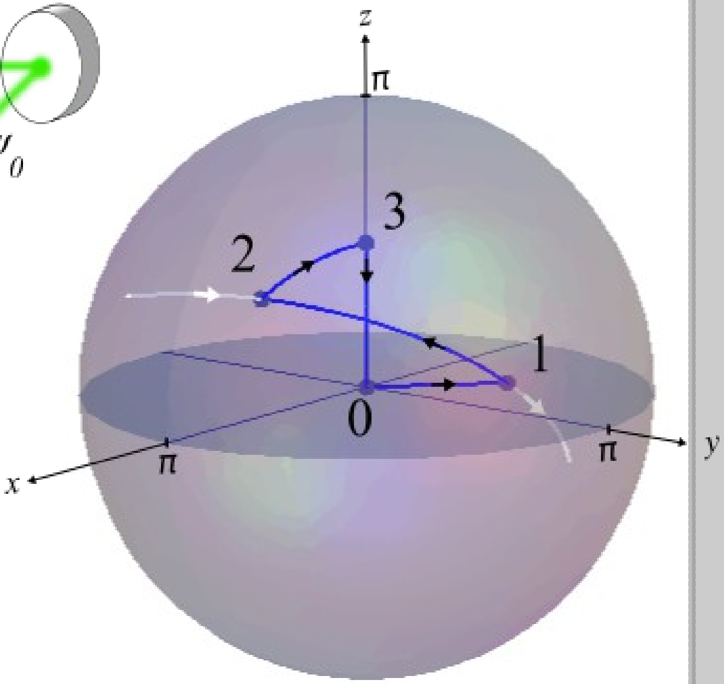
Astigmatic Mode Converters $C(\phi,\theta) = \begin{bmatrix} \cos(\phi/2) & \imath e^{-2\imath\theta}\sin\phi/2 \\ \imath e^{2\imath\theta}\sin\phi/2 & \cos(\phi/2) \end{bmatrix}$

Topological Phase in Spin-Orbit modes

Geometrical phase in Spin-Orbit modes: two qubit pure state maximally entangled

$$\vec{E}_{rad} = \frac{1}{\sqrt{2}}\{\varphi_H \vec{e}_H + \varphi_V \vec{e}_V\}$$

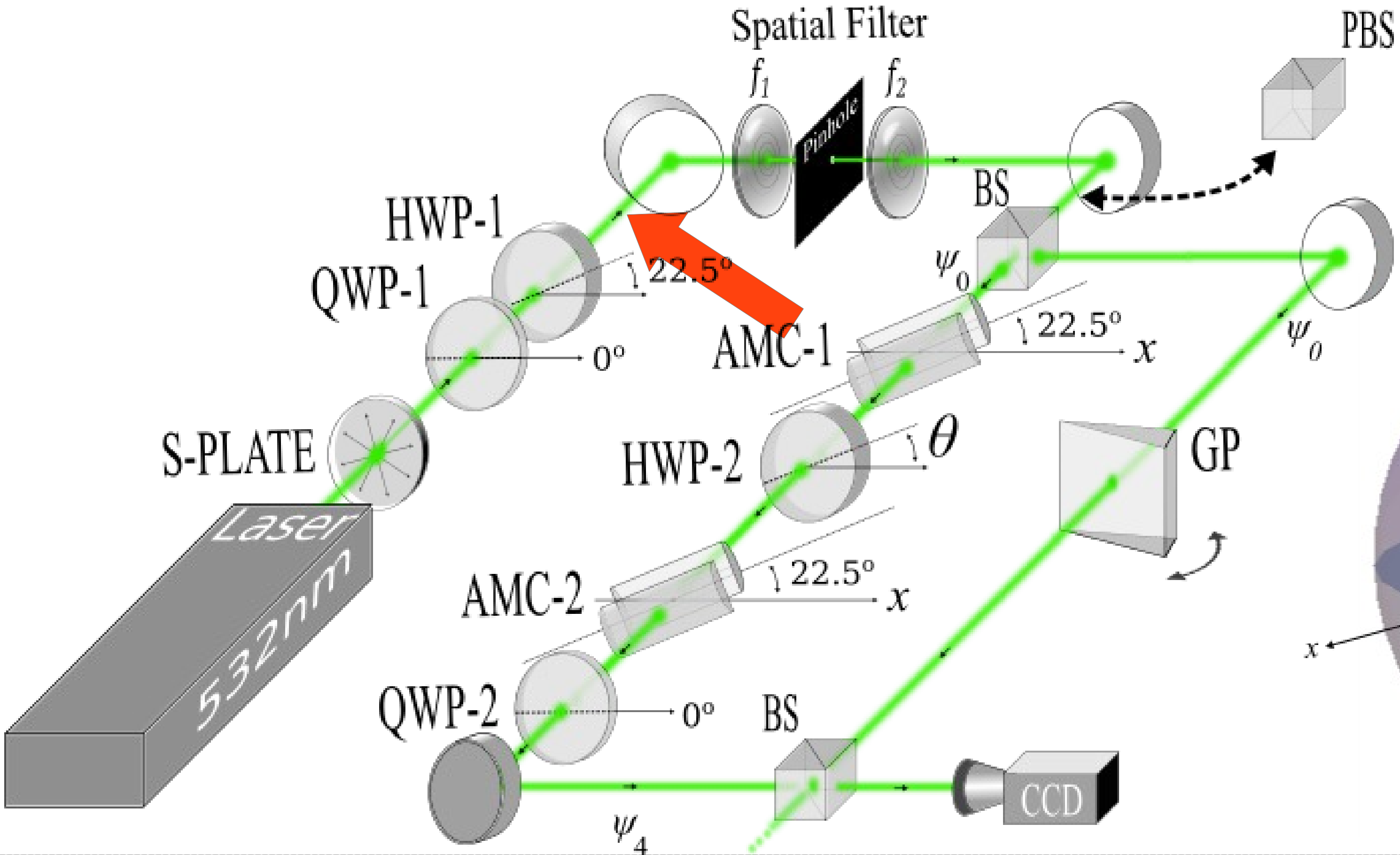


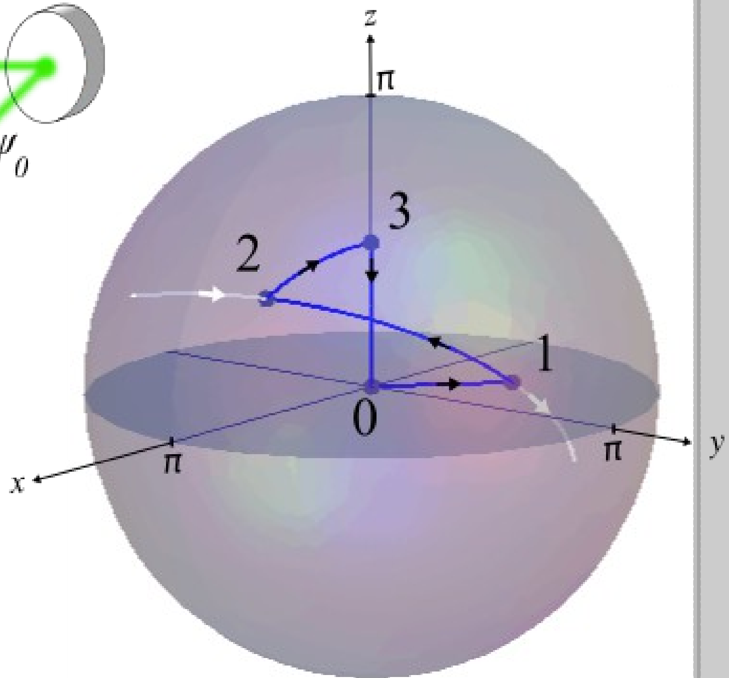


C. E. R. Souza, J. A. O. Huguenin and A. Z. Khoury - JOSA-A **31** No.5 1007 (2014)

Geometrical phase in Spin-Orbit modes: two qubit pure state maximally entangled

$$\vec{E}_0 = \frac{1}{\sqrt{2}}\{\psi_+ \vec{e}_H + \psi_- \vec{e}_V\}$$





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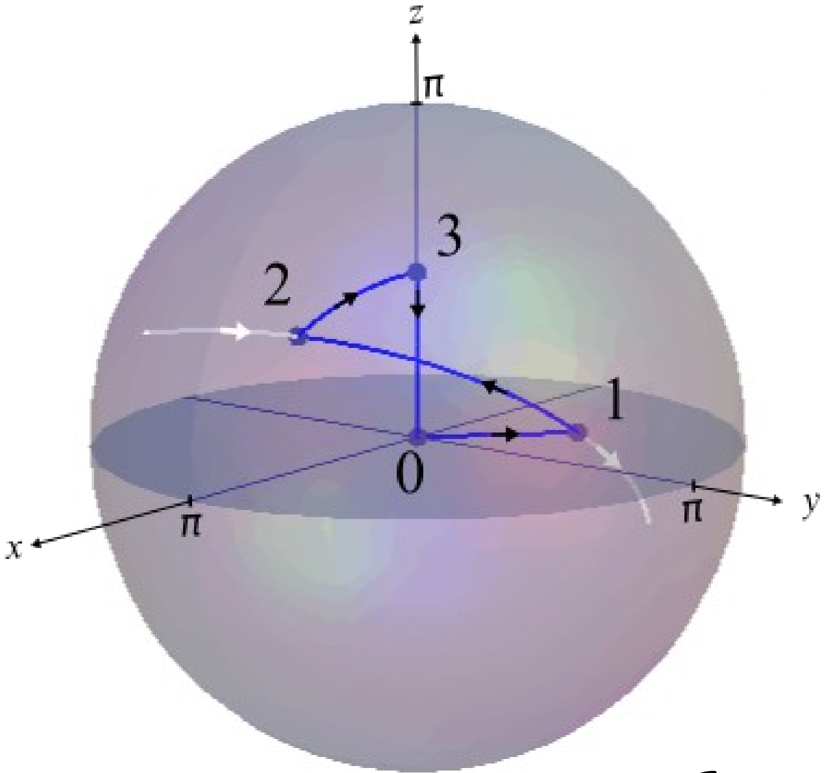
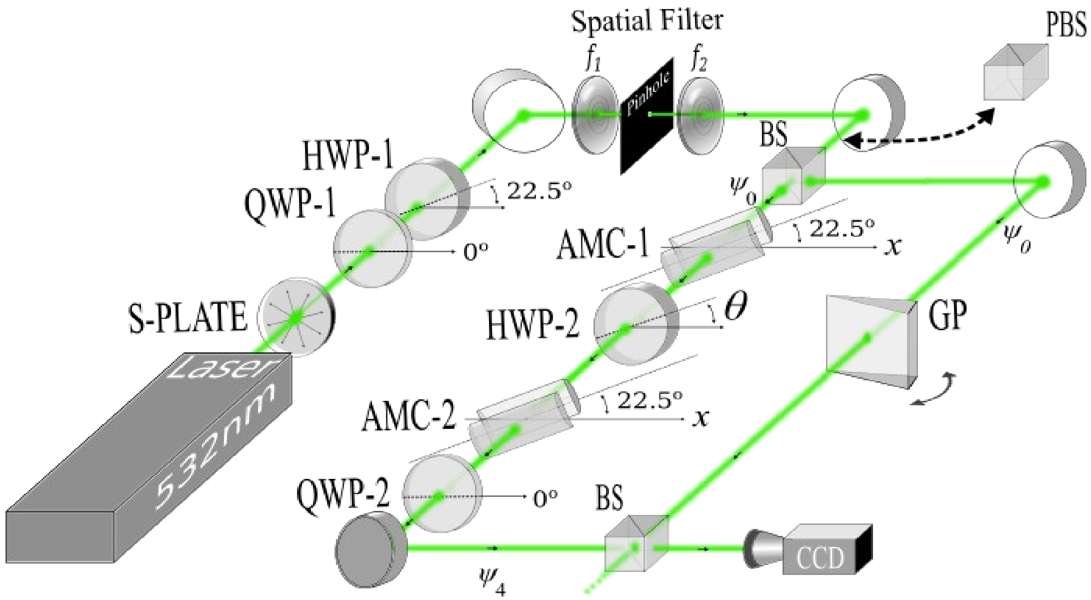
Topological Phase in Spin-Orbit modes

Geometrical phase in Spin-Orbit modes: two qubit pure state maximally entangled



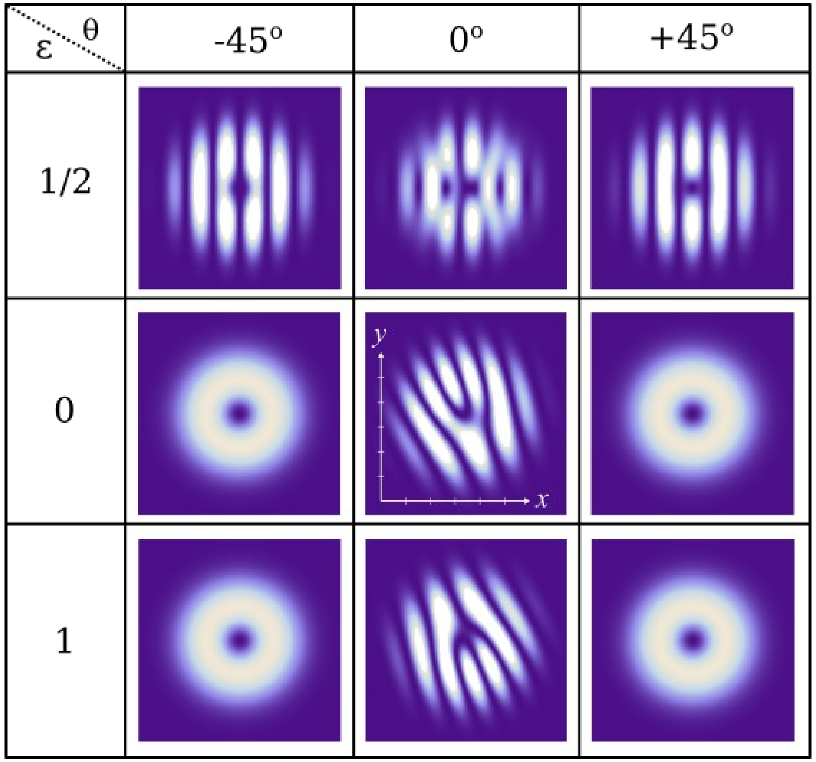


$$\vec{E}_0 \longrightarrow \vec{E}_1 \begin{cases} \longrightarrow \vec{E}_2 \longrightarrow \vec{E}_3 \longrightarrow \vec{E}_0 & \text{(1→2 Path White)} \\ \longrightarrow \vec{E}_2 \longrightarrow \vec{E}_3 \longrightarrow -\vec{E}_0 & \text{(1→2 Path Blue)} \end{cases}$$

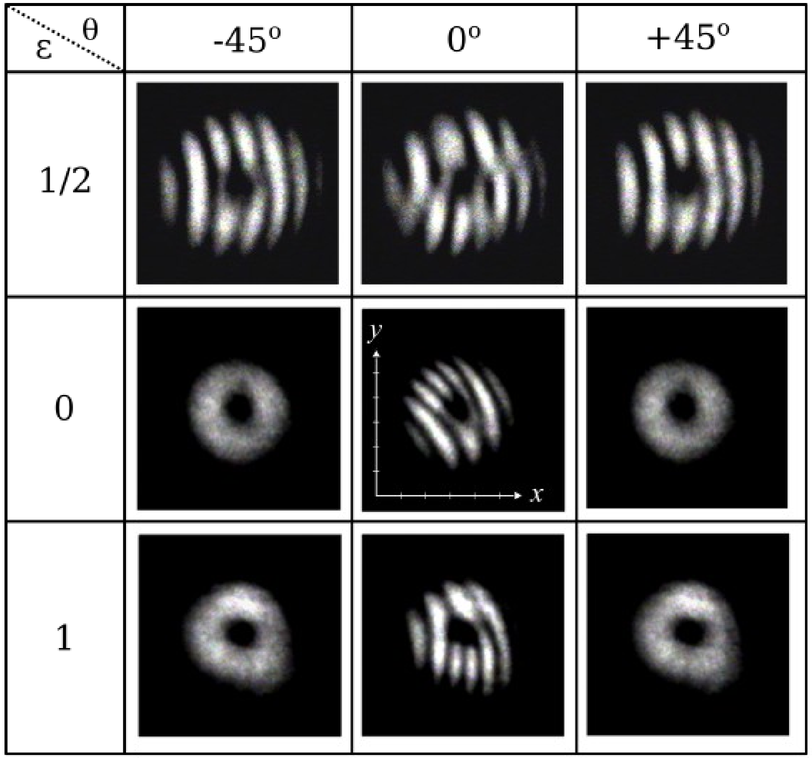
Topological Phase in Spin-Orbit modes

Results and Analysis

Teoretical Results



Experimental Results



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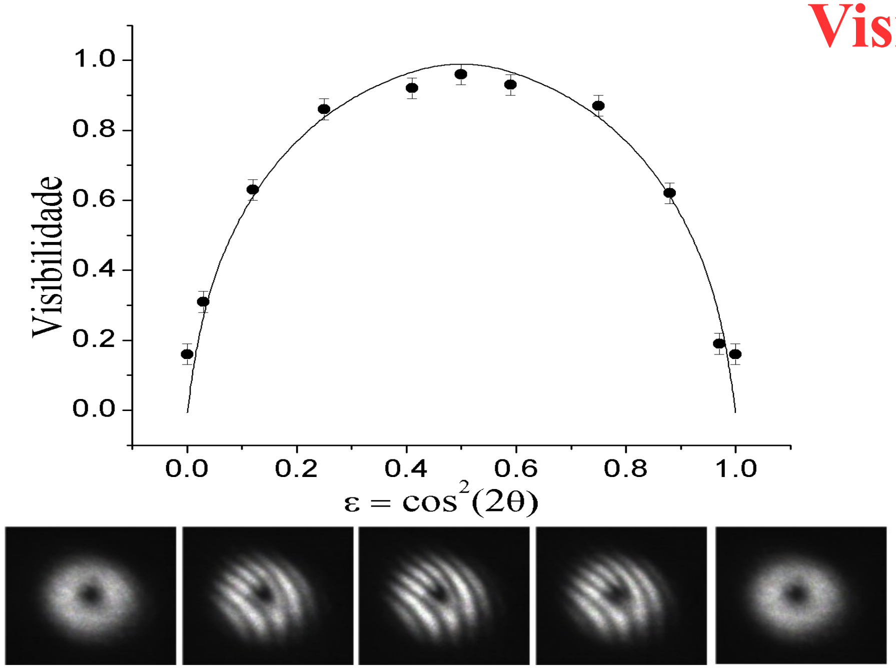
Topological Phase in Spin-Orbit modes

Results and Analysis: ***Visibility*** *vs* ***Separability***

$$\vec{E}_0 = \sqrt{\epsilon}\psi_+\vec{e}_H + \sqrt{1-\epsilon}\psi_-\vec{e}_V$$

Interferency patern on CCD camera

$$I(\mathbf{r}) = 2|\psi(\mathbf{r})|^2[1 + \underbrace{2\sqrt{\epsilon(1-\epsilon)}}_{\text{Visibility}}\sin 2\phi \sin(\delta k x)]$$



$$C = 2\sqrt{\epsilon(1-\epsilon)}$$

$$C = 2|\alpha\delta - \beta\gamma|$$

Some works in classical entanglement at UFF...

PHYSICAL REVIEW A **77**, 032345 (2008)

Quantum key distribution without a shared reference frame

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(Received 12 September 2007; published 27 March 2008)

We report a simple quantum-key-distribution experiment in which Alice and Bob do not need to share a common polarization direction in order to send information. Logical qubits are encoded into nonseparable states of polarization and first-order transverse spatial modes of the same photon.

BB84 quantum cryptografy

Classical Entanglement

Some works in classical entanglement at UFF...

3210 J. Opt. Soc. Am. B / Vol. 30, No. 12 / December 2013 Pinheiro *et al.*

Vector vortex implementation of a quantum game

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Quantum Game

Classical Entanglement

Some works in classical entanglement at UFF...

A Michelson controlled-not gate with a single-lens astigmatic mode converter

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26 April 2010 / Vol. 18, No. 9 / OPTICS EXPRESS 9207

Optical devices to
Quantum Information

Classical Entanglement

Some works in classical entanglement at UFF...

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DOI 10.1007/s13538-014-0250-6

SOCIEDADE BRASILEIRA DE FÍSICA

CONDENSED MATTER

Using Polarization to Control the Phase of Spatial Modes for Application in Quantum Information

W. F. Balthazar · D. P. Caetano · C. E. R. Souza · J. A. O. Huguenin

Phase-gate

PHYSICAL REVIEW A **82**, 033833 (2010)

Bell-like inequality for the spin-orbit separability of a laser beam

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In analogy with Bell's inequality for two-qubit quantum states, we propose an inequality criterion for the nonseparability of the spin-orbit degrees of freedom of a laser beam. A definition of separable and nonseparable spin-orbit modes is used in consonance with the one presented in Phys. Rev. Lett. **99**, 160401 (2007). As the usual Bell's inequality can be violated for entangled two-qubit quantum states, we show both theoretically and experimentally that the proposed spin-orbit inequality criterion can be violated for nonseparable modes. The inequality is discussed in both the classical and quantum domains.

Violating the Bell's inequality using classical spin-orbit modes

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Arbitrary orbital angular momentum addition in second harmonic generation

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Optical Vortices in non-linear process

Conclusions

- The use of transverse modes increases the computacional power of the light.

- We can use classical Spin-Orbit modes to study some particularities of the quantum systems. It constitutes an important tool for the Quantum Computing and the Quantum Information.

- Applied devices for Quantum Computation with Spin-Orbit modes - CNOT gates, Q-Phase gates and the BB84 setup - can be built and tested.

Conclusions

Thank you!!
(Obrigado)