

University of Virginia  
4 Dec 2015

# Three Dimensional Imaging of the Proton and Atomic Nuclei

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Old Dominion University  
Norfolk VA

- The Challenge of Imaging
- Elastic and Deep Inelastic Scattering
- Two Solutions to the Challenge:
  - Deep Virtual Exclusive Scattering:
    - Spatial Imaging
  - Semi-Inclusive Deep Inelastic Scattering
    - Momentum Imaging (someone else's talk)
- The Future

# The Challenge

- The construction of an image implies that the object being observed is unaffected by the measurement
- The proton rms charge radius  $\sim 10^{-15}$  m (1 fm)
  - To image something this small requires that it absorb momenta of the order  $pc > \hbar c/(1 \text{ fm}) = 200 \text{ MeV}$
  - But the proton mass  $Mc^2 = 938 \text{ MeV}$
  - Imaging the proton requires disturbing the proton
    - Is it even physically sensible to talk about imaging the proton?

# Elastic Electron Scattering on the proton, 1950s – 2010s

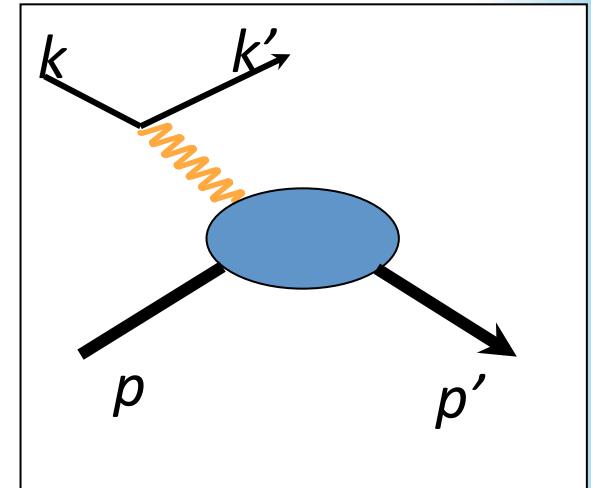
- Wave equation  $\square A^\mu = J^\mu$

- Interaction

$$\int dx A(x) \cdot J(x)$$

- An electron makes a transition from momentum state  $k$  to  $k'$ :
  - Current  $j^\mu(q)$  generates a vector potential  $A^\mu(x) \sim e^{-iq \cdot x} j^\mu(q)/q^2$
  - This vector potential then interacts with the current density  $J^\mu(x)$  of the proton.

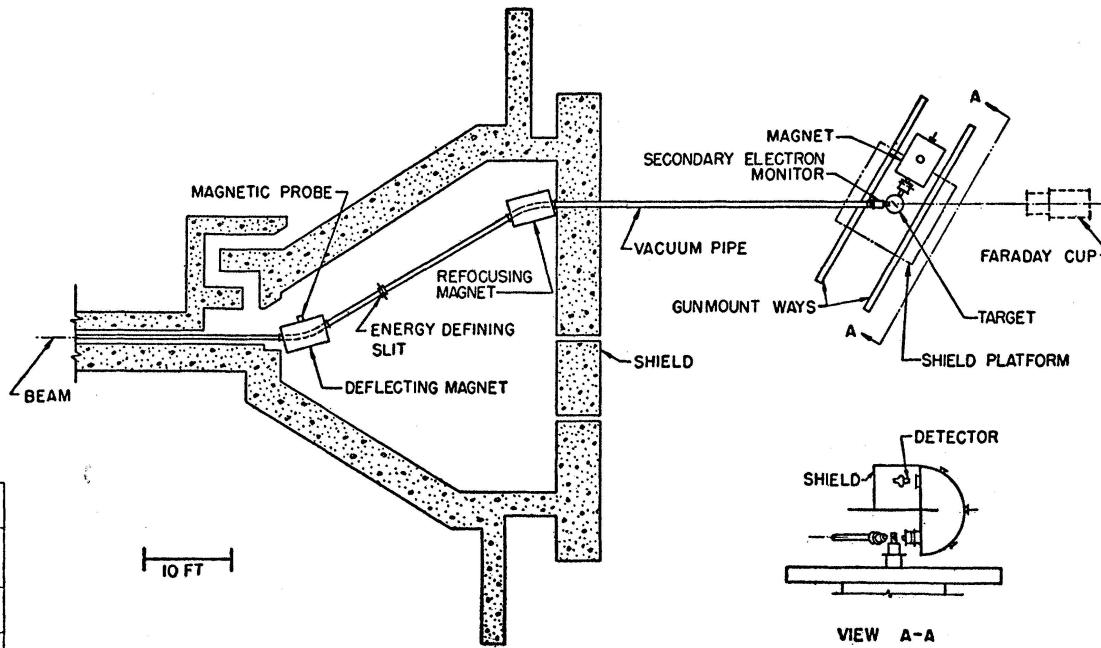
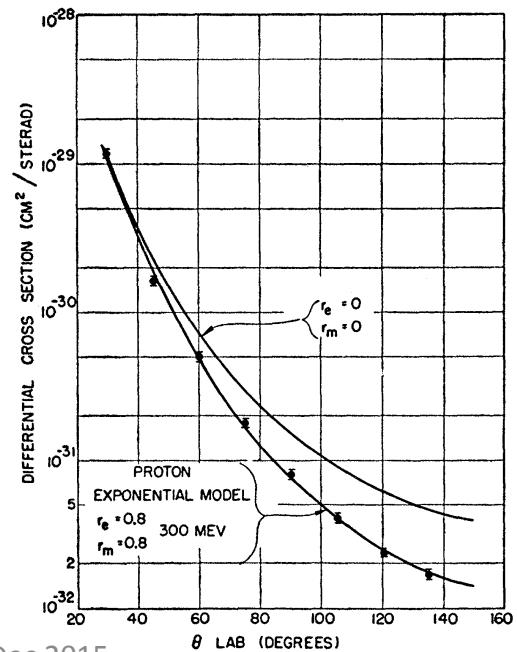
$$q^\mu = (k - k')^\mu$$



# R. Hofstadter, et al., Phys Rev 1956

- Nobel Prize, 1961

$$\sqrt{\langle r^2 \rangle_{Ch}} = 0.7 \pm 0.1 \text{ fm}$$



2014 PDG review:

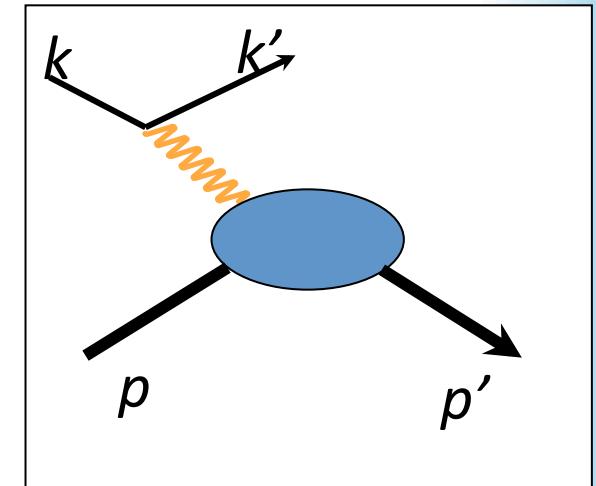
- $(e, e')$  (Mainz...):  $r_{rms} = 0.879(8) \text{ fm}$
- H Atomic levels:  $r_{rms} = 0.877(5) \text{ fm}$
- $\mu p$  'Hydrogen':  $r_{rms} = 0.8409(4) \text{ fm}$

# The Proton is not an Elementary Particle:

- Anomalous Magnetic moment,  
Charge and Current Densities,
  - General EM current for a Dirac spin-1/2 nucleon to make a transition from a state  $(p, s)$  to  $(p', s')$  with  $q = p' - p$  ( $Q^2 = -q^2 > 0$ ):

$$J^\mu(q) = \bar{U}(p', s') \left[ \gamma^\mu F_1(Q^2) - \frac{[\gamma^\mu, \gamma^\nu] q_\nu}{4M} F_2(Q^2) \right] U(p, s)$$

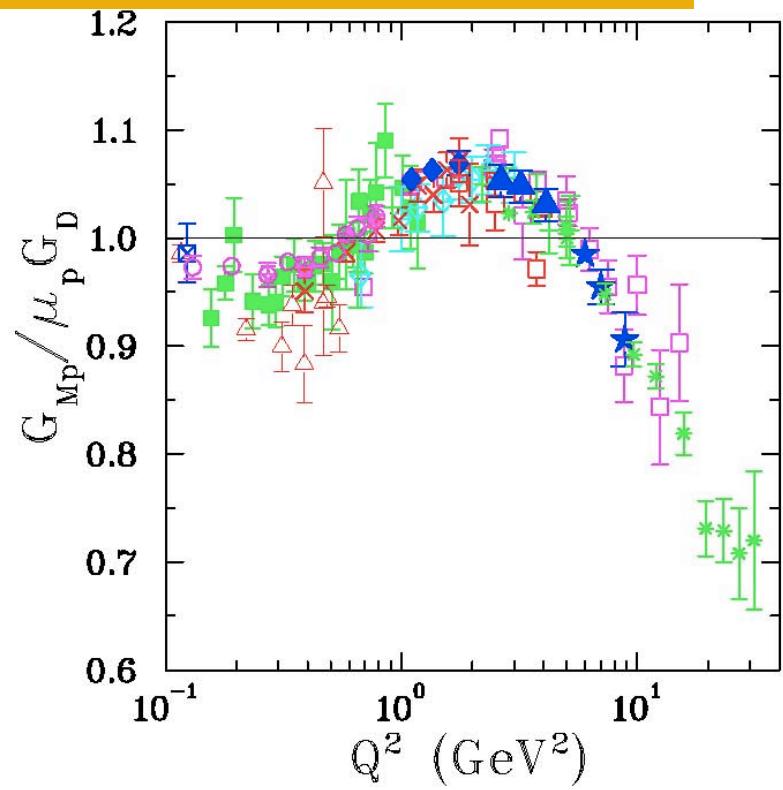
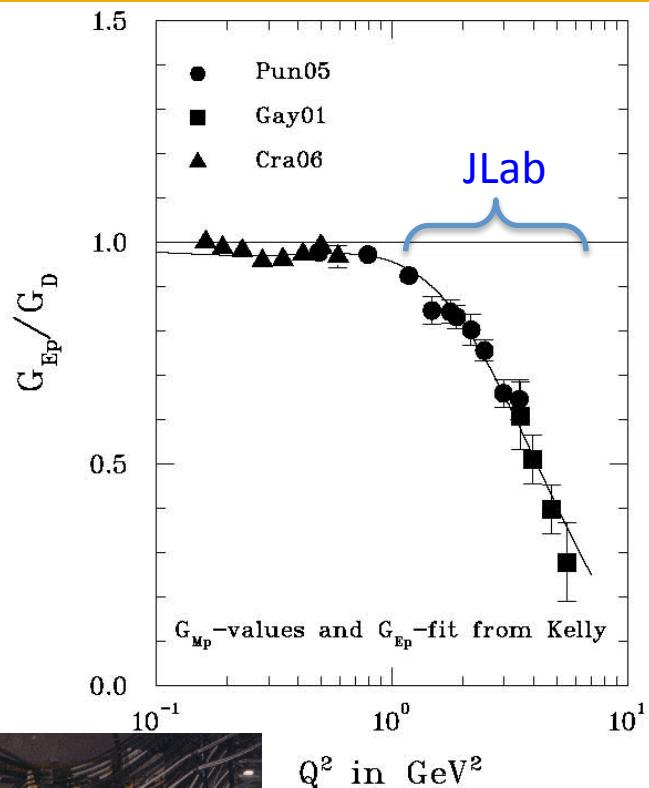
- Macroscopic Limits
  - $F_1(0) = 1$      $F_2(0) = \kappa$ 
    - O.Stern 1933:  $\kappa_p = 1.5 \pm 0.2$
    - 2014 PDG review  $\kappa_p = 1.792847356(023)$
- $G_M(Q^2) = F_1 + F_2$
- $G_E(Q^2) = F_1 - Q^2/(4M^2)F_2$



# Elastic Electron Scattering Today

- Ratios to ‘Dipole’  $G_D = [1+Q^2/\Lambda^2]^{-2}$ ,  $\Lambda^2 = 0.71 \text{ GeV}^2$

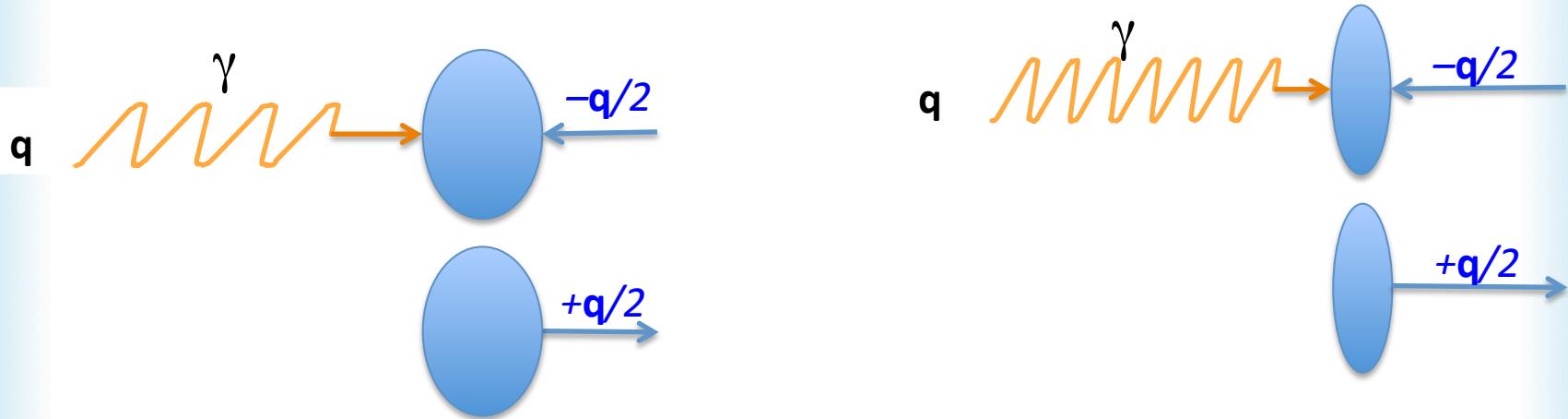
- Extensive new low- $Q^2$  data (Mainz, MIT, JLab)
- Experiments at JLab 12 GeV (2016+) looking for zero crossing in  $G_E$ .



# Form Factors and Densities

- Naively,  $G_E(Q^2)$  is the Fourier transform of the charge density.  
But this only works for  $Q^2 \ll M_p^2$
- Consider  $H(e,e)p$  in the 'Breit' Frame:  $q^\mu_{\text{Breit}} = [0, (Q^2)^{1/2}]$   
 $P=-q/2, P'=+q/2$  (zero energy transfer)
- At each  $|q|=[Q^2]^{1/2}$ ,  $G_E(Q^2)$  samples the charge distribution of a differently boosted proton.

Lorentz contracted protons:

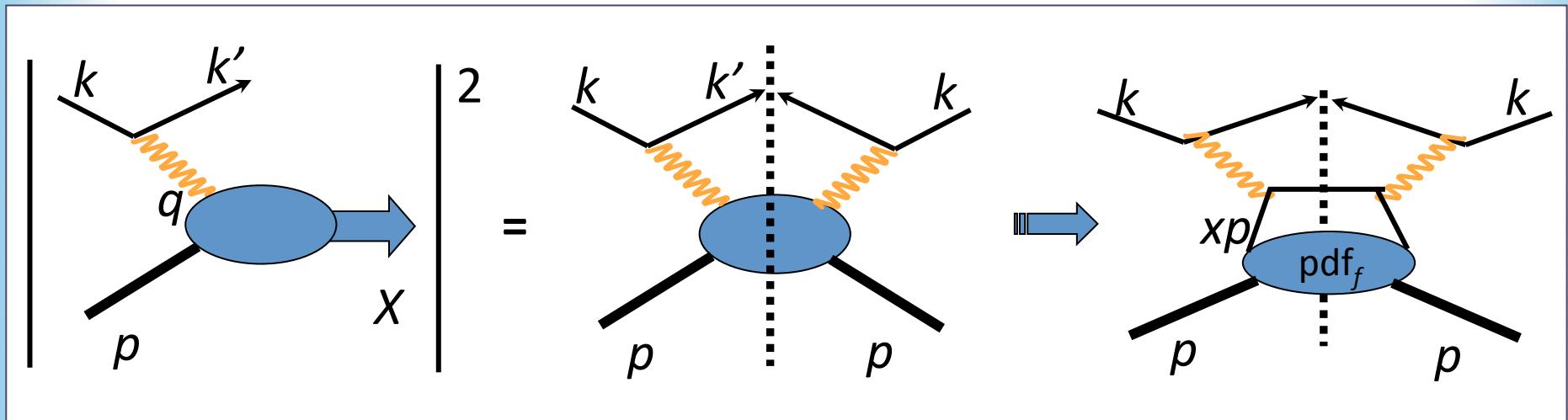


# Lepton Scattering

II. Deep Inelastic Scattering:  $e + p \rightarrow e' + X$

$$Q^2 = -q^2 = (k-k')^2 \quad x_{Bj} = Q^2/(2p \cdot q)$$

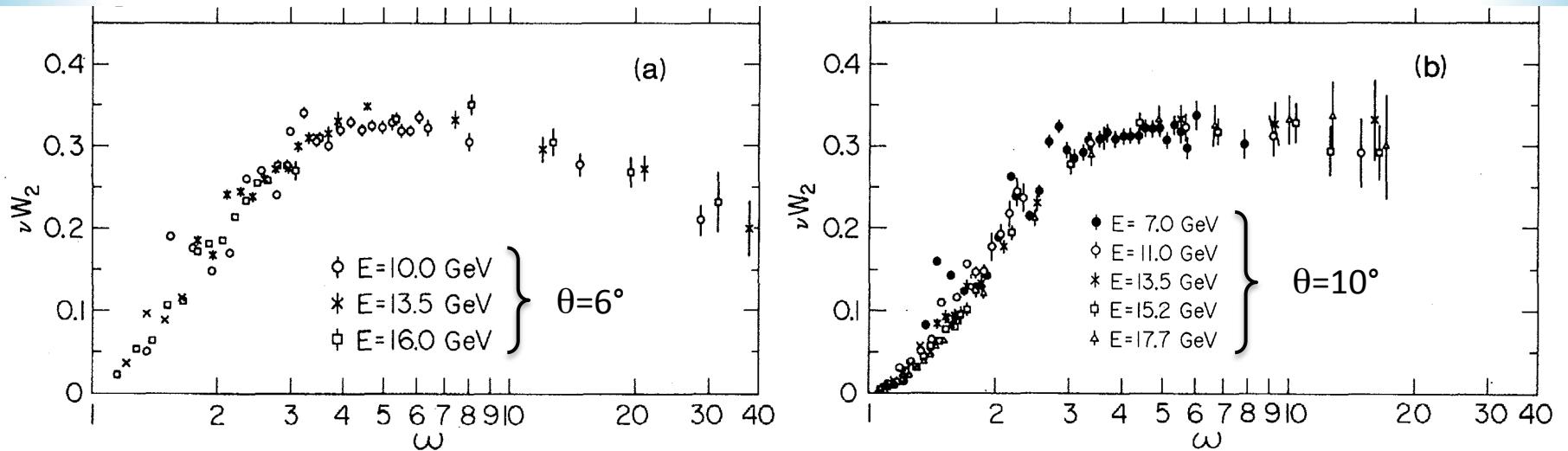
$$d\sigma =$$



$$\frac{d\sigma}{dx_{Bj} dQ^2} \rightarrow \frac{4\pi\alpha^2}{x_{Bj} Q^4} \left[ 1 - y + \frac{y^2}{2} \right] \sum_f [2x_{Bj} q_f^2 pdf_f(x_{Bj}, \ln Q^2)]$$

# The proton is made of charged spin-1/2 constituents

- M. Breidenbach *et al* PRL **23** (1969) 935  
Friedman, Kendal, Taylor, Nobel Prize 1990

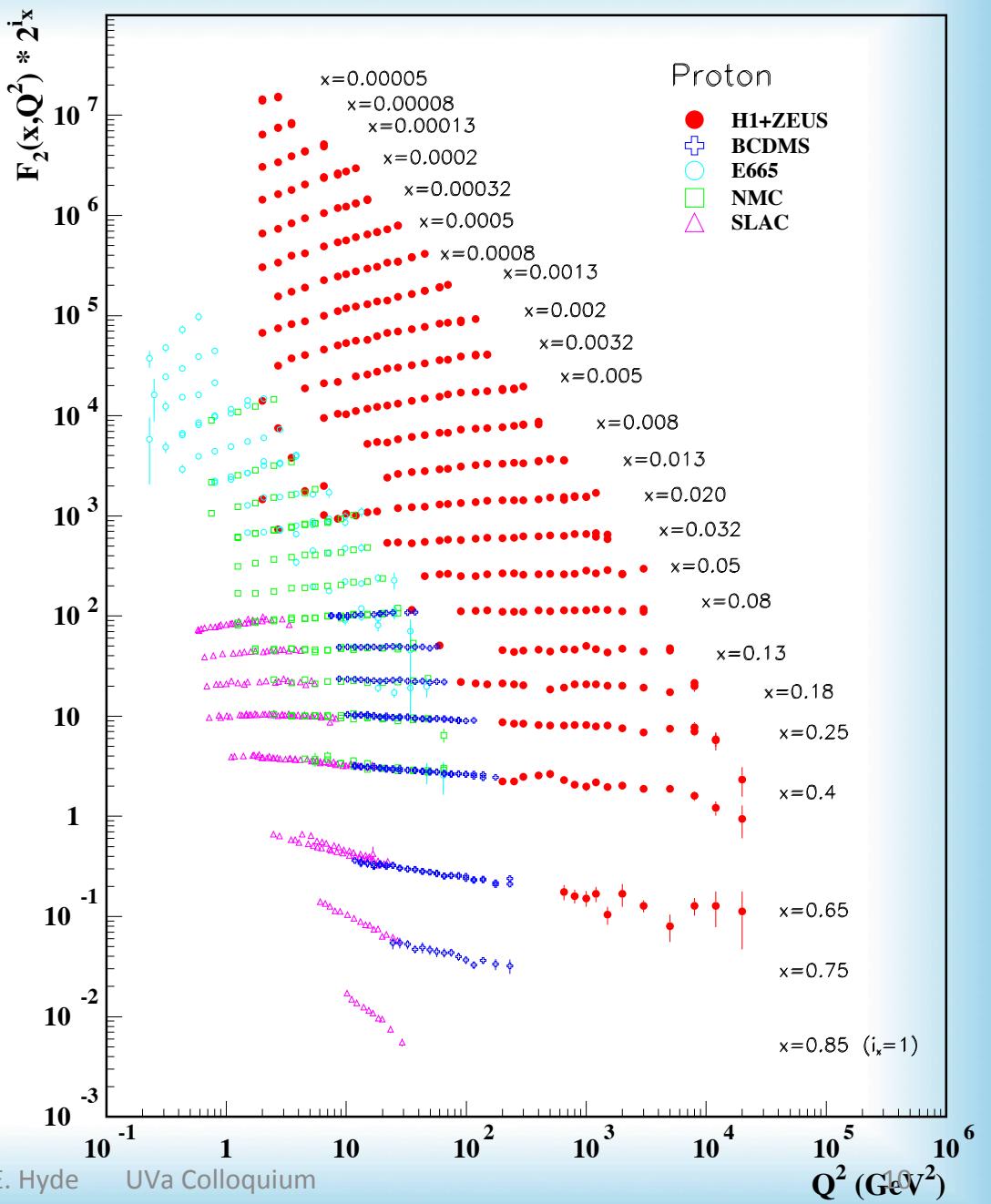
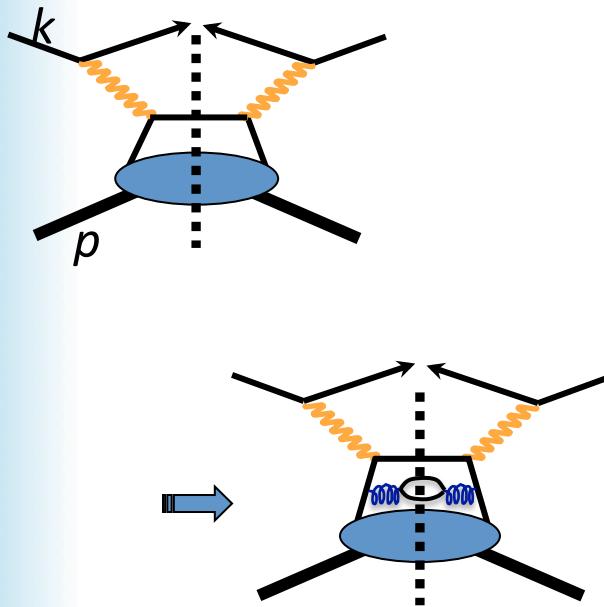


- Universal behavior of the cross section as a function of single variable  $\omega = 1/x = 2q \bullet P/Q^2$

# Protons are made of Quarks and Gluons, described by QCD.

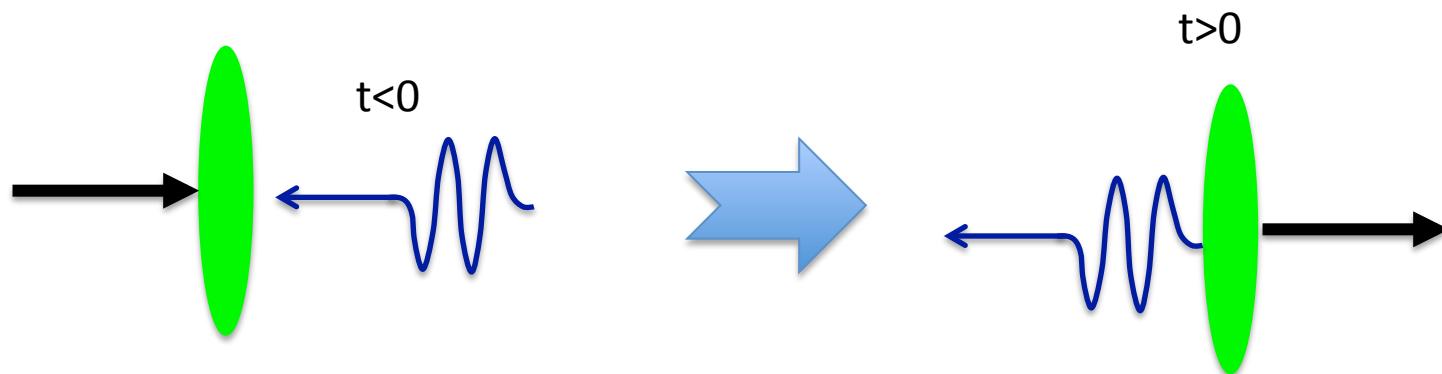
- Scaling violations:

As  $Q^2$  increases:  
 $q \rightarrow q + g$   
 $g \rightarrow q + \bar{q}$



# How to Image the Proton?

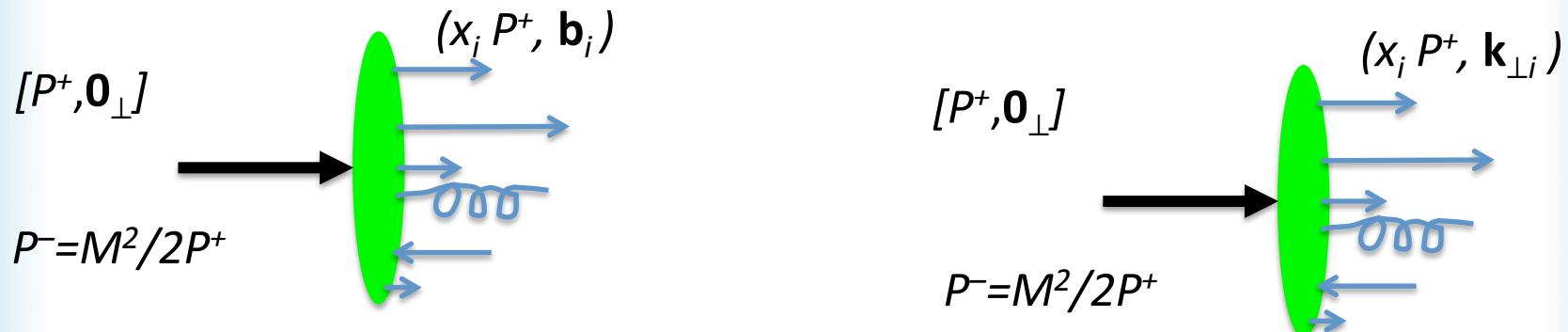
- A relativistic proton moving in the  $+z$ -direction.
- Illuminate it with a photon moving in  $-z$ -direction.
  - Photon-quark scattering samples the proton at equal light-cone times  $x^+ = ct + z$
  - Astronomy, the farther we look in distance ( $+z$ ), the farther back in time we are observing ( $-t$ ).



Quantize at equal light-cone times  $x^+$ , Dirac's Front-form dynamics.

- Hamiltonian = ' $P^-$ ',  $M^2 = P^2 \rightarrow M^2 = 2P^+P^- - \mathbf{P}_\perp^2$
- Proton has definite  $P^+ = (E + P_z)/\sqrt{2}$ ,
- Proton is spatially localized in transverse plane

# Proton structure in transverse impact parameter or momentum space



Partonic Fluctuations

$$\sum_i x_i = 1$$

$$\sum_i \mathbf{b}_i x_i = 0$$

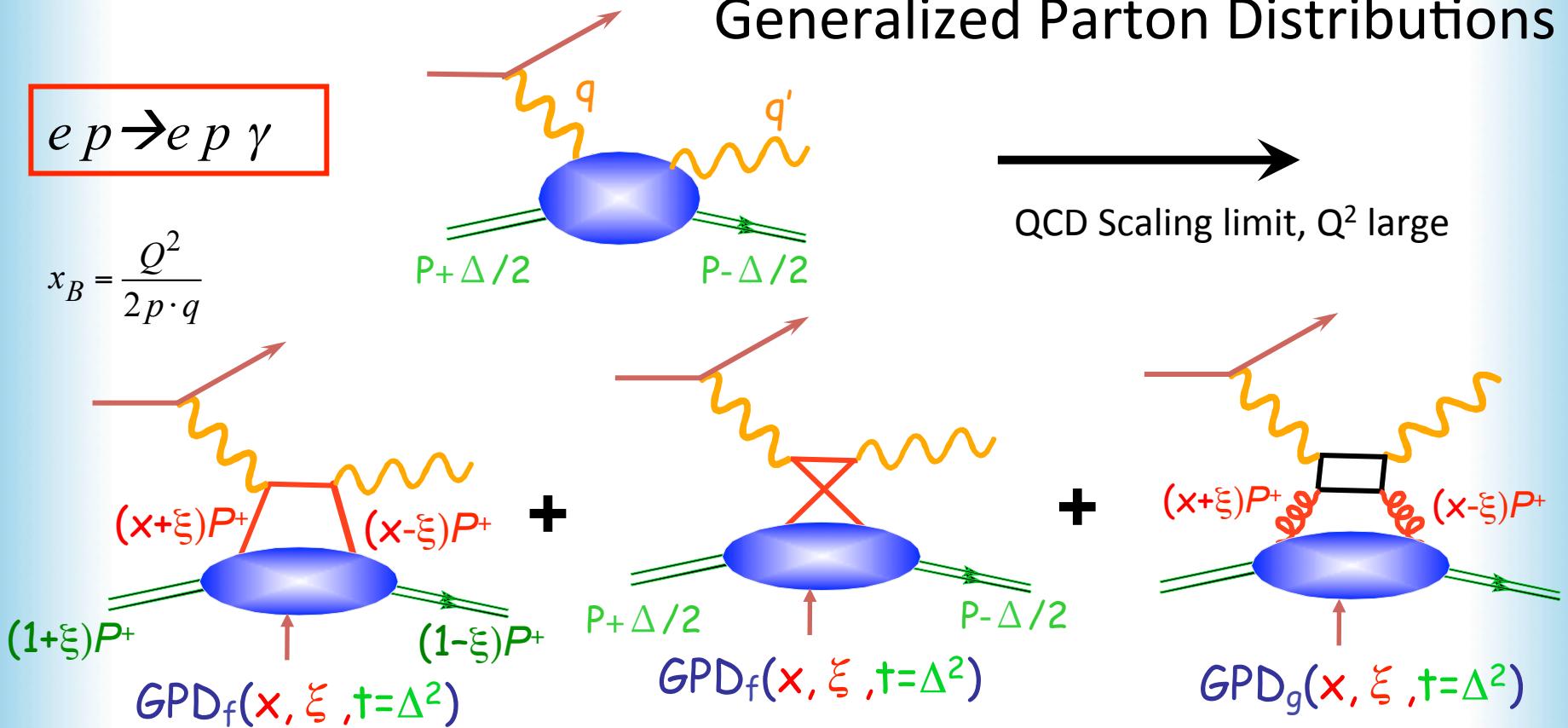
$$\sum_i x_i = 1$$

$$\sum_i \mathbf{k}_{\perp,i} = 0$$

Virtual 'Energy'

$$P^- = \frac{1}{2P^+} \sum_i \frac{(\mathbf{k}_{\perp,i})^2}{x_i} > \frac{M^2}{2P^+}$$

# Deeply Virtual Compton Scattering Generalized Parton Distributions



- Symmetrized Bjorken variable:  $\xi = \frac{-(q+q')^2}{2(q+q') \cdot P} \xrightarrow{\Delta^2 \ll Q^2} \frac{x_B}{2 - x_B}$
- $\Delta = (q-q')$ , Fourier conjugate to impact parameter **b** of active parton

# Imaging Quarks and Gluons: Deeply Virtual Exclusive Reactions

- $ep \rightarrow e p \gamma$ ,     $ep \rightarrow e N \pi$ ,     $ep \rightarrow e p V$ :  $V = \rho \omega \phi \dots$ 
  - ( $e, e'$ ) kinematics in DIS regime:
    - Incoherent sum of partons (quarks and gluons) of momentum fraction fixed by electron kinematics ( $x_{Bj} = Q^2/(W^2 - M^2)$ )
    - Proton  $\rightarrow$  proton (or neutron) elastic transition:
      - Momentum transfer to target  $\Delta^\mu = (q - q')^\mu$
      - Fourier transform of spatial density in transverse plane
  - (1+2) Tomographic image:
    - Distribution of quark momenta along one axis and the spatial density in the plane transverse to this axis  
[Generalized Parton Distributions = GPDs]
    - J. Xi, A. Radyushkin 1997, D. Mueller et al, 1994
    - Hermes, HERA, JLab, COMPASS (CERN) experiments
- Image gluons via  $ep \rightarrow e p J/\Psi$ ,    $ep \rightarrow e p \phi$

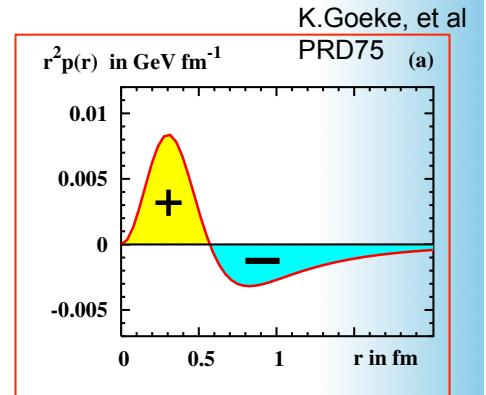
# GPDs: Correlations of Spatial, Momentum, and Spin d.o.f.

Vector:  $H_f(x, \xi, t), E_f(x, \xi, t) \Leftarrow \int e^{iP^+ z^-} dz^- \langle P + \Delta/2 | \bar{\Psi}(+z^-) \gamma^+ \Psi(-z^-) | P - \Delta/2 \rangle$

Axial:  $\tilde{H}_f(x, \xi, t), \tilde{E}_f(x, \xi, t) \Leftarrow \int e^{iP^+ z^-} dz^- \langle P + \Delta/2 | \bar{\Psi}(+z^-) \gamma^+ \gamma_5 \Psi(-z^-) | P - \Delta/2 \rangle$

**Forward limits**     $H_f(x, 0, 0) = q_f(x)$   
 $\tilde{H}_f(x, 0, 0) = \Delta q_f(x)$

**First Moments**     $\int_{-1}^1 dx [H, E]_f(x, \xi, \Delta^2) = [F_1, F_2]_f(-\Delta^2)$



**Second Moments:**     $\int x dx H_f(x, \xi, t) = +M_{2f}(t) + \frac{4}{5} \xi^2 d_{1f}(t)$

                                        Forces

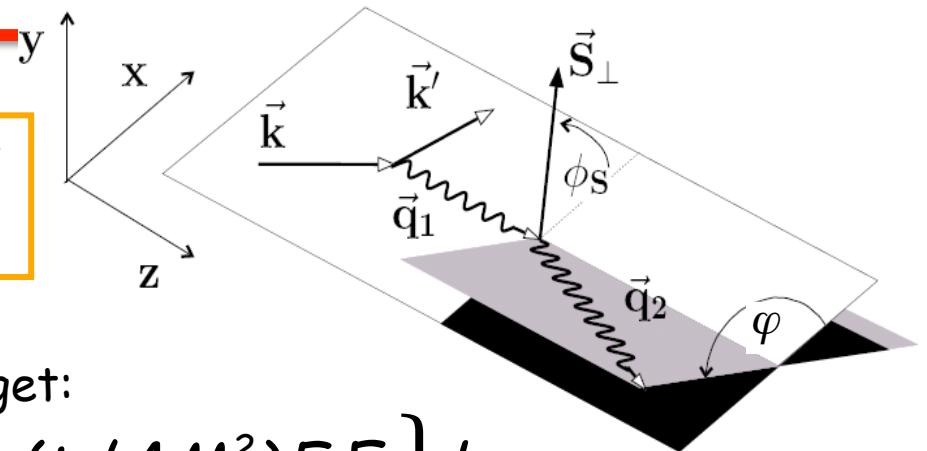
$\int x dx E_f(x, \xi, t) = -M_{2f}(t) + \frac{4}{5} \xi^2 d_{1f}(t) + 2J_f(t)$

                                        Proton Spin

X. Ji: Angular Momentum Sum Rule     $\int_{-1}^1 x dx [H_f(x, \xi, 0) + E_f(x, \xi, 0)] = 2J_f$

# Exploiting the harmonic structure of DVCS with polarization

The difference of cross-sections is a key observable to extract GPDs



With **polarized beam** and unpolarized target:

$$\Delta\sigma_{LU} \sim \sin\varphi \left\{ F_1 H + \xi(F_1 + F_2) \tilde{H} + (t/4M^2) F_2 E \right\} d\varphi$$

With unpolarized beam and **Long. polarized target**:

$$\Delta\sigma_{UL} \sim \sin\varphi \left\{ F_1 \tilde{H} + \xi(F_1 + F_2) H + (t/4M^2) F_2 E \right\} d\varphi$$

With unpolarized beam and **Transversely polarized target**:

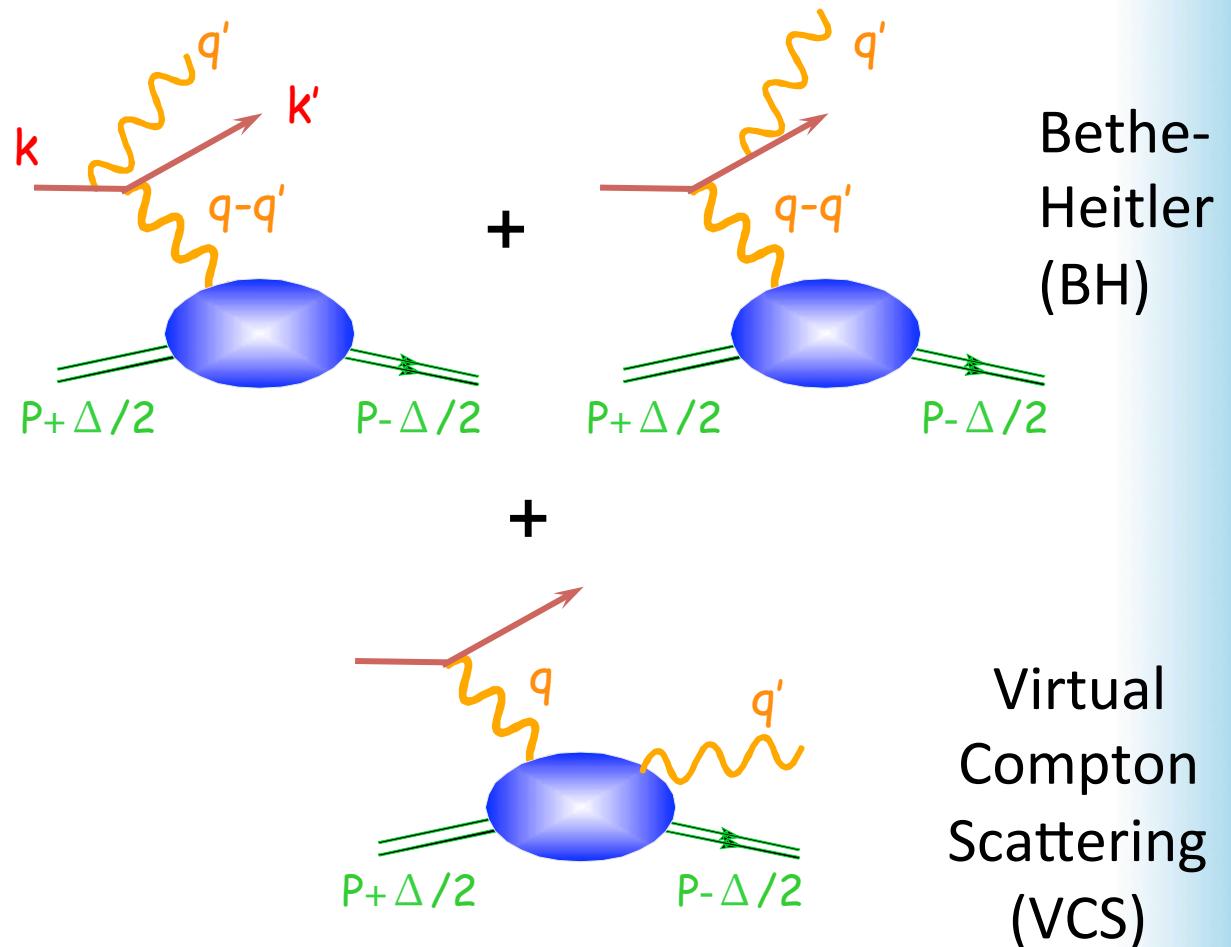
$$\Delta\sigma_{UT} \sim \cos\varphi \sin(\phi_S - \varphi) \left\{ (t/4M^2) F_2 H - (t/4M^2) F_1 E + \dots \right\} d\varphi$$

Separations of CFFs  **$H(\pm\xi, \xi, t)$ ,  $\tilde{H}(\pm\xi, \xi, t)$ ,  $E(\pm\xi, \xi, t)$ , ...**

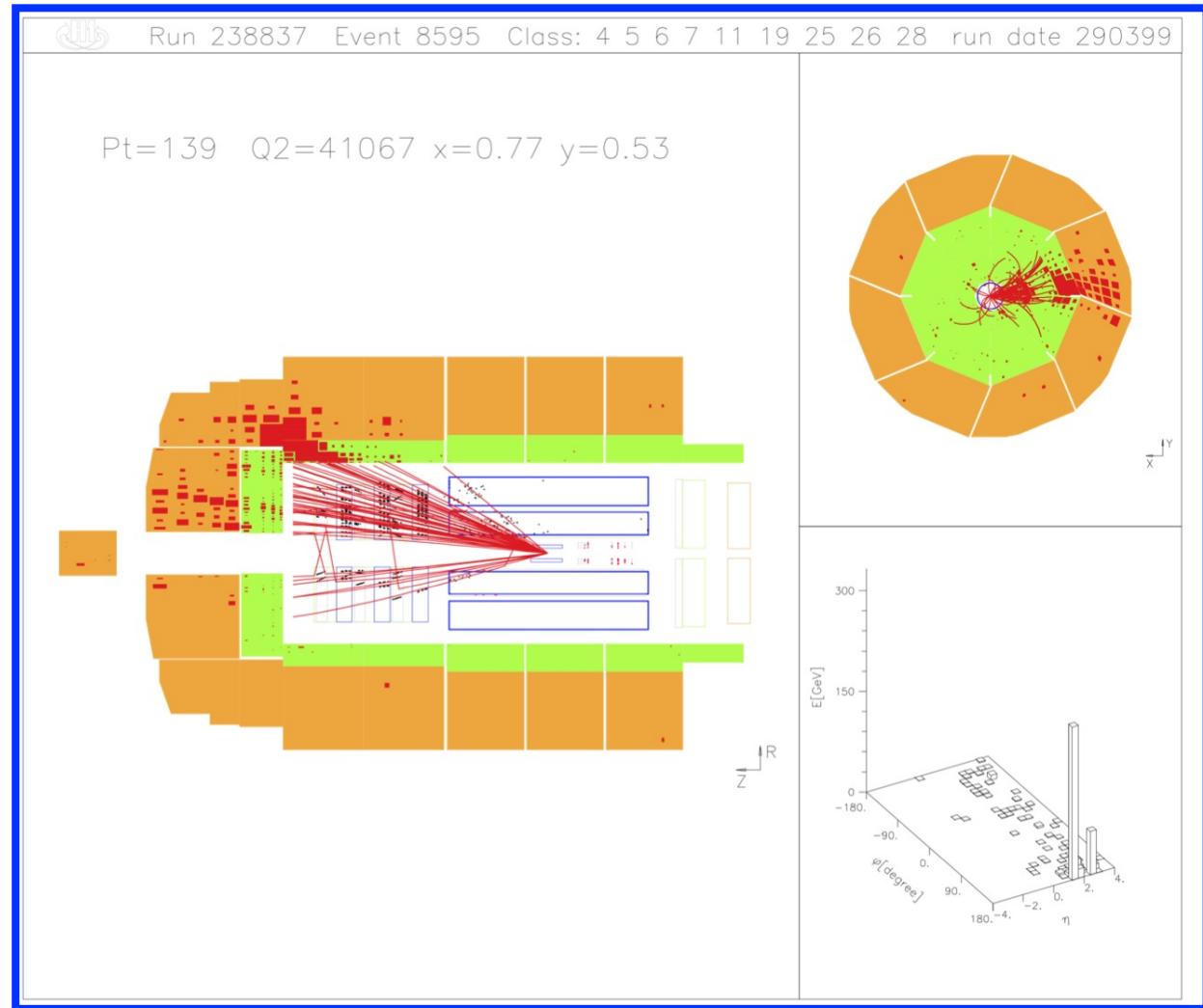
# Measuring GPDs

$e p \rightarrow e p \gamma$

- HERA  
(2001 – 2007)
- HERMES  
(2001 – 2007)
- JLab 6 GeV  
(2001 – 2012)
- JLab 12 GeV  
(2014 –
- COMPASS  
(2016 –
- EIC  
(2025+?)



# HERA DIS Event $e p \rightarrow \nu X$

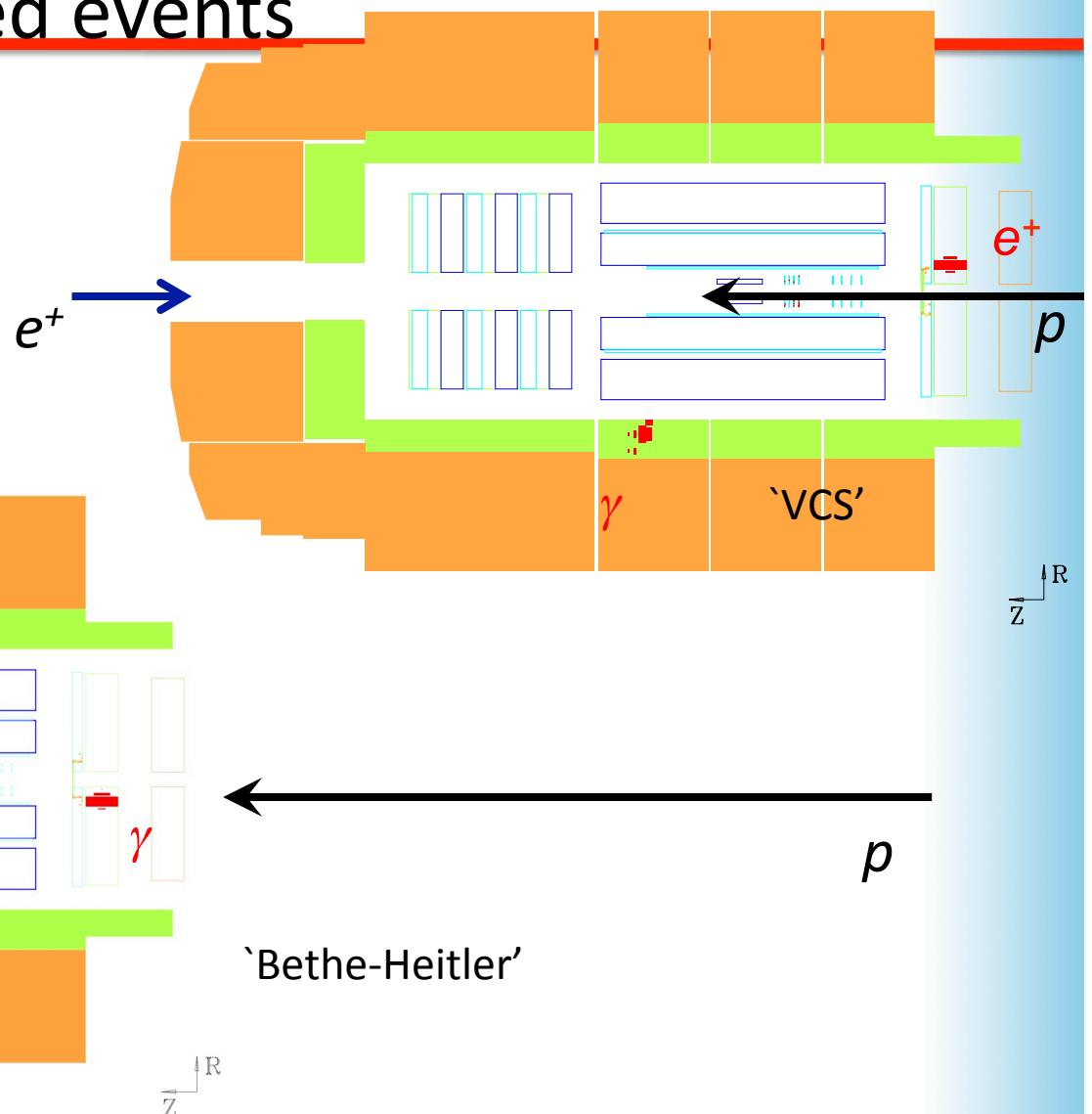
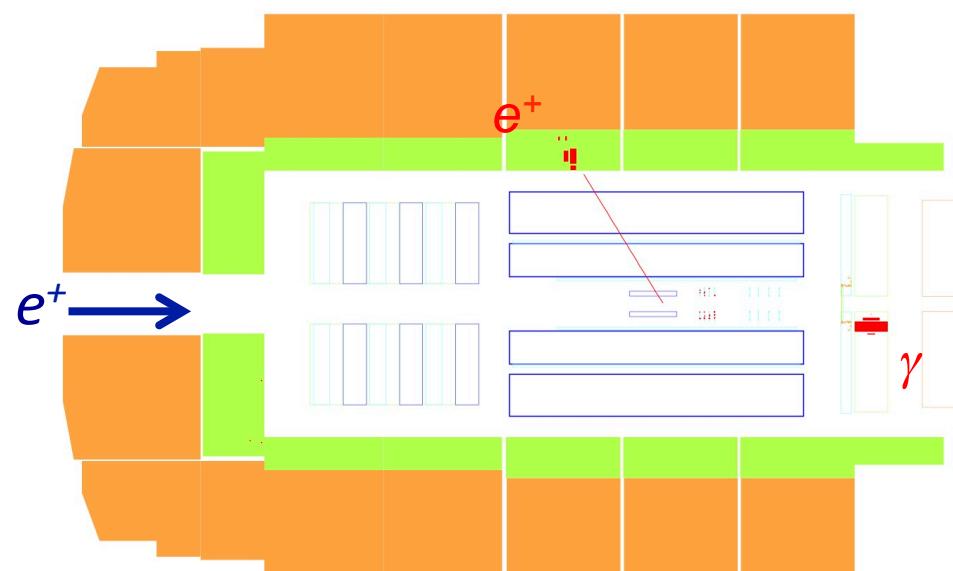


# HERA-H1: Sample VCS-dominated; and BH-dominated events

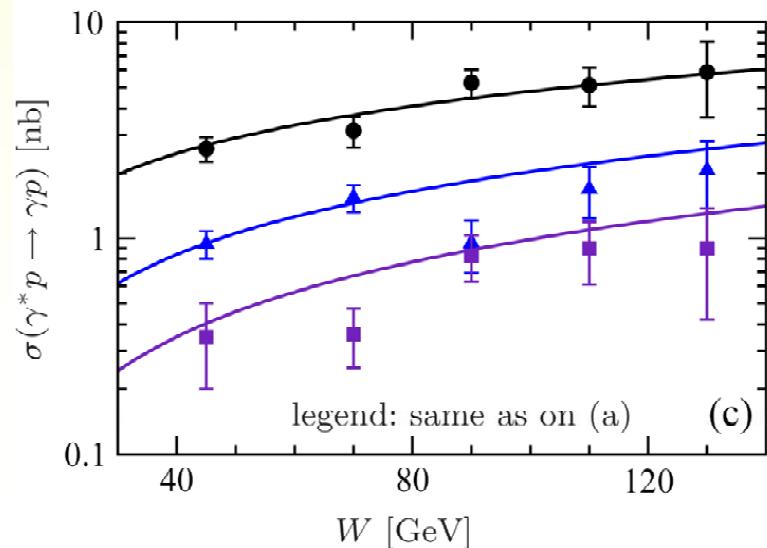
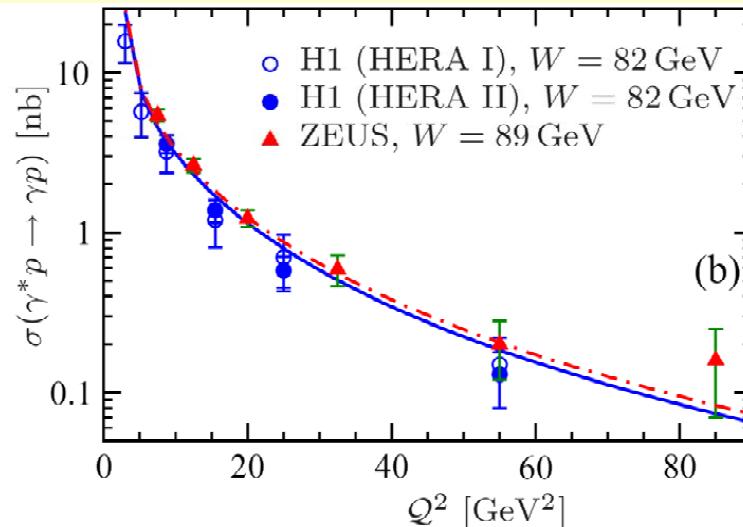
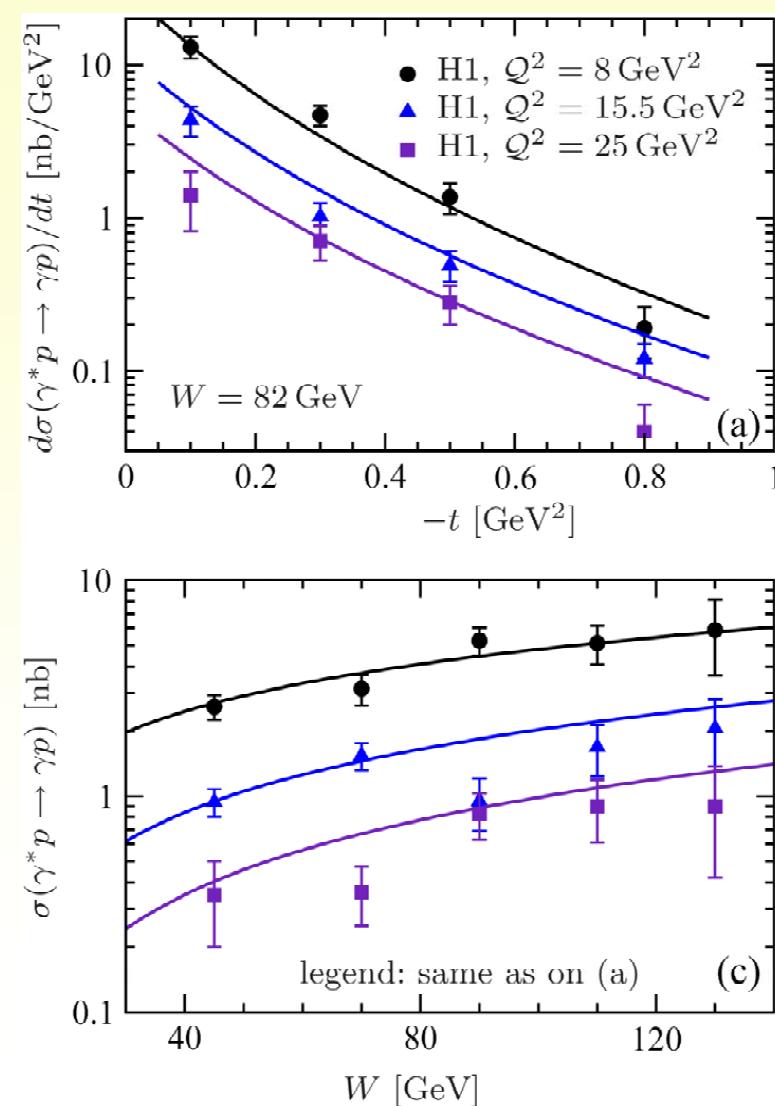
$e p \rightarrow e \gamma X$

$X$  is ultra-forward (left  $\leftarrow$ )

No visible energy:  
dominated by exclusive



# HERA DVCS, fits by D.Müller *et al.*, 2012

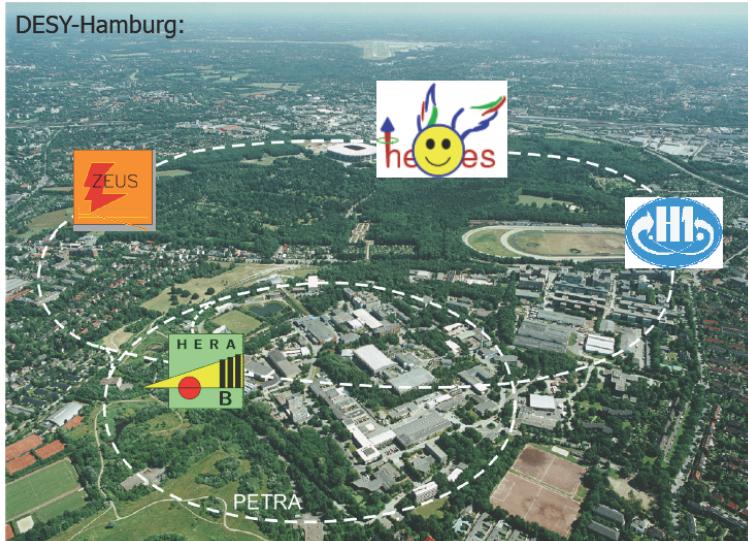


- DVCS fits at LO, NLO, NNLO
  - Flexible GPD ansatz
  - Dominated by  $H_{glue}$
- a) Spatial imaging
- b)  $Q^2$ -Dependence: Validation of QCD factorization
- c) Longitudinal momentum fraction  $x = Q^2/(W^2 - M^2)$

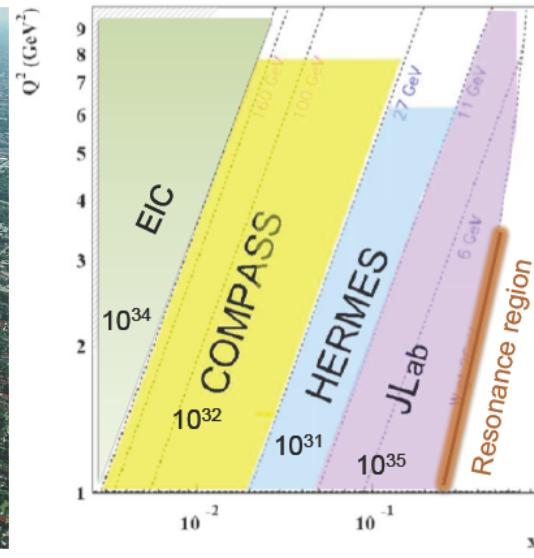
# HERMES overview

27.6 GeV e+/e- HERA beam

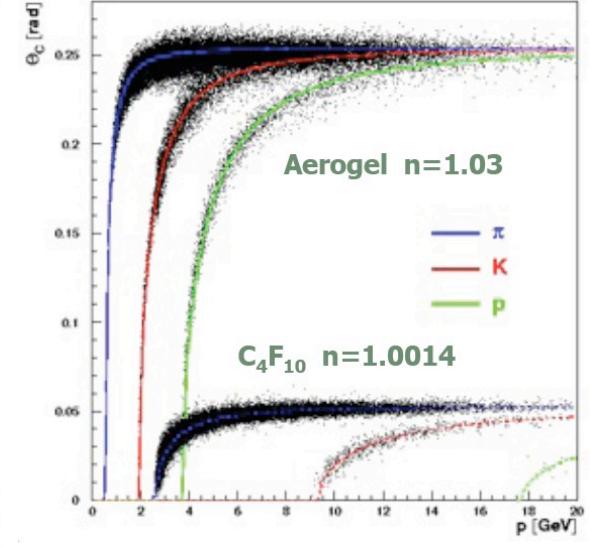
DESY-Hamburg:



Access to valence and sea



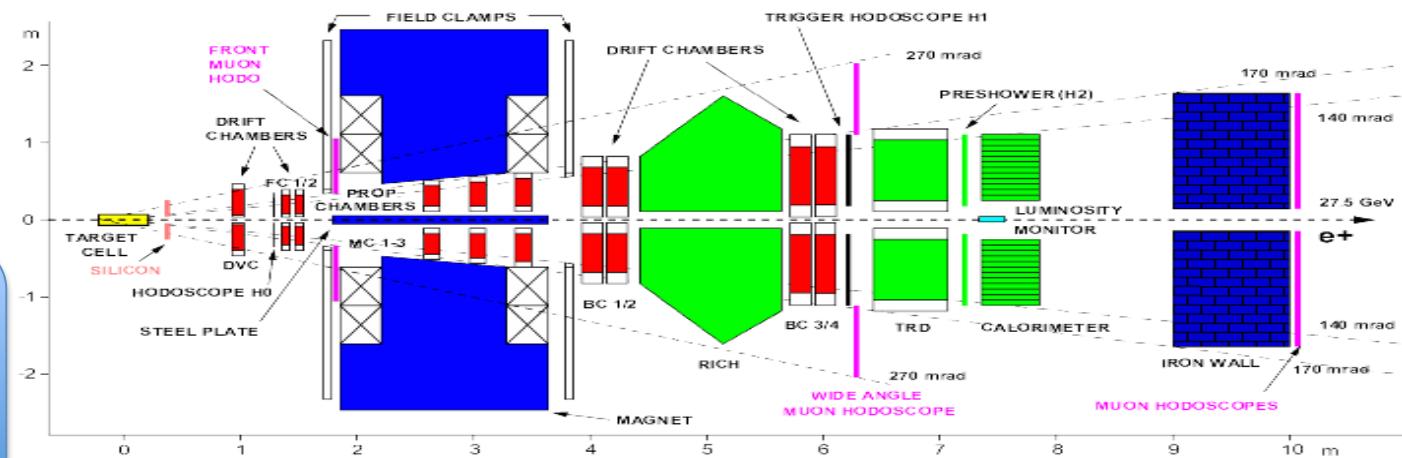
Electron and Hadron ID



Data taking: 95-07

Internal gaseous target  
(no nuclear effects)

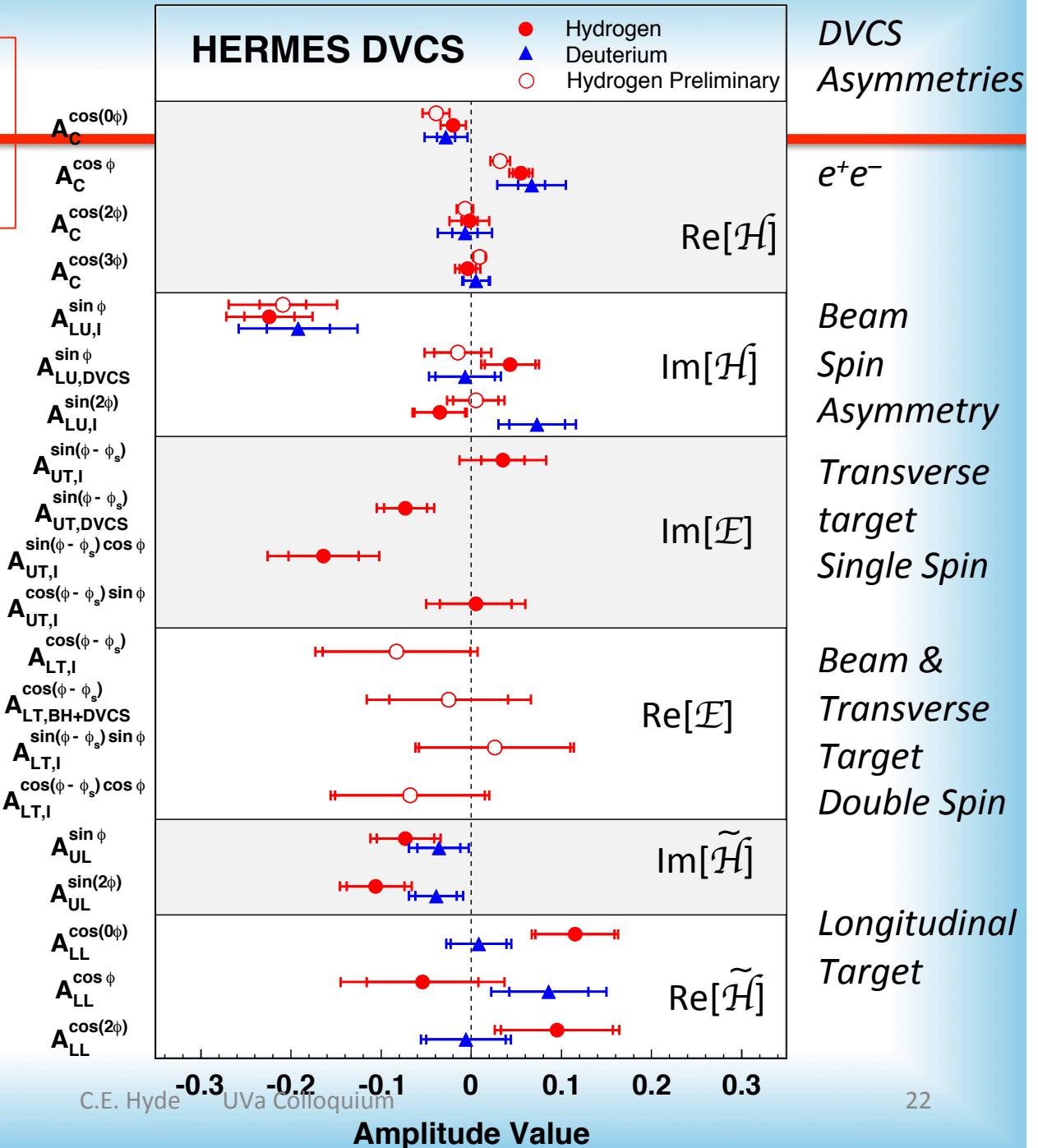
96-00 (H/D) Lpol + Upol  
02-05 (H) Tpol + Upol  
06-07 (H/D) Upol+Recoil



# HERMES

## summary

- averaged over  $Q^2$  and  $t$
- Transversely polarized  $H^-$  target → sensitivity to  $E(\xi, \xi, \Delta^2)$ ,  $\xi \approx 0.1$

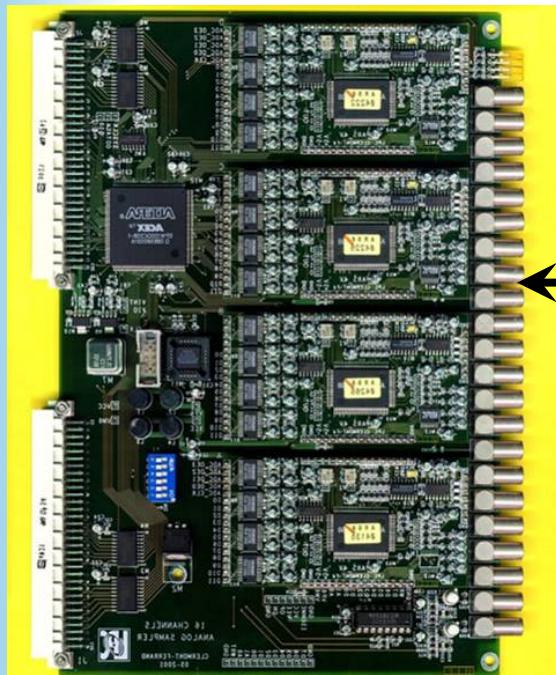


# DVCS: JLab Hall A 2004, 2010, 2014-2016

$L \geq 10^{37} \text{ cm}^2/\text{s}$

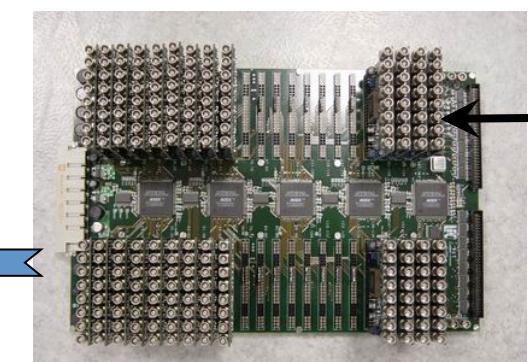
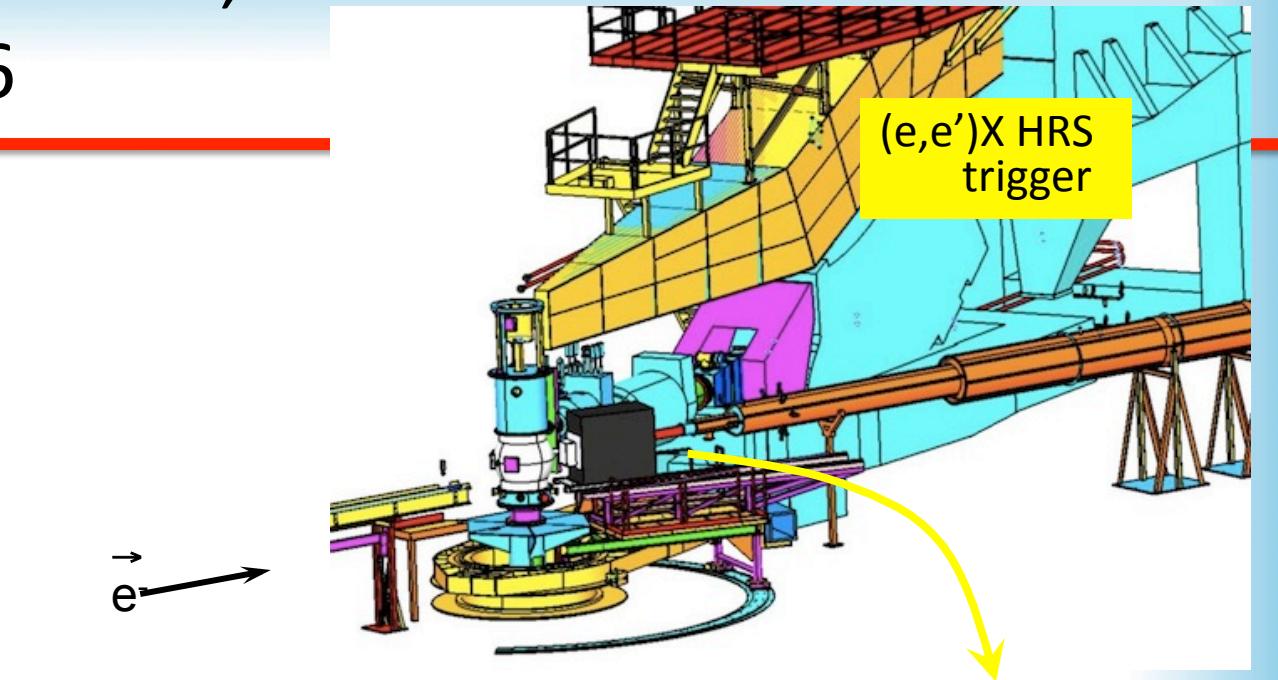
Precision cross sections

- Test factorization
- Calibrate Asymmetries

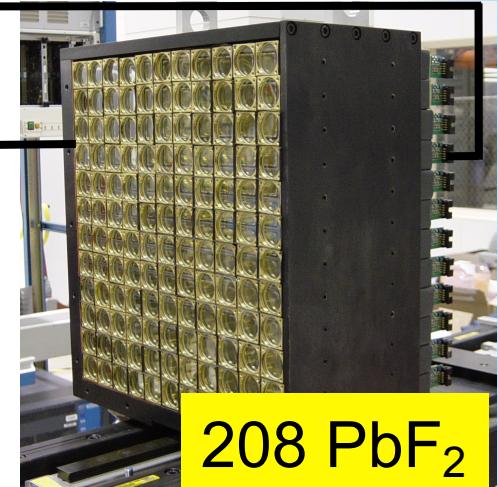


16chan VME6U: ARS  
128 samples@1GHz

4 Dec 2015



Digital Trigger  
Validation

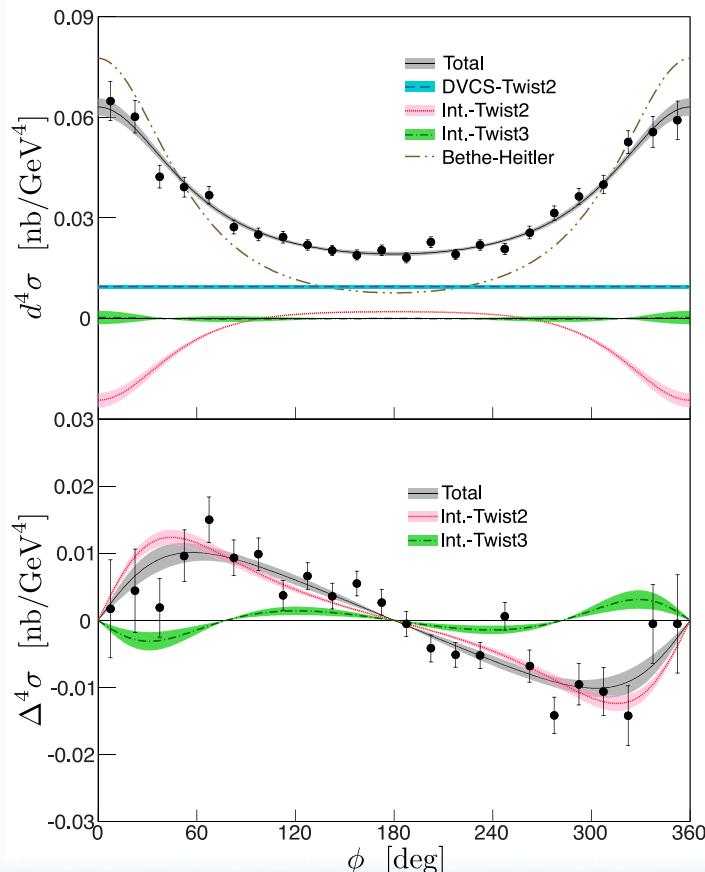


208 PbF<sub>2</sub>

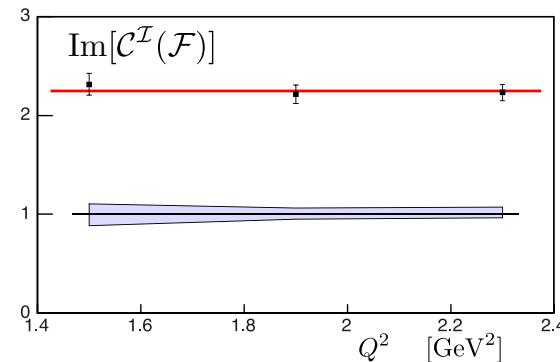
# Hall A Results: Scaling Tests

- $Q^2 = 2.3 \text{ GeV}^2, x_{Bj} = 0.36, t = -0.23 \text{ GeV}^2$

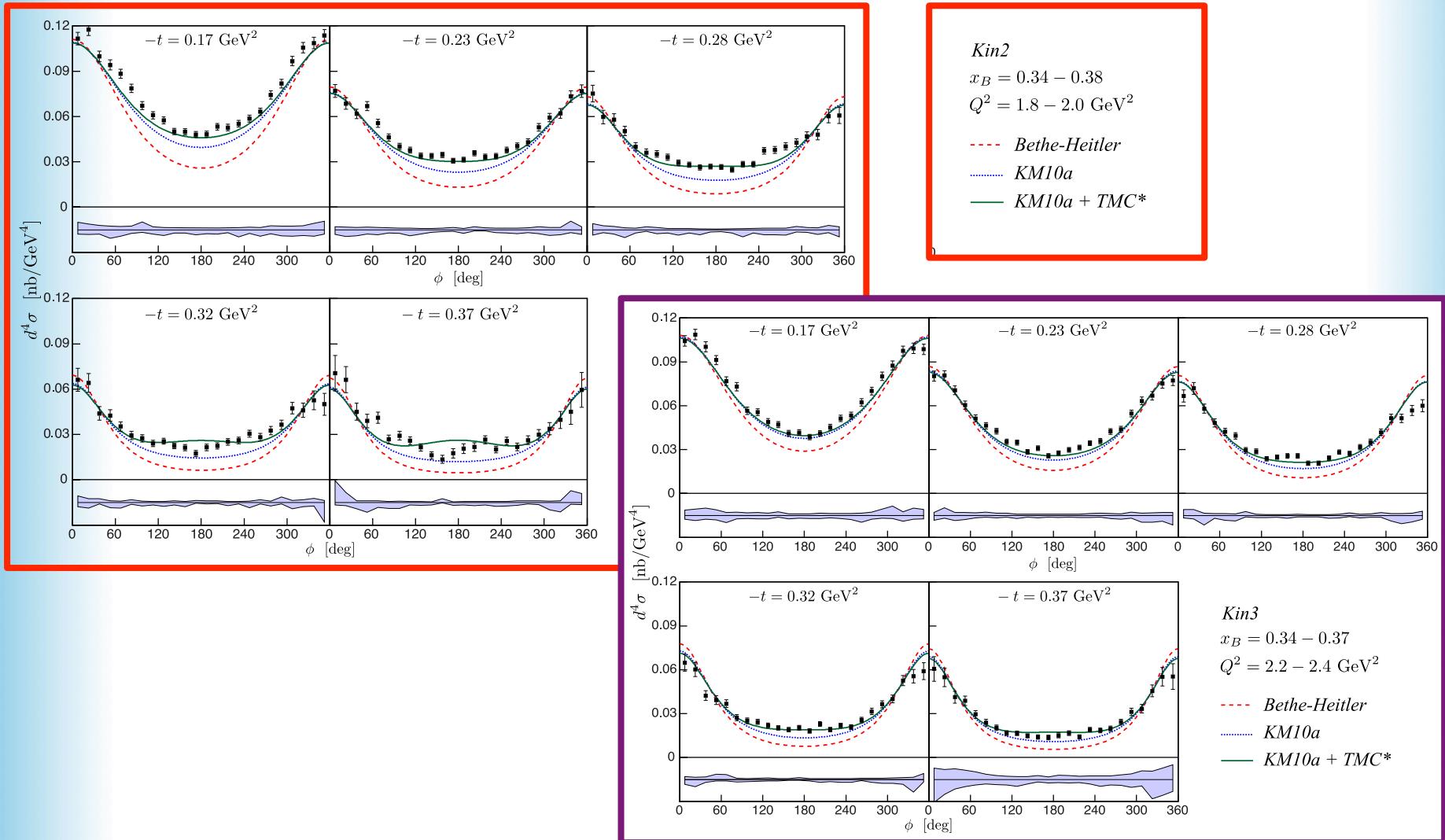
PRL 97:262002 (2006)  
*C. Muñoz Camacho, et al.*,  
PRC 92, 055202 (2015)  
M. Defurne, et al.,



- Empirical extraction
  - Leading-twist (GPD);
  - Higher-twist terms
- Test  $Q^2$ -independence of GPD terms

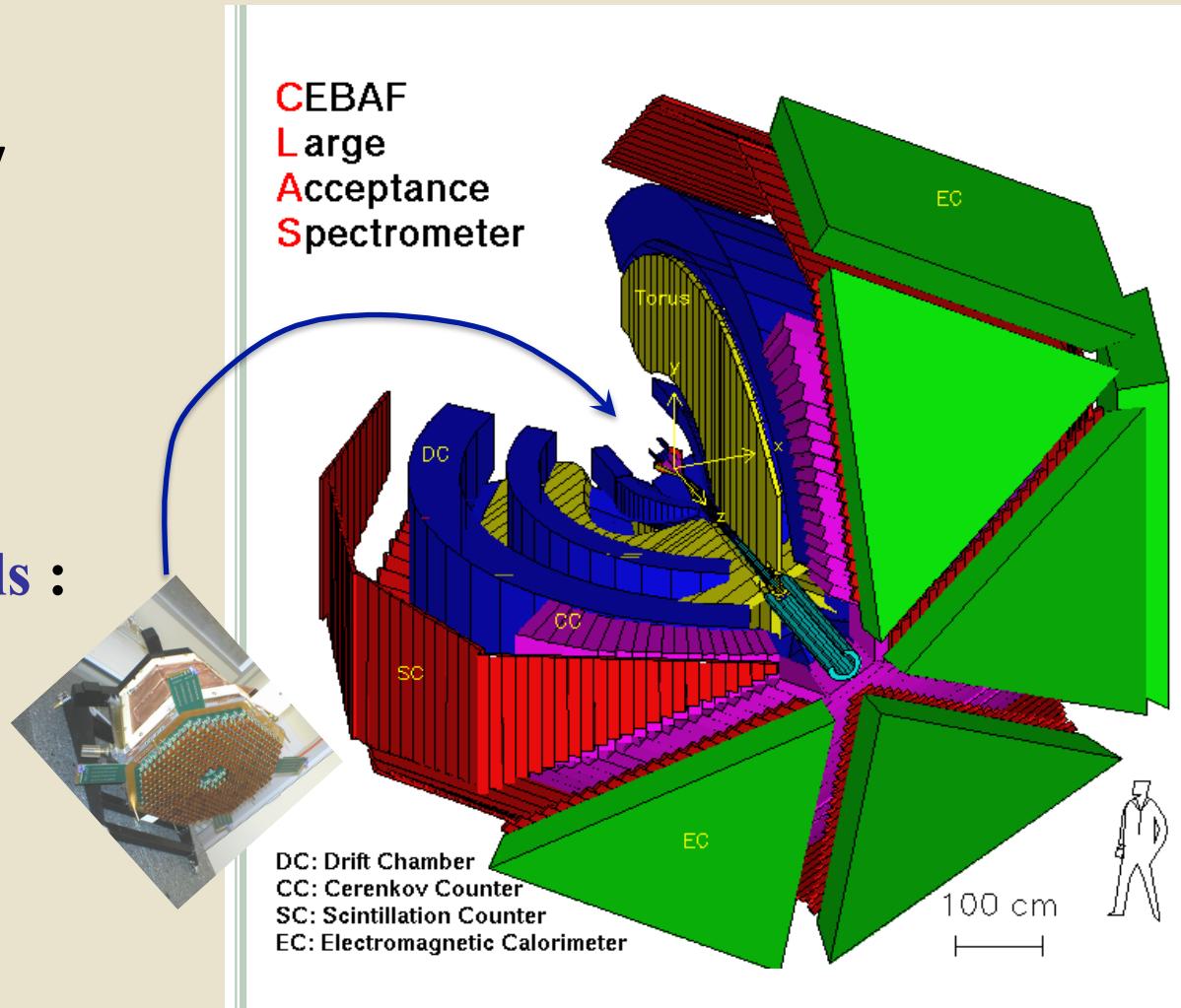


# Unpolarized Cross Sections: Model Consistency → Scaling

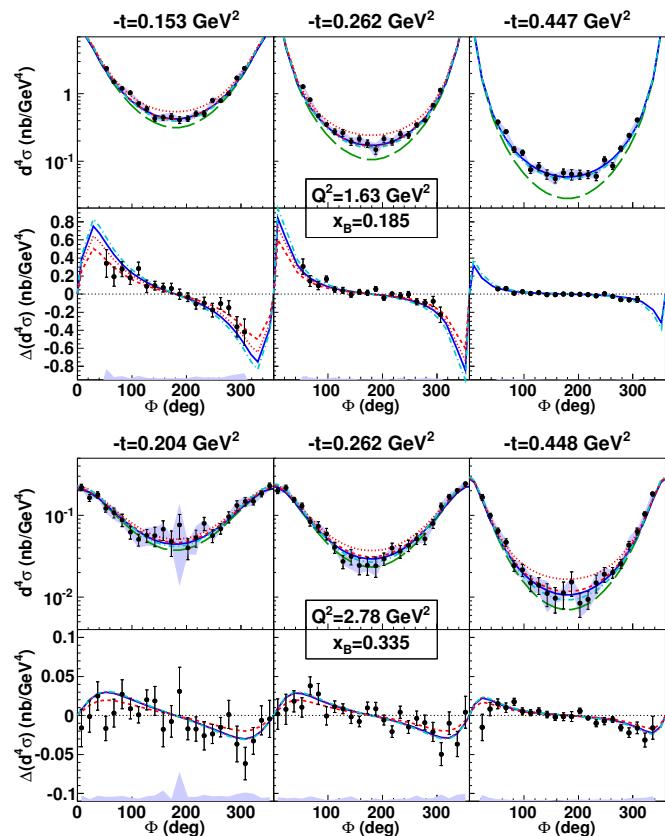


# DVCS in CLAS @ 6 GeV

- $H(e, e'\gamma p)$
- Longitudinally polarized  $\text{NH}_3$  target.
- Add:  
**5 Tesla Solenoid**  
**420  $\text{PbWO}_4$  crystals :**  
 $\sim 10 \times 10 \times 160 \text{ mm}^3$   
**APD+preamp readout**  
**Orsay / Saclay / ITEP / Jlab**



# CLAS DVCS (unpolarized Target)



H.S. Jo, F.-X Girod, *et al.*,  
Phys.Rev.Lett. 115  
(2015) 21, 212003

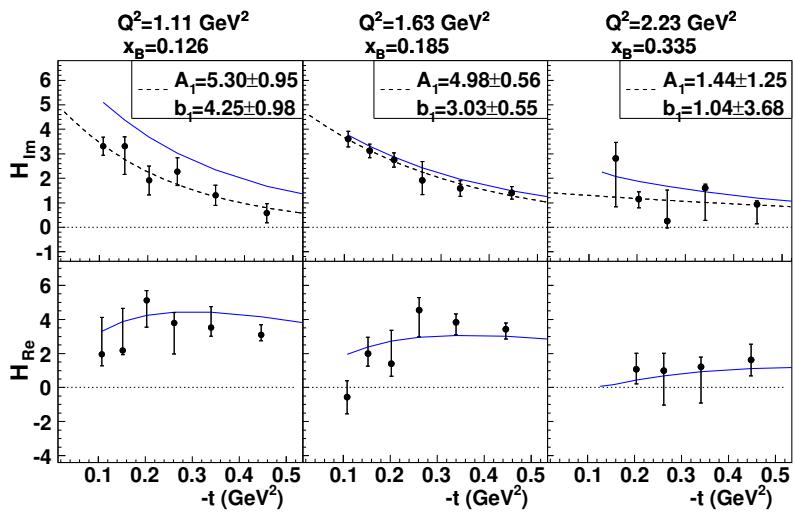
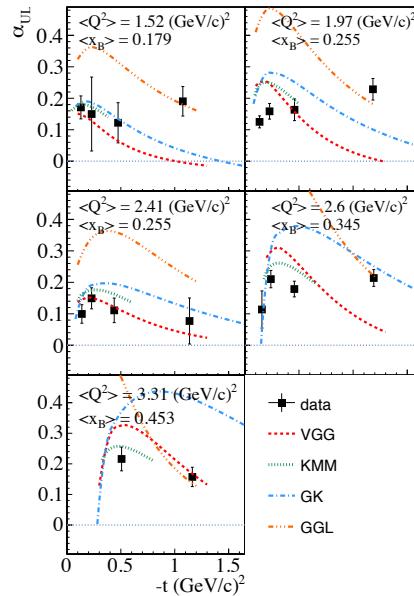
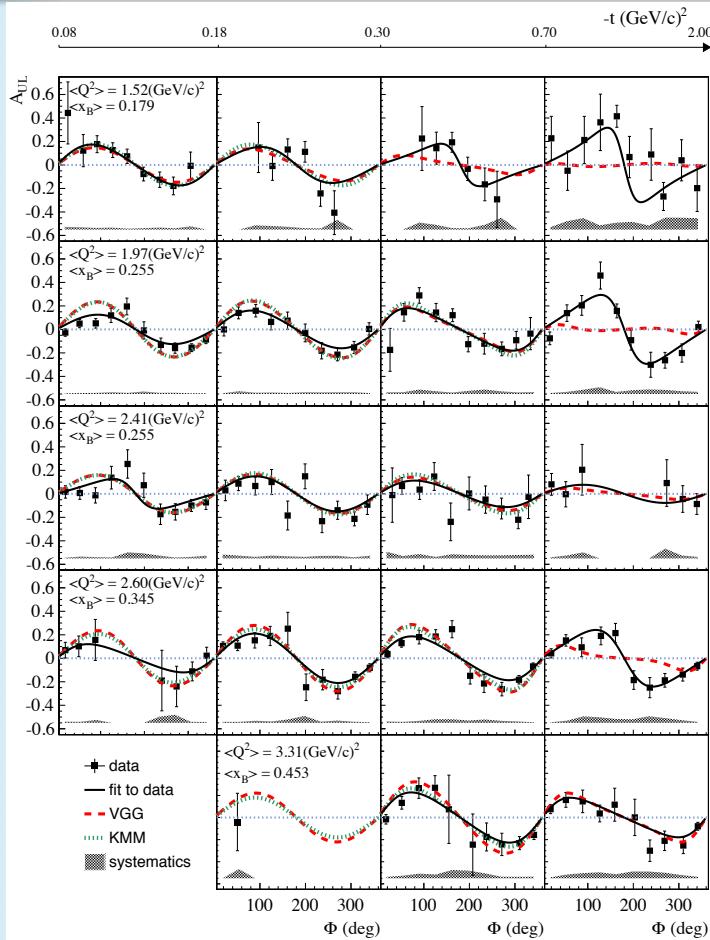


FIG. 5. (Color online) Results of the CFF fit of our data

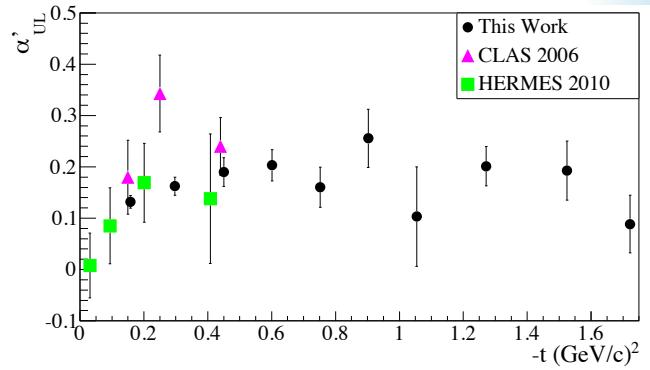
Model-dependent extraction of  
Re and Im parts of the  $H(\xi, \xi, t)$   
Compton form Factor  
(unpolarized GPD)

# CLAS: Longitudinally Polarized Protons Target-Spin Asymmetries

$A_{UL}$



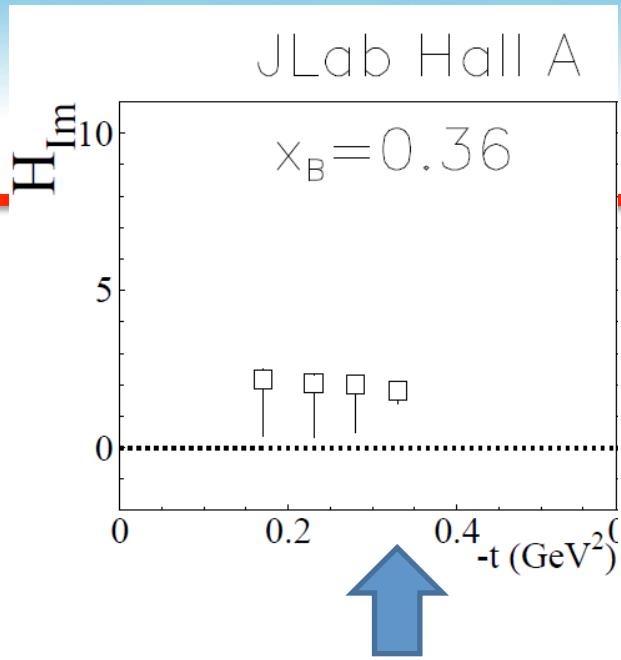
S. Pisano, *et al.*,  
Phys. Rev. D91  
(2015) 5, 052014



- Spatial distribution of quark helicity
- On to to 11 GeV!

# Global analyses of GPD data

- K. Kumericki, D. Mueller, M. Murray,
  - arXiv:1301.1230 hep-ph, arXiv:1302.7308 hep-ph
- M. Guidal, H. Moutarde,
  - EPJA **42** (2009) 71.
- P. Kroll, M. Diehl
  - Eur.Phys.J. **C73** (2013) 2397
- M. Guidal,
  - PLB **689** (2010) 159, PLB **693** (2010) 17.
- S. Liutti, G. Goldstein,
  - Phys.Rev. D84 (2011) 034007
- LO or NLO implemented
- Dynamic twist-3 formalism known, not implemented in global analysis yet.
  - Finite  $-t/Q^2$ ,  $M^2/Q^2$  corrections up to kinematic twist-4.  
V. Braun, *et al*, Phys.Rev. D89 (2014) 074022.



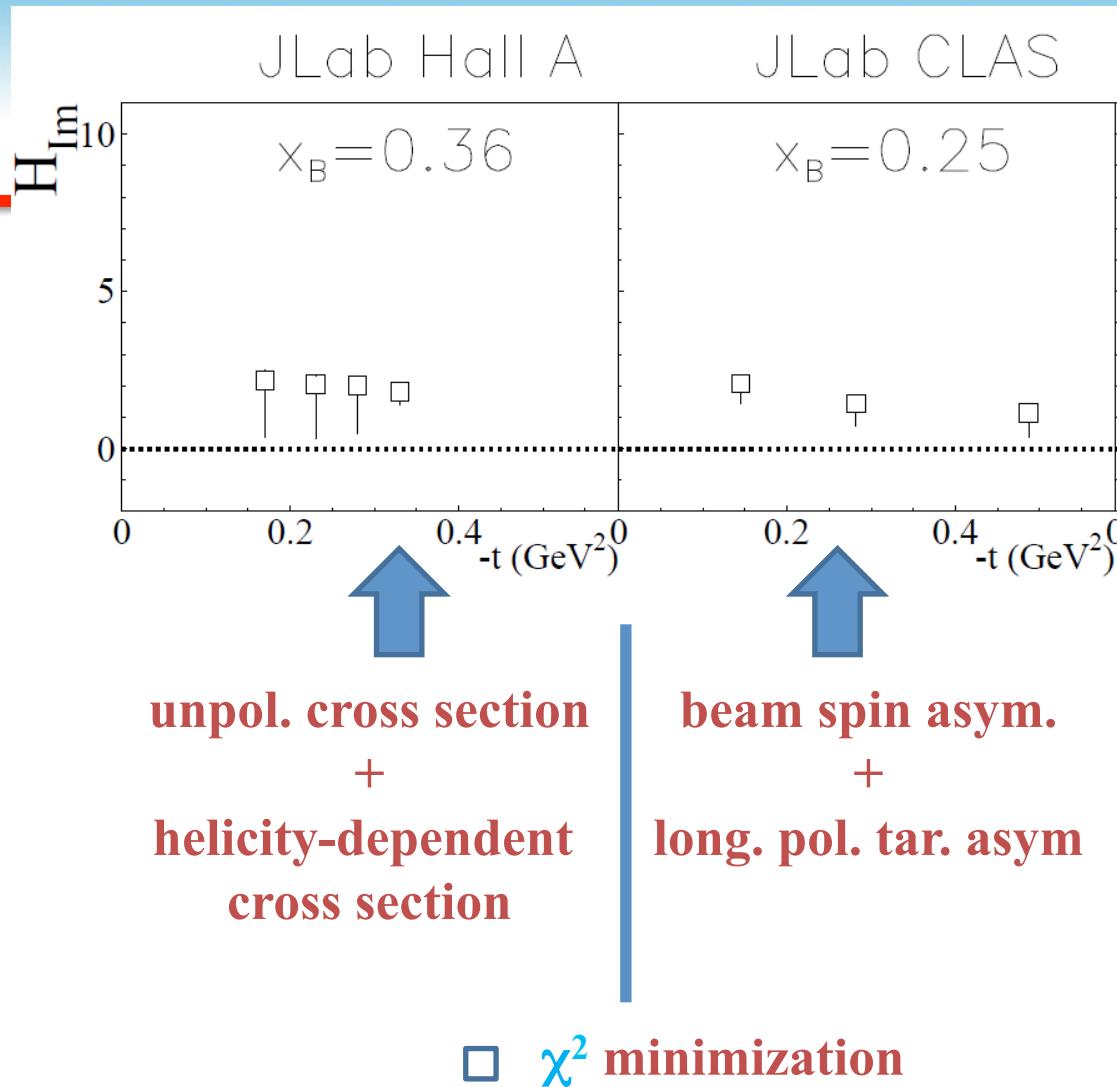
unpol. cross section

+

helicity-dependent  
cross section

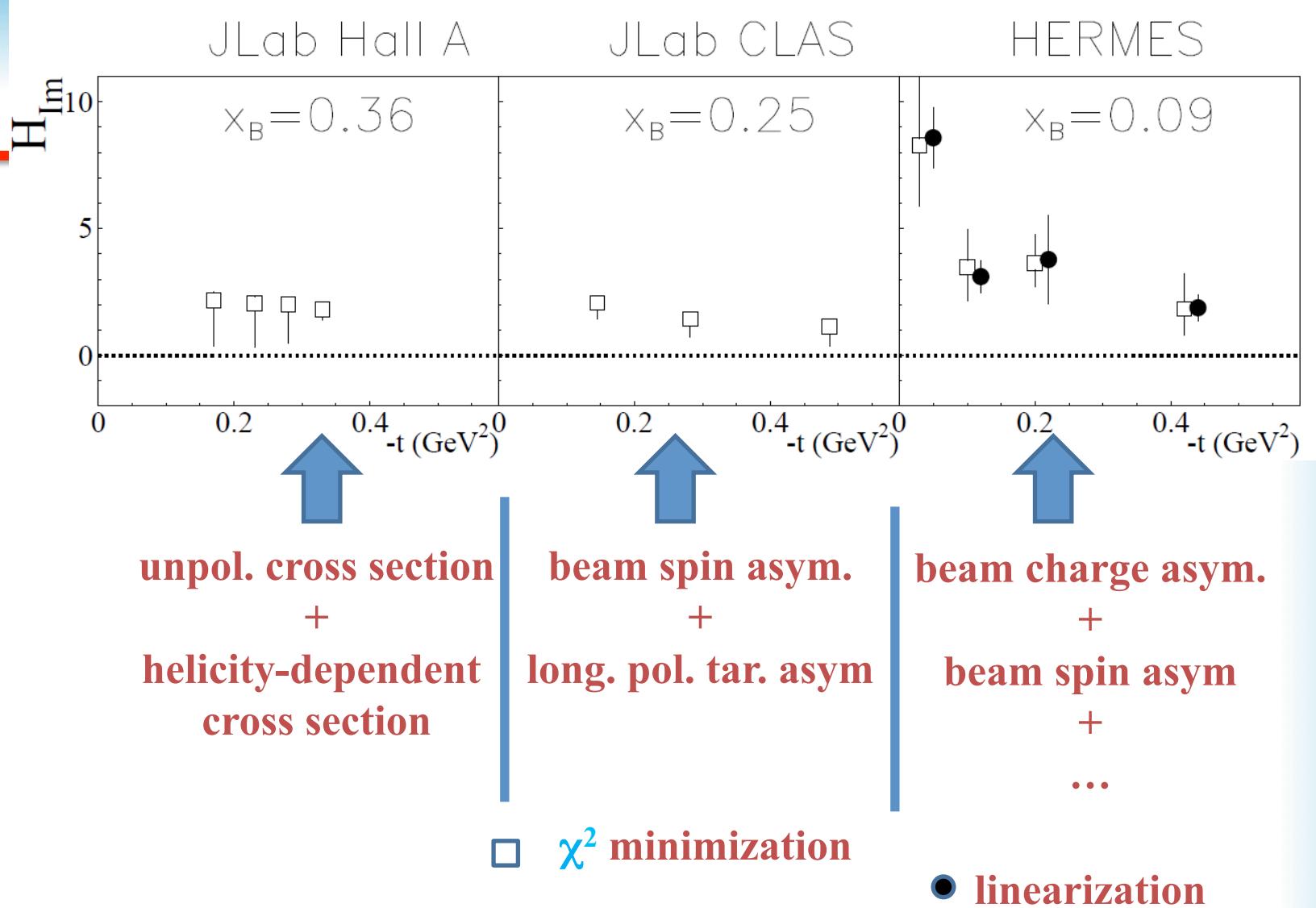
$\square \chi^2$  minimization

$$\Delta\sigma_{LU} \sim \sin\phi \operatorname{Im}\{F_1 \mathcal{H} + \xi(F_1 + F_2) \tilde{\mathcal{H}} - kF_2 \mathcal{E}\} d\phi$$



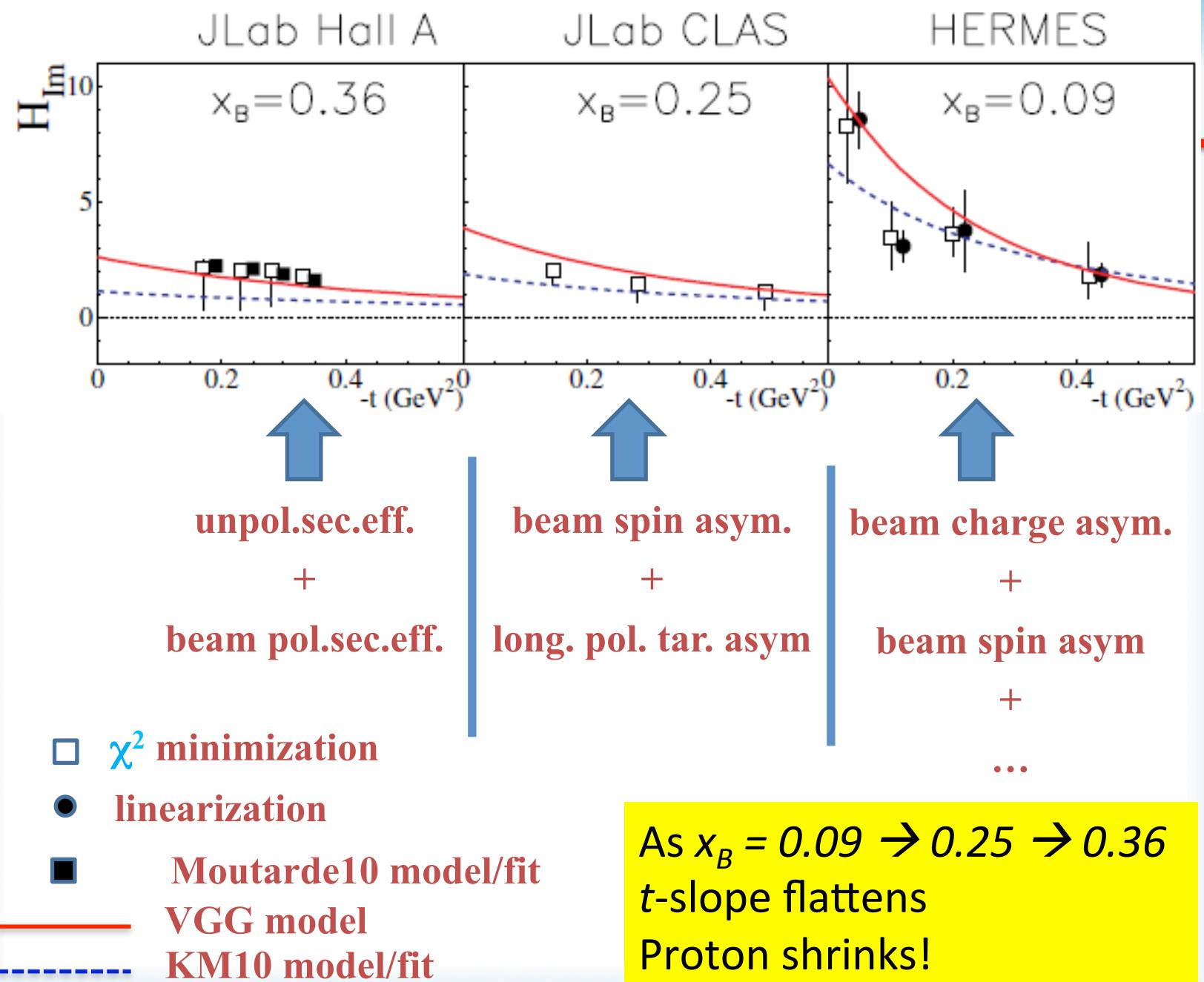
$$\Delta\sigma_{LU} \sim \sin\phi \operatorname{Im}\{F_1\mathcal{H} + \xi(F_1+F_2)\widetilde{\mathcal{H}} - kF_2\mathcal{E}\} d\phi$$

$$\Delta\sigma_{UL} \sim \sin\phi \operatorname{Im}\{F_1\widetilde{\mathcal{H}} + \xi(F_1+F_2)(\mathcal{H} + x_B/2\mathcal{E}) - \xi k F_2 \widetilde{\mathcal{E}} + \dots\} d\phi$$



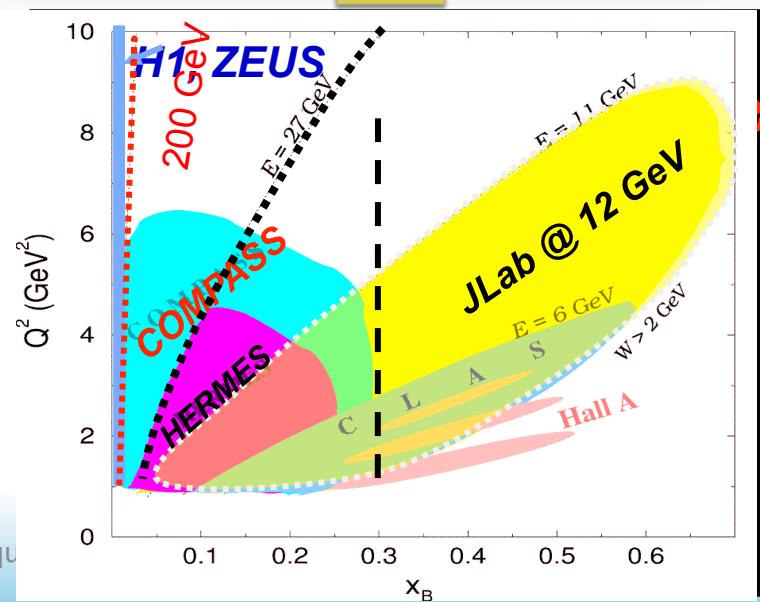
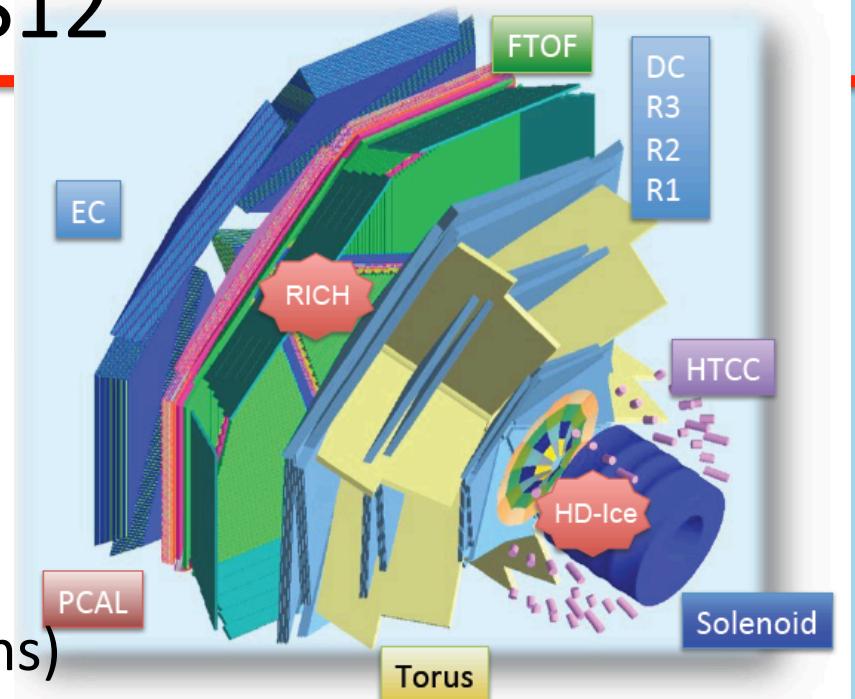
$$\Delta\sigma_{LU} \sim \sin\phi \operatorname{Im}\{F_1 \mathcal{H} + \xi(F_1 + F_2) \mathcal{L} - kF_2 \mathcal{E}\} d\phi$$

$$\Delta\sigma_{UL} \sim \sin\phi \operatorname{Im}\{F_1 \mathcal{H} + \xi(F_1 + F_2)(\mathcal{H} + x_B/2\mathcal{E}) - \xi kF_2 \mathcal{E} + \dots\} d\phi$$



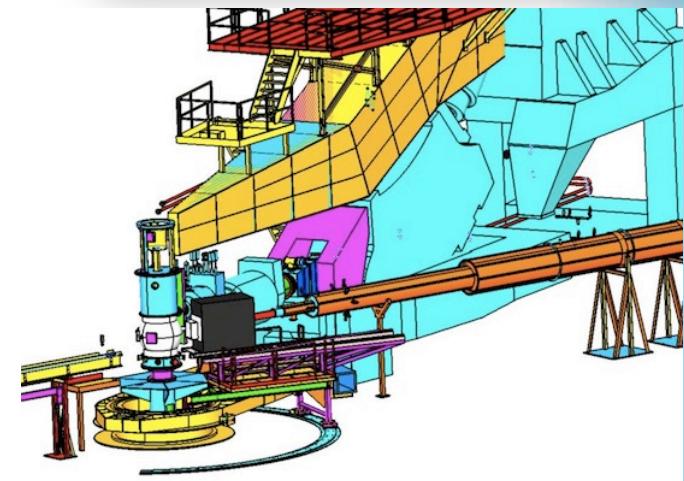
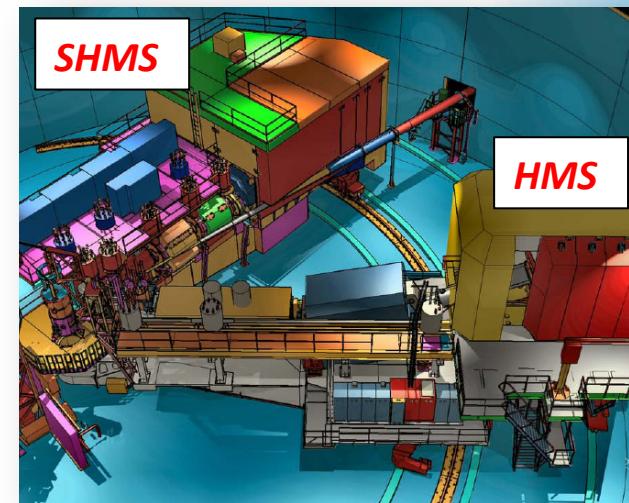
# Hall B @ 11 GeV: CLAS12

- Wide kinematic coverage
- Over-complete exclusivity
- High multiplicity mesonic final states
  - $\rho \rightarrow \pi\pi$ ,  $\omega \rightarrow \pi\pi\pi$ ,  $\phi \rightarrow KK$
- Timelike Compton Scattering (TCS)
  - $\gamma p \rightarrow p e^+ e^-$  (Quasi-real Photons)
  - $\gamma p \rightarrow p J/\Psi$
- Cross section systematic errors 5-10%
- Longitudinally polarized  $\text{NH}_3$ ,  $\text{ND}_3$
- Transversely polarized HD-ice target in development



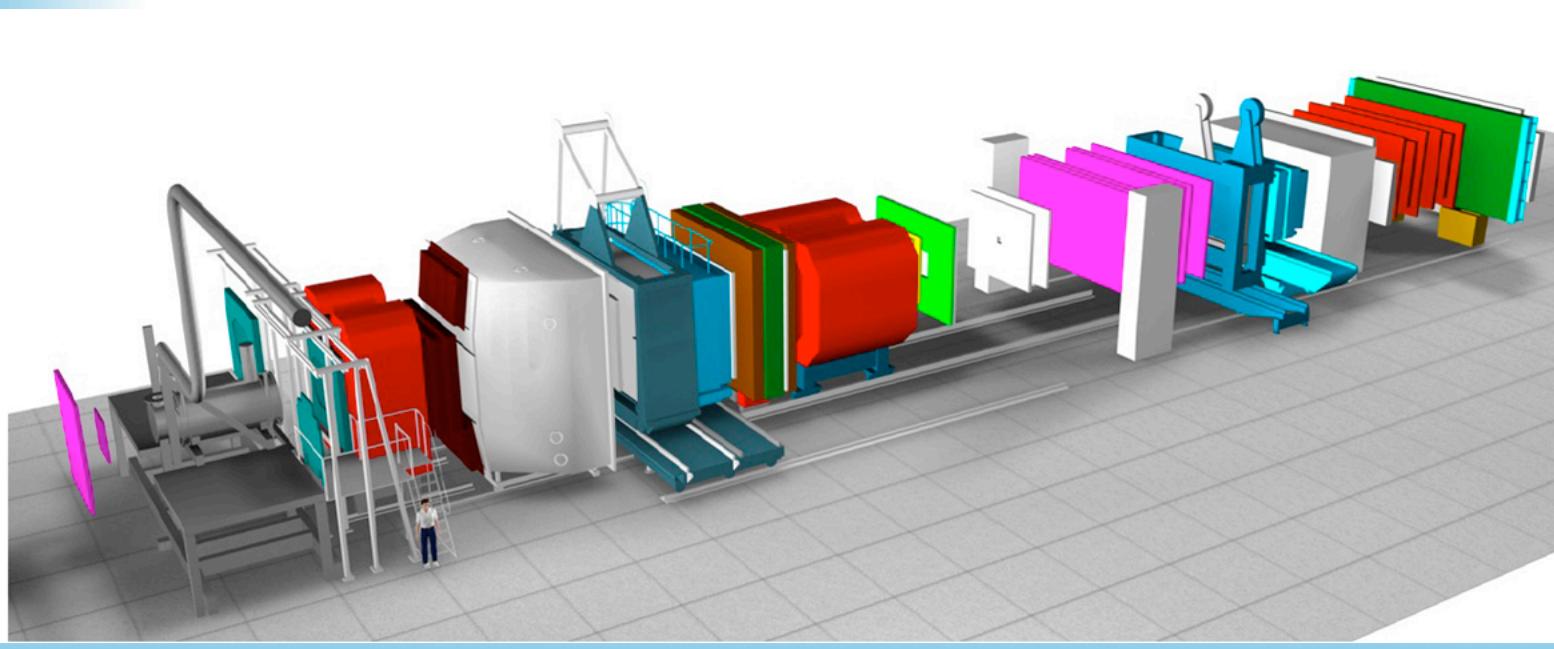
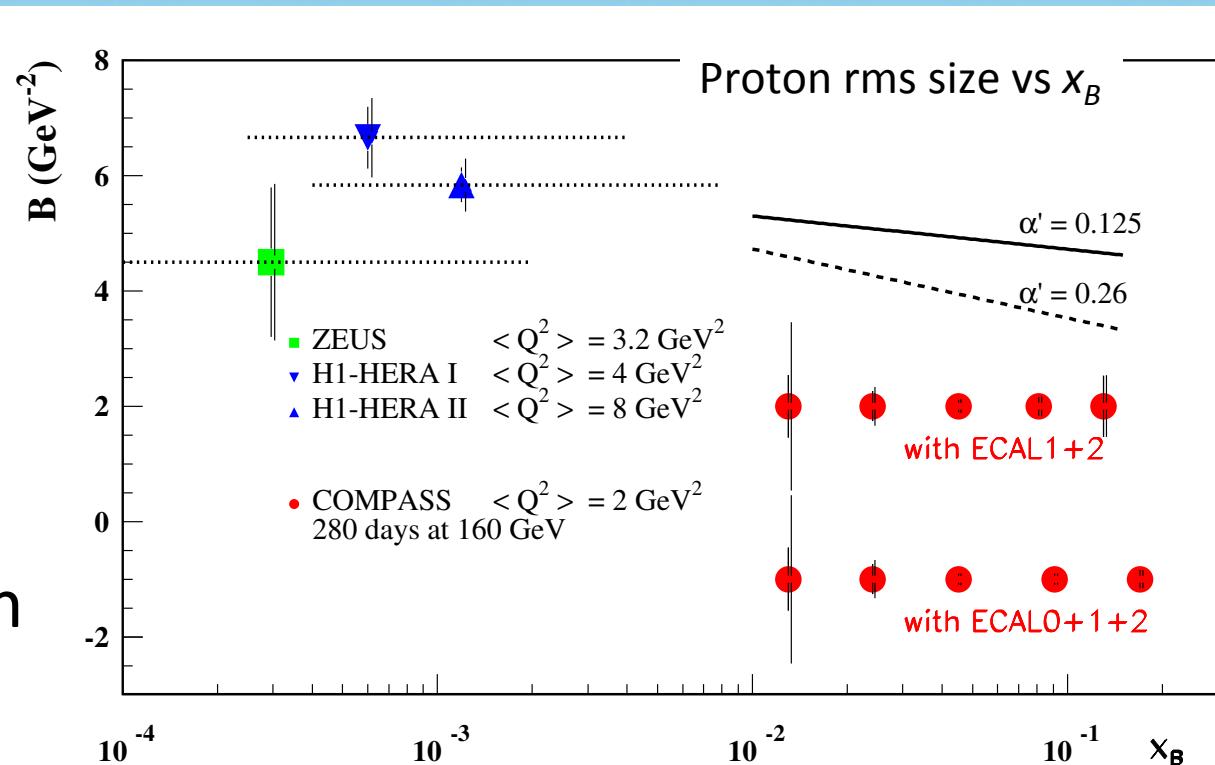
# GPDs at JLab: Halls A & C @ 11 GeV

- Hall C: 12 GeV: HMS x SHMS
  - L/T separations:  
 $H(e, e' \pi^+)n$ ,     $H(e, e' K^+)\Lambda$
- Halls A & C:  
Spectrometer x Calorimeter
  - DVCS & Exclusive  $\pi^0$ .
    - Hall A 2014-2015
  - $H(e, e' \gamma)p$        $H(e, e' \gamma\gamma)p$ 
    - Exclusivity by missing mass
  - $d\sigma$  systematic errors  $\leq 4\%$
  - Polarized  ${}^3\text{He}$  (L & T) possible



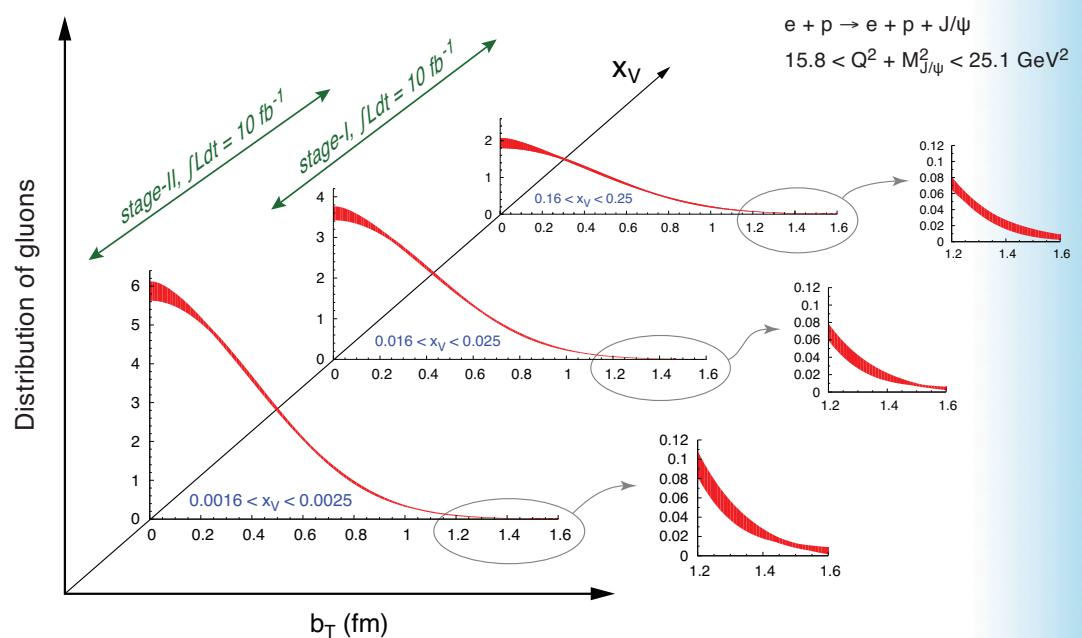
# COMPASS

- 200 GeV  $\mu^+ \uparrow, \mu^- \downarrow$
  - $\mathcal{L} = 10^{32} / \text{cm}^2/\text{s}$
  - 2016+
- After Drell-Yan run



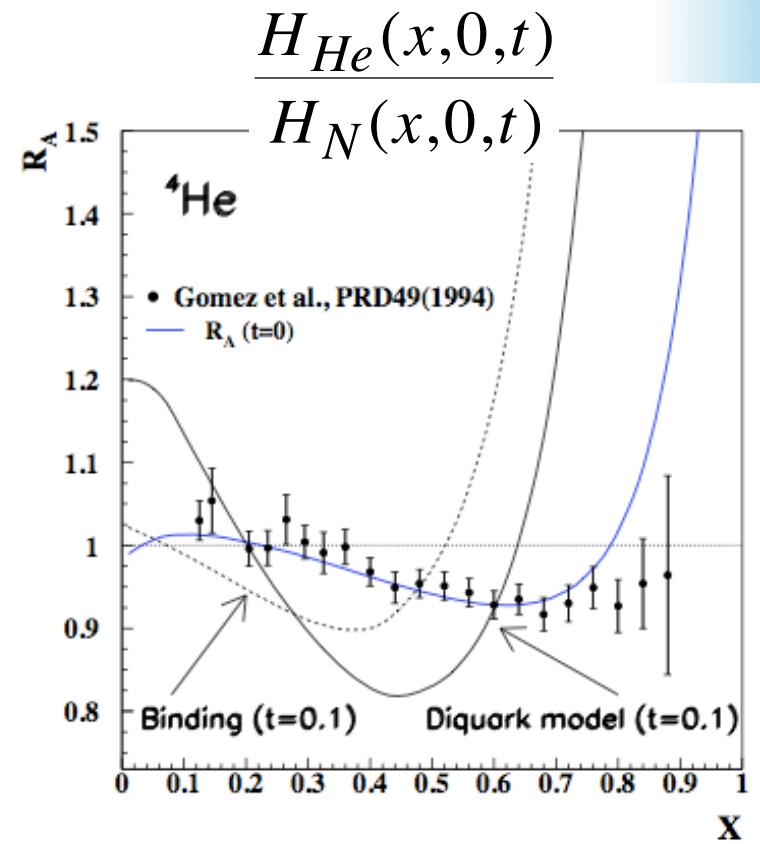
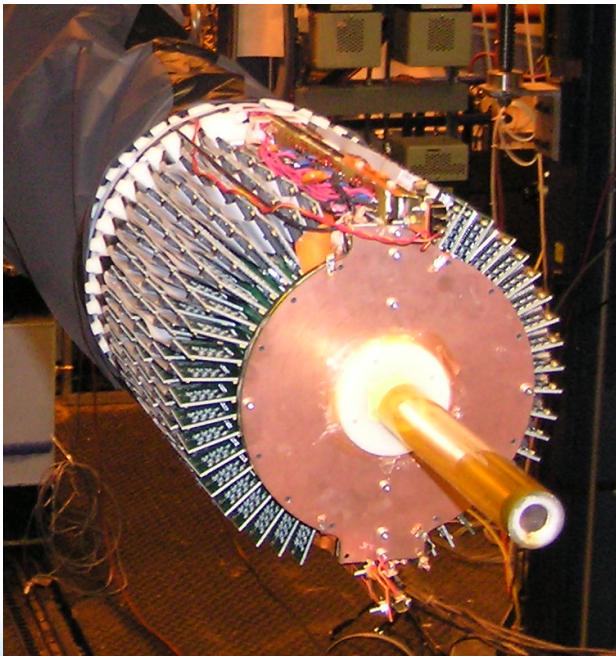
# Electron-Ion Collider

- JLab and BNL designs
  - $s = 2k(E+P) \geq 4000 \text{ GeV}^2$
  - $\mathcal{L} > 10^{33} / \text{cm}^2/\text{sec}$  (proton)
- Longitudinal and transversely polarized light ion beams
- Unpolarized from H to U
- Spectator tagging: D,  ${}^3\text{He}$ 
  - Inclusive and Deep virtual
- ArXiv 1212.1701



# Nuclei Too! CLAS: ${}^4\text{He}(e,e'\gamma\alpha)$

- Only one GPD on spin 0 target
  - $H(\xi, \xi, t) = (4/9)H_u + (1/9) H_{u'}$
  - $G_E = \int dx [(2/9)H_u - (1/9)H_{u'}]$ .
  - EMC effect on GPDs
  - BoNuS GEM radial TPC



$[t=0.0] \rightarrow$  EMC effect,  
 $[t=-0.1] \rightarrow$  GPD  
 (Liuti & Taneja, Guzey &  
 Strickman)

# Conclusions

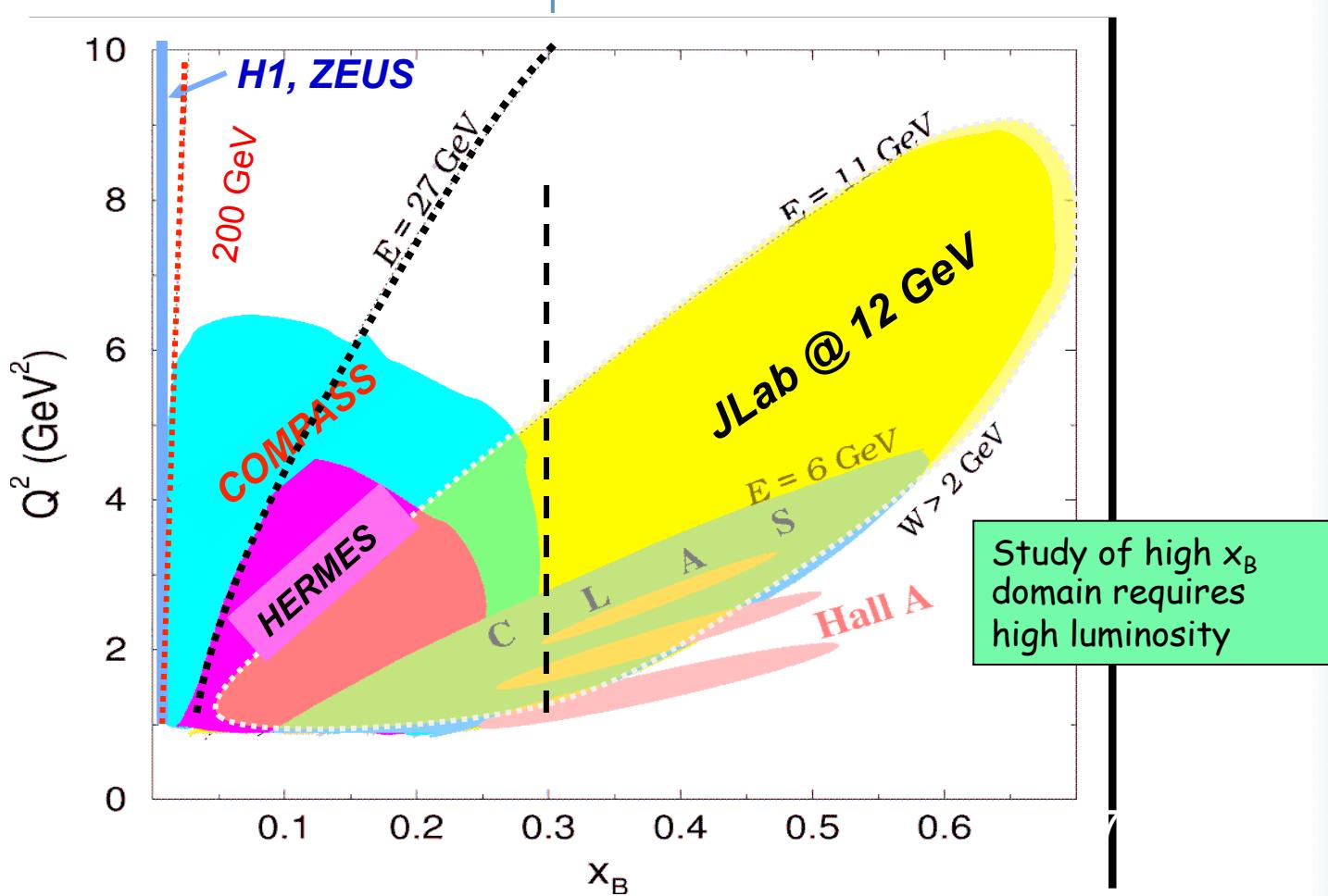
- Spatial Imaging *is* possible (in 1+2 dimensions)
- New experimental and theoretical tools are helping us to understand how QCD generates
  - The mass of ordinary matter (98%)
  - The spin of the hadrons: proton, neutron, vector mesons...
    - proton spin ~25% from spin of quarks
    - How much is gluons? How much is Orbital Angular Momentum.
  - Spatial distribution of charge and matter in hadrons.  
(non-trivial flavor, momentum-fraction dependence)
  - Nuclear Binding
    - Why is the deuteron ( $np$ ) bound but  $nn$  not?
    - Why are  ${}^4\text{He}$ ,  ${}^6\text{He}(\beta^- \text{ 1sec})$ ,  ${}^8\text{He}(\beta^- \text{ 0.1sec})$  bound, but not  ${}^5\text{He}$  ?

# Backup Slides

TCS

Deep Virtual Meson Production

# Deeply Virtual Exclusive Processes - Kinematic Coverage



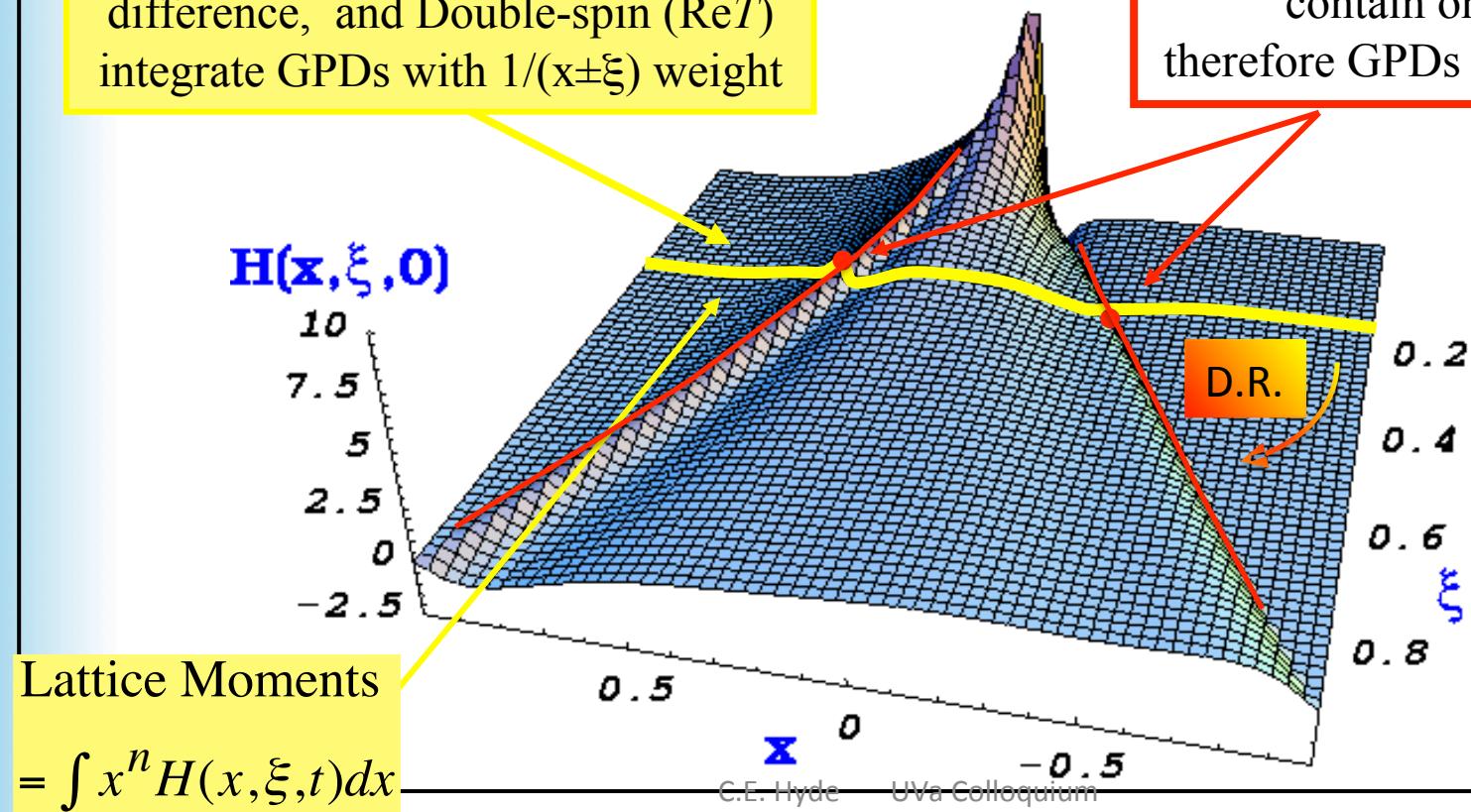
## DVCS, GPDs, Compton Form Factors(CFF), and Lattice QCD

(at leading order:)

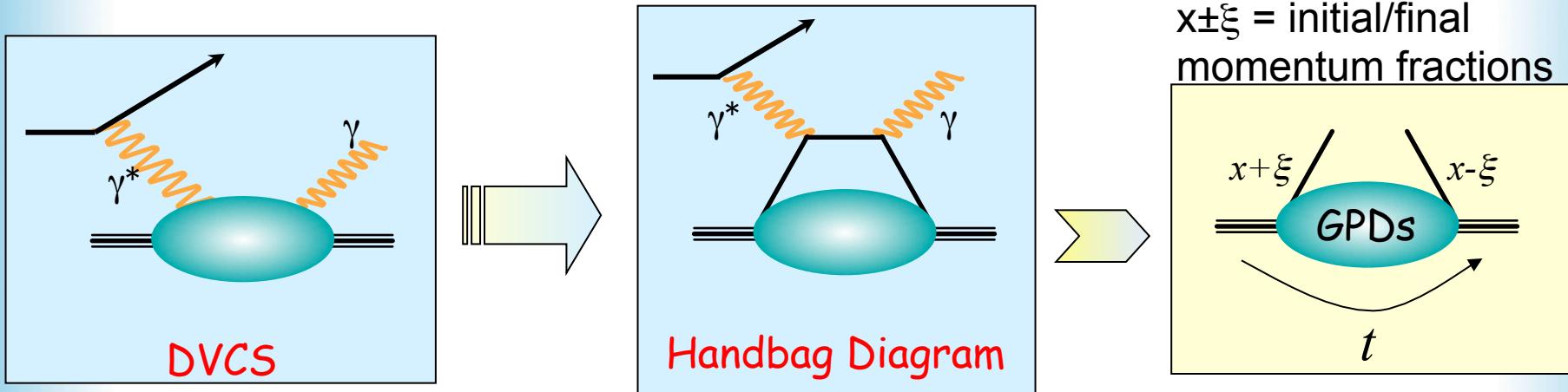
$$T^{DVCS} \sim \int_{-1}^{+1} \frac{H(x, \xi, t)}{x \pm \xi + i\epsilon} dx + \dots \sim P \int_{-1}^{+1} \frac{H(x, \xi, t)}{x \pm \xi} dx - i\pi H(\pm \xi, \xi, t) + \dots$$

Cross-section ( $\sigma$ ), Beam-charge-difference, and Double-spin ( $\text{Re}T$ ) integrate GPDs with  $1/(x \pm \xi)$  weight

Beam or target spin  $\Delta\sigma$  contain only  $\text{Im}T$ , therefore GPDs at  $x = \xi$  and  $-\xi$



# Spatial distributions and orbital angular momenta of quarks: Deeply Virtual Compton Scattering (DVCS): $N(e, e'\gamma N)$



$$\begin{aligned} T_{DVCS}^{\mu\nu} &= \int dz e^{i(q+q')z} \langle p' | T \left[ J^\mu \left( \frac{z}{2} \right), J^\nu \left( -\frac{z}{2} \right) \right] | p \rangle \\ &\Rightarrow \int dz^- e^{i\bar{q}^+ z^- x} \left[ t^{\mu\nu} \langle p' | \bar{\psi} \left( \frac{z}{2} \right) \gamma^+ \psi \left( -\frac{z}{2} \right) | p \rangle + i \epsilon^{\mu\nu\rho\sigma} q_\rho q'_\sigma \langle p' | \bar{\psi} \left( \frac{z}{2} \right) \gamma^+ \gamma_5 \psi \left( -\frac{z}{2} \right) | p \rangle \right] \end{aligned}$$

$$q^+ = (q^0 + q^3)/\sqrt{2}$$

$$\gamma^+ = (\gamma^0 + \gamma^3)/\sqrt{2}$$

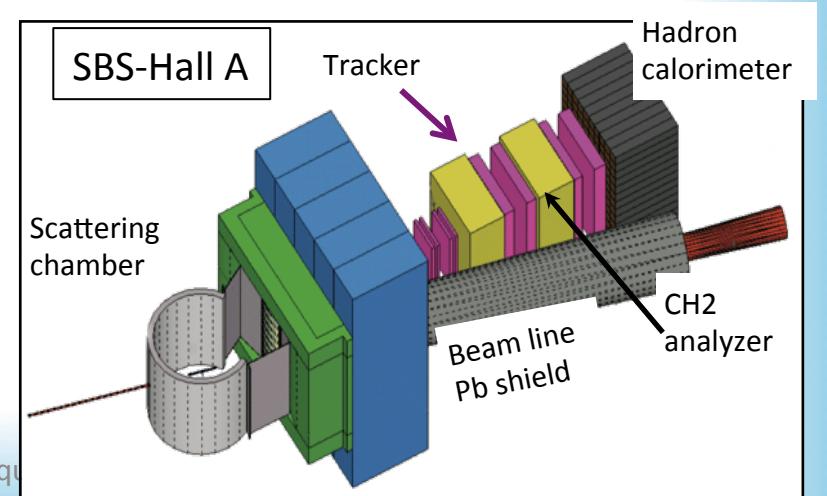
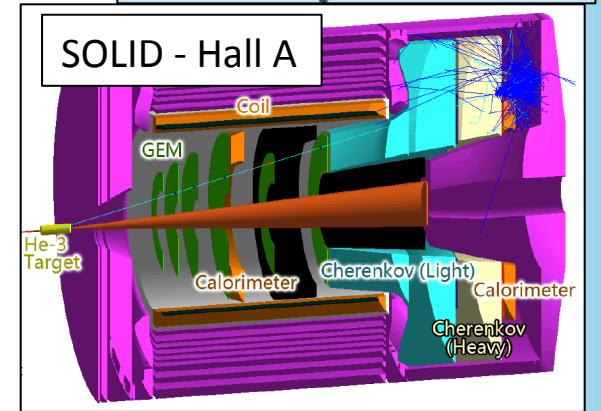
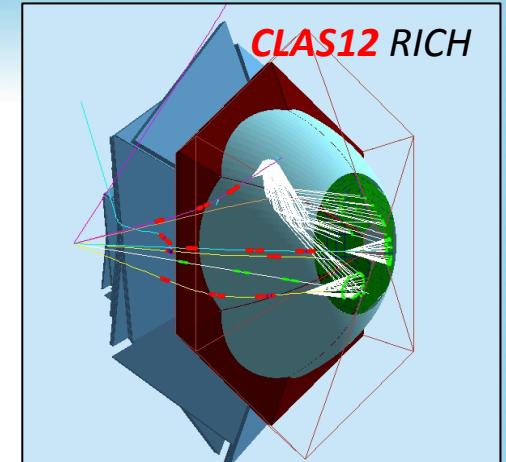
$$z^- = (t c - z^3)/\sqrt{2}$$

Generalized Parton Distributions: Mueller, Ji, Radyushkin,  
Factorization: Ji, Osborne, Collins, Freund...

# GPDs at JLab: Future Upgrades

(Mostly motivated by non-GPD topics)

- RICH Detector (partial) in CLAS 12:  
 $\pi/K$  id
  - INFN participation
- **Solenoidal Large Intensity Detector (SoLID)** in Hall A (CLEO Solenoid)
  - TCS,  $J/\Psi$
  - Chinese participation
- **Super BigBite Spectrometer**
  - Dipole from BNL
  - Funded, under construction
  - GEM trackers for high rates



# Time-Like Compton Scattering: $\gamma + p \rightarrow e^+ e^- p$

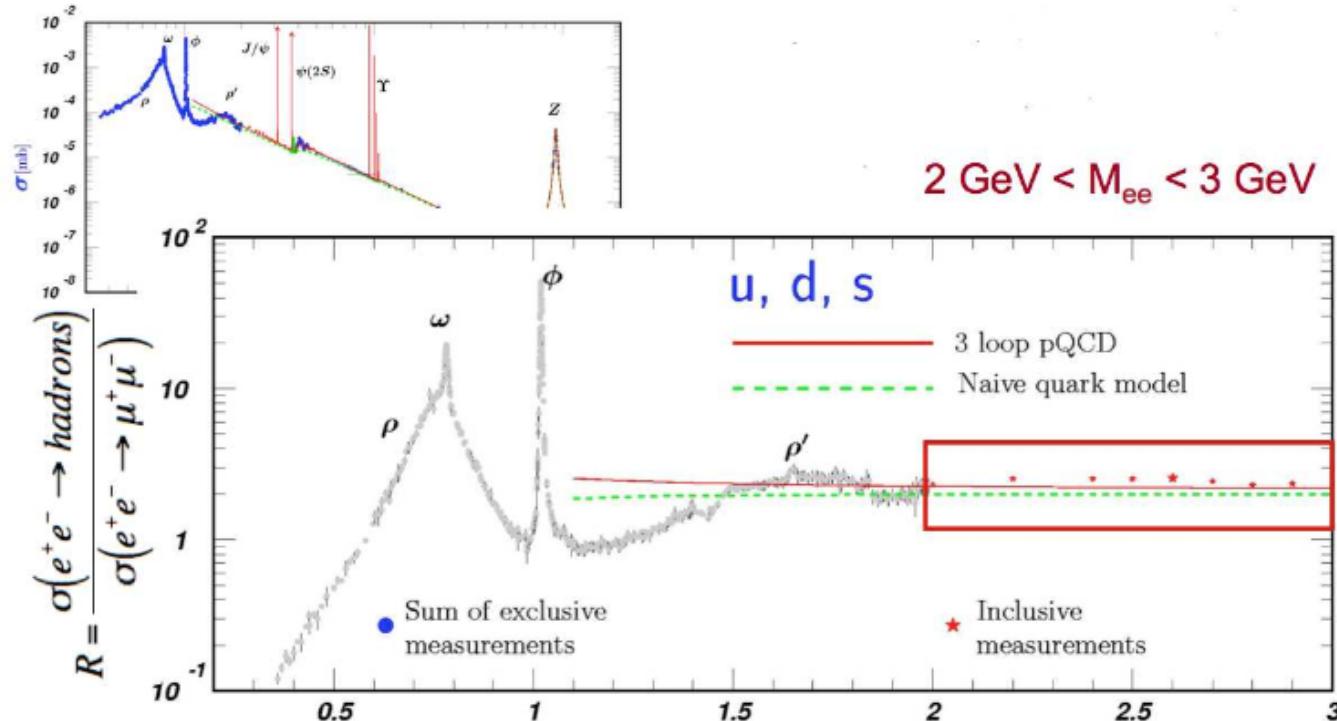
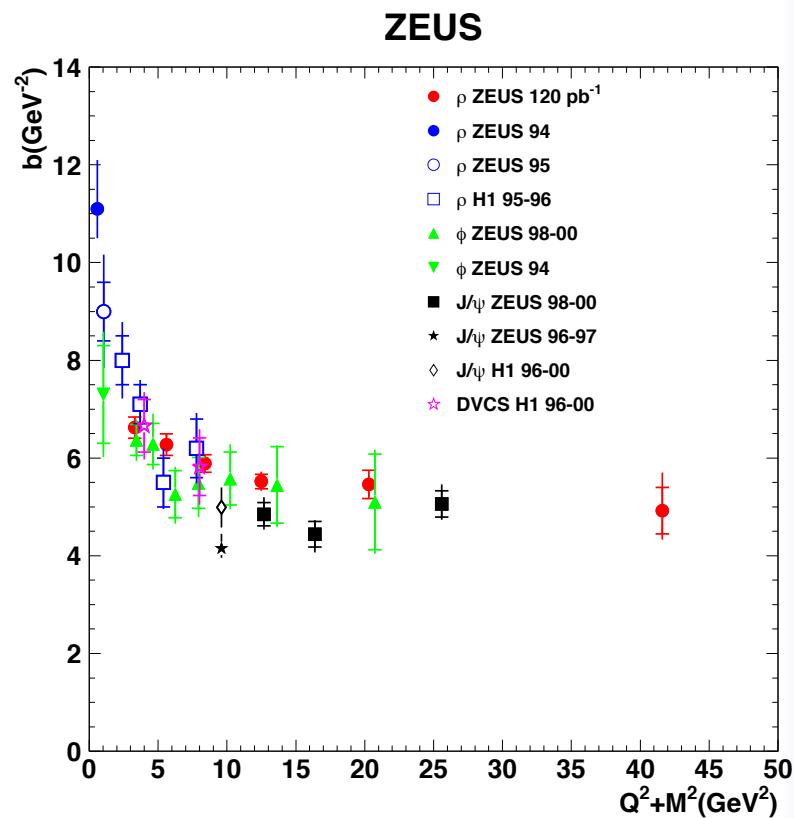


FIG. 4: Measurements of  $e^+ e^-$  annihilation into hadrons show a resonance-free window between the  $\rho'$  and the  $J/\psi$ , which is ideal for TCS studies at 12 GeV.

# Semi Universal behavior of exclusive reactions at high $W^2$

- Two views:
  - Extracting leading twist information is hopeless for  $Q^2+q'^2 < 10 \text{ GeV}^2$
  - Perturbative  $t$ -channel exchange even for modest  $Q^2$ , but convolution of finite size of nucleon and probe.
- Fitting data requires setting scale of gluon pdf  $\mu^2 \ll Q^2$ 
  - Finite transverse spatial size  $b \approx 1/\mu$  of  $\gamma \rightarrow V$  amplitude



GWolf 0907.1217

# Tomography with Generalized Parton Distributions (M. Burkardt)

- $H(x,t)\gamma^{\mu} + E(x,t)\sigma^{\mu\nu}\Delta_{\nu}$ 
  - Proton size shrinks as  $x \rightarrow 1$ .
  - Spatial separation of up- and down-quarks in a transversely polarized proton
- Spin-Flavor dependence to Proton size & profile.
  - M. Burkardt
  - up and down quarks separate in transversely polarized proton

$$\varepsilon_f(x, b_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} e^{i \Delta_{\perp} \cdot b_{\perp}} E_f(x, \Delta_{\perp})$$

$$q_X(x, b_{\perp}) = h_q(x, b_{\perp}) + \frac{1}{2M} \frac{\partial}{\partial y} \varepsilon_q(x, b_{\perp})$$

