Helicity Evolution at Small x

Matthew D. Sievert

BROOKHAVEN NATIONAL LABORATORY

High-Energy Physics Seminar

University of Virginia

Wednesday Jan. 27, 2016

References: 1511.06737
1505.01176

Collaborators: Y. Kovchegov
D. Pitonyak

Outline

<u>Introduction: Studying Proton Structure</u>

- Deep Inelastic Scattering and the Parton Model
- Quantum evolution and the small-x limit
- The Proton Spin Crisis: Is there spin at small x?

Outline

Introduction: Studying Proton Structure

- Deep Inelastic Scattering and the Parton Model
- Quantum evolution and the small-x limit
- The Proton Spin Crisis: Is there spin at small x?

The Toolbox: Quarks and the Small-x Limit

- TMD quark distributions at large and small x
- Coherence and quasi-classical initial conditions
- Small-x evolution

Outline

Introduction: Studying Proton Structure

- Deep Inelastic Scattering and the Parton Model
- Quantum evolution and the small-x limit
- The Proton Spin Crisis: Is there spin at small x?

The Toolbox: Quarks and the Small-x Limit

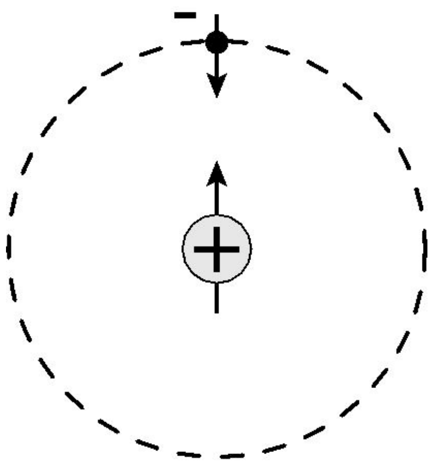
- TMD quark distributions at large and small x
- Coherence and quasi-classical initial conditions
- Small-x evolution

The Calculation: Helicity at Small x

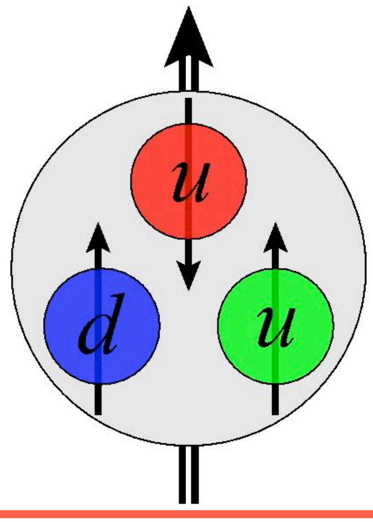
- Polarized initial conditions
- Evolving spin to small x
- The added complexity: Non-Ladder Diagrams

Introduction:
Studying Proton Structure

The Hydrogen Atom

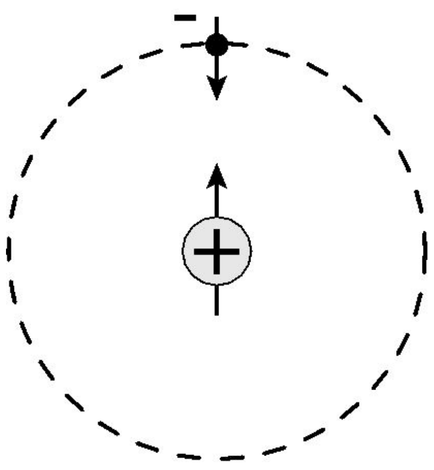


The Proton



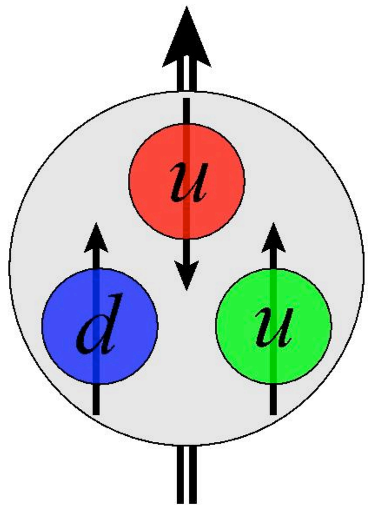
The Hydrogen Atom

- Elementary bound state of a proton and electron.
- Bound by QED interactions.



The Proton

- Elementary bound state of three quarks.
- Bound by QCD interactions.

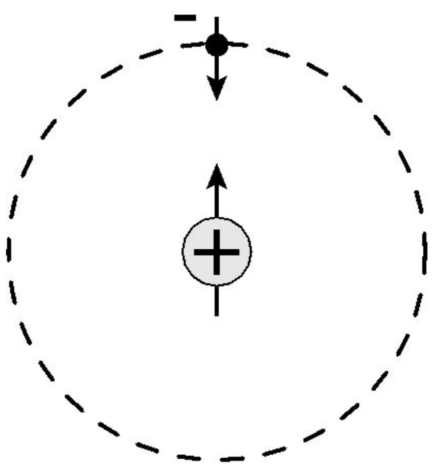


An Analogy: The Proton and the Atom

The Hydrogen Atom

- Elementary bound state of a proton and electron.
- Bound by QED interactions.
- Ground state is spherically symmetric with zero net angular momentum.

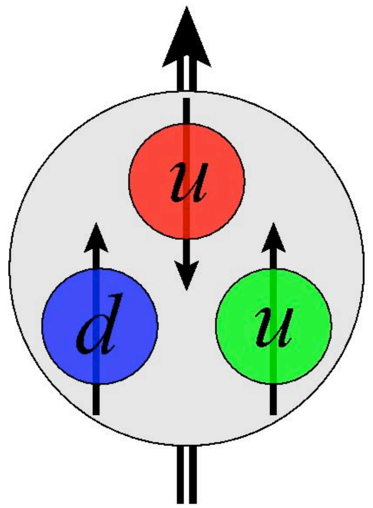
$$J, L, F = 0$$



The Proton

- Elementary bound state of three quarks.
- Bound by QCD interactions.
- Spin $\frac{1}{2}$ fermion can be accommodated by quark spin pairing.

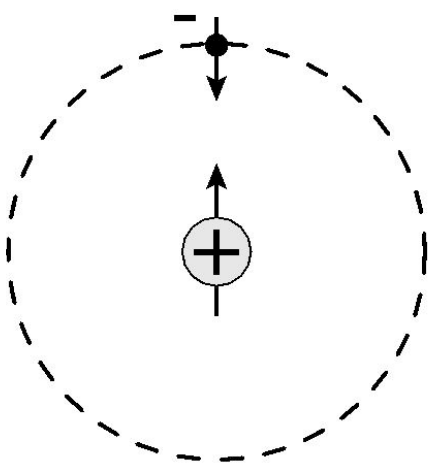
$$J = \frac{1}{2}$$



The Hydrogen Atom

- Hydrogen has complex structure: fine, hyperfine...

... but it is well described by the Bohr model because QED is a perturbative theory.

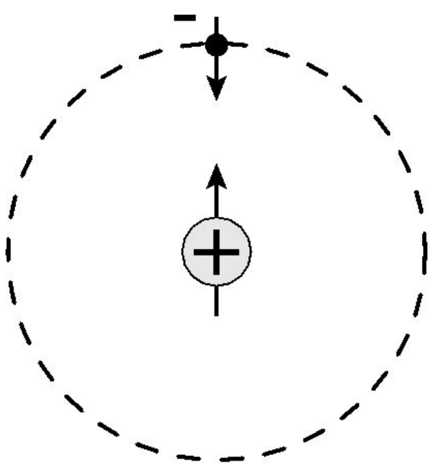


The Importance of Proton Structure

The Hydrogen Atom

- Hydrogen has complex structure: fine, hyperfine...

... but it is well described by the Bohr model
because QED is a perturbative theory.



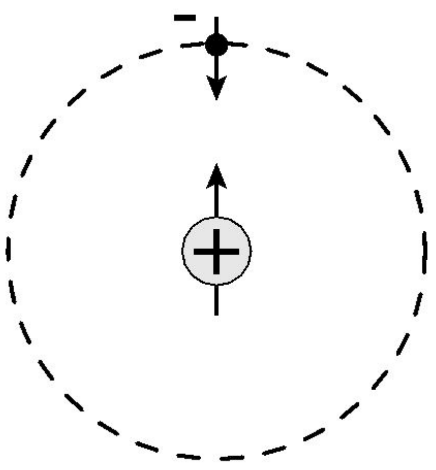
- Atomic structure led to chemistry, electronics, and the nanotech revolution.

The Hydrogen Atom

- Hydrogen has complex structure: fine, hyperfine...

... but it is well described by the Bohr model because QED is a perturbative theory.

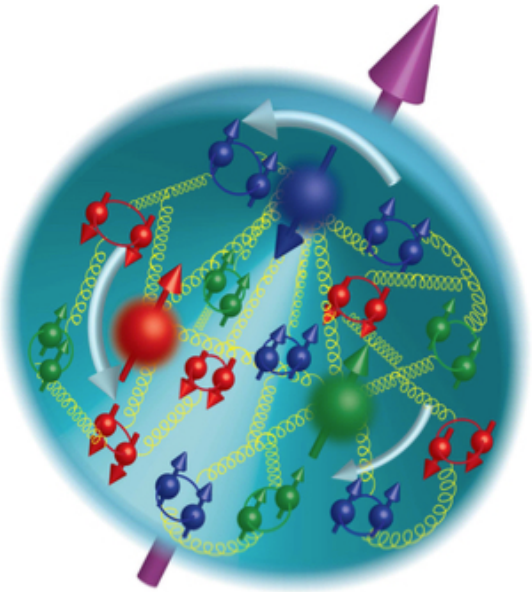
- Atomic structure led to chemistry, electronics, and the nanotech revolution.



The Proton

- QCD is only perturbative at short distances...

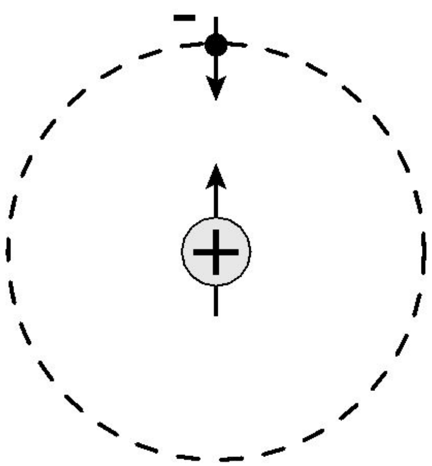
... so the proton structure embodies all the non-perturbative complexity of the field theory.



The Hydrogen Atom

- Hydrogen has complex structure: fine, hyperfine…

… but it is well described by the Bohr model because QED is a perturbative theory.

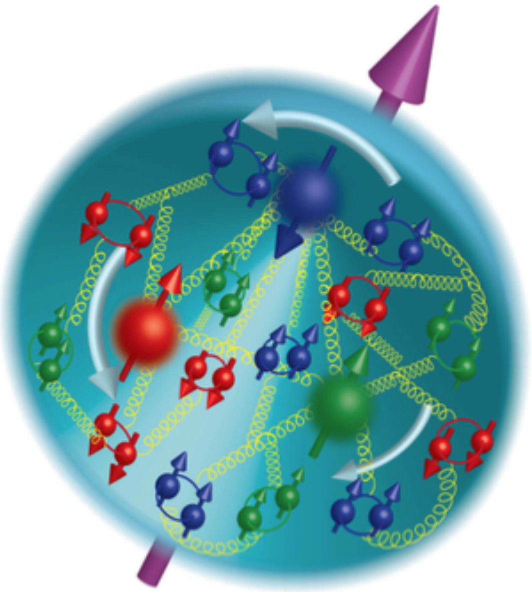


- Atomic structure led to chemistry, electronics, and the nanotech revolution.

The Proton

- QCD is only perturbative at short distances…

… so the proton structure embodies all the non-perturbative complexity of the field theory.

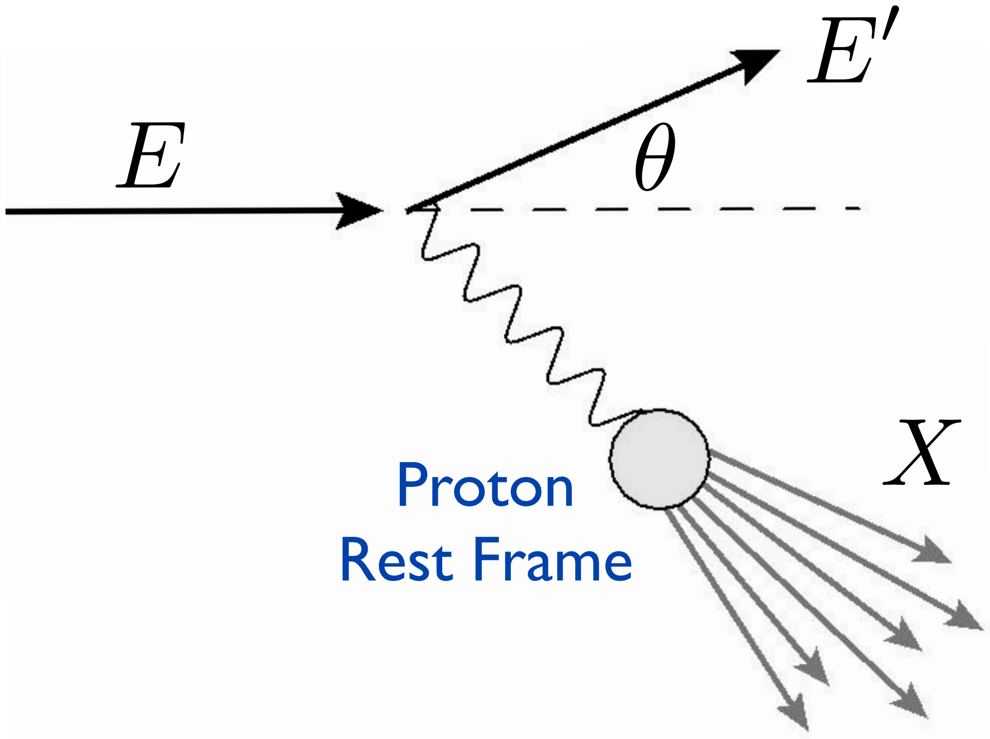


- Proton structure will tell us about the nature of QCD and a future femtoscale revolution.

The DIS "Femto-scope"

- Deep Inelastic Scattering (DIS)

$$e + p \to e' + X$$

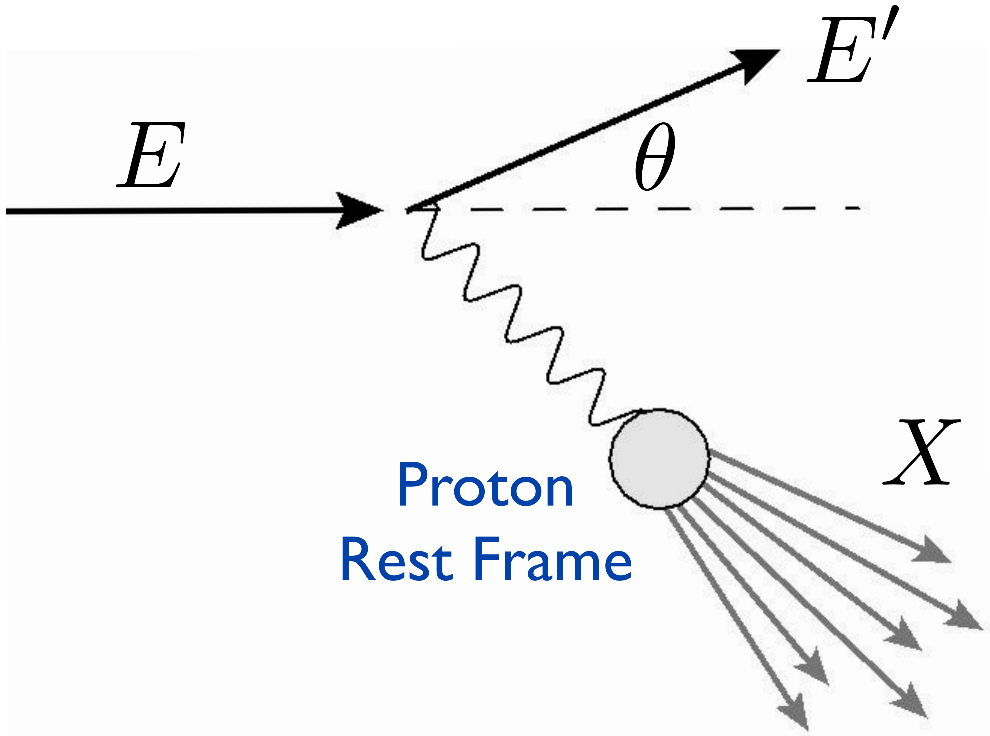


The DIS "Femto-scope"

- Deep Inelastic Scattering (DIS)
- Kinematic variables:

$$\boxed{\begin{matrix} E \\ E' \\ \theta \end{matrix}} \longrightarrow \boxed{\begin{aligned} Q^2 &= 4EE' \sin^2 \tfrac{\theta}{2} \\ x &= \tfrac{Q^2}{2m_N (E-E')} \\ s &= 2Em_N \end{aligned}}$$

$$e + p \to e' + X$$

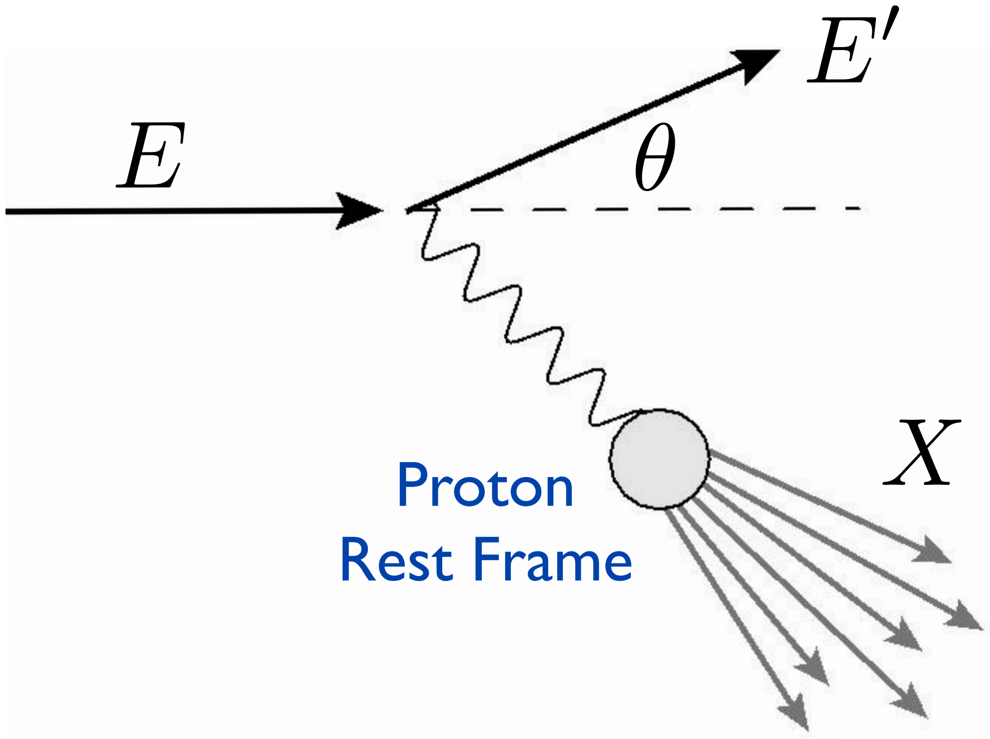


The DIS "Femto-scope"

- Deep Inelastic Scattering (DIS)
- Kinematic variables:

$$e + p \to e' + X$$

$$\boxed{\begin{matrix} E \\ E' \\ \theta \end{matrix}} \longrightarrow \boxed{\begin{aligned} Q^2 &= 4EE' \sin^2 \tfrac{\theta}{2} \\ x &= \tfrac{Q^2}{2m_N(E-E')} \\ s &= 2Em_N \end{aligned}}$$



Q^2 = "Resolution" $\qquad \Delta x_\perp^2 < \frac{1}{Q^2}$

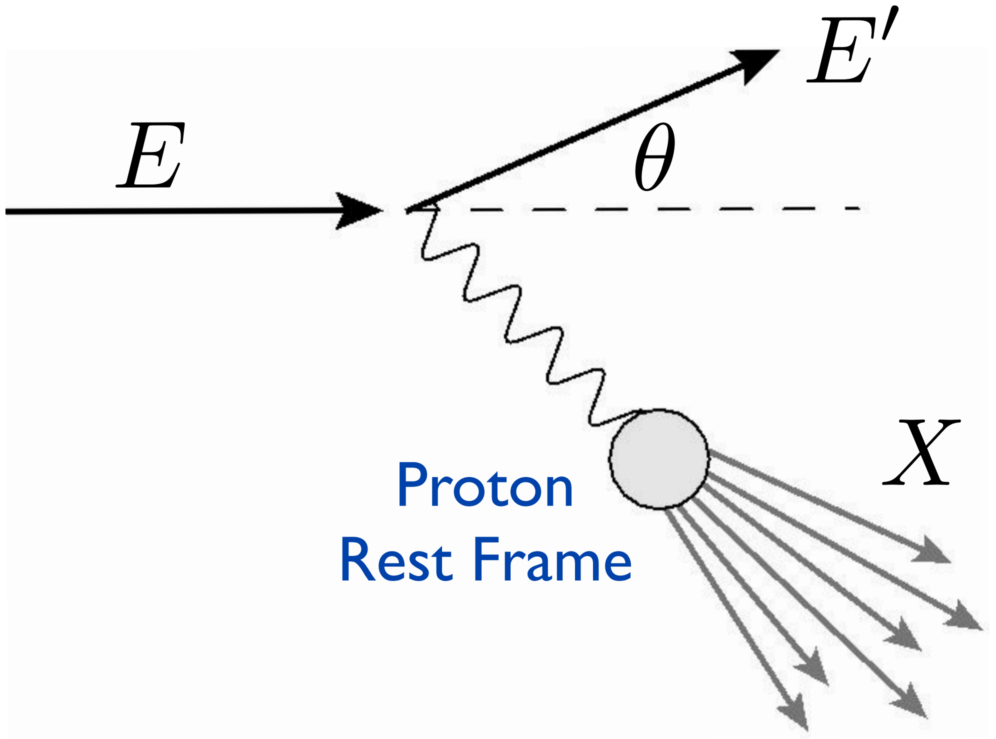
x = "Exposure" $\qquad \Delta t < \frac{1}{m_N x}$

The DIS "Femto-scope"

- Deep Inelastic Scattering (DIS)
- Kinematic variables:

$$e + p \to e' + X$$

$$\boxed{\begin{matrix} E \\ E' \\ \theta \end{matrix}} \longrightarrow \boxed{\begin{aligned} Q^2 &= 4EE' \sin^2 \tfrac{\theta}{2} \\ x &= \frac{Q^2}{2m_N(E-E')} \\ s &= 2Em_N \end{aligned}}$$



Q^2 = "Resolution" $\quad \Delta x_\perp^2 < \frac{1}{Q^2}$

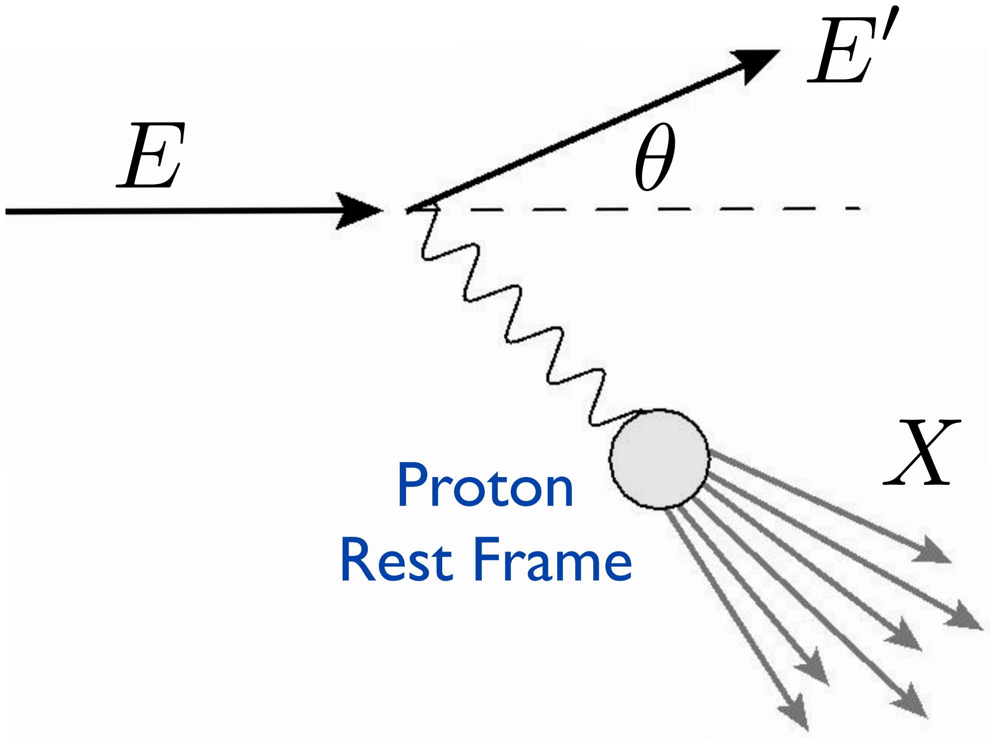
x = "Exposure" $\quad \Delta t < \frac{1}{m_N x}$

- Resolve proton substructure: $Q^2 \gg m_N^2$

- Deep Inelastic Scattering (DIS)
- Kinematic variables:

$$e + p \to e' + X$$

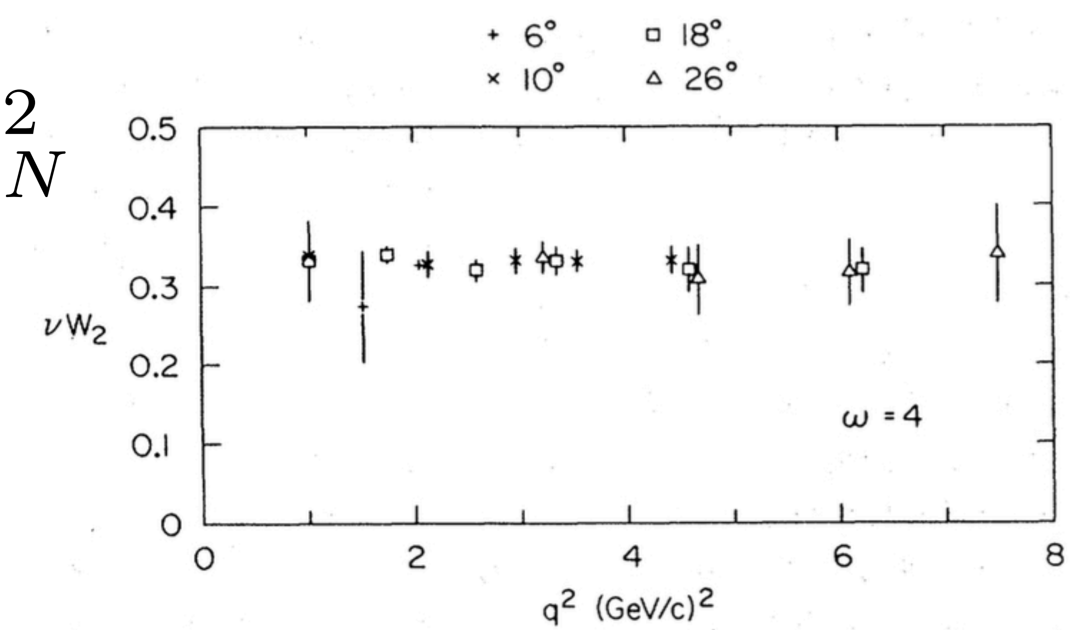
$$\boxed{\begin{matrix} E \\ E' \\ \theta \end{matrix}} \longrightarrow \boxed{\begin{aligned} Q^2 &= 4EE' \sin^2 \tfrac{\theta}{2} \\ x &= \frac{Q^2}{2m_N(E-E')} \\ s &= 2Em_N \end{aligned}}$$



Q^2 = "Resolution" $\quad \Delta x_\perp^2 < \frac{1}{Q^2}$

x = "Exposure" $\quad \Delta t < \frac{1}{m_N x}$

- Resolve proton substructure: $Q^2 \gg m_N^2$
- Bjorken scaling: asymptotic freedom!



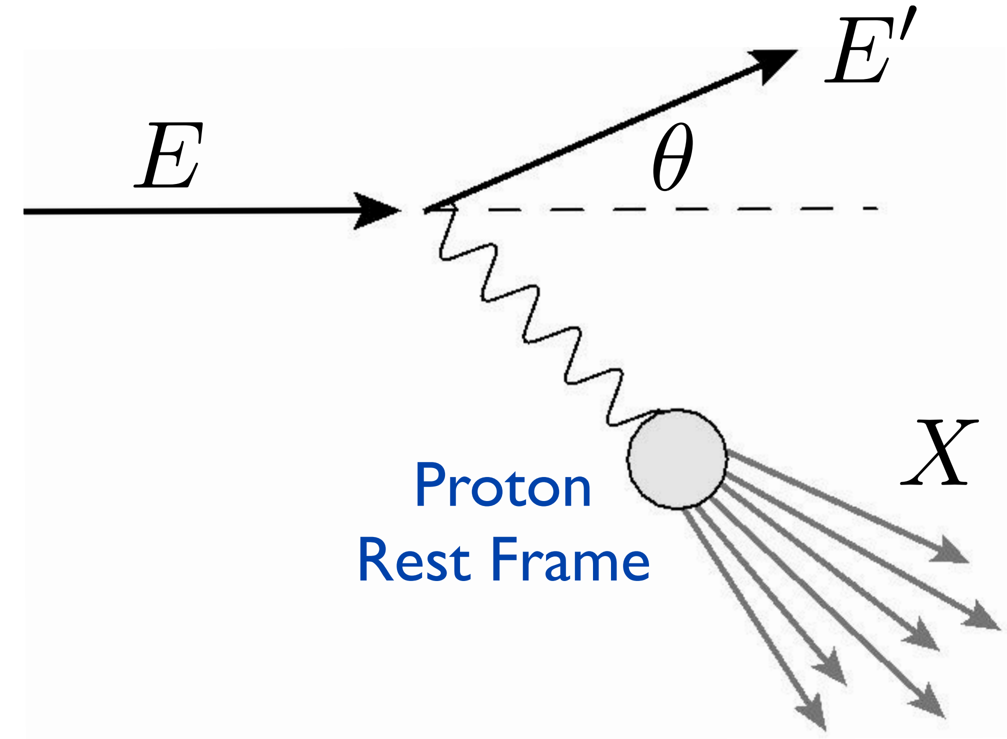
- Deep Inelastic Scattering (DIS)
- Kinematic variables:

$$\boxed{\begin{matrix} E \\ E' \\ \theta \end{matrix}} \longrightarrow \boxed{\begin{aligned} Q^2 &= 4EE' \sin^2 \tfrac{\theta}{2} \\ x &= \frac{Q^2}{2m_N(E-E')} \\ s &= 2Em_N \end{aligned}}$$

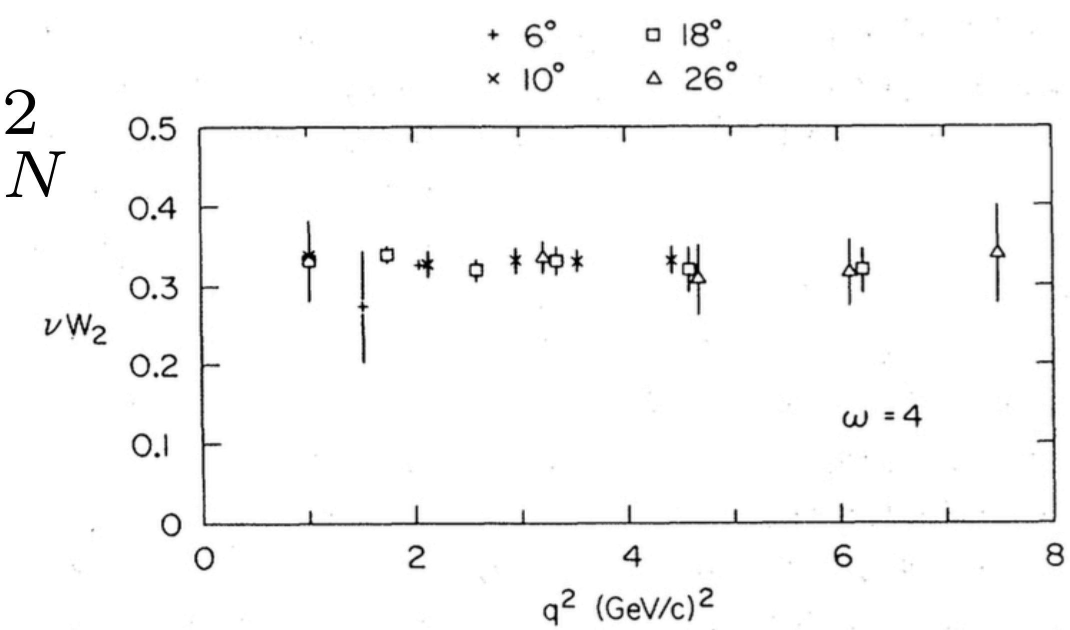
Q^2 = "Resolution" $\qquad \Delta x_\perp^2 < \frac{1}{Q^2}$

x = "Exposure" $\qquad \Delta t < \frac{1}{m_N x}$

$$e + p \to e' + X$$

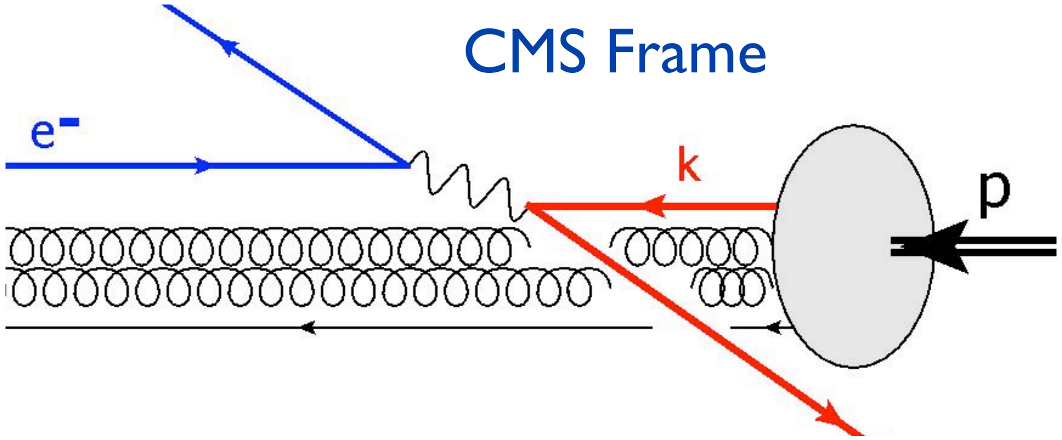


- Resolve proton substructure: $Q^2 \gg m_N^2$
- Bjorken scaling: asymptotic freedom!
- Identified QCD as the fundamental theory of the strong nuclear force.

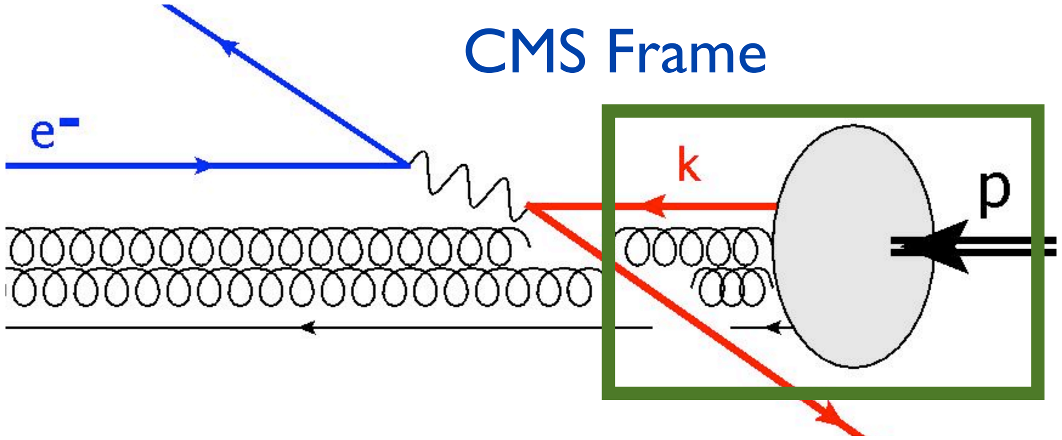


Parton Distributions

- Parton model: At short distances, virtual photon strikes one quark



- Parton model: At short distances, virtual photon strikes one quark



- Structure functions interpreted as parton distribution functions

$$F_2(x, Q^2) = \sum_f Z_f^2 \, \underline{x f(x, Q^2)}$$

Parton Distributions

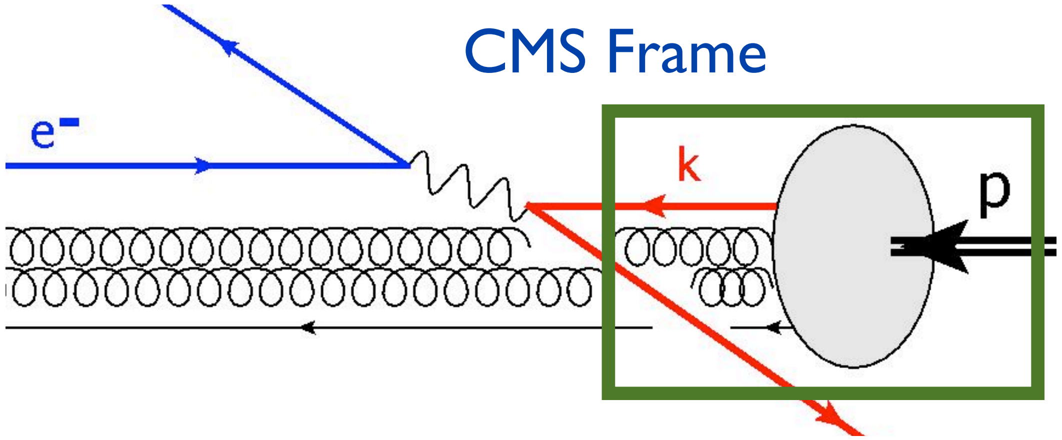
- Parton model: At short distances, virtual photon strikes one quark

- Structure functions interpreted as parton distribution functions

$$F_2(x,Q^2) = \sum_f Z_f^2 \, \underline{x f(x,Q^2)}$$

- x has the interpretation as the quark momentum fraction

$$x = \frac{k^+}{p^+} \qquad 0 \leq x \leq 1$$

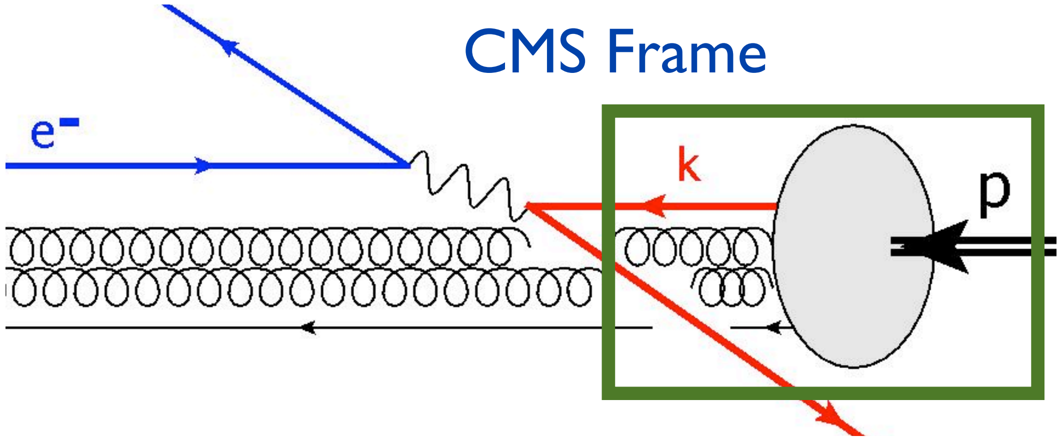


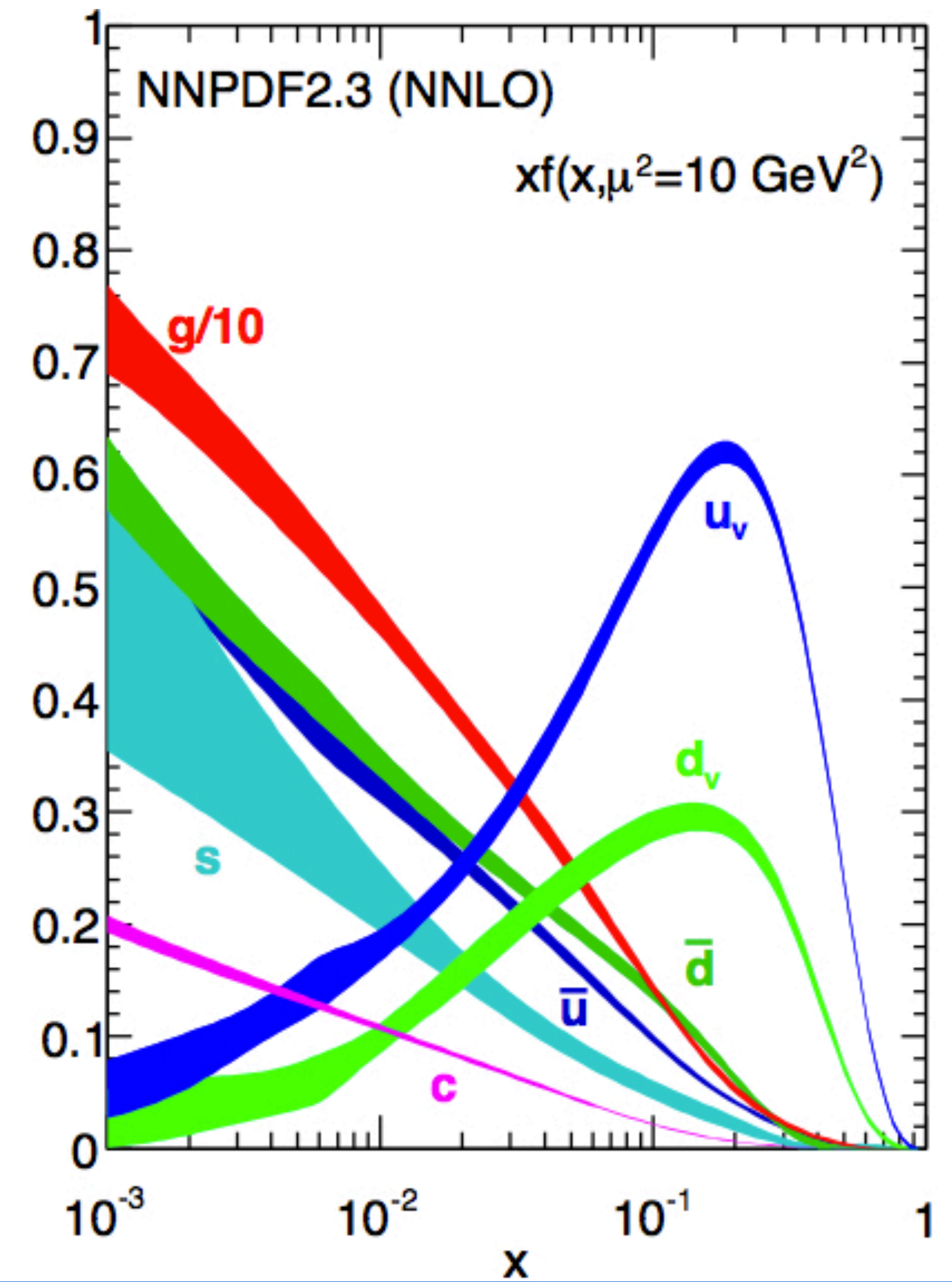
- Parton model: At short distances, virtual photon strikes one quark
- Structure functions interpreted as parton distribution functions

$$F_2(x,Q^2) = \sum_f Z_f^2 \, \underline{x f(x,Q^2)}$$

- x has the interpretation as the quark momentum fraction

$$x = \frac{k^+}{p^+} \qquad 0 \le x \le 1$$





Parton Distributions

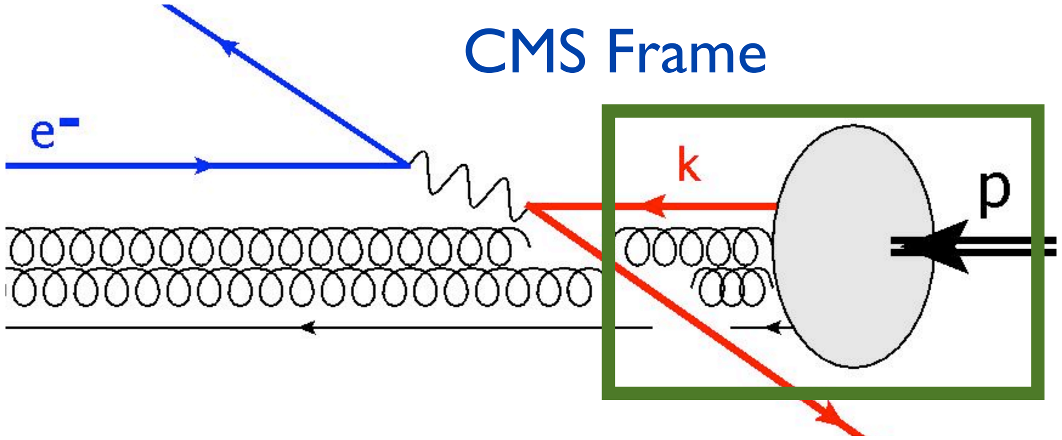
- Parton model: At short distances, virtual photon strikes one quark
- Structure functions interpreted as parton distribution functions

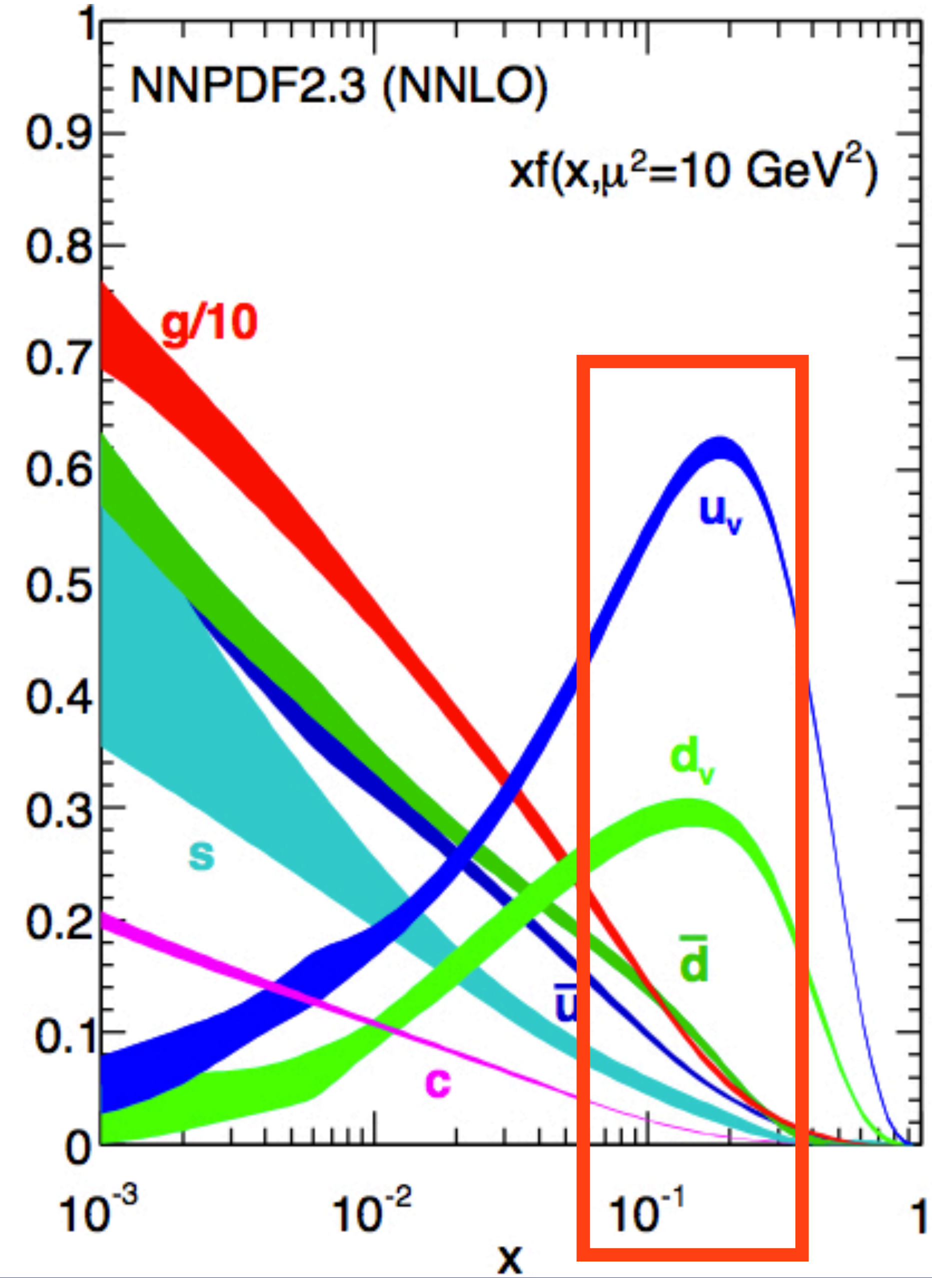
$$F_2(x, Q^2) = \sum_f Z_f^2 \, \underline{x f(x, Q^2)}$$

- x has the interpretation as the quark momentum fraction

$$x = \frac{k^+}{p^+} \qquad 0 \leq x \leq 1$$

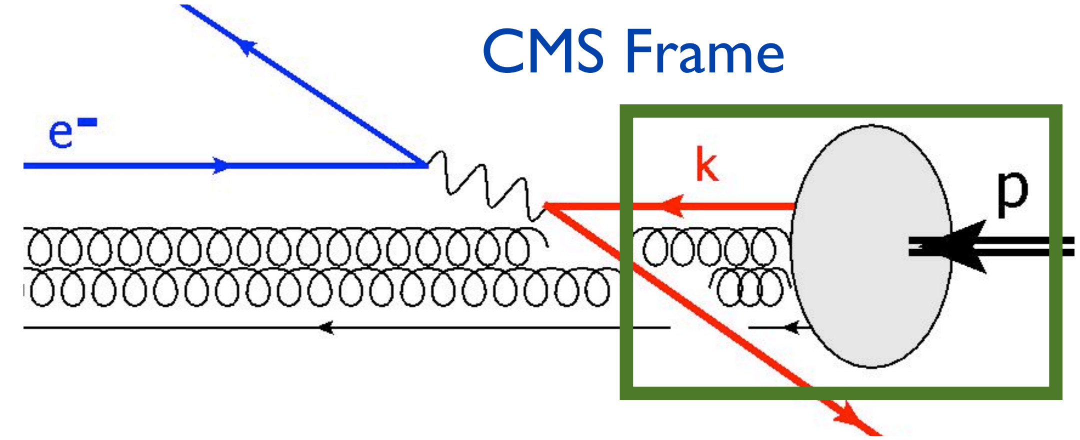
Large x: $x \approx \frac{1}{3}$ Valence Quarks





Parton Distributions

- Parton model: At short distances, virtual photon strikes one quark

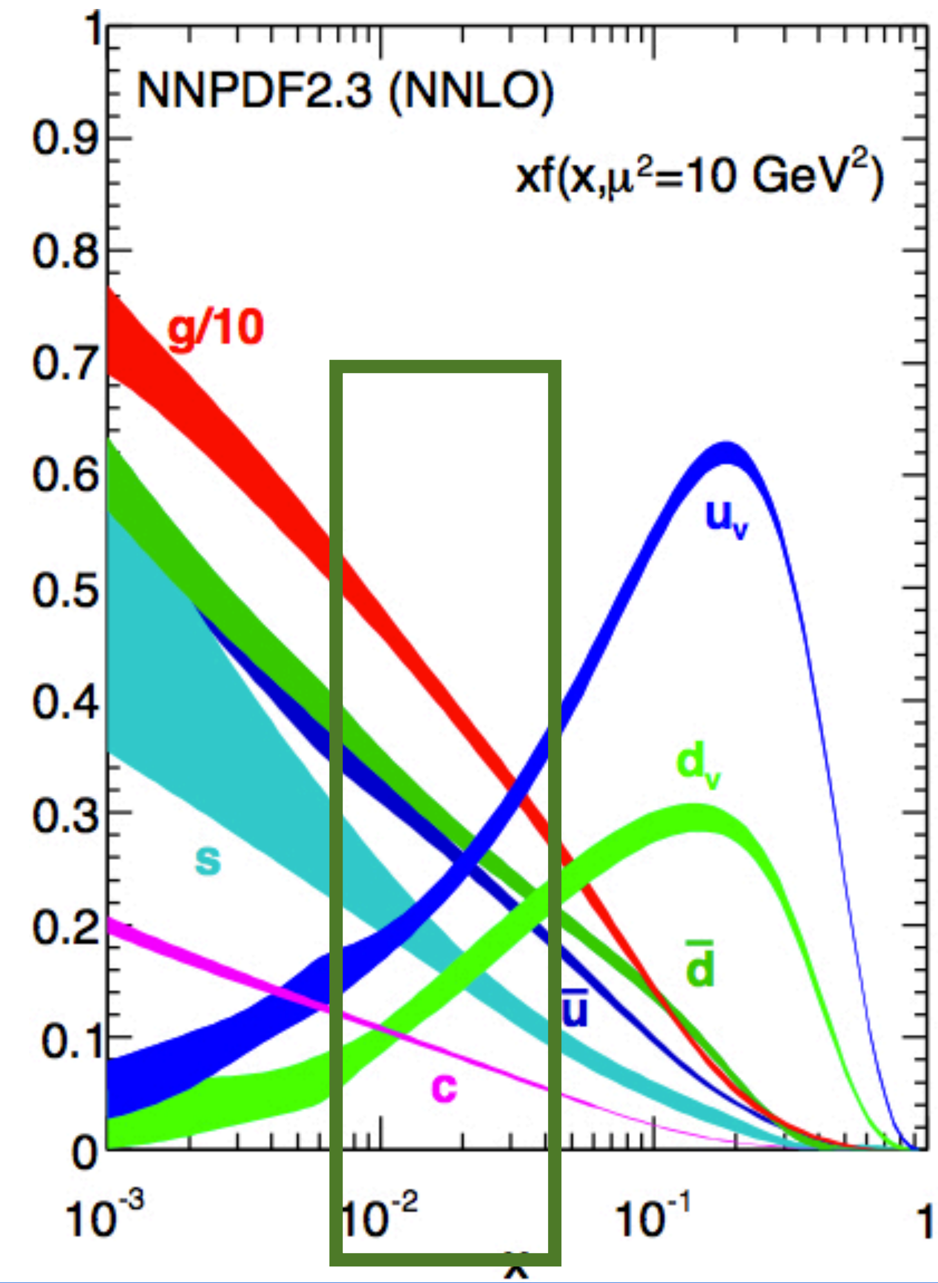


- Structure functions interpreted as parton distribution functions

$$F_2(x, Q^2) = \sum_f Z_f^2 \, \underline{x f(x, Q^2)}$$

- x has the interpretation as the quark momentum fraction

$$x = \frac{k^+}{p^+} \qquad 0 \leq x \leq 1$$



Large x:	$x \approx \frac{1}{3}$	Valence Quarks
Small x:	$x \approx 1\%$	Bremsstrahlung

Parton Distributions

- Parton model: At short distances, virtual photon strikes one quark

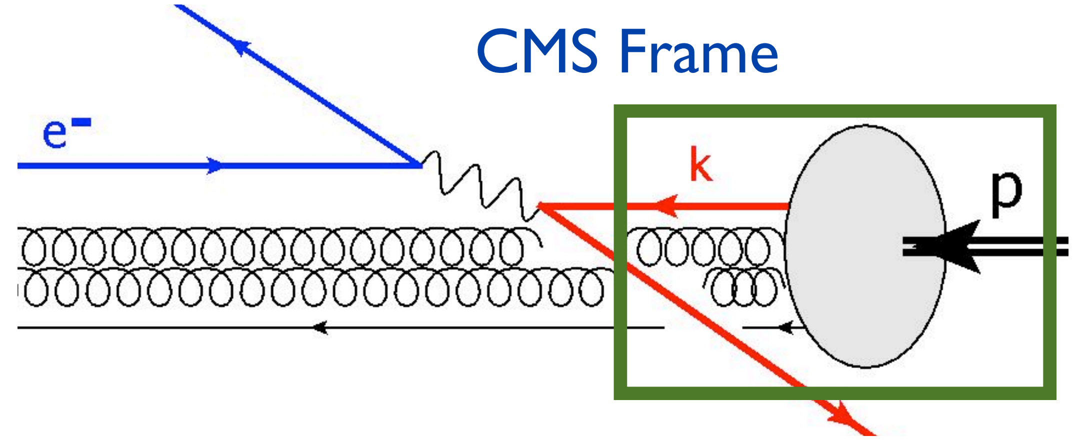
- Structure functions interpreted as parton distribution functions

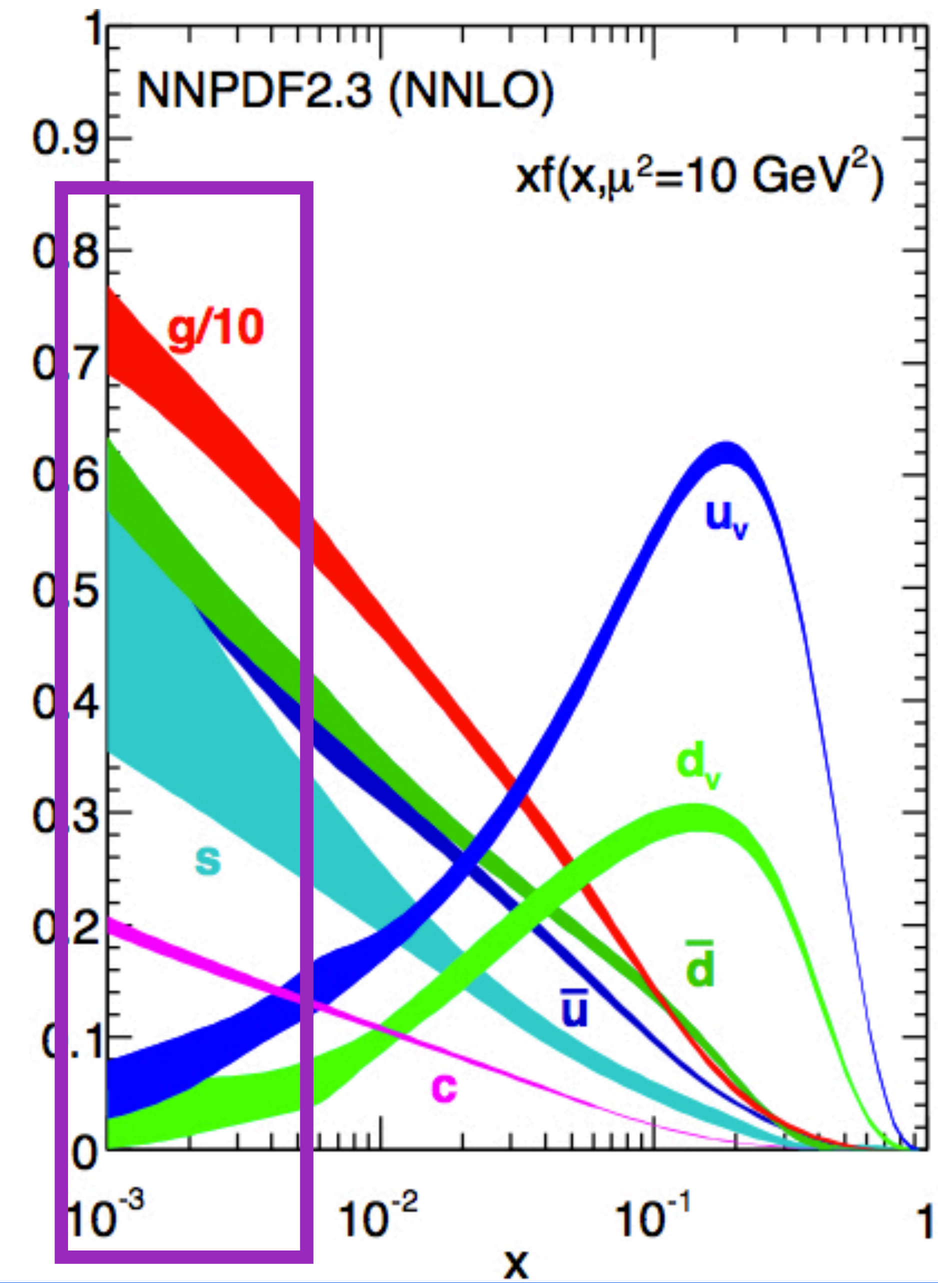
$$F_2(x, Q^2) = \sum_f Z_f^2 \, \underline{x f(x, Q^2)}$$

- x has the interpretation as the quark momentum fraction

$$x = \frac{k^+}{p^+} \qquad 0 \leq x \leq 1$$

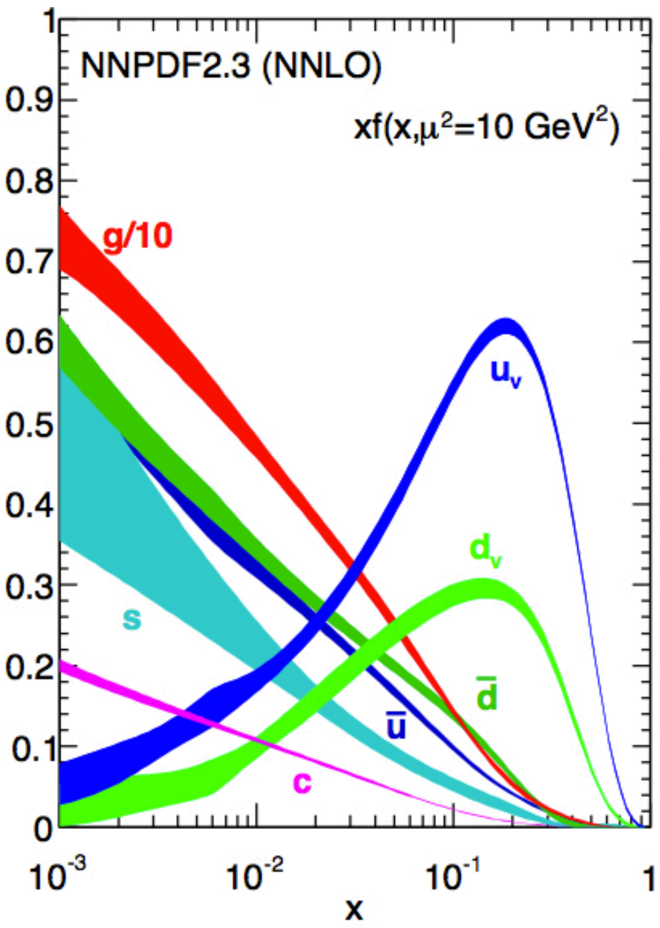
Large x:	$x \approx \frac{1}{3}$	Valence Quarks
Small x:	$x \approx 1\%$	Bremsstrahlung
Smaller x:	$x < 1\%$	Gluon Explosion!





- Bjorken scaling occurs over a range of Q^2

➡ Smaller-scale quantum fluctuations are suppressed by asymptotic freedom.

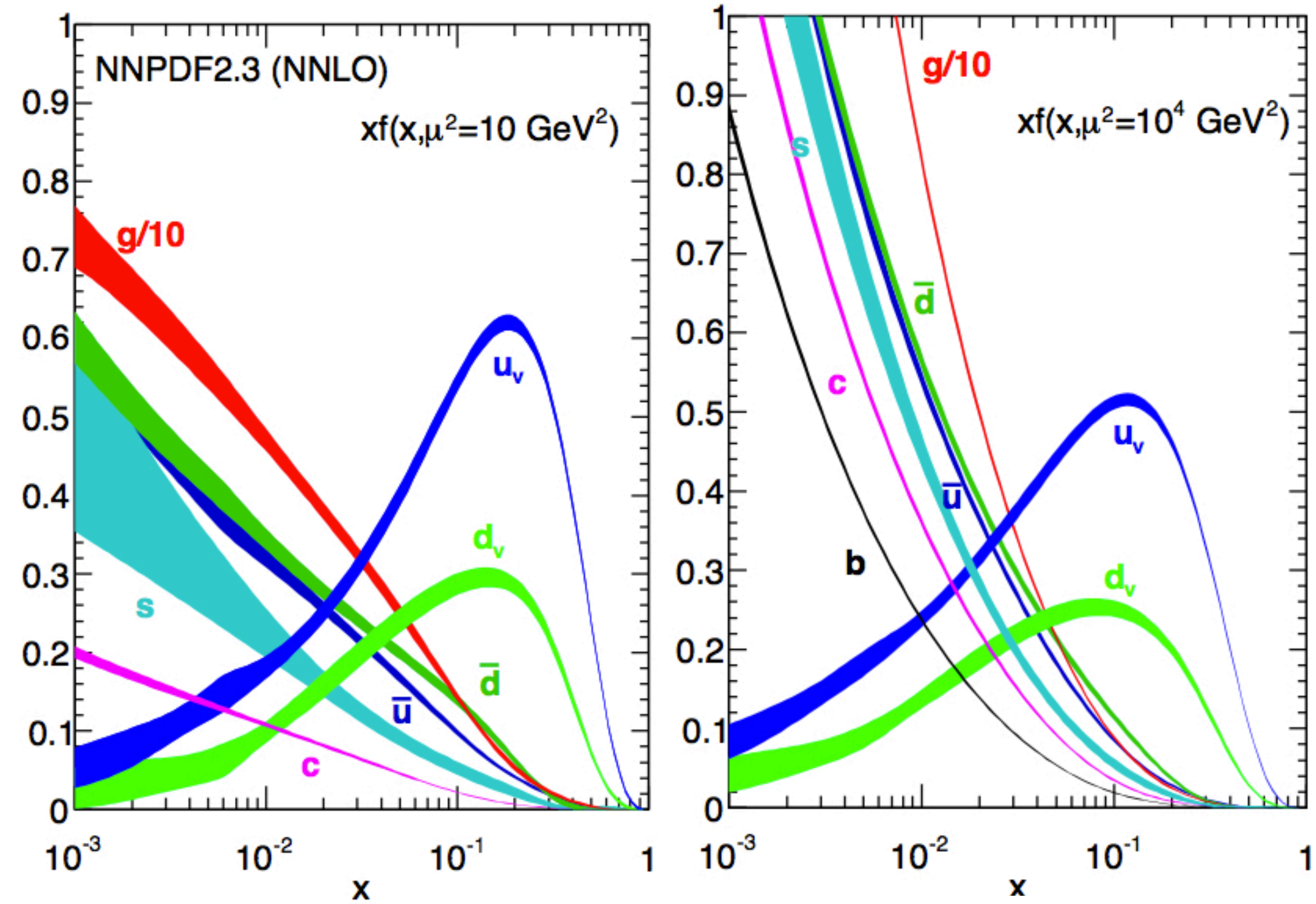


Scaling Violations: DGLAP Evolution

- Bjorken scaling occurs over a range of Q^2

➡ Smaller-scale quantum fluctuations are suppressed by asymptotic freedom.

- But large increases in Q^2 **do** resolve small-scale quantum fluctuations.



- Bjorken scaling occurs over a range of Q^2

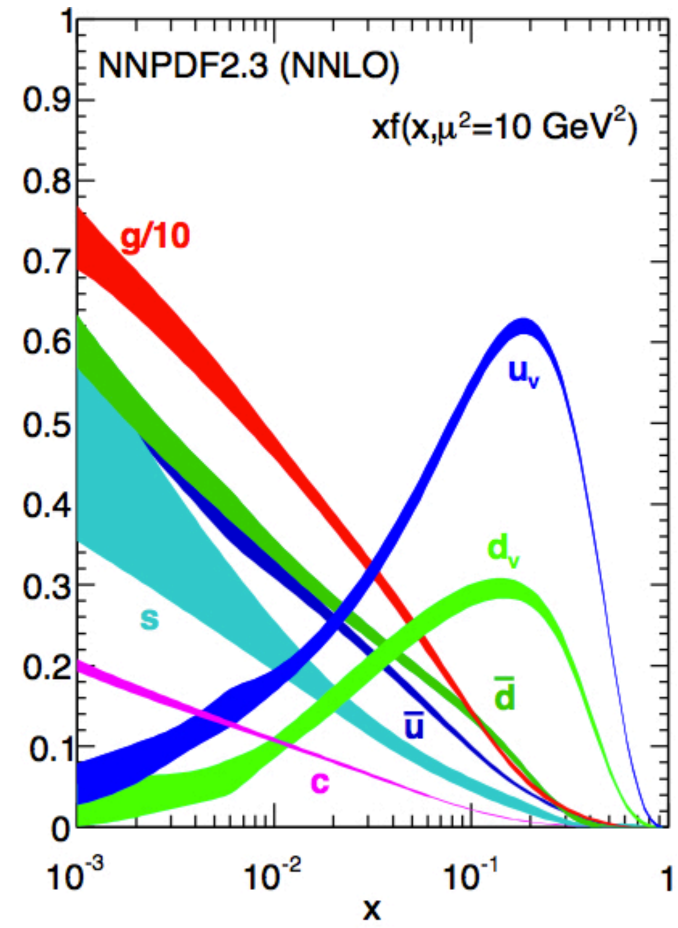
  ➡ Smaller-scale quantum fluctuations are suppressed by asymptotic freedom.

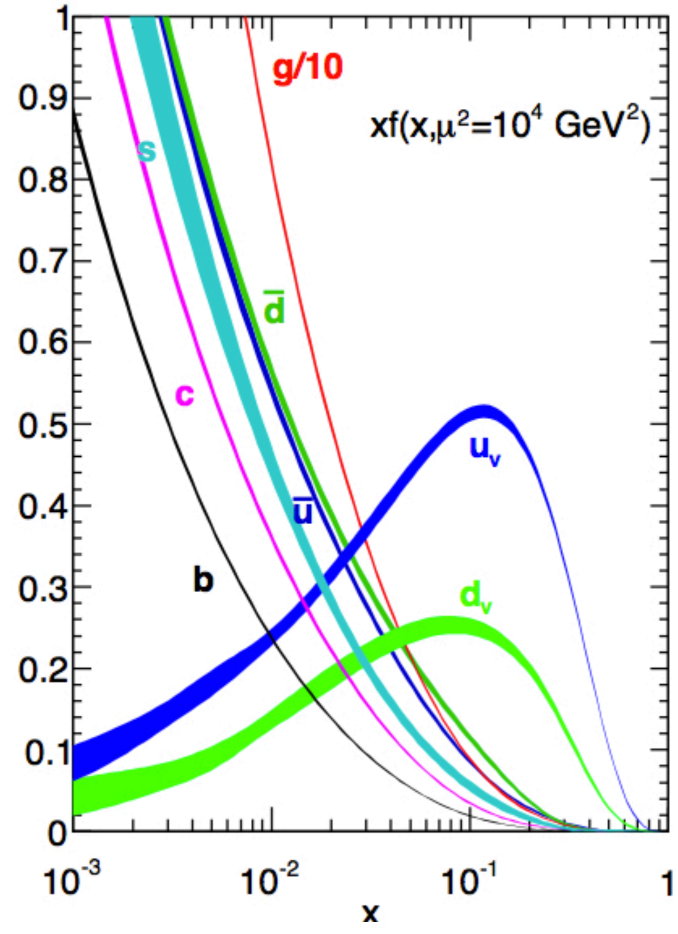
- But large increases in Q^2 **do** resolve small-scale quantum fluctuations.

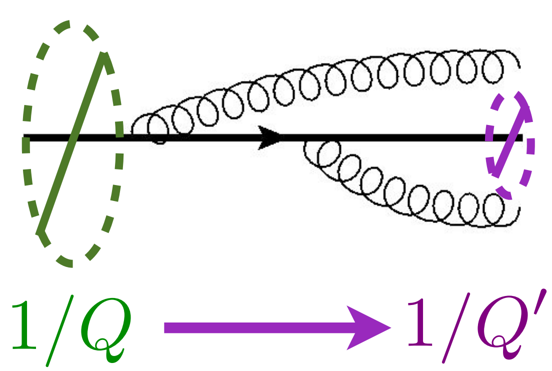
- Short-distance fluctuations are suppressed…

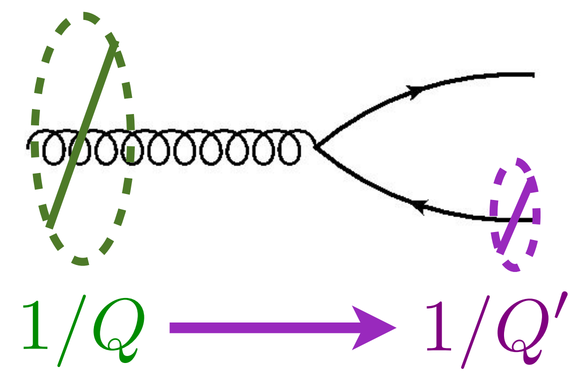
  …but some are enhanced by logarithms of Q^2

  ➡ "Quantum Evolution" of the parton distributions!





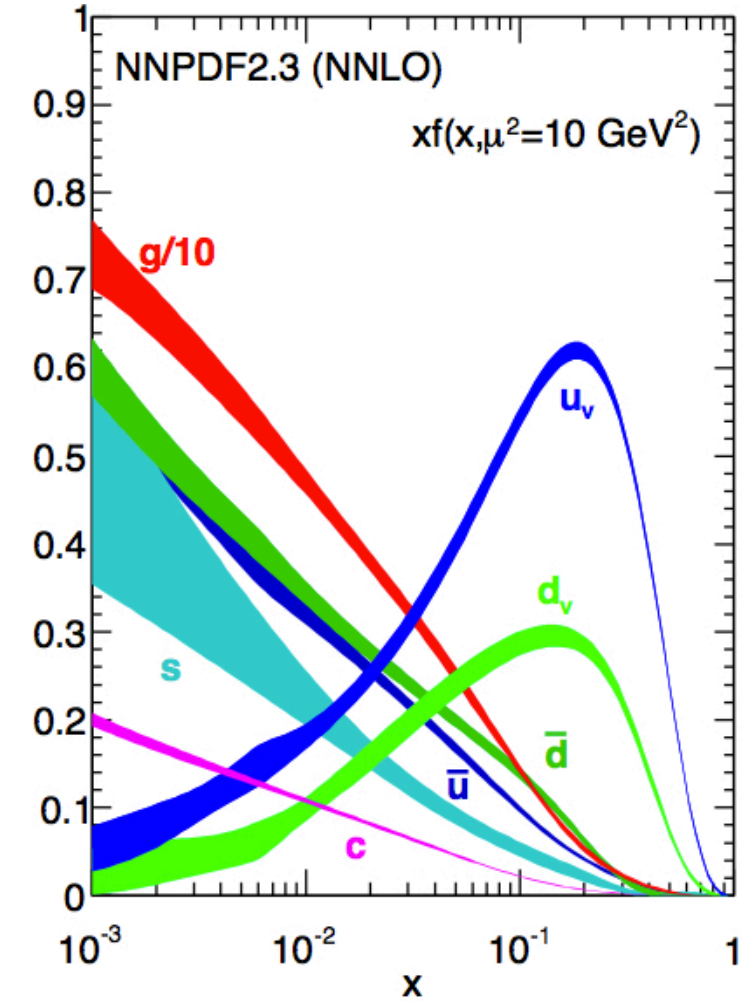




$$\alpha_s(Q^2) \ln \frac{Q^2}{\Lambda^2} \sim 1$$

What's So Special about Small x?

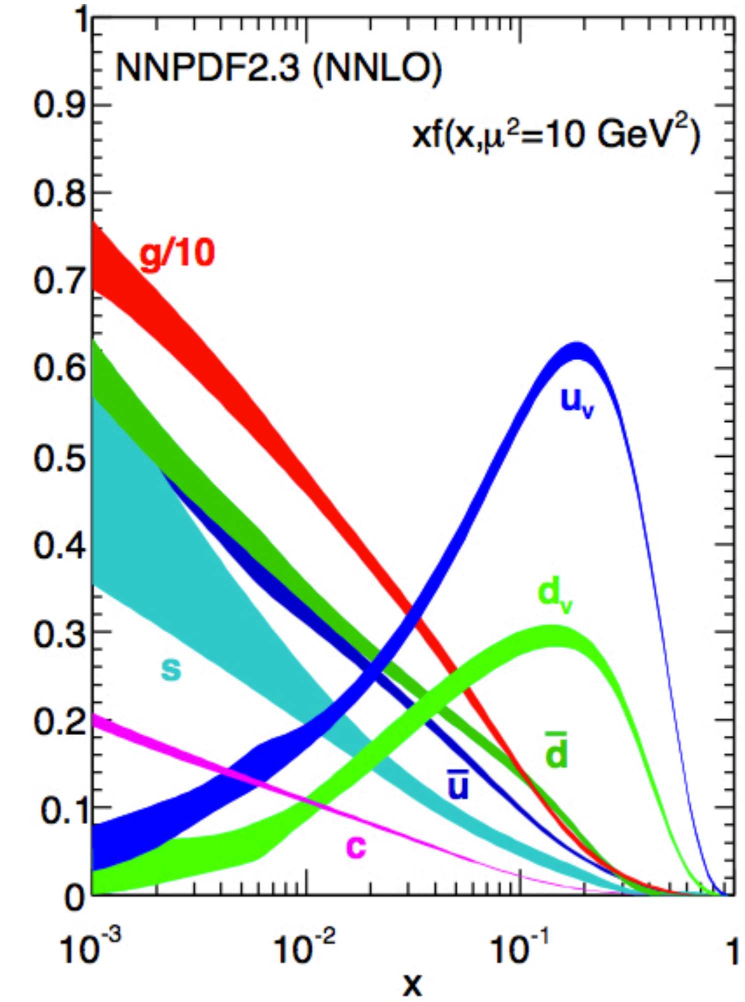
- Multiple bremsstrahlung leads to an explosion of gluon density at small x.



- Multiple bremsstrahlung leads to an explosion of gluon density at small x.

- Quantum Evolution: soft bremsstrahlung enhanced by logarithms of $\frac{1}{x}$

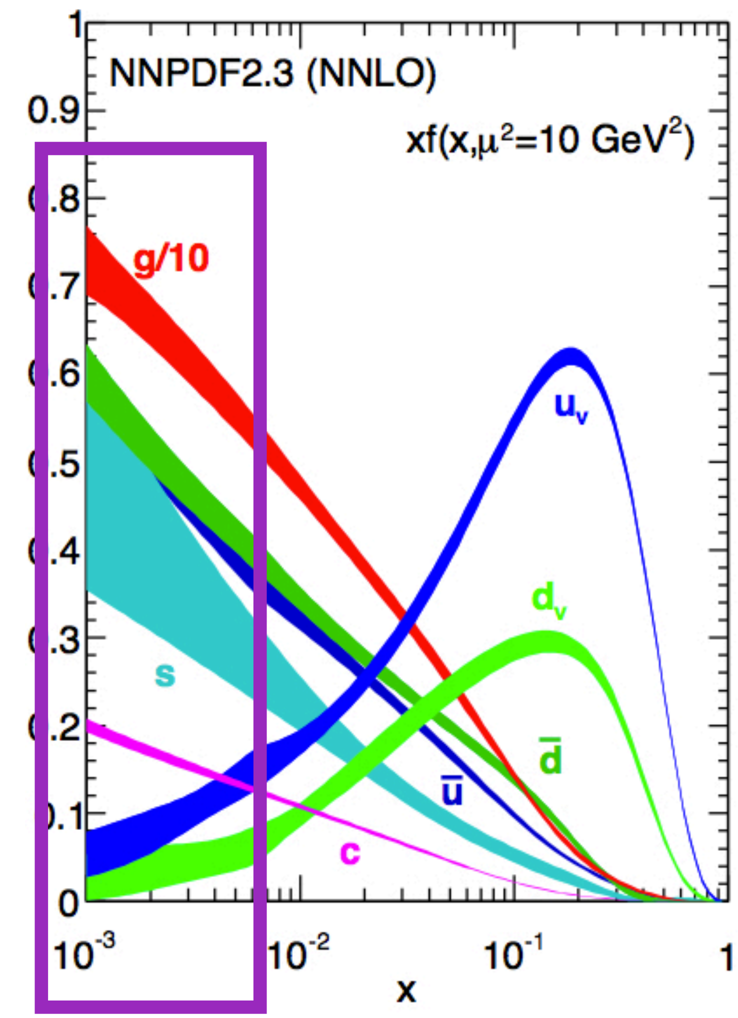
$$\alpha_s \ln \frac{1}{x} \sim 1$$

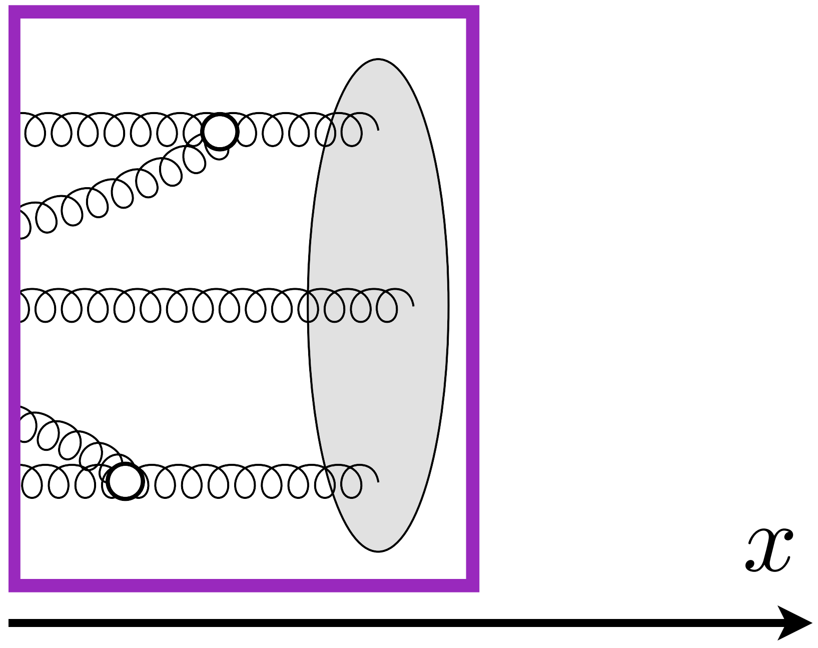


- Multiple bremsstrahlung leads to an explosion of gluon density at small x.
- Quantum Evolution: soft bremsstrahlung enhanced by logarithms of $\frac{1}{x}$

$$\alpha_s \ln \frac{1}{x} \sim 1$$

- The gluon density increases so quickly it would violate unitarity (Froissart bound)



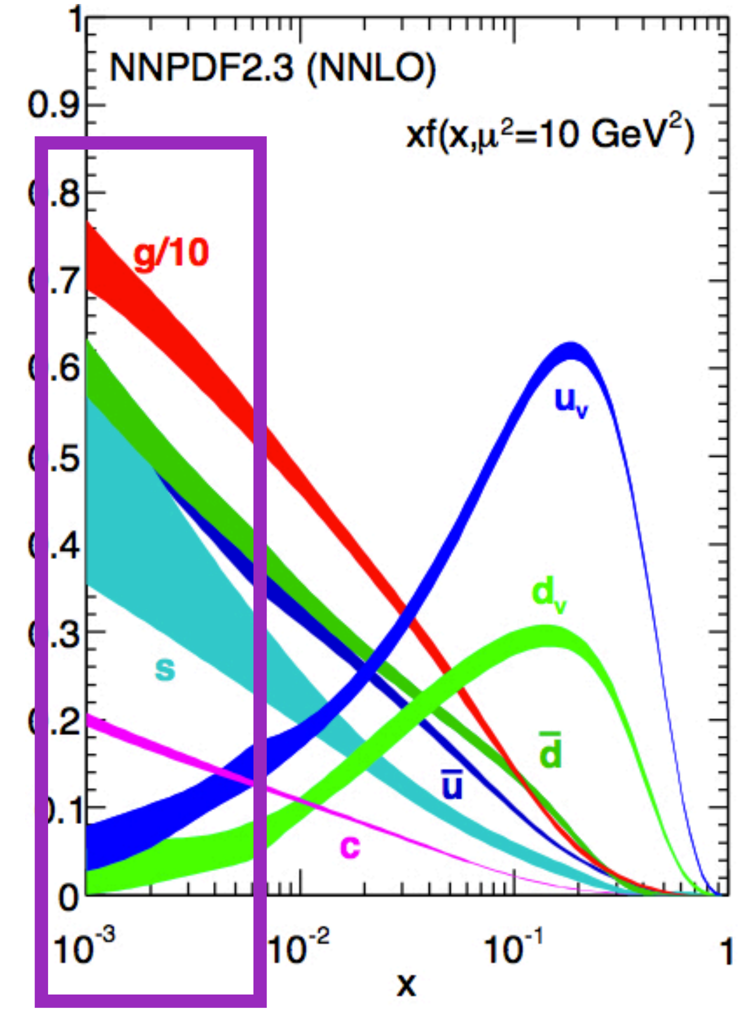


What's So Special about Small x?

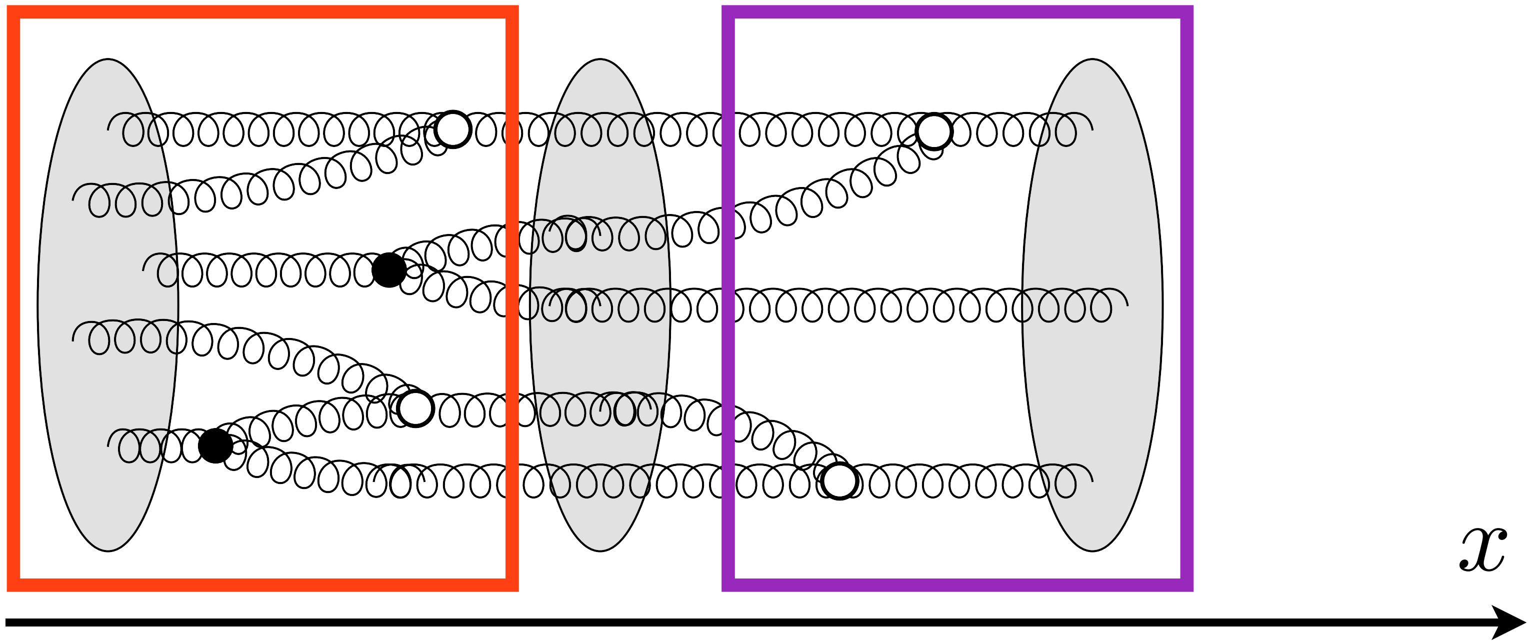
- Multiple bremsstrahlung leads to an explosion of gluon density at small x.
- Quantum Evolution: soft bremsstrahlung enhanced by logarithms of $\frac{1}{x}$

$$\alpha_s \ln \frac{1}{x} \sim 1$$

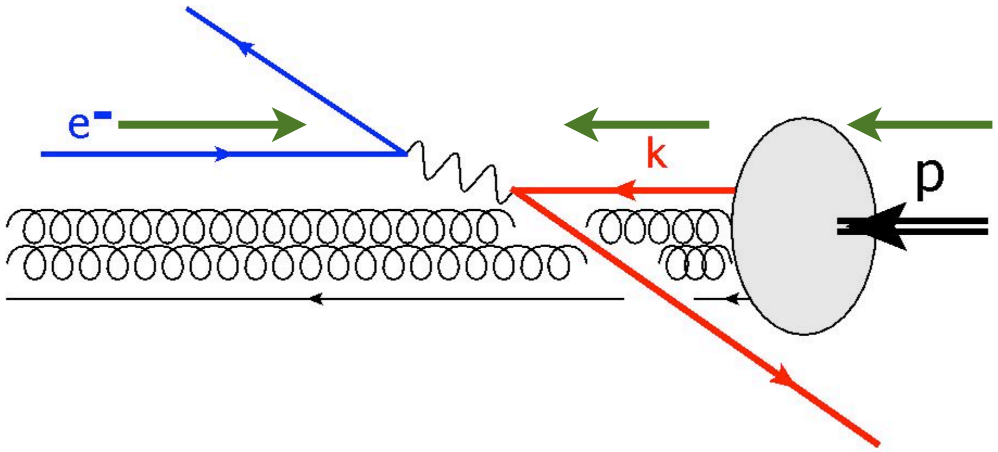
- The gluon density increases so quickly it would violate unitarity (Froissart bound)



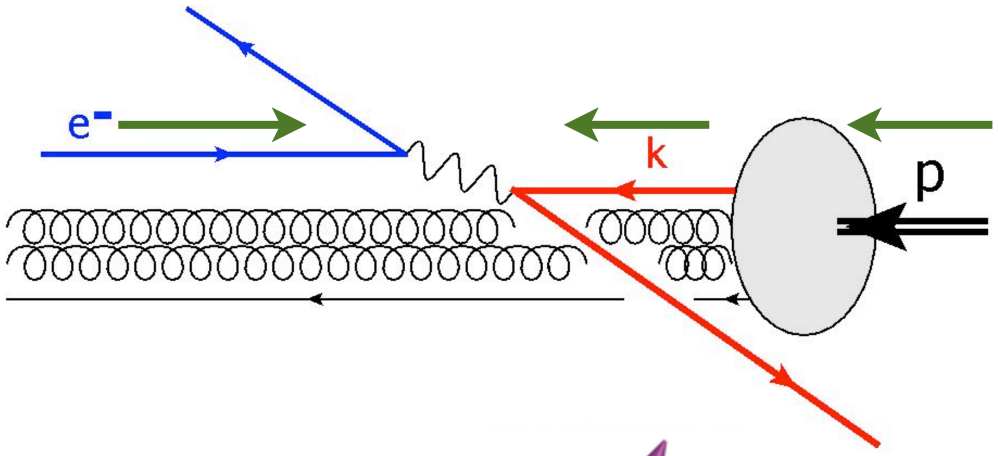
- At very small x, nonlinear gluon fusion must lead to a **saturation** of the gluon density.



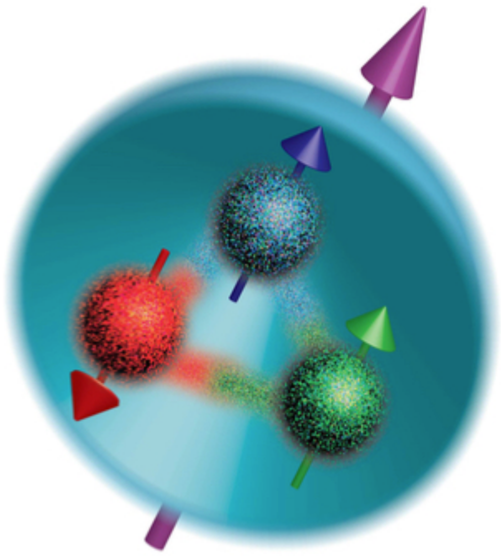
- If you can do DIS with polarized protons and electrons, you can measure the spin of the quarks



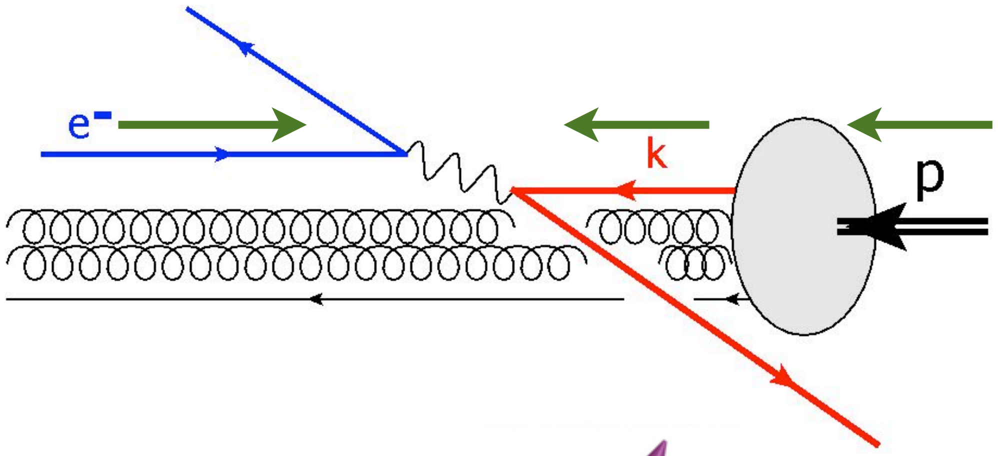
- If you can do DIS with polarized protons and electrons, you can measure the spin of the quarks

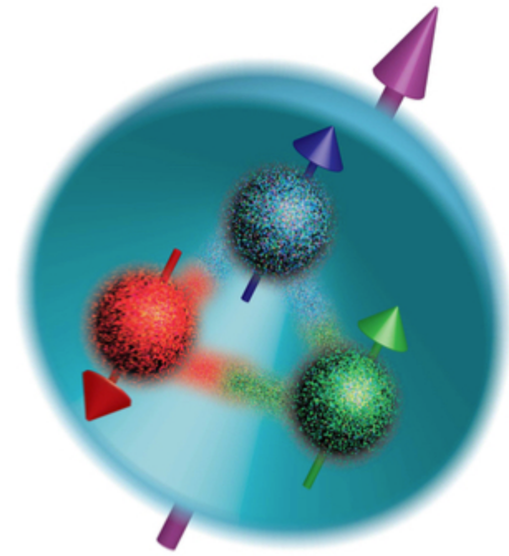


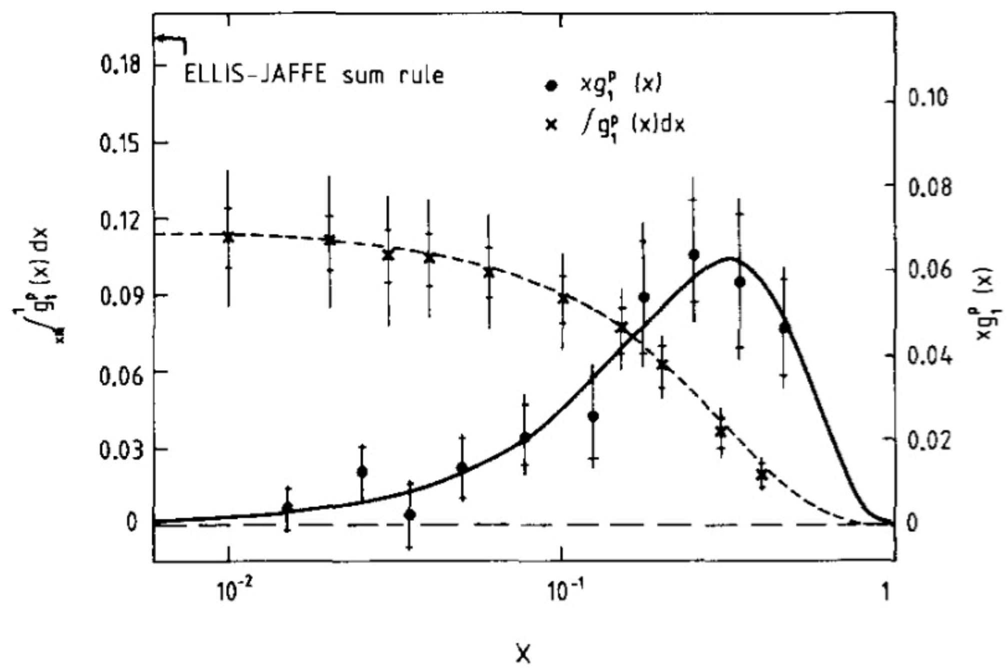
- From a naive constituent quark picture, one expects the valence quarks to accommodate the proton spin.



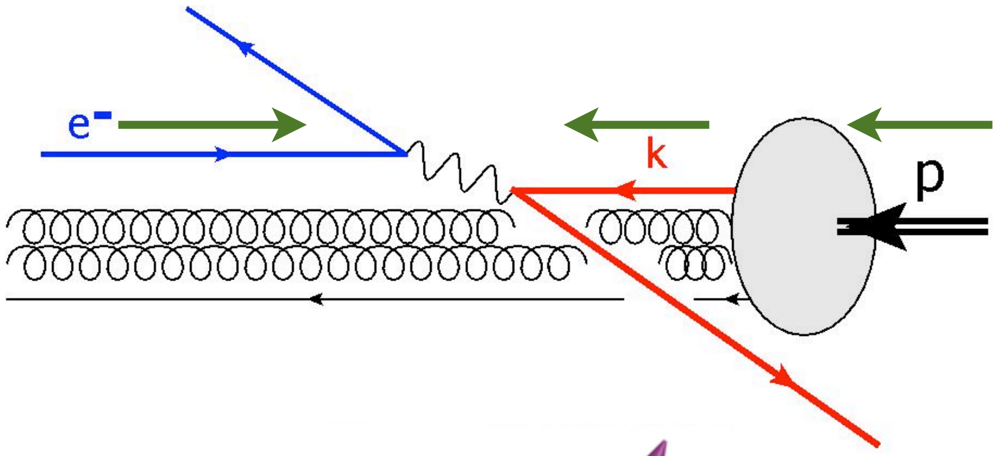
- If you can do DIS with polarized protons and electrons, you can measure the spin of the quarks
- From a naive constituent quark picture, one expects the valence quarks to accommodate the proton spin.
- But in 1988 the EMC Collaboration found that "only $14 \pm 9 \pm 21\%$ of the proton spin is carried by the spin of the quarks"!



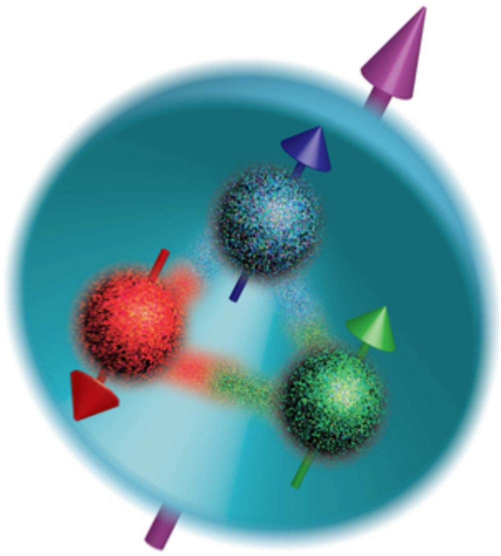




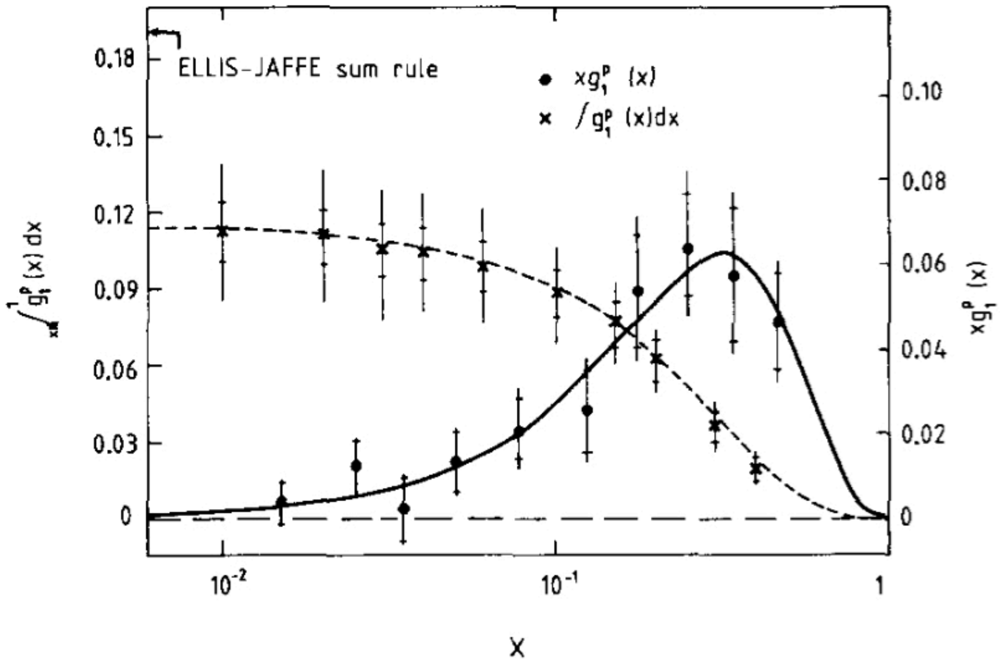
- If you can do DIS with polarized protons and electrons, you can measure the spin of the quarks



- From a naive constituent quark picture, one expects the valence quarks to accommodate the proton spin.



- But in 1988 the EMC Collaboration found that "only $14 \pm 9 \pm 21\%$ of the proton spin is carried by the spin of the quarks"!



- If the quark spins don't account for the proton spin... **what does?**

The Proton Spin Crisis

- The "Proton Spin Budget" is described by the Jaffe-Manohar Sum Rule.

$$\boxed{\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g}$$

The Proton Spin Crisis

- The "Proton Spin Budget" is described by the Jaffe-Manohar Sum Rule.

$$\boxed{\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g}$$

- Modern measurements still cannot account for the spin of the proton!
  - ➡ Quark spins from polarized DIS
  - ➡ Gluon spins from in polarized proton-proton collisions

Data from

$$0.001 < x < 1$$

$$\boxed{\Delta\Sigma \approx 0.25 \ \ (25\%)}$$

$$\boxed{\Delta G \approx 0.2 \ \ (40\%)}$$

The Proton Spin Crisis

- The "Proton Spin Budget" is described by the Jaffe-Manohar Sum Rule.

$$\boxed{\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g}$$

- Modern measurements still cannot account for the spin of the proton!
  - ➡ Quark spins from polarized DIS
  - ➡ Gluon spins from in polarized proton-proton collisions

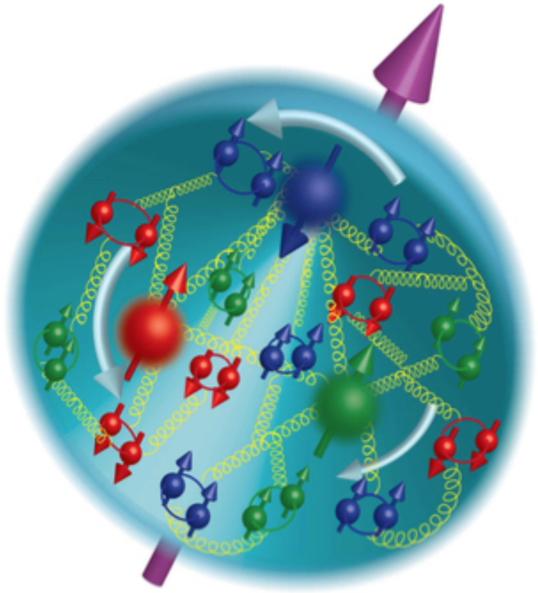
Data from

$$0.001 < x < 1$$

$$\boxed{\Delta\Sigma \approx 0.25 \ \ (25\%)}$$

$$\boxed{\Delta G \approx 0.2 \ \ (40\%)}$$

- Proton structure is much more complex than previously believed!
  - ➡ Orbital angular momentum?
  - ➡ Polarization at very small x?



The Toolbox:
Quarks and the Small-x Limit

Definition: TMD Quark Distribution

$$\phi_{\alpha\beta}(x, \vec{k}_\perp) = \int \frac{d^{2-}r}{(2\pi)^3} \, e^{ik \cdot r} \langle h(p,S)| \, \bar{\psi}_\beta(0) \, \mathcal{U}[0,r] \, \psi_\alpha(r) \, |h(p,S)\rangle$$

Definition: TMD Quark Distribution

$$\boxed{\phi_{\alpha\beta}(x, \vec{k}_\perp)} = \int \frac{d^{2-}r}{(2\pi)^3} \, e^{ik\cdot r} \langle h(p,S)| \, \bar{\psi}_\beta(0) \, \mathcal{U}[0,r] \, \psi_\alpha(r) \, |h(p,S)\rangle$$

Transverse Momentum Dependent
Parton Distribution Functions

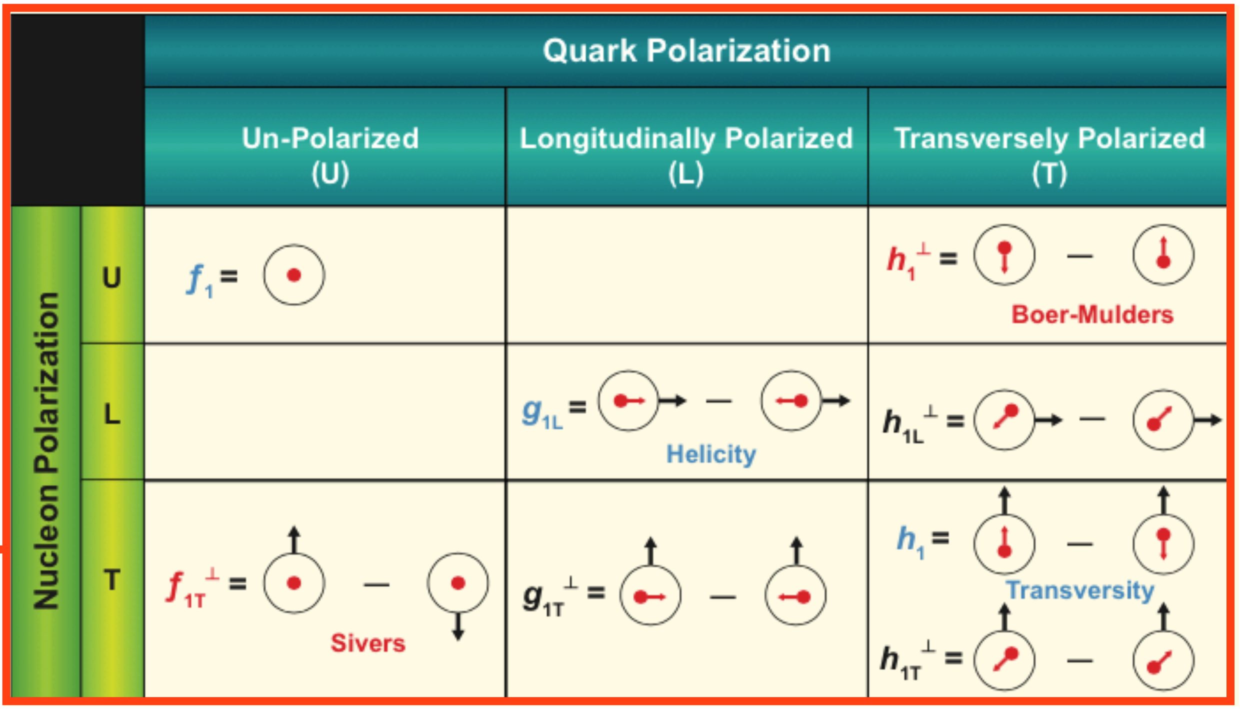
Definition: TMD Quark Distribution

$$\boxed{\phi_{\alpha\beta}(x, \vec{k}_\perp)} = \int \frac{d^{2-}r}{(2\pi)^3} \, e^{ik\cdot r} \langle h(p,S)| \boxed{\bar{\psi}_\beta(0)} \, \mathcal{U}[0,r] \, \boxed{\psi_\alpha(r)} \, |h(p,S)\rangle$$

Transverse Momentum Dependent
Parton Distribution Functions

$$\sum_{\sigma\lambda} \langle h(p)| b^\dagger_{k\sigma} b_{k\lambda} |h(p)\rangle \;\; [\bar{U}_\sigma(k)]_\beta [U_\lambda(k)]_\alpha$$

Quark number operator + Dirac spinors



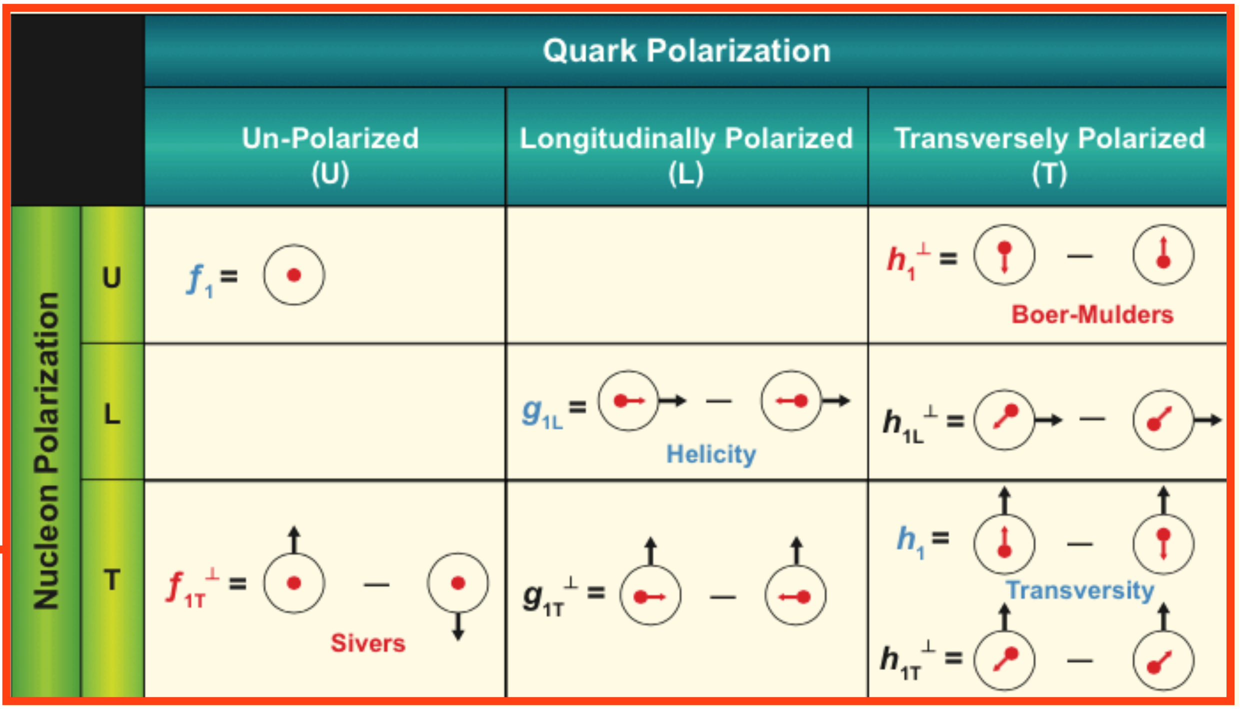
Γ	γ^+	$\gamma^+\gamma^5$	$\gamma^+\gamma^i_\perp\gamma^5$

$$\boxed{\phi_{\alpha\beta}(x,\vec{k}_\perp)} = \int \frac{d^{2-}r}{(2\pi)^3}\, e^{ik\cdot r} \langle h(p,S)| \boxed{\bar{\psi}_\beta(0)}\boxed{\mathcal{U}[0,r]}\boxed{\psi_\alpha(r)}\, |h(p,S)\rangle$$

Transverse Momentum Dependent
Parton Distribution Functions

$$\sum_{\sigma\lambda} \langle h(p)| b^\dagger_{k\sigma} b_{k\lambda} |h(p)\rangle \;\; [\bar{U}_\sigma(k)]_\beta [U_\lambda(k)]_\alpha$$

Quark number operator + Dirac spinors



Γ	γ^+	$\gamma^+\gamma^5$	$\gamma^+\gamma_\perp^i\gamma^5$

Gauge Link: momentum redistribution due to final-state interactions

$$\boxed{\phi_{\alpha\beta}(x,\vec{k}_\perp)} = \int \frac{d^{2-}r}{(2\pi)^3}\, e^{ik\cdot r} \langle h(p,S)| \boxed{\bar{\psi}_\beta(0)} \boxed{\mathcal{U}[0,r]} \boxed{\psi_\alpha(r)} |h(p,S)\rangle$$

Transverse Momentum Dependent
Parton Distribution Functions

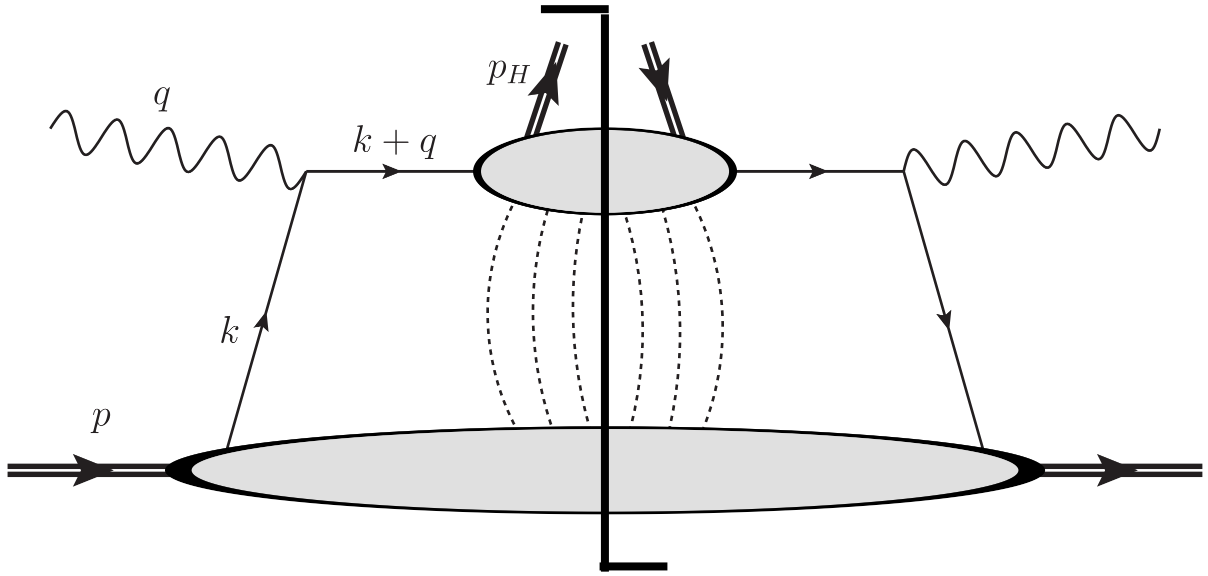
$$\sum_{\sigma\lambda} \langle h(p)| b^\dagger_{k\sigma} b_{k\lambda} |h(p)\rangle \;\; [\bar{U}_\sigma(k)]_\beta [U_\lambda(k)]_\alpha$$

Quark number operator + Dirac spinors

Semi-Inclusive

Deep Inelastic Scattering (SIDIS)

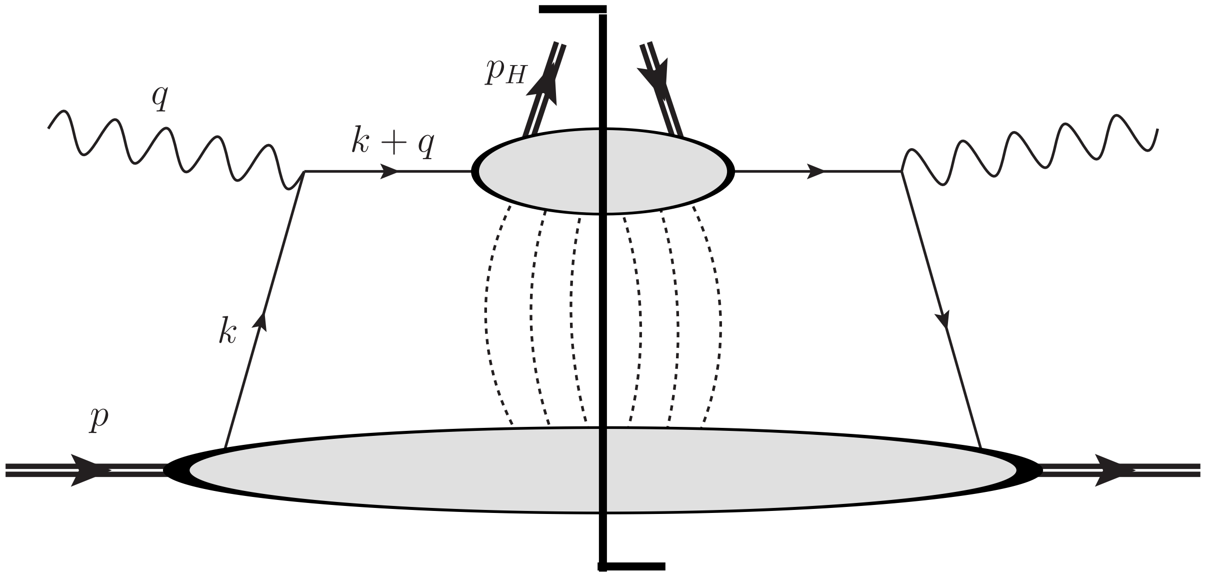
$$e + p \to e' + \boxed{h} + X$$



Semi-Inclusive

Deep Inelastic Scattering (SIDIS)

$$e + p \to e' + \boxed{h} + X$$



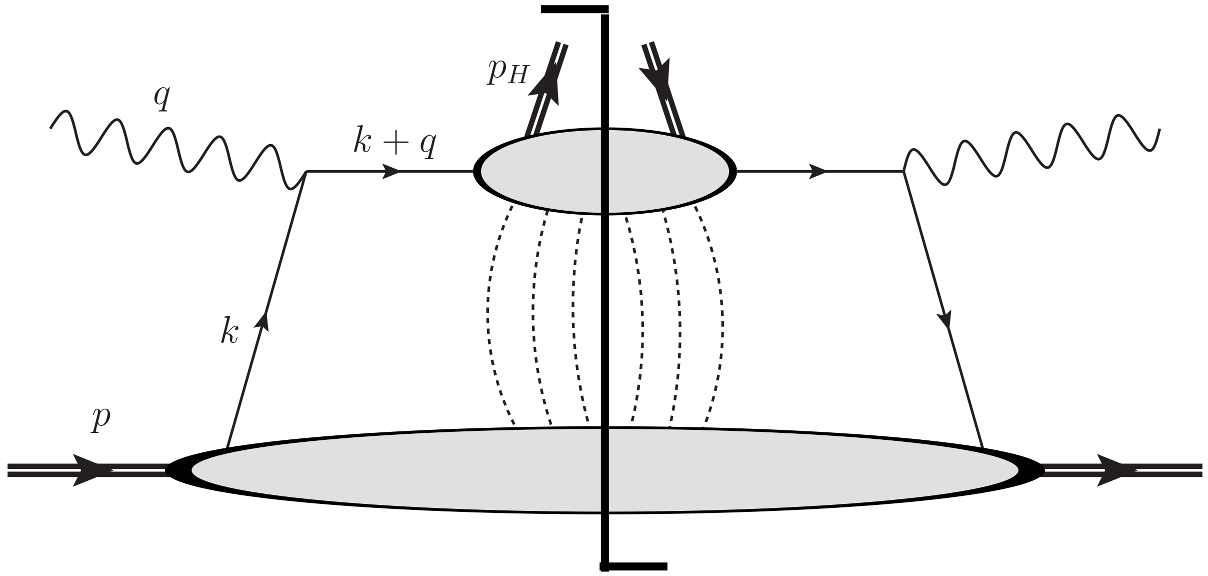
Large-x Kinematics:

$$\hat{s} \sim Q^2 \gg k_T^2$$
$$x = \frac{Q^2}{\hat{s}+Q^2} \sim \mathcal{O}(1)$$

Semi-Inclusive

Deep Inelastic Scattering (SIDIS)

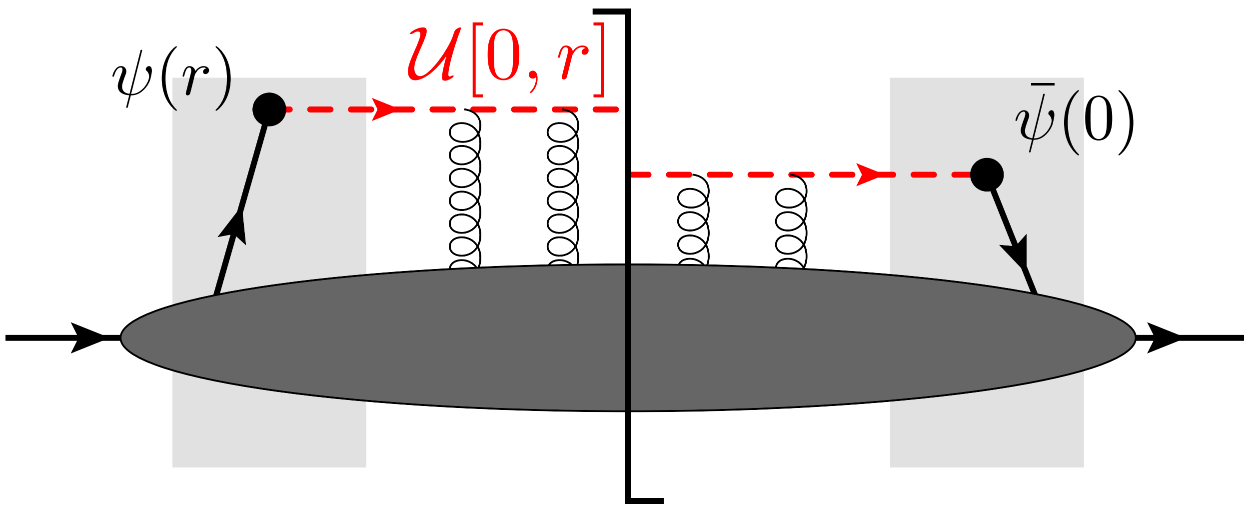
$$e + p \to e' + \boxed{h} + X$$



Large-x Kinematics:

$$\boxed{\begin{array}{c} \hat{s} \sim Q^2 \gg k_T^2 \\ x = \frac{Q^2}{\hat{s}+Q^2} \sim \mathcal{O}(1) \end{array}}$$

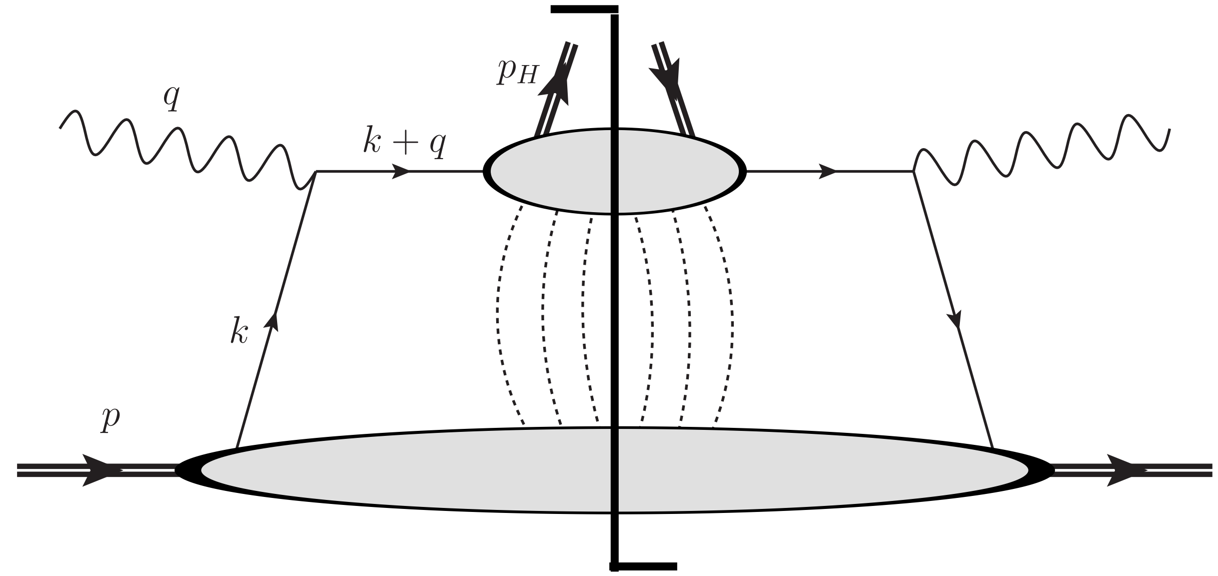
- Photon knocks out a quark from the proton.



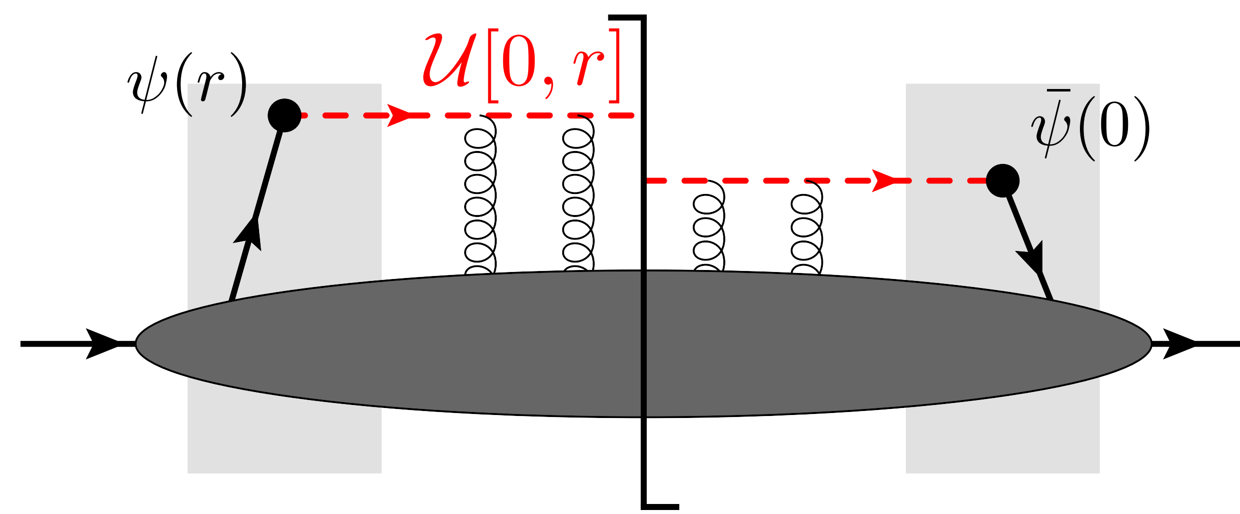
Quark Distributions at Large x

Semi-Inclusive
Deep Inelastic Scattering (SIDIS)

$$e + p \to e' + \boxed{h} + X$$



Large-x Kinematics: $\boxed{\begin{array}{c} \hat{s} \sim Q^2 \gg k_T^2 \\ x = \frac{Q^2}{\hat{s}+Q^2} \sim \mathcal{O}(1) \end{array}}$

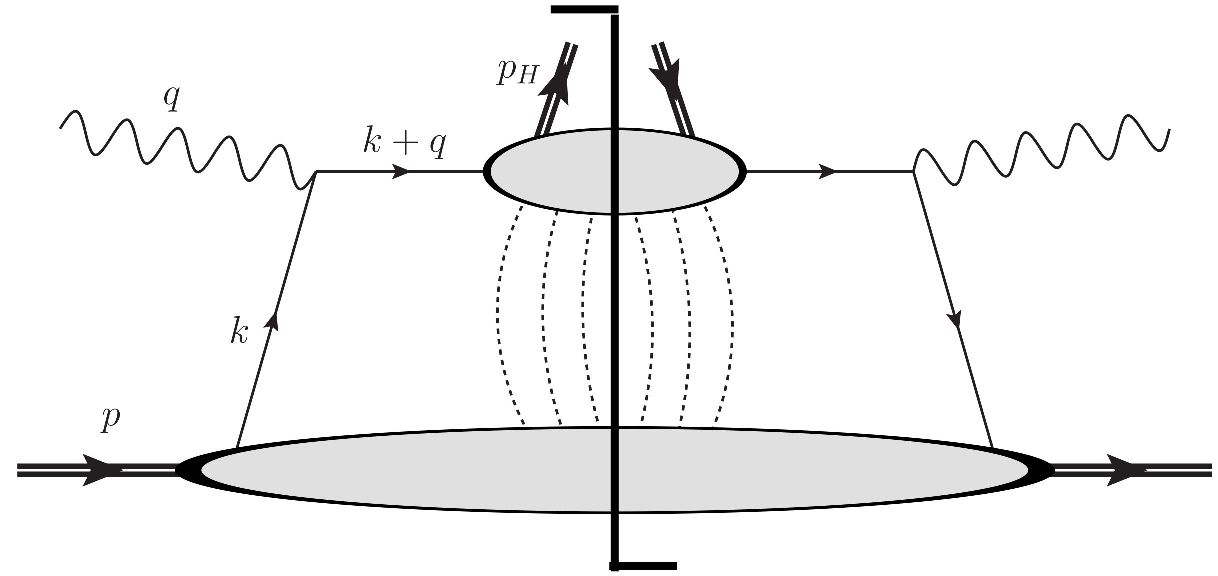


- Photon knocks out a quark from the proton.
- Propagates through the gauge field before escaping

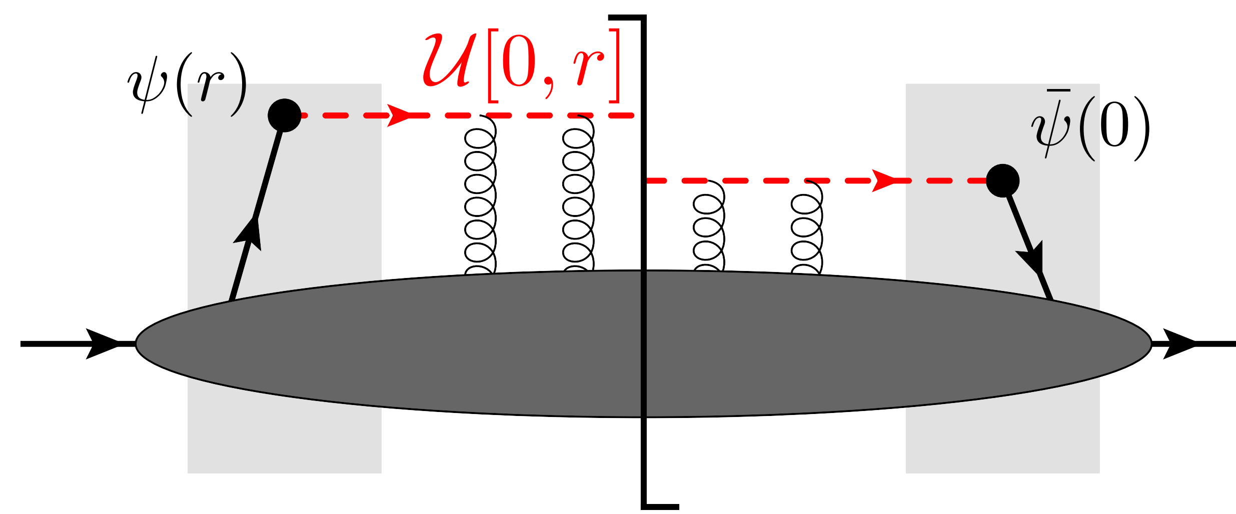
Semi-Inclusive

Deep Inelastic Scattering (SIDIS)

$$e + p \to e' + \boxed{h} + X$$



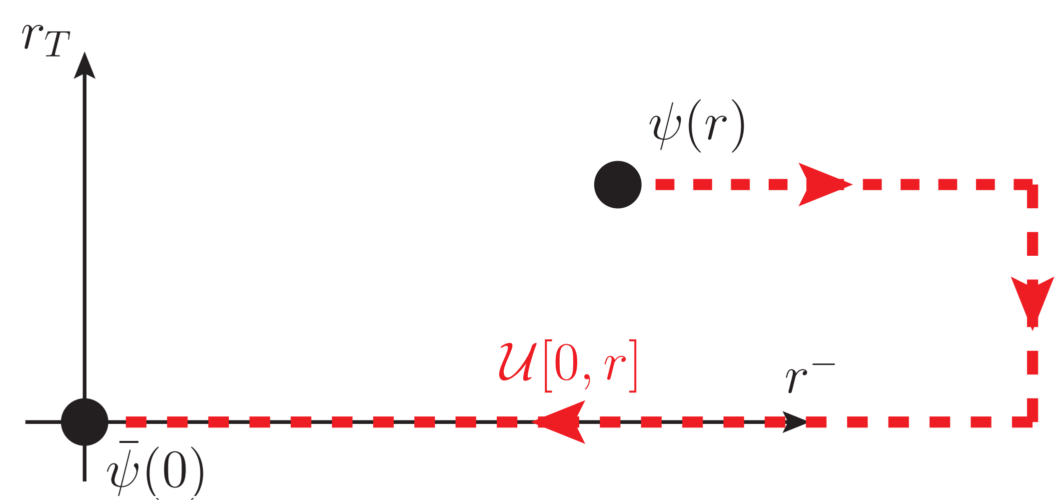
Large-x Kinematics:
$$\hat{s} \sim Q^2 \gg k_T^2$$
$$x = \frac{Q^2}{\hat{s}+Q^2} \sim \mathcal{O}(1)$$



- Photon knocks out a quark from the proton.

- Propagates through the gauge field before escaping

- Staple-shaped gauge link encodes final-state interactions



Quarks Distributions at Small x

Small-x Kinematics:

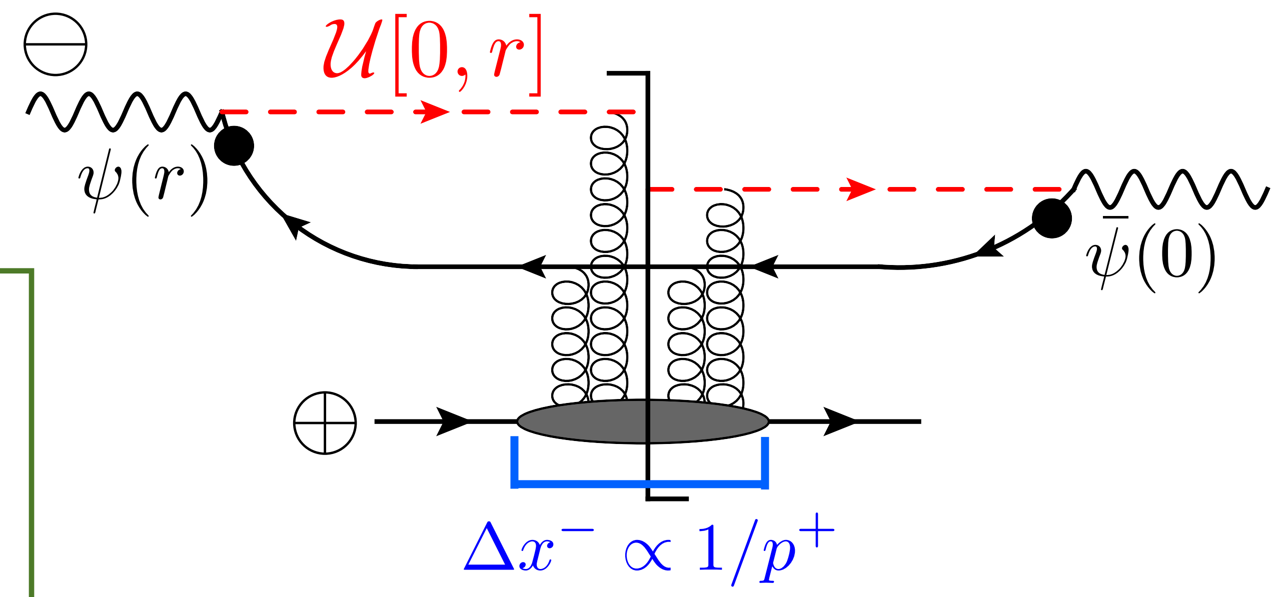
$$\boxed{\begin{array}{l} \hat{s} \gg Q^2 \gg k_T^2 \\ x = \frac{Q^2}{\hat{s}} \ll 1 \end{array}}$$

$$\Delta t < \frac{1}{m_N x}$$

Small-x Kinematics:

$$\hat{s} \gg Q^2 \gg k_T^2$$
$$x = \frac{Q^2}{\hat{s}} \ll 1$$

$$\Delta t < \frac{1}{m_N x}$$



- Photon creates a quark / antiquark pair which propagates through the proton.

➡ Quark transport is x-suppressed.

Quark Distributions at Small x

Small-x Kinematics: $\boxed{\begin{array}{l}\hat{s} \gg Q^2 \gg k_T^2 \\ x = \frac{Q^2}{\hat{s}} \ll 1\end{array}}$

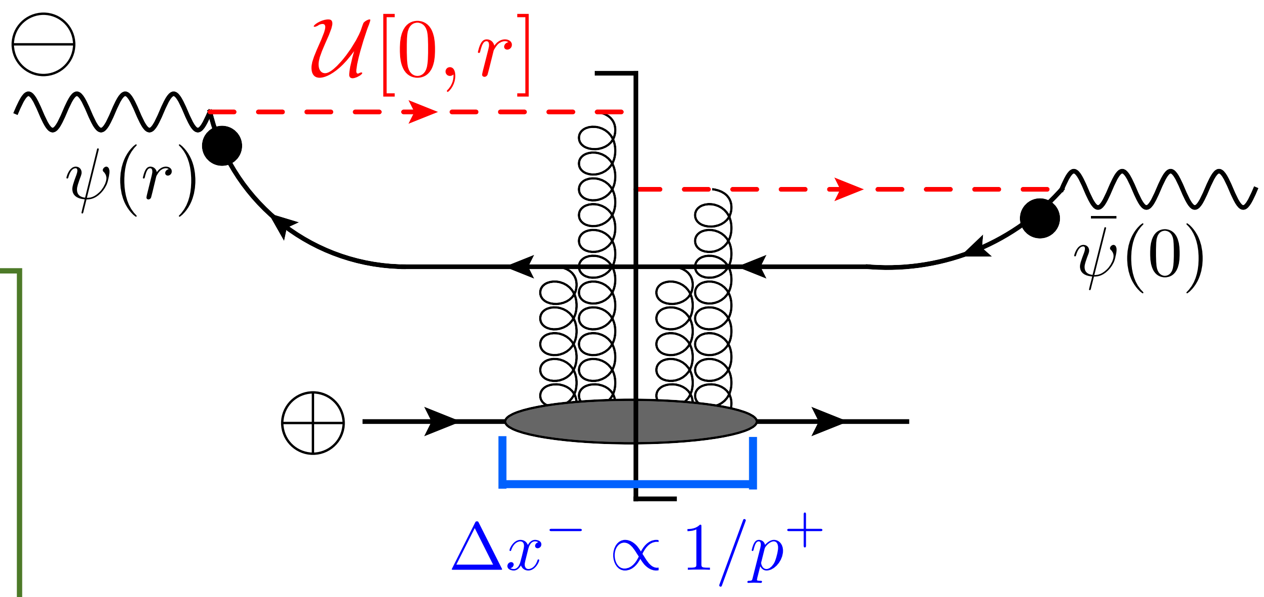
$$\Delta t < \frac{1}{m_N x}$$

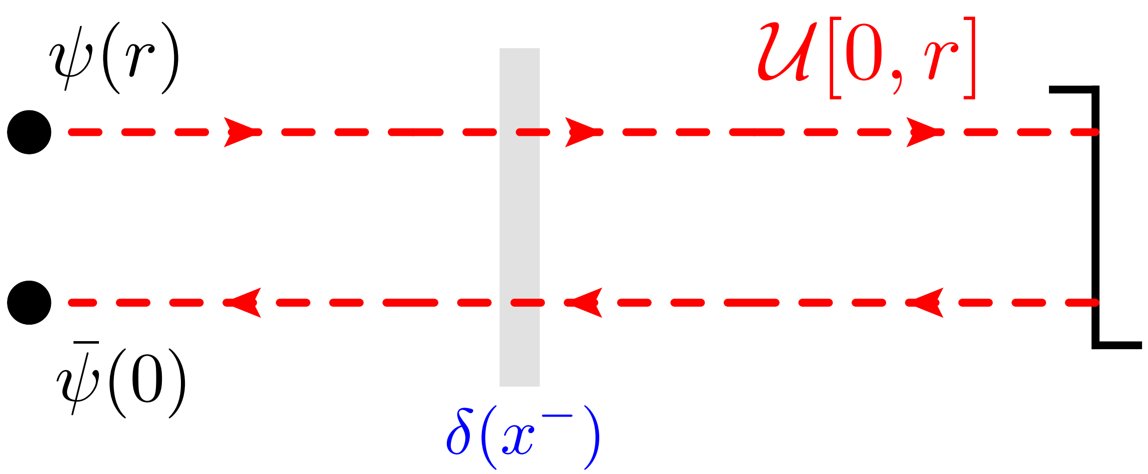
- Photon creates a quark / antiquark pair which propagates through the proton.

➡ Quark transport is x-suppressed.

- Proton is Lorentz-contracted to a "shockwave".

➡ Gauge link covers the entire proton.



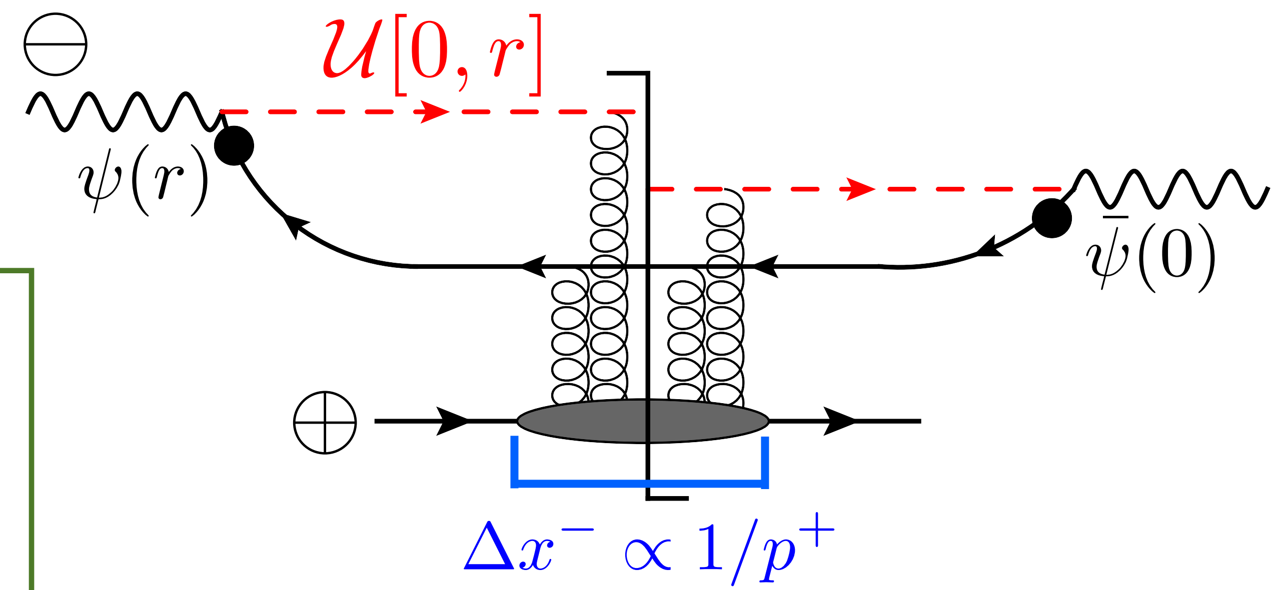


Quark Distributions at Small x

Small-x Kinematics: $\hat{s} \gg Q^2 \gg k_T^2$

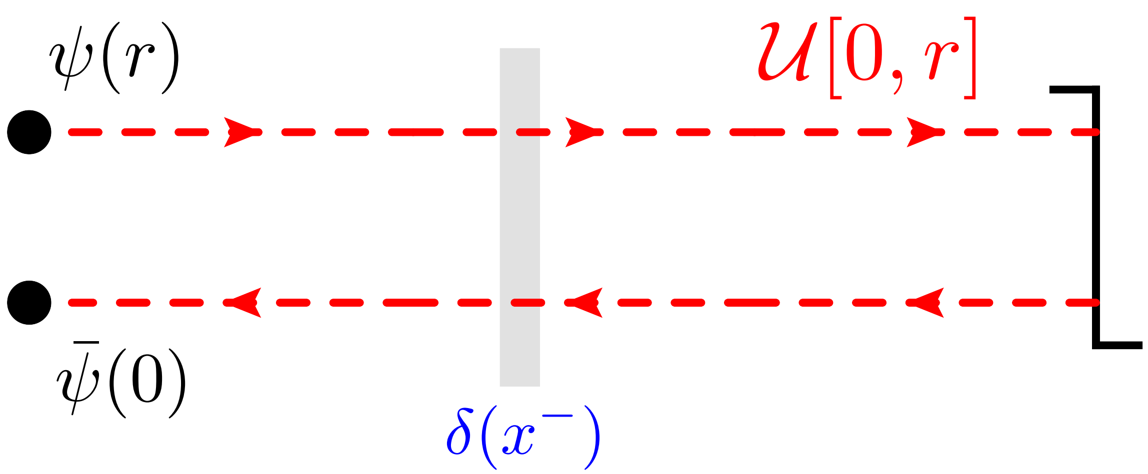
$x = \frac{Q^2}{\hat{s}} \ll 1$

$\Delta t < \frac{1}{m_N x}$



- Photon creates a quark / antiquark pair which propagates through the proton.

➡ Quark transport is x-suppressed.



- Proton is Lorentz-contracted to a "shockwave".

➡ Gauge link covers the entire proton.

➡ Infinite dipole degrees of freedom at small x

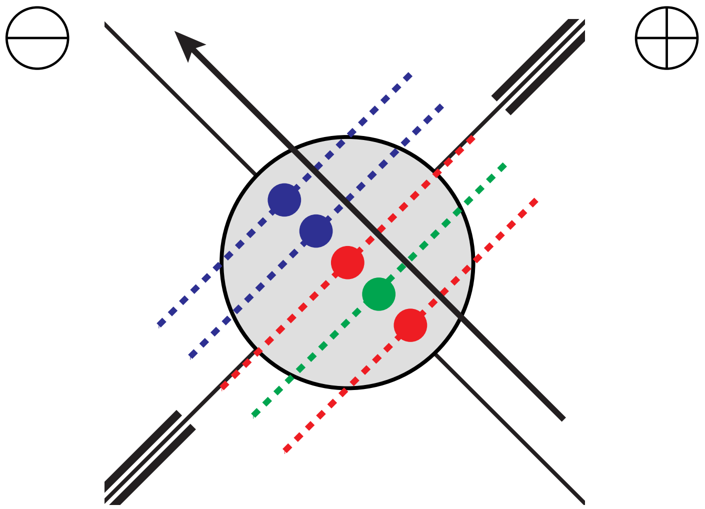
$$S_{xy} = \frac{1}{N_c} \mathrm{Tr}\left[V_x V_y^\dagger\right]$$

Initial Conditions at Small x

- Long-lived projectile sees whole target coherently.

➡ High gluon density at small x enhances multiple scattering

$$\Delta t < \frac{1}{m_N x}$$

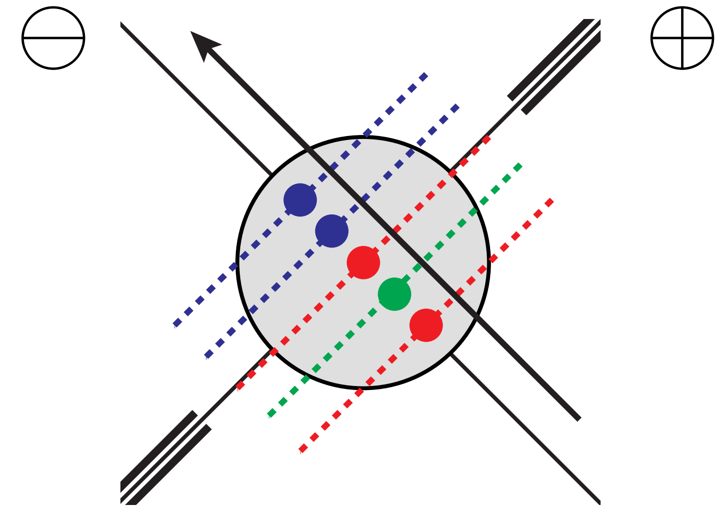


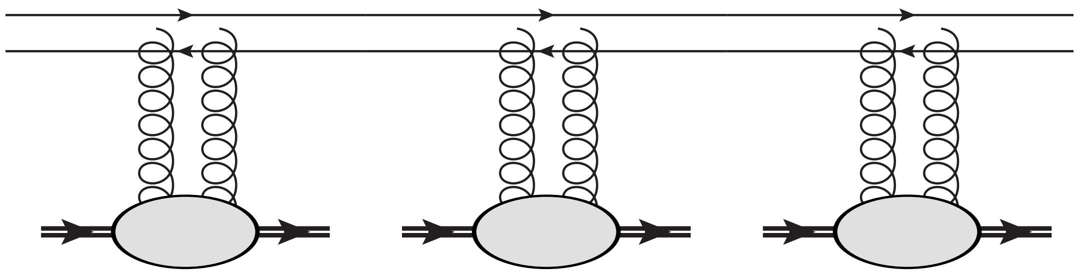
Initial Conditions at Small x

- Long-lived projectile sees whole target coherently.

➡ High gluon density at small x enhances multiple scattering

$$\Delta t < \frac{1}{m_N x}$$





- High density rescattering can be systematically re-summed

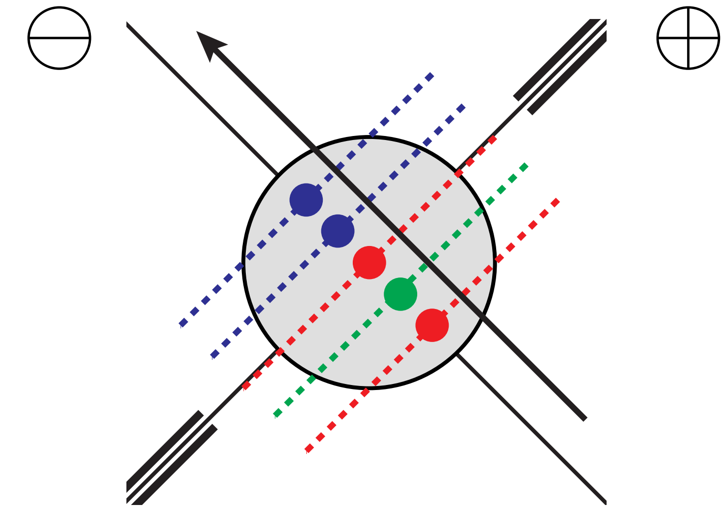
➡ Classical gluon fields!

Nucleus: $\alpha_s^2 A^{1/3} \sim 1$ Proton: $\alpha_s \rho \sim 1$

- Long-lived projectile sees whole target coherently.

➡ High gluon density at small x enhances multiple scattering

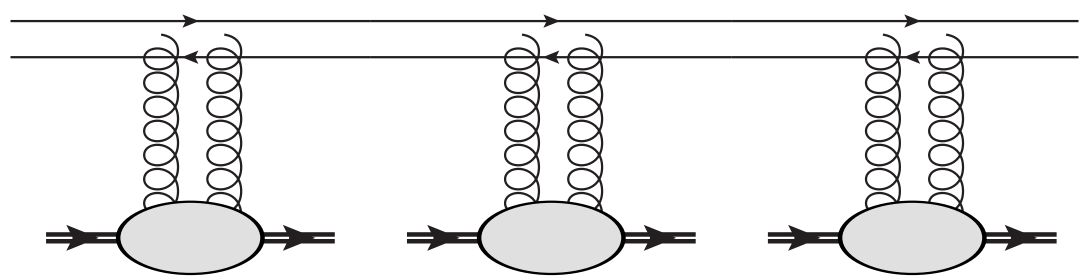
$$\Delta t < \frac{1}{m_N x}$$



- High density rescattering can be systematically re-summed

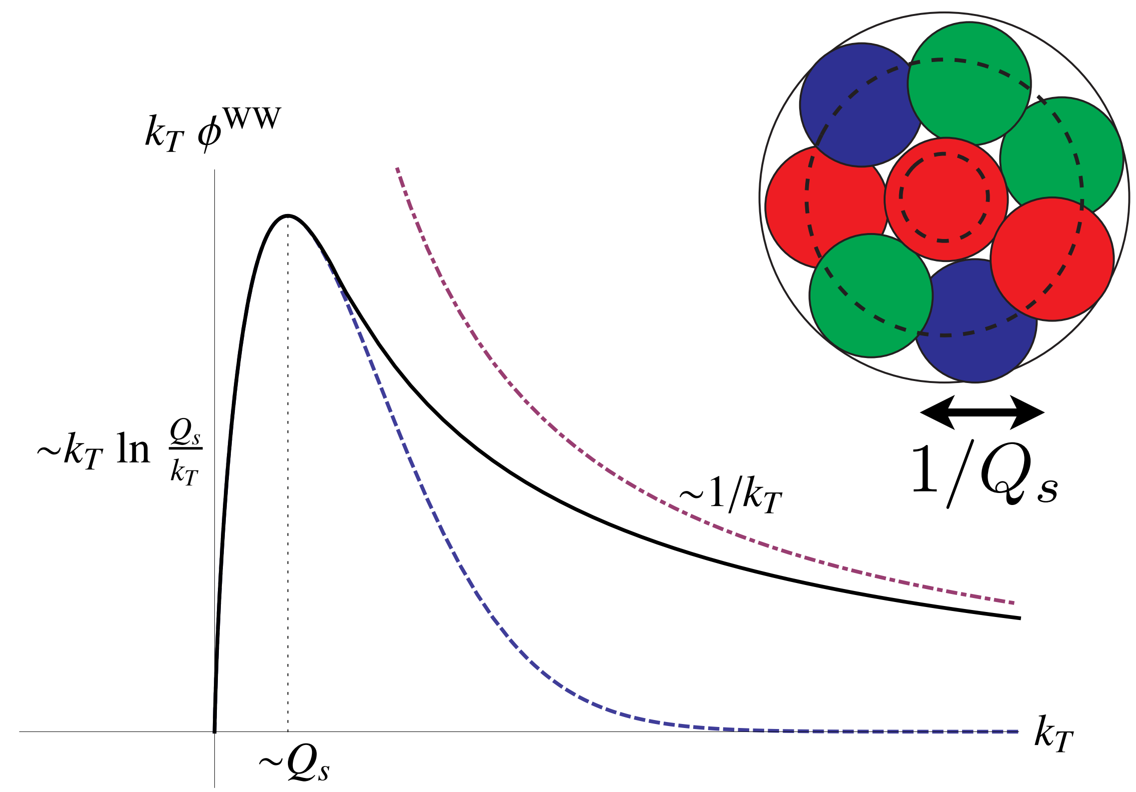
➡ Classical gluon fields!

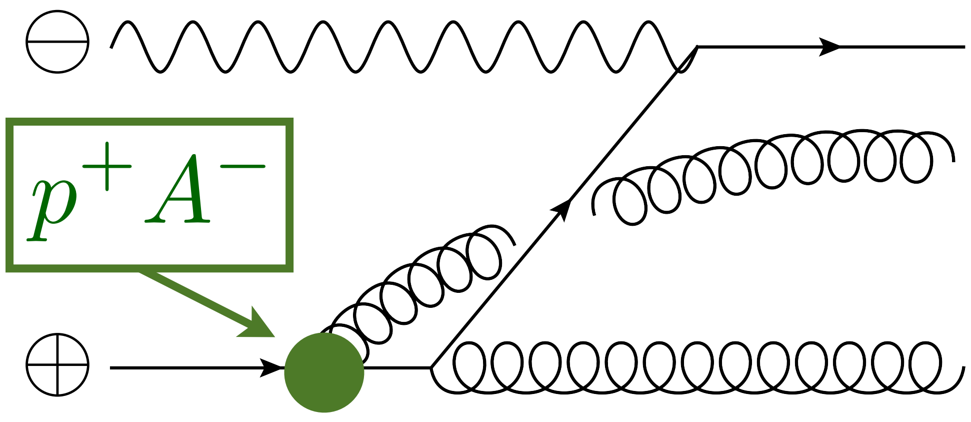
Nucleus: $\alpha_s^2 A^{1/3} \sim 1$ Proton: $\alpha_s \rho \sim 1$

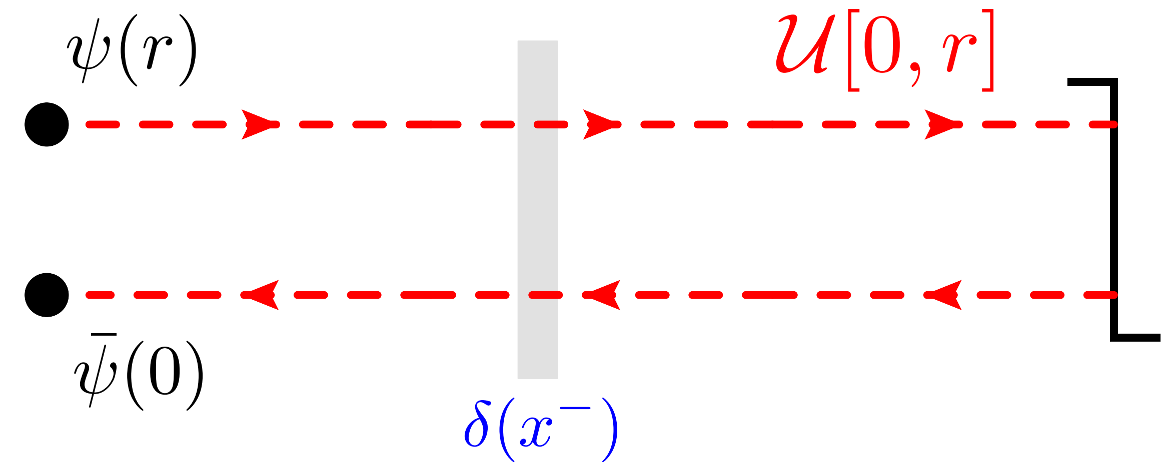


- Charge density defines a hard momentum scale which screens the IR gluon field.

Both:
$$Q_s^2 \propto \alpha_s^2 A^{1/3} \propto \alpha_s \rho$$
$$Q_s^2 \gg \Lambda^2$$



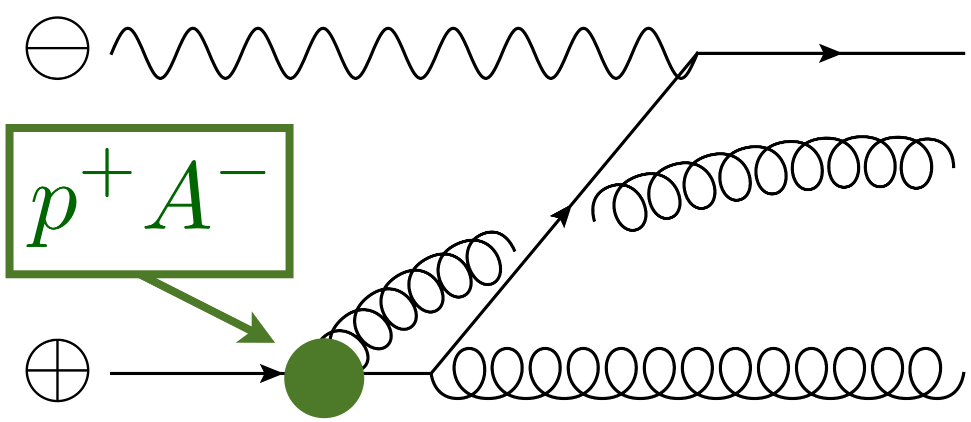


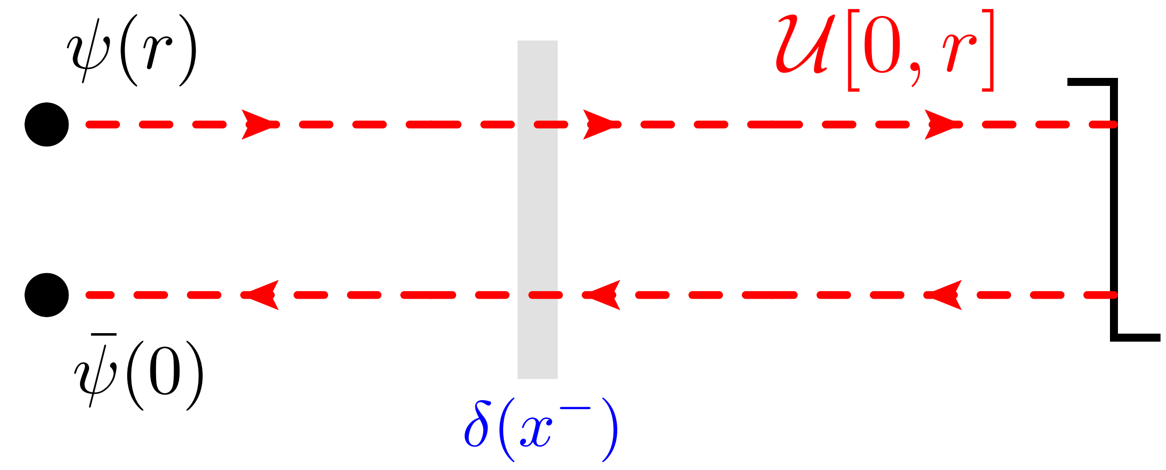


- High-energy radiation from a $\oplus$ moving particle couples to A^-

➡ In $A^- = 0$ gauge this radiation is suppressed.

Quantum Evolution in the Light-Cone Gauge





- High-energy radiation from a $\oplus$ moving particle couples to A^-

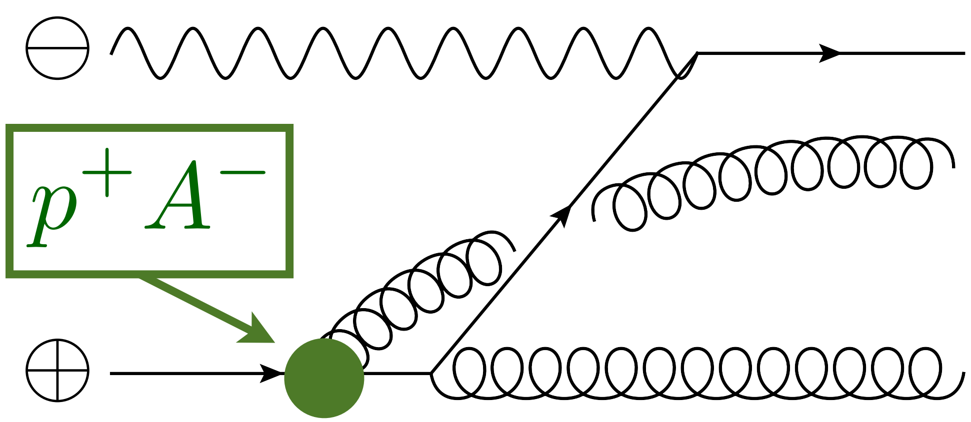
➡ In $A^- = 0$ gauge this radiation is suppressed.

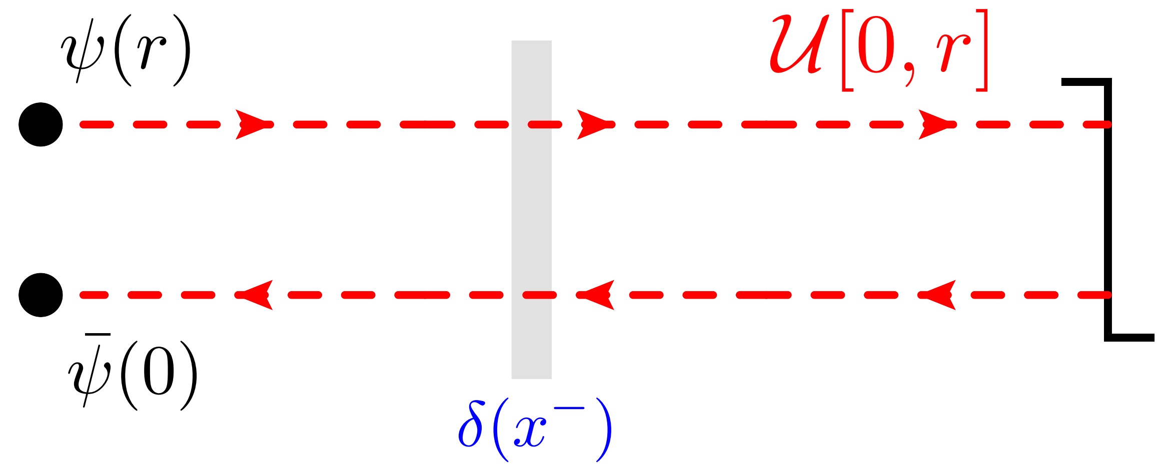
- Quantum evolution requires long lifetimes to generate logarithms of a large phase space.

➡ Instantaneous t-channel particles do not evolve either.

➡ All evolution takes place within the $\ominus$ moving particles.

Quantum Evolution in the Light-Cone Gauge





- High-energy radiation from a $\oplus$ moving particle couples to A^-

➡ In $A^- = 0$ gauge this radiation is suppressed.

- Quantum evolution requires long lifetimes to generate logarithms of a large phase space.

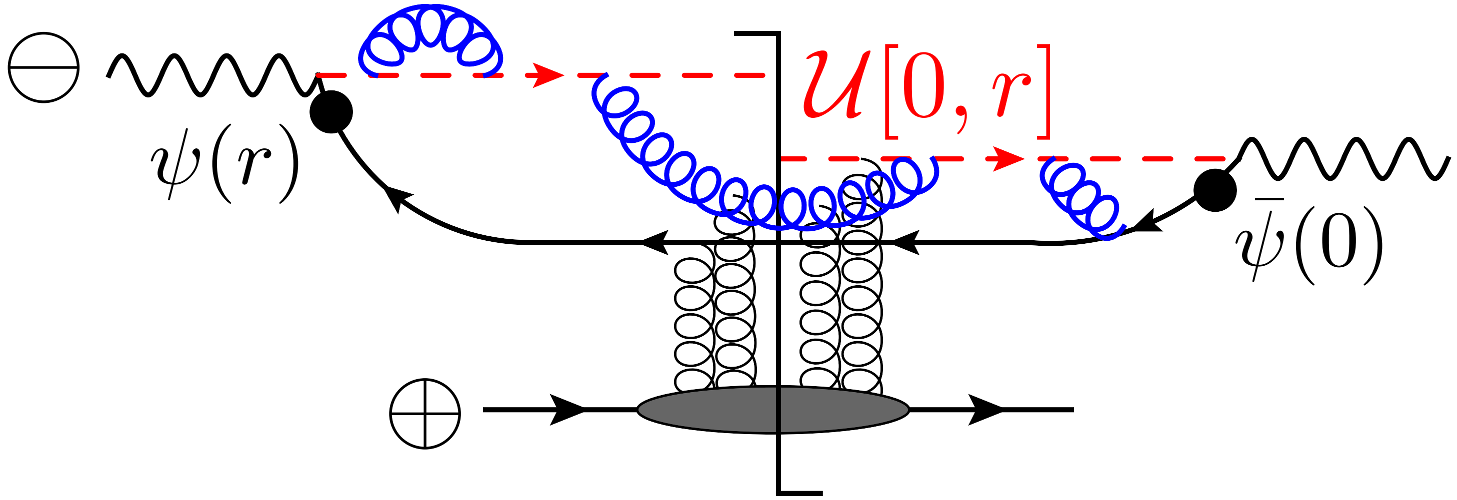
➡ Instantaneous t-channel particles do not evolve either.

➡ All evolution takes place within the $\ominus$ moving particles.

- For classical fields and leading-log evolution, $A_\perp = 0$ as well.

➡ The transverse part of the gauge link does not contribute.

BK Evolution: The Small-x Gluon Cascade



$$S_{xy} = \frac{1}{N_c} \mathrm{Tr}\left[V_x V_y^\dagger\right]$$

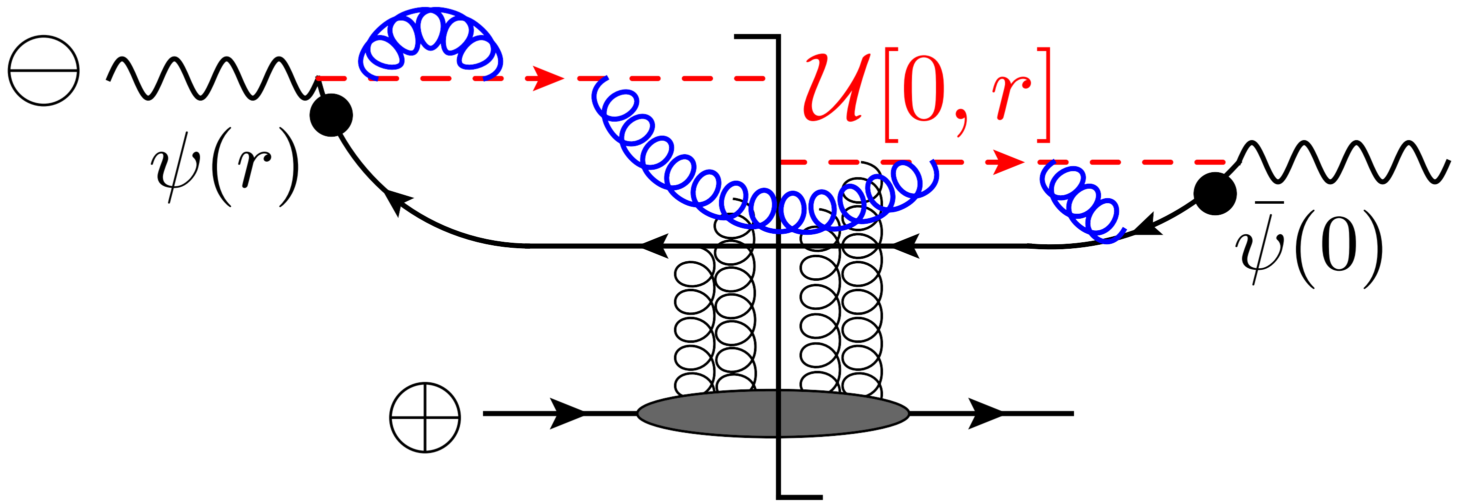
- The quark dipole radiates soft gluons before and after scattering.

➡ Evolution of the dipole scattering amplitude

➡ Re-sums single logarithms of x

$$\alpha_s \ln \frac{1}{x} \sim 1$$

BK Evolution: The Small-x Gluon Cascade



$$S_{xy} = \frac{1}{N_c}\mathrm{Tr}\left[V_x V_y^\dagger\right]$$

$$\frac{\partial}{\partial \ln s}\langle S_{xy}\rangle_{(s)} = \bar{\alpha}_s \int d^2 z \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2}\left[\langle S_{xz} S_{zy}\rangle_{(s)} - \langle S_{xy}\rangle_{(s)}\right]$$

- The quark dipole radiates soft gluons before and after scattering.

➡ Evolution of the dipole scattering amplitude

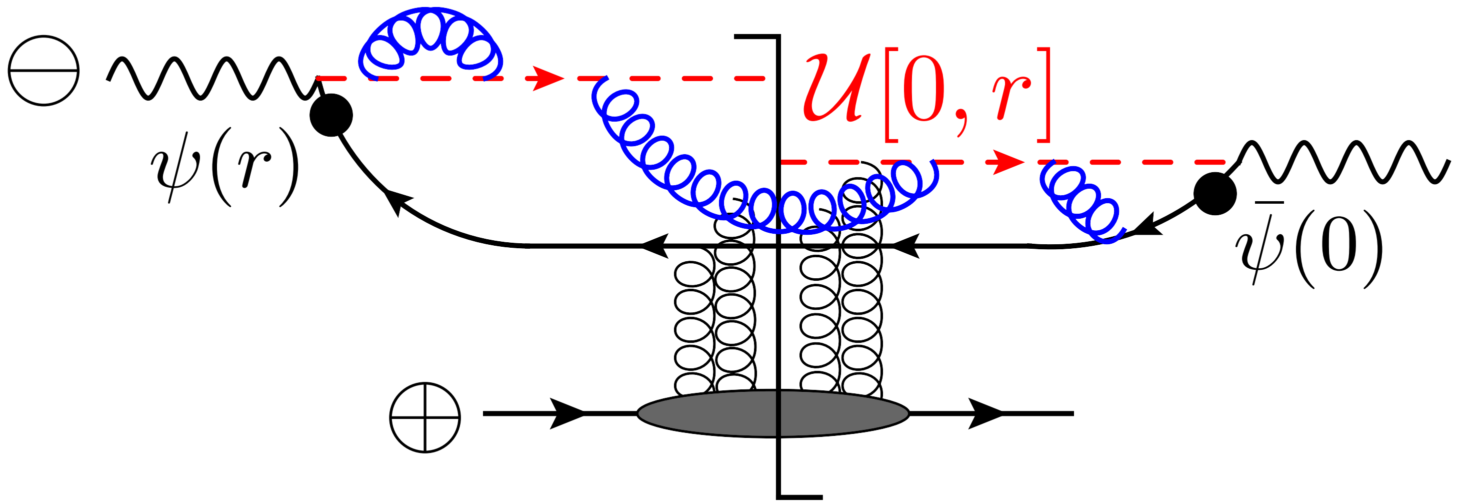
➡ Re-sums single logarithms of x

$$\alpha_s \ln\frac{1}{x} \sim 1$$

- Some radiated gluons also rescatter in the target gauge field.

➡ Non-linear evolution with a hierarchy of operators

BK Evolution: The Small-x Gluon Cascade



$$S_{xy} = \frac{1}{N_c} \mathrm{Tr}\left[V_x V_y^\dagger\right]$$

$$\frac{\partial}{\partial \ln s} \langle S_{xy} \rangle_{(s)} = \bar{\alpha}_s \int d^2 z \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2} \left[\langle S_{xz} \rangle_{(s)} \langle S_{zy} \rangle_{(s)} - \langle S_{xy} \rangle_{(s)} \right]$$

- The quark dipole radiates soft gluons before and after scattering.

➡ Evolution of the dipole scattering amplitude

➡ Re-sums single logarithms of x

$$\alpha_s \ln \frac{1}{x} \sim 1$$

- Some radiated gluons also rescatter in the target gauge field.

➡ Non-linear evolution with a hierarchy of operators

- Evolution closes in the large N_c limit (BK eqn.)

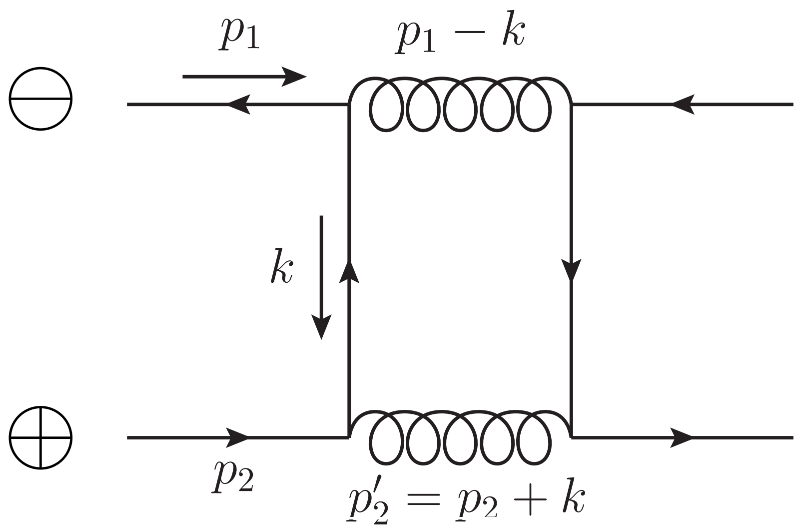
$$Q_s^2(x) \propto \left(\frac{1}{x}\right)^{1/3}$$

The Calculation:
Helicity at Small x

Digging for Spin Structure

- High energy (small x) scattering is predominantly spin independent.

➡ BK evolution: total cross section, unpolarized quark distribution.



$$\boxed{\frac{d\Delta\sigma_q}{d^2k} = -2\frac{\alpha_s^2 C_F^2}{N_c}\frac{1}{s}\frac{1}{k_T^2}}$$

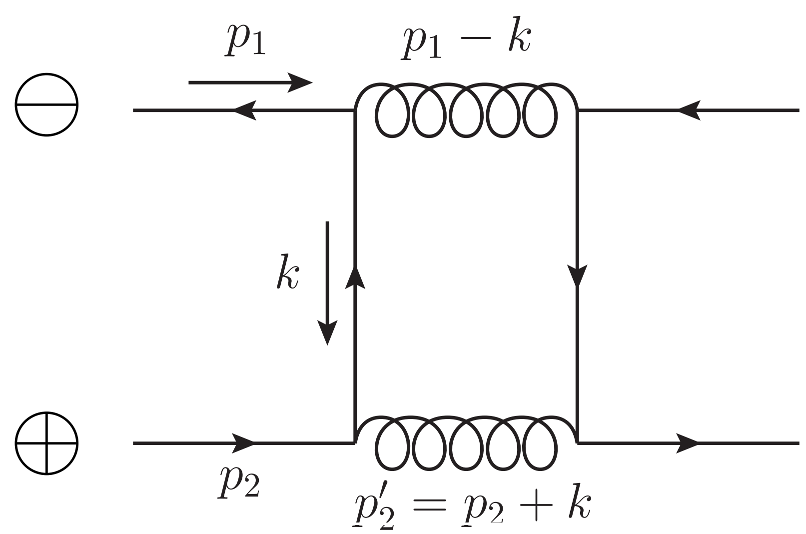
- High energy (small x) scattering is predominantly spin independent.

➡ BK evolution: total cross section, unpolarized quark distribution.

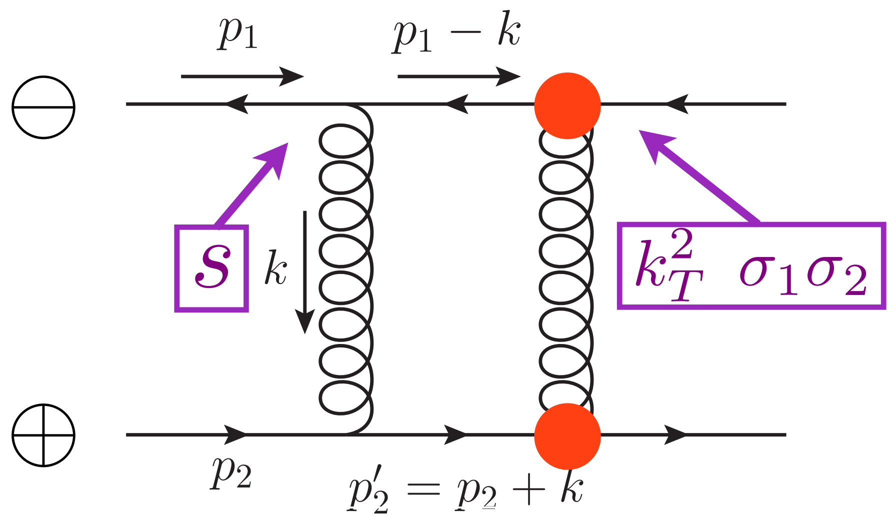
- Transporting quark polarization to small x is suppressed!

➡ Spin asymmetries, polarized quarks are suppressed at small x.

Digging for Spin Structure



$$\boxed{\frac{d\Delta\sigma_q}{d^2k} = -2\frac{\alpha_s^2 C_F^2}{N_c}\ \frac{1}{s}\ \frac{1}{k_T^2}}$$



$$\boxed{\frac{d\Delta\sigma_G}{d^2k} = +\frac{\alpha_s^2 C_F}{N_c}\ \frac{1}{s}\ \frac{1}{k_T^2}}$$

- High energy (small x) scattering is predominantly spin independent.

➡ BK evolution: total cross section, unpolarized quark distribution.

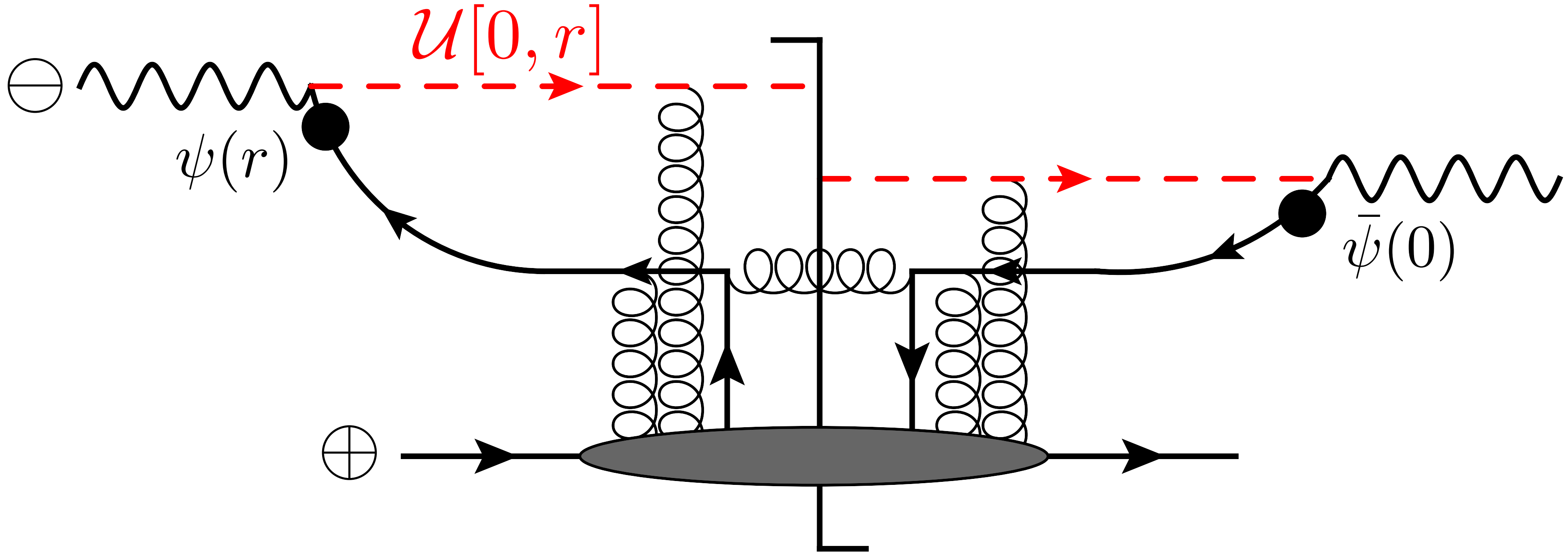
- Transporting quark polarization to small x is suppressed!

➡ Spin asymmetries, polarized quarks are suppressed at small x.

- Sub-leading gluon exchange can also transfer spin dependence.

➡ Gluon exchange can mix with quark exchange.

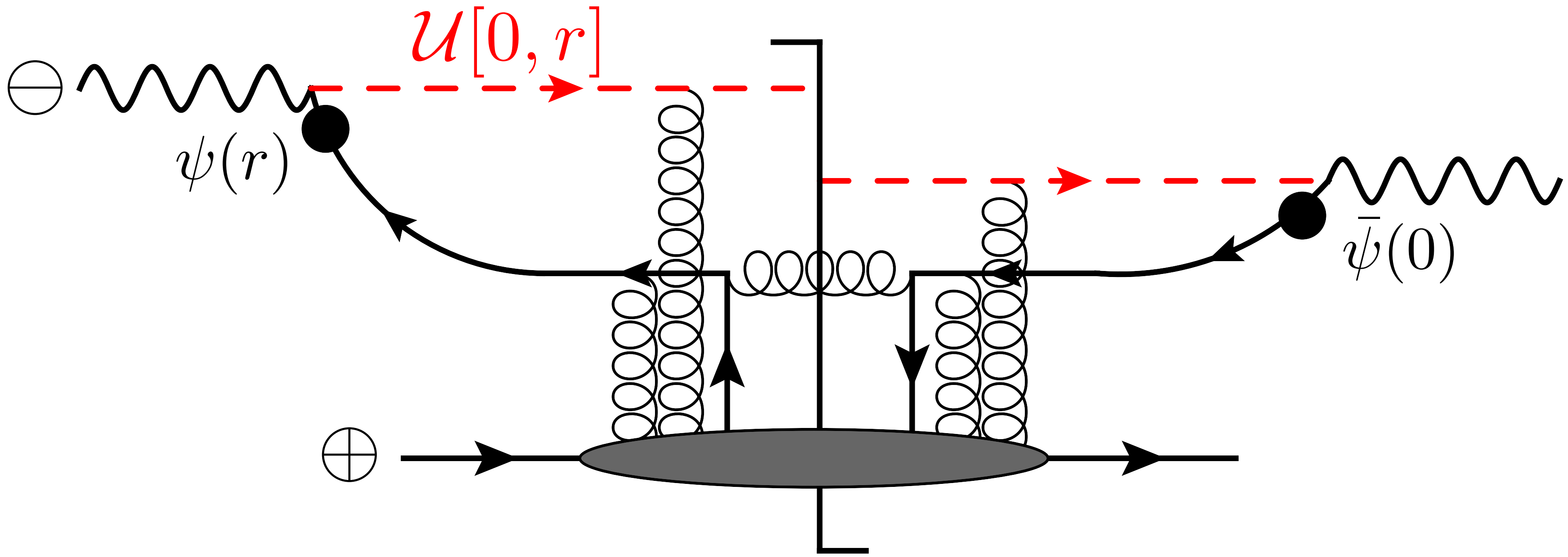
Polarized Initial Conditions



- "Polarized Wilson Line" - Coherent, spin-dependent scattering.

➡ One spin-dependent exchange (more are suppressed)

➡ Dressed by multiple unpolarized scattering



- "Polarized Wilson Line" - Coherent, spin-dependent scattering.

➡ One spin-dependent exchange (more are suppressed)

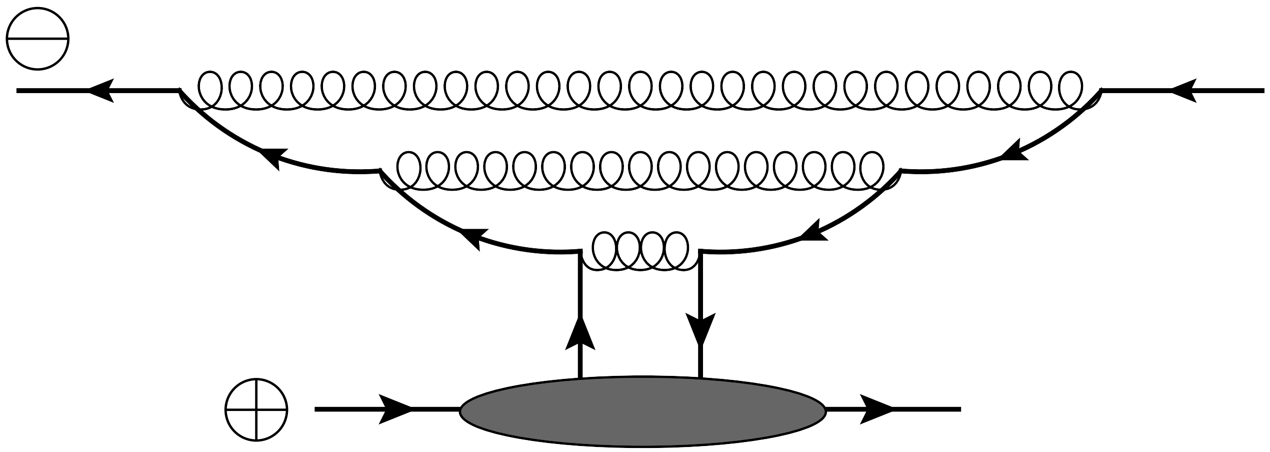
➡ Dressed by multiple unpolarized scattering

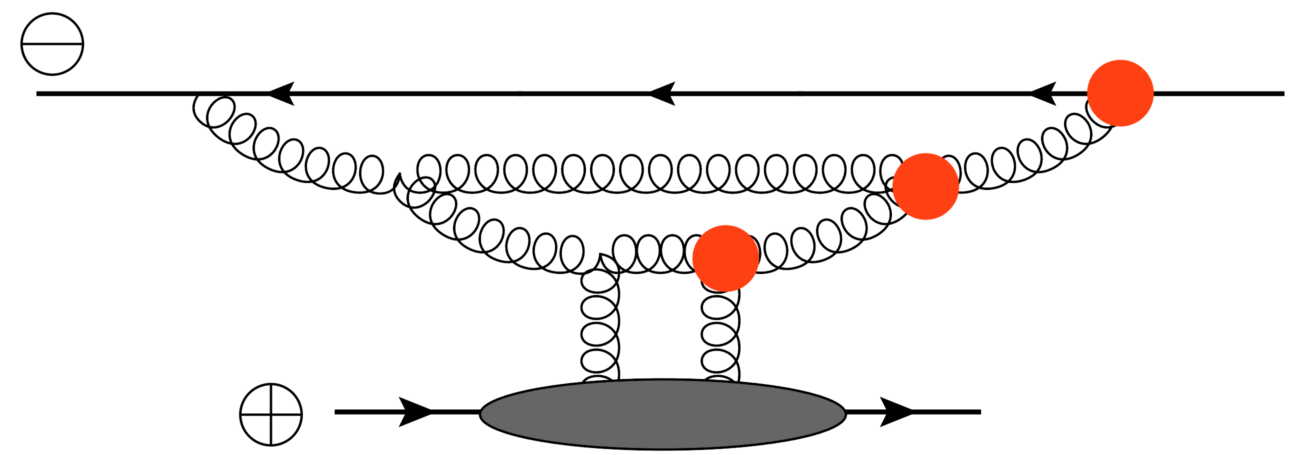
- "Polarized Dipole Amplitude":

➡ Quark (gauge link) scatters by an unpolarized Wilson line.

➡ Fermion (antiquark) scatters by a polarized Wilson line.

$$G_{xy} \equiv \frac{1}{2N_c} \mathrm{Tr} \left[V_x V_y^\dagger(\sigma) + V_y(\sigma) V_x^\dagger \right]$$

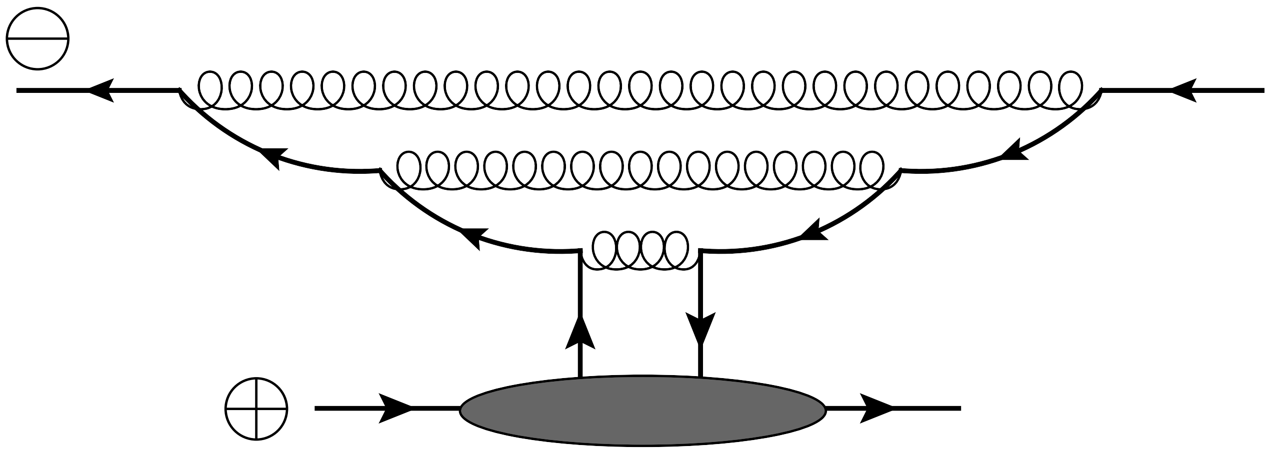


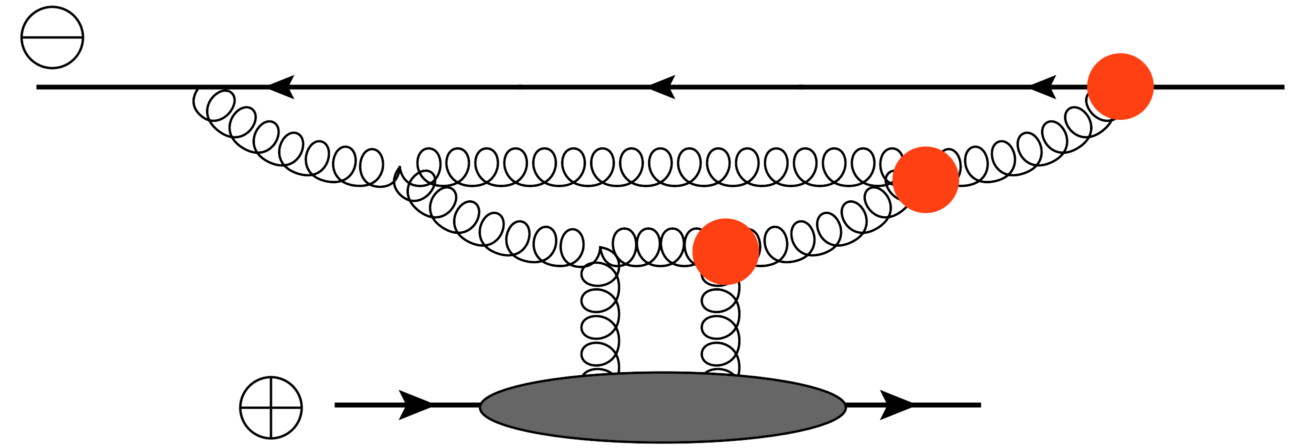


- Kernels: Spin-dependent quark / gluon wave functions

➡ Soft quarks and soft gluons can mix (same order)

Evolving Spin to Small x





- Kernels: Spin-dependent quark / gluon wave functions

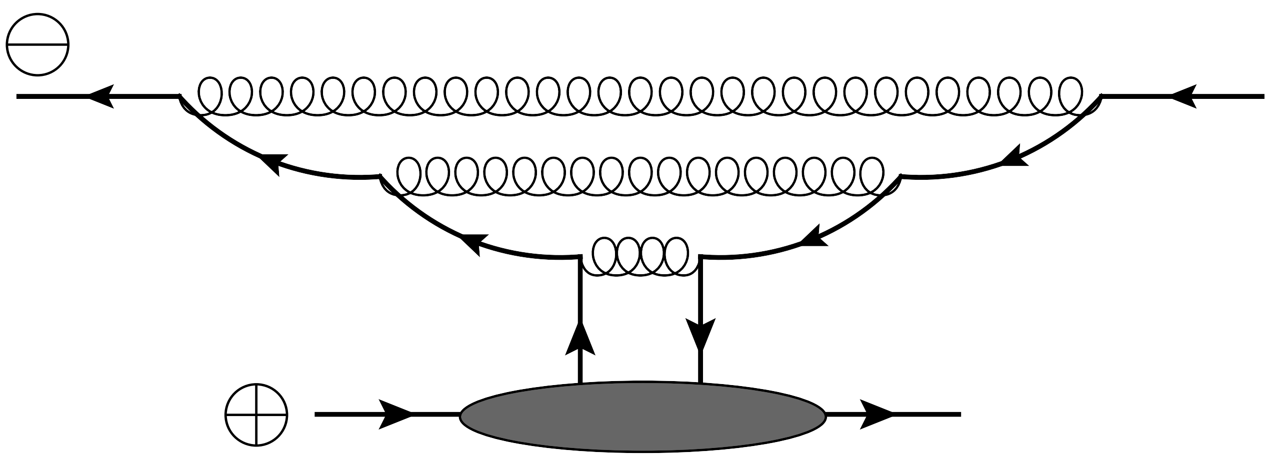
➡ Soft quarks and soft gluons can mix (same order)

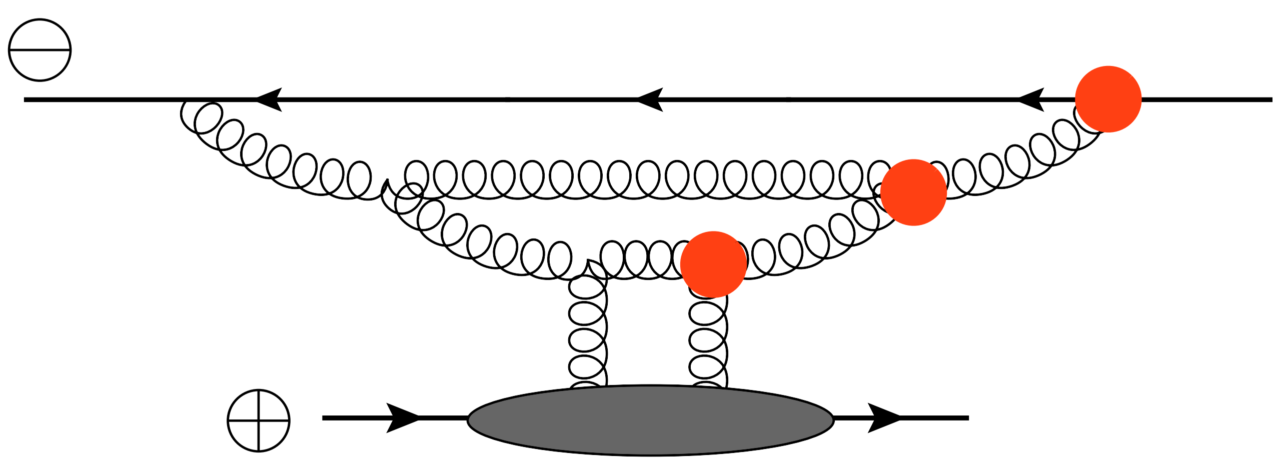
- Requires longitudinal and transverse momentum ordering

$$1 \gg z_1 \gg z_2 \gg \cdots \gg \frac{Q^2}{s} \qquad\qquad Q^2 \ll \frac{k_{1T}^2}{z_1} \ll \frac{k_{2T}^2}{z_2} \ll \cdots$$

➡ Includes "infrared" phase space: $k_{1T}^2 \gg k_{2T}^2 \gg k_{1T}^2 \frac{z_2}{z_1}$

Evolving Spin to Small x





- Kernels: Spin-dependent quark / gluon wave functions

➡ Soft quarks and soft gluons can mix (same order)

- Requires longitudinal and transverse momentum ordering

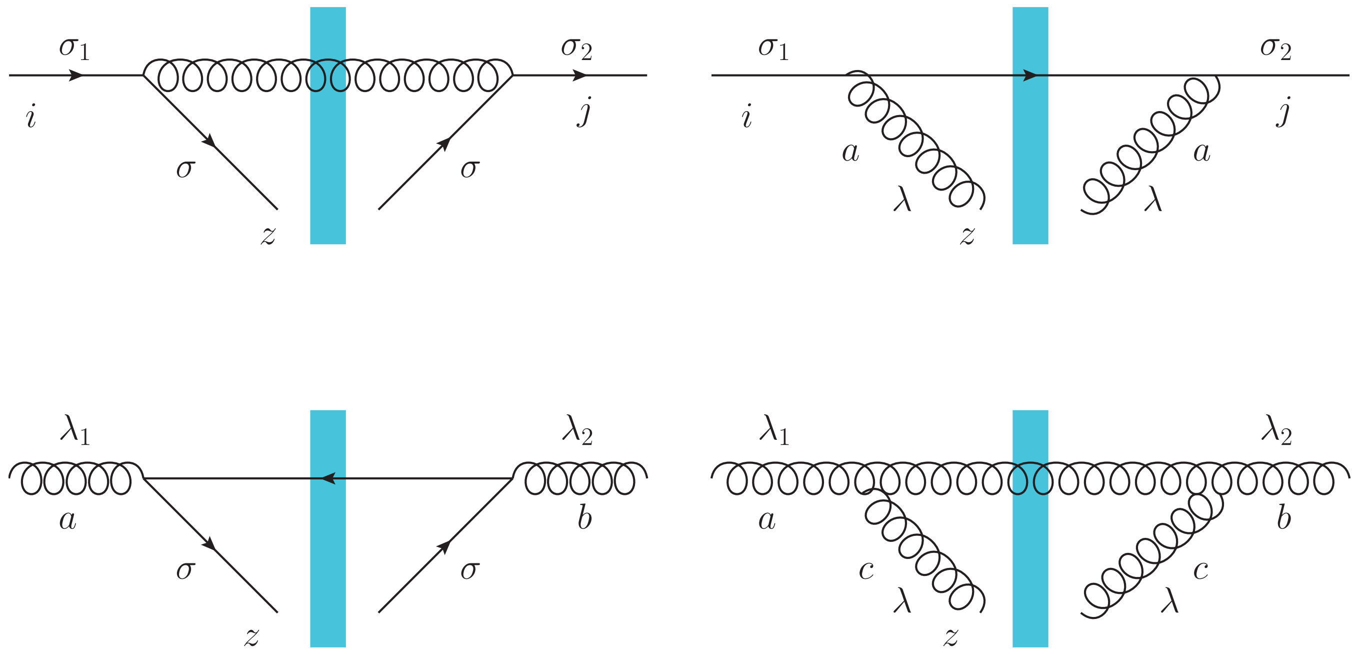
$$1 \gg z_1 \gg z_2 \gg \cdots \gg \frac{Q^2}{s} \qquad\qquad Q^2 \ll \frac{k_{1T}^2}{z_1} \ll \frac{k_{2T}^2}{z_2} \ll \cdots$$

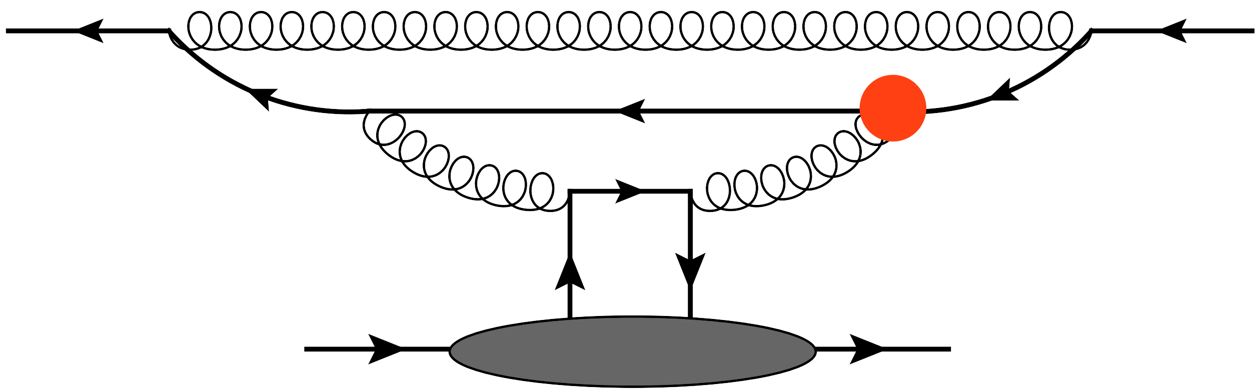
➡ Includes "infrared" phase space: $k_{1T}^2 \gg k_{2T}^2 \gg k_{1T}^2 \frac{z_2}{z_1}$

- Leads to double-log evolution.

➡ Faster evolution than unpolarized BK!

$$\alpha_s \ln^2 \frac{1}{x} \sim 1$$

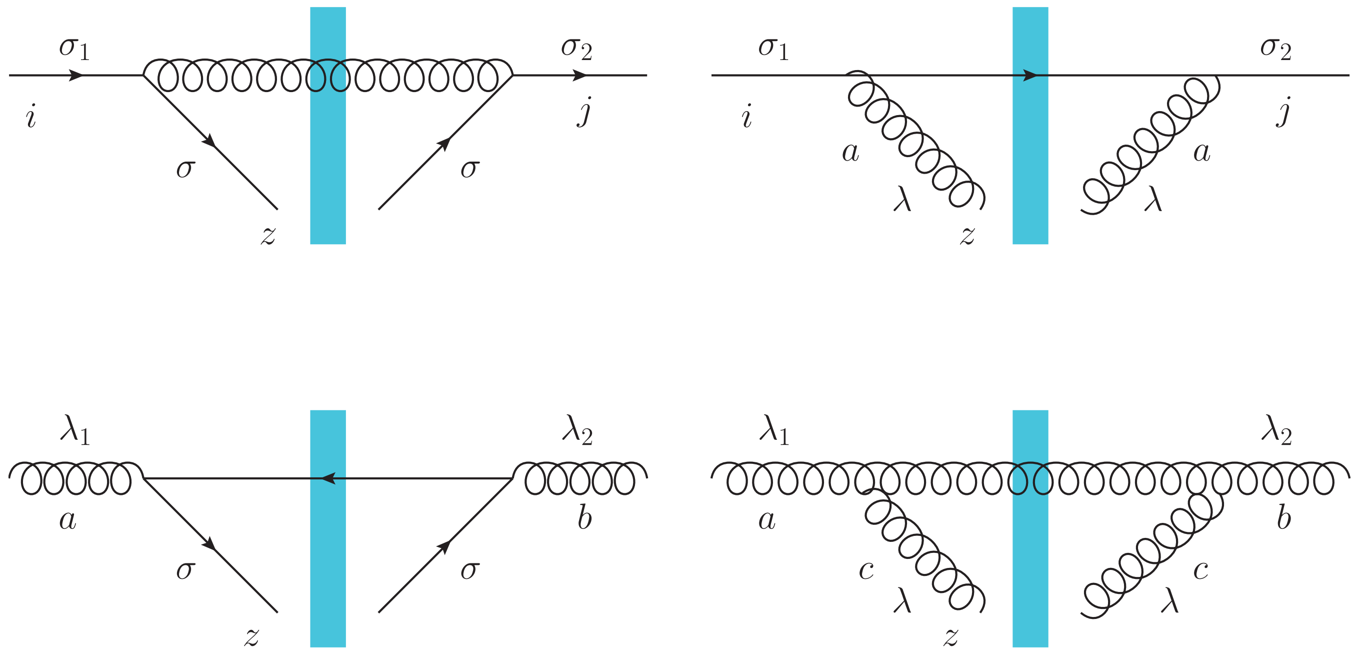


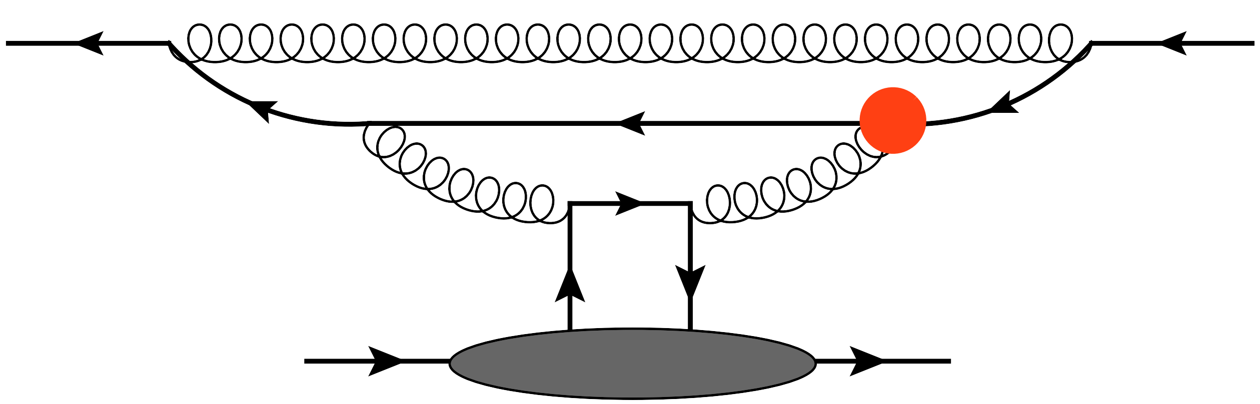


- To solve, first keep only the kernels without unpolarized rescattering.

$$\frac{\alpha_s}{2\pi}\int\frac{dz}{z}\int\frac{dk_T^2}{k_T^2}\begin{pmatrix} C_F & 2C_F \\ -N_f & 4N_c \end{pmatrix}$$

Solution: Ladder Evolution





- To solve, first keep only the kernels without unpolarized rescattering.

$$\frac{\alpha_s}{2\pi}\int\frac{dz}{z}\int\frac{dk_T^2}{k_T^2}\begin{pmatrix} C_F & 2C_F \\ -N_f & 4N_c \end{pmatrix}$$

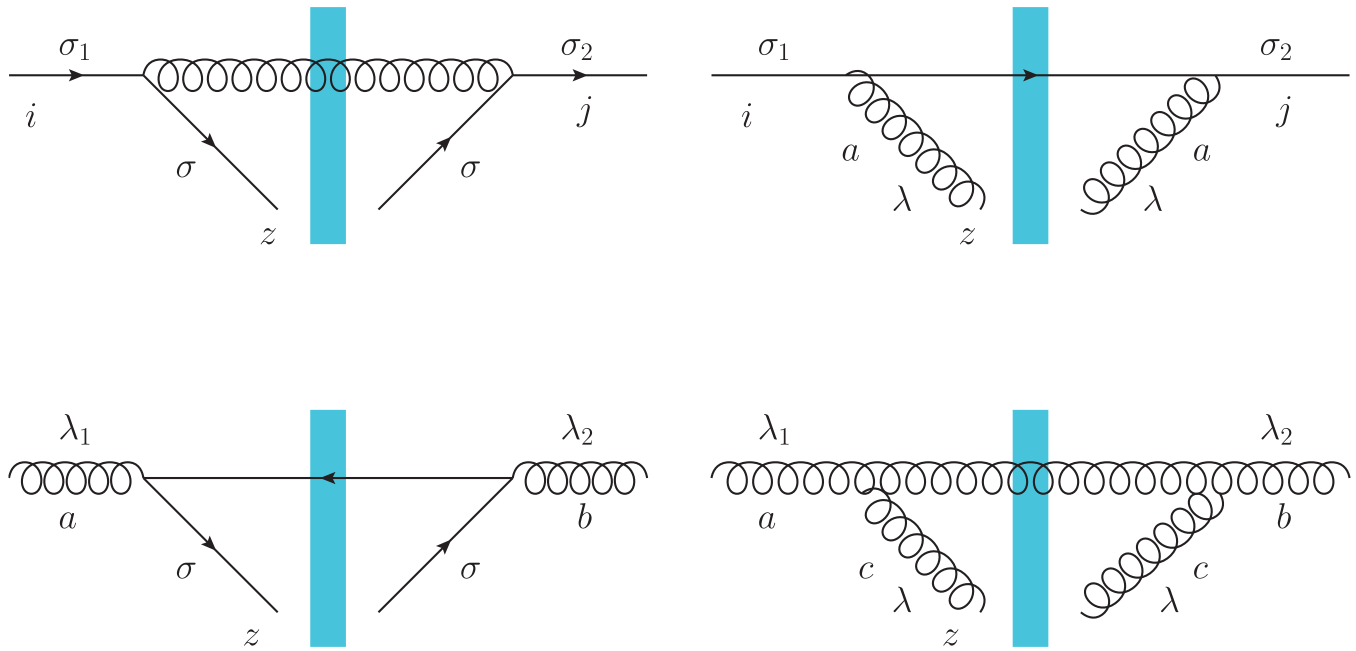
- Solve by Mellin transform and saddle point approximation.

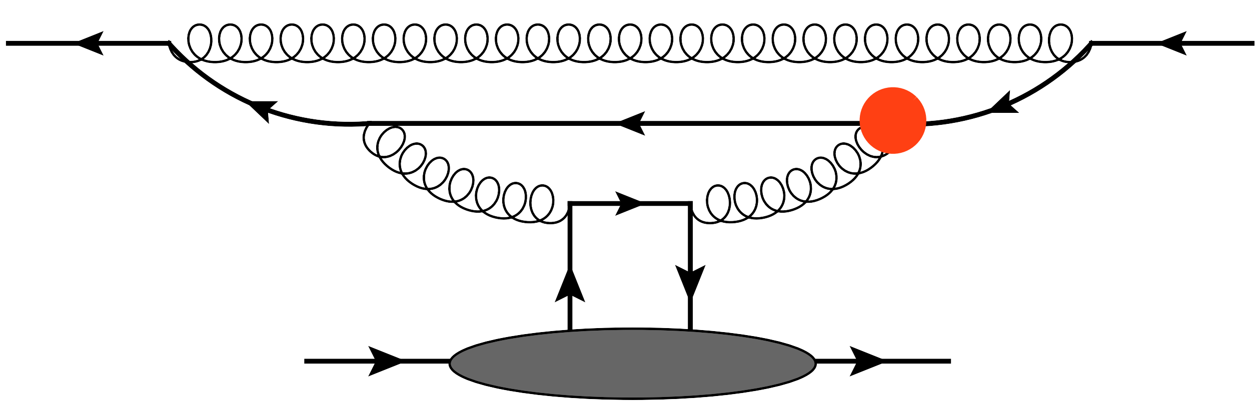
$$\alpha_s = 0.3$$

$$N_c = N_f = 3$$

$$G_{xy}(s) \sim \left(\frac{s}{Q^2}\right)^{1.46}$$

Solution: Ladder Evolution





- To solve, first keep only the kernels without unpolarized rescattering.

$$\frac{\alpha_s}{2\pi}\int\frac{dz}{z}\int\frac{dk_T^2}{k_T^2}\begin{pmatrix} C_F & 2C_F \\ -N_f & 4N_c \end{pmatrix}$$

- Solve by Mellin transform and saddle point approximation.

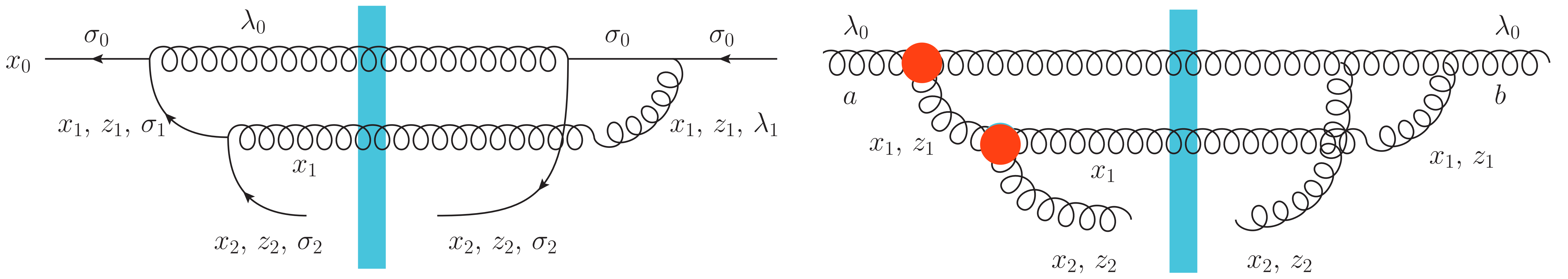
$$\alpha_s = 0.3$$
$$N_c = N_f = 3$$
$$G_{xy}(s) \sim \left(\frac{s}{Q^2}\right)^{1.46}$$

- Fast growth of quark polarization at small x!

➡ Large contribution to the proton spin?

$$S_{xy}(s) \sim \left(\frac{s}{Q^2}\right)^{0.3}$$

The Catch: Non-Ladder Diagrams

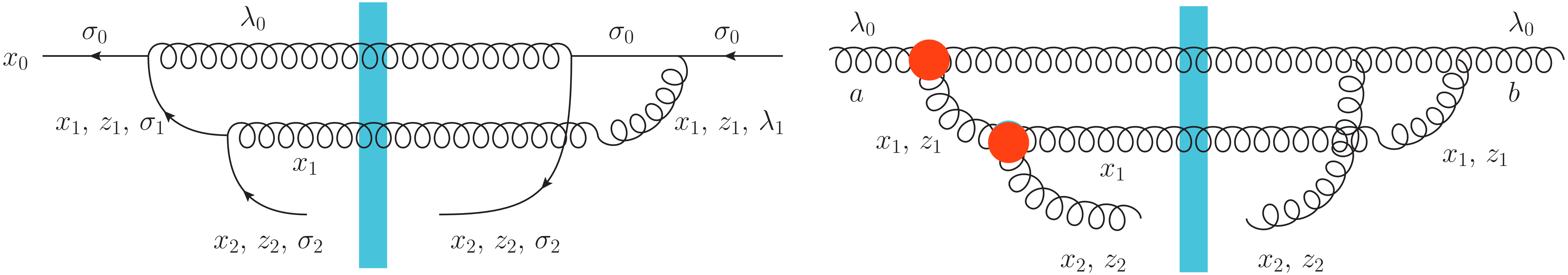


- Unlike BK or DGLAP, leading-log evolution is also generated by non-ladder graphs

➡ Arises uniquely from the IR sector.

$$k_{1T}^2 \gg k_{2T}^2 \gg k_{1T}^2 \frac{z_2}{z_1}$$

The Catch: Non-Ladder Diagrams



- Unlike BK or DGLAP, leading-log evolution is also generated by non-ladder graphs

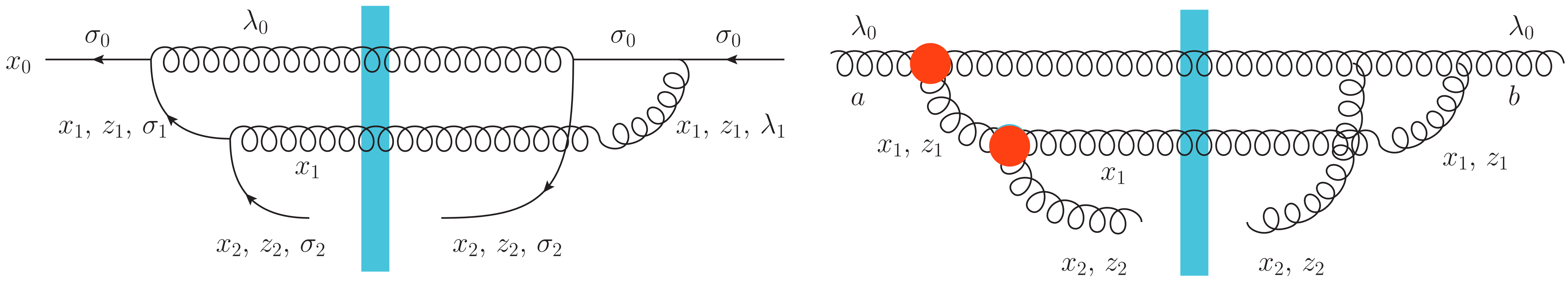
➡ Arises uniquely from the IR sector.

$$k_{1T}^2 \gg k_{2T}^2 \gg k_{1T}^2 \frac{z_2}{z_1}$$

- Quark and antiquark non-ladder graphs cancel

$$G_{xy} \equiv \frac{1}{2N_c} \mathrm{Tr}\left[V_x V_y^\dagger(\sigma) + V_y(\sigma) V_x^\dagger\right]$$

The Catch: Non-Ladder Diagrams



- Unlike BK or DGLAP, leading-log evolution is also generated by non-ladder graphs

➡ Arises uniquely from the IR sector.

$$k_{1T}^2 \gg k_{2T}^2 \gg k_{1T}^2 \frac{z_2}{z_1}$$

- Quark and antiquark non-ladder graphs cancel

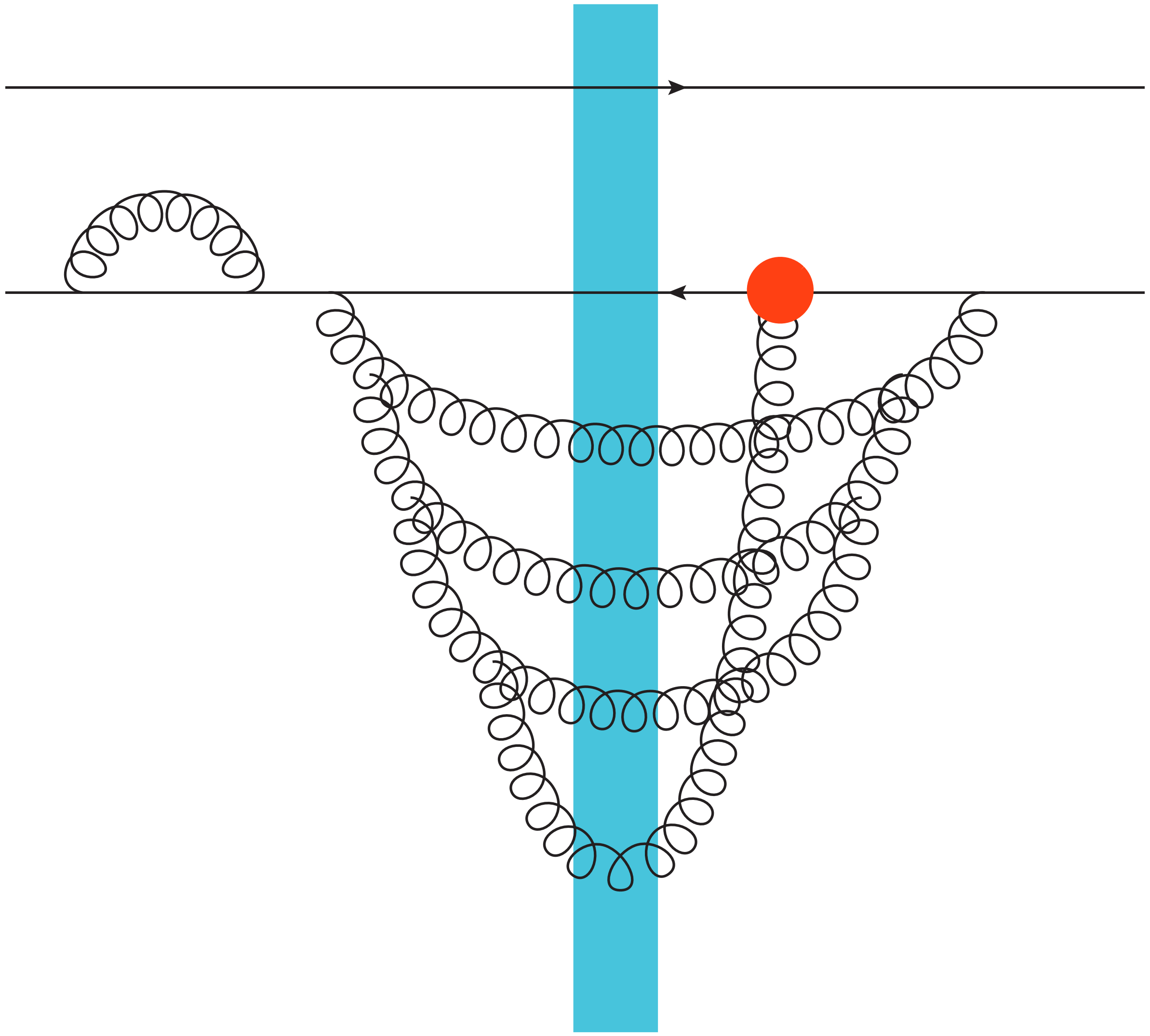
$$G_{xy} \equiv \frac{1}{2N_c} \mathrm{Tr} \left[V_x V_y^\dagger(\sigma) + V_y(\sigma) V_x^\dagger \right]$$

- Complication: Gluon non-ladder graphs do not cancel.

➡ Ladder evolution is an unjustified truncation

Knots of Non-Ladder Gluons

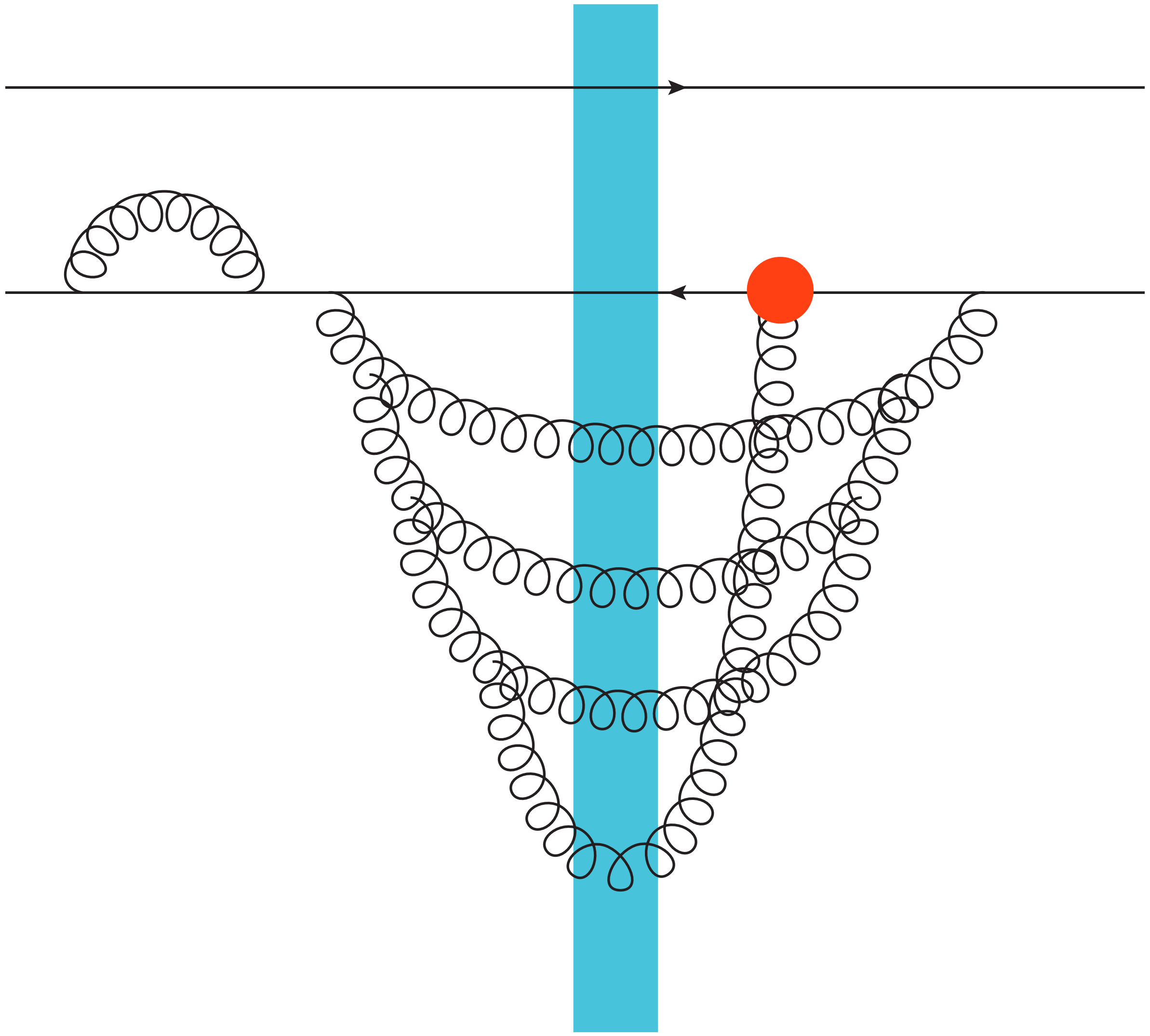
- Non-ladder gluons can stack in complex ways which still generate leading logarithms.



Knots of Non-Ladder Gluons

- Non-ladder gluons can stack in complex ways which still generate leading logarithms.

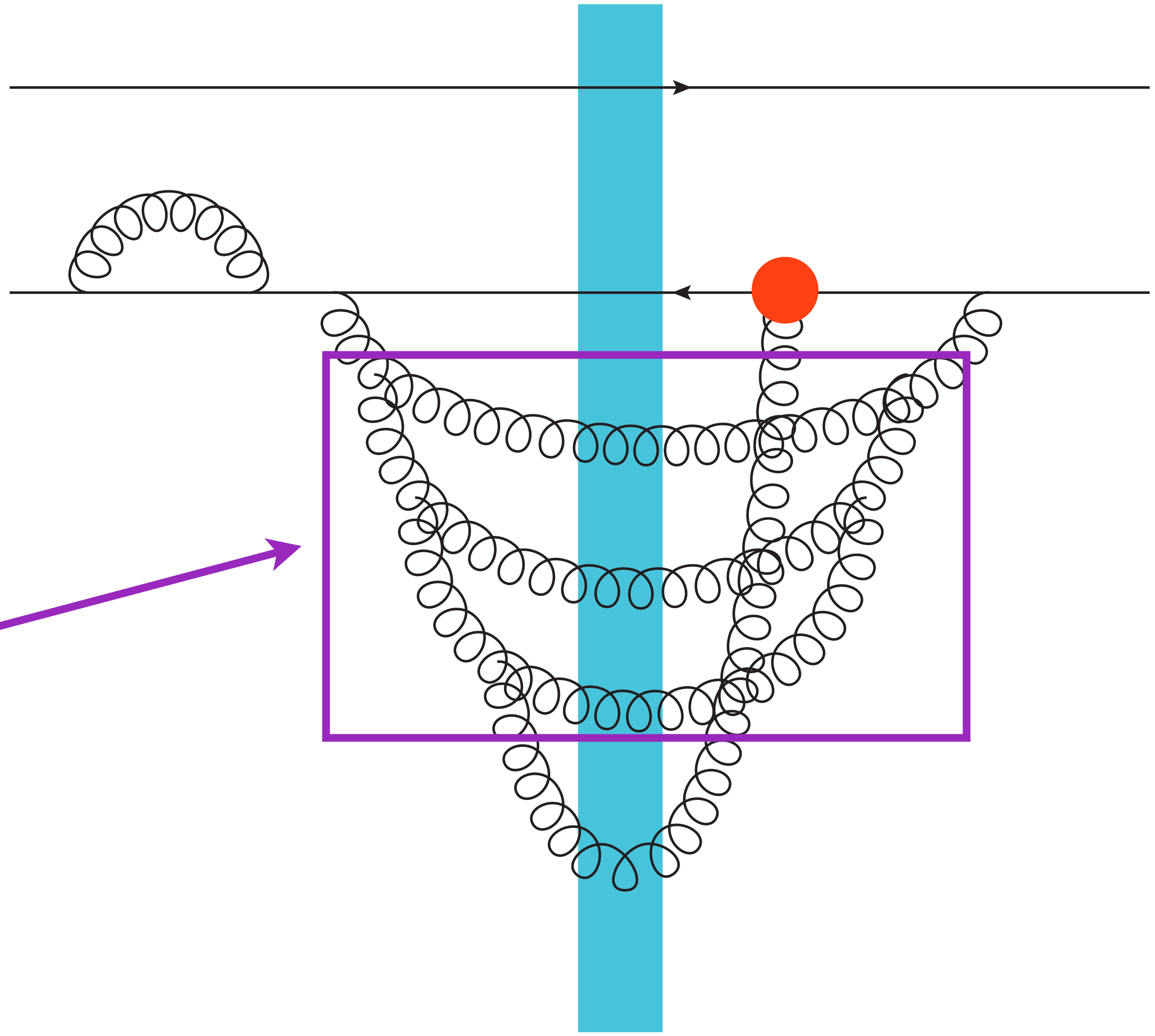
➡ Polarized gluons can "jump a rung" of evolution



- Non-ladder gluons can stack in complex ways which still generate leading logarithms.

➡ Polarized gluons can "jump a rung" of evolution

➡ Even unpolarized BK evolution in an intermediate step can contribute!



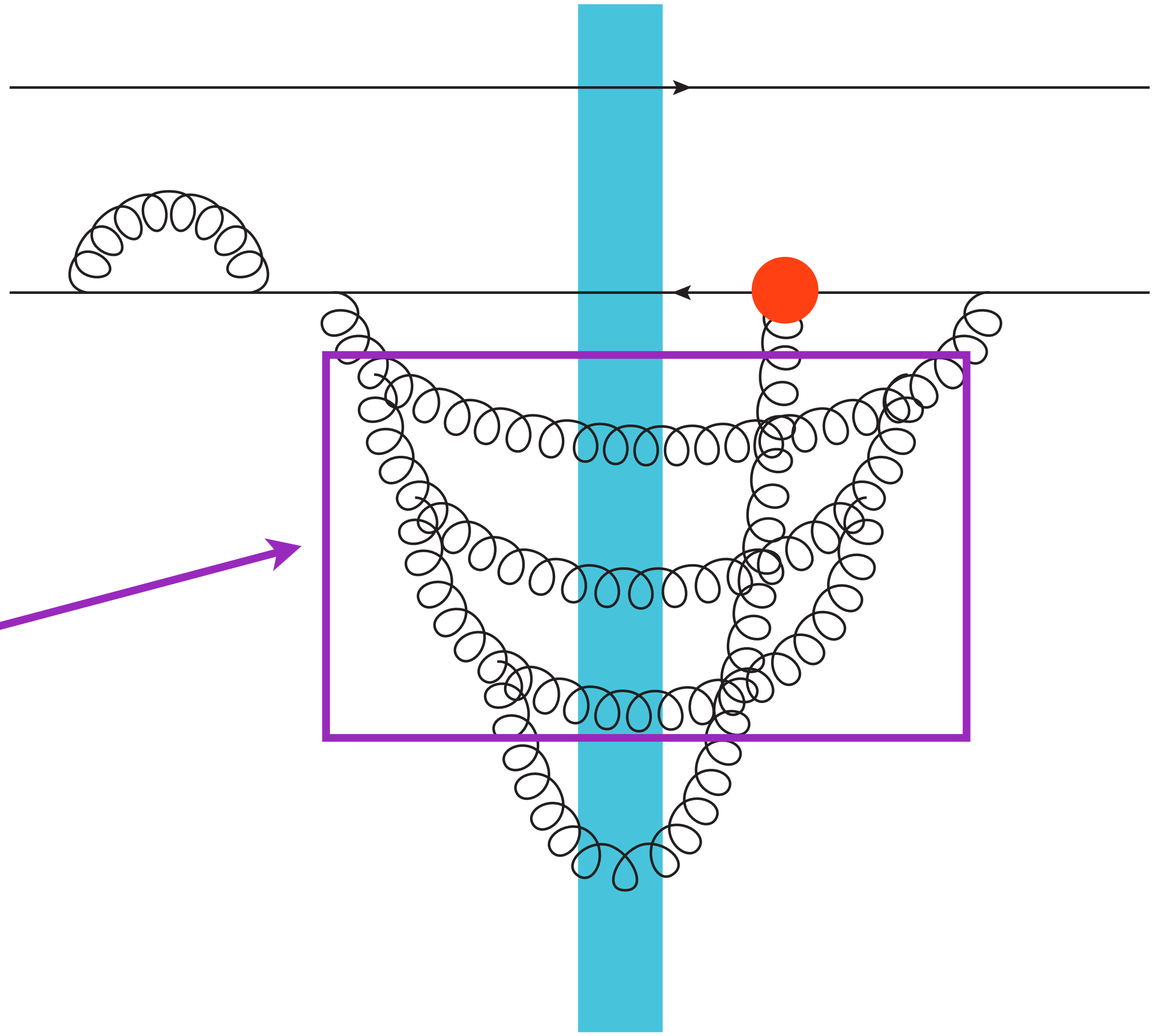
Knots of Non-Ladder Gluons

- Non-ladder gluons can stack in complex ways which still generate leading logarithms.

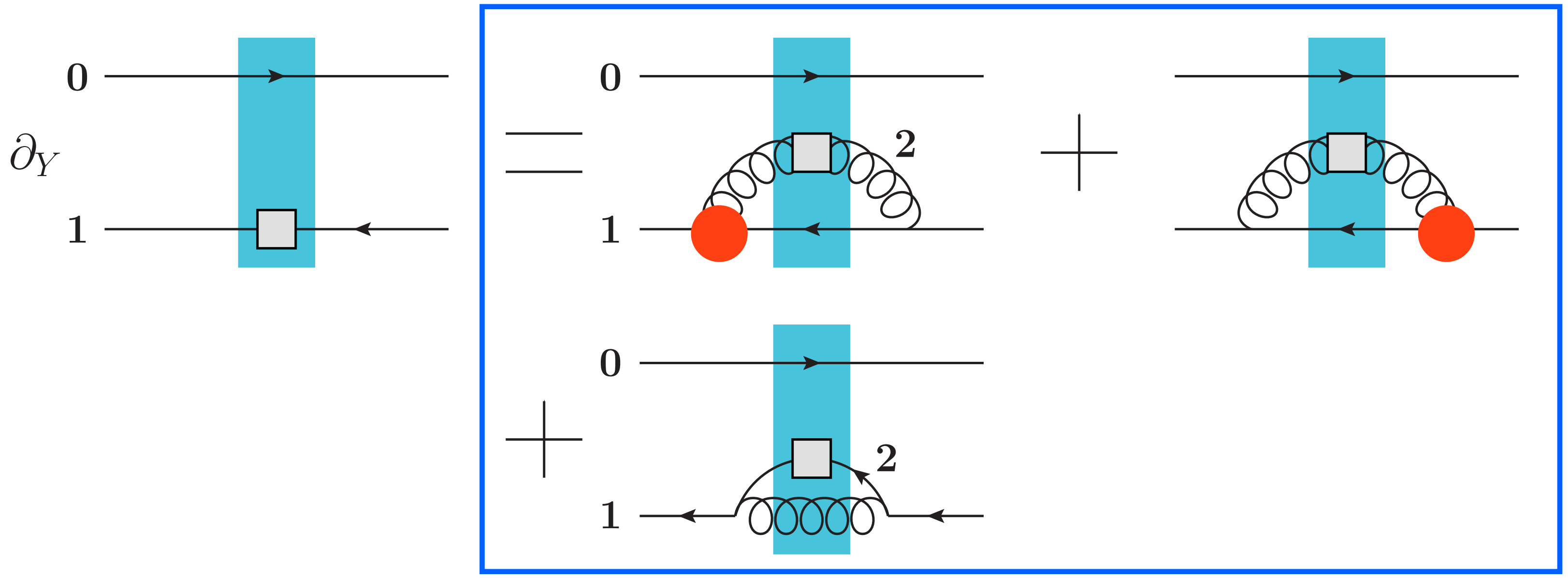
➡ Polarized gluons can "jump a rung" of evolution

➡ Even unpolarized BK evolution in an intermediate step can contribute!

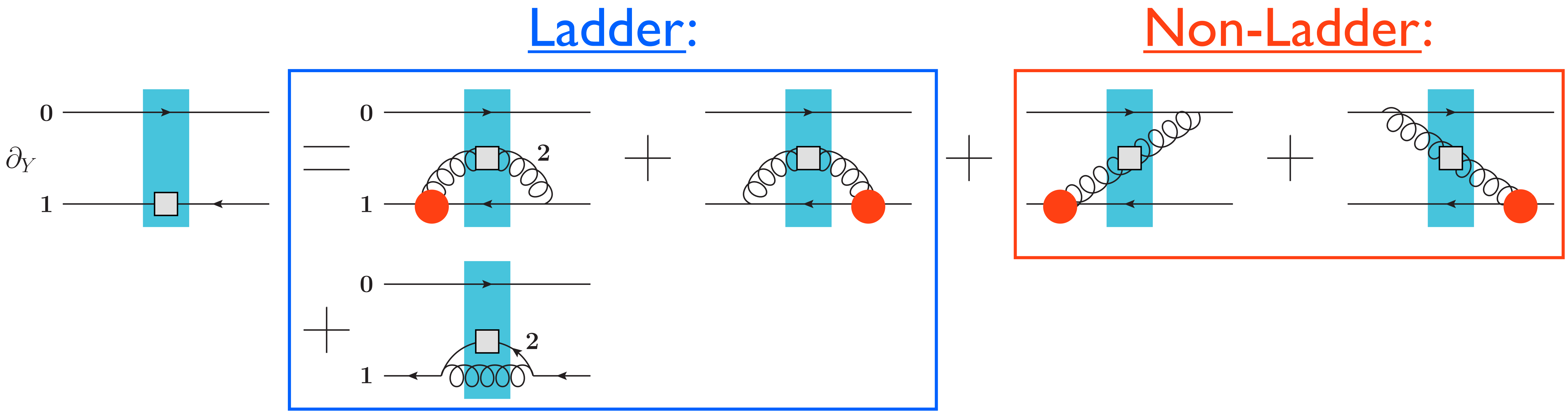
➡ Unpolarized evolution is in a color-octet state (unlike ordinary BK evolution)



Ladder:



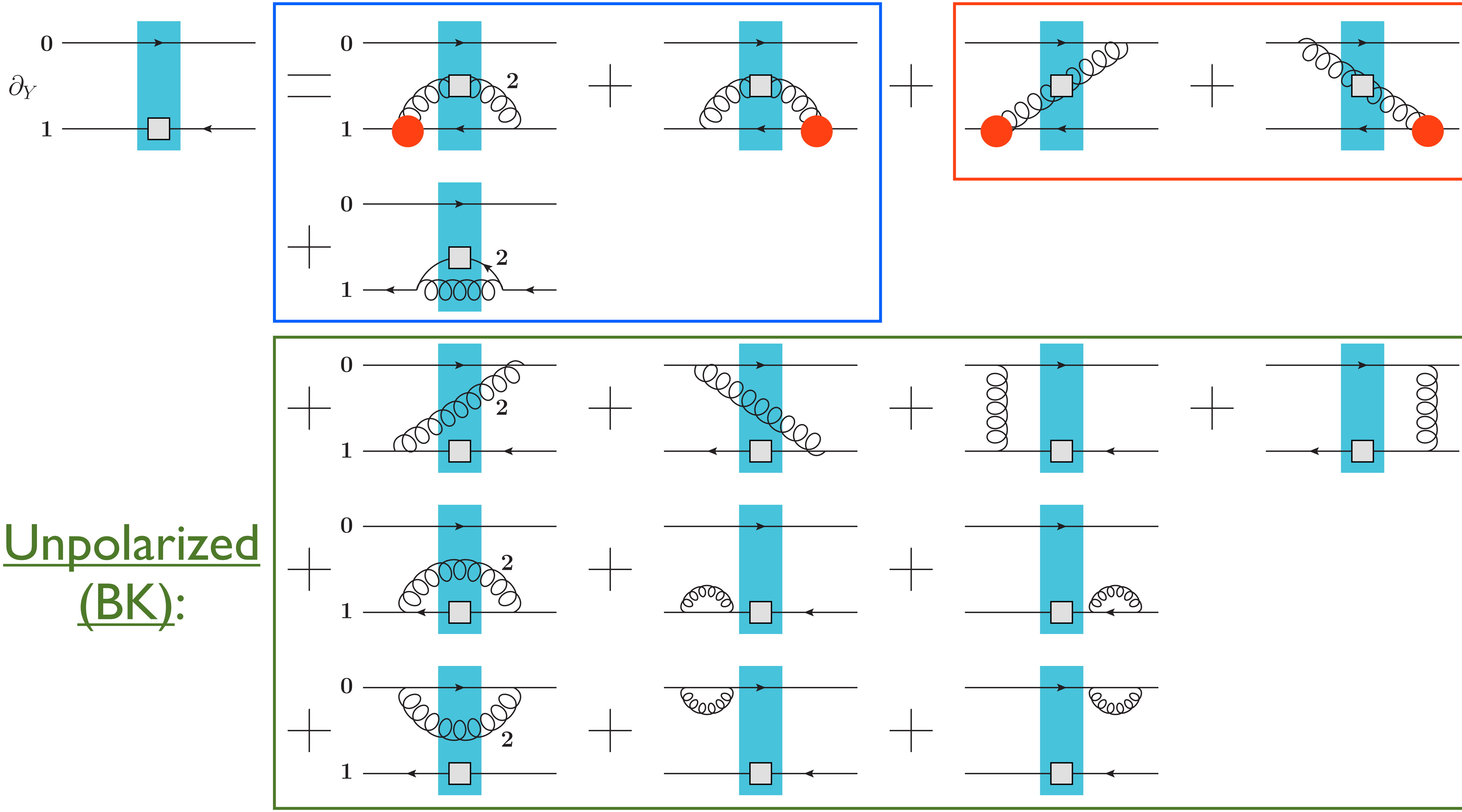
Operator Evolution of the Polarized Dipole

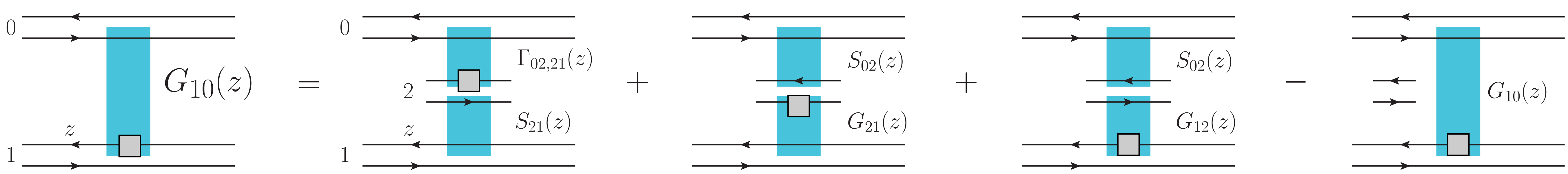


Operator Evolution of the Polarized Dipole

Ladder:

Non-Ladder:



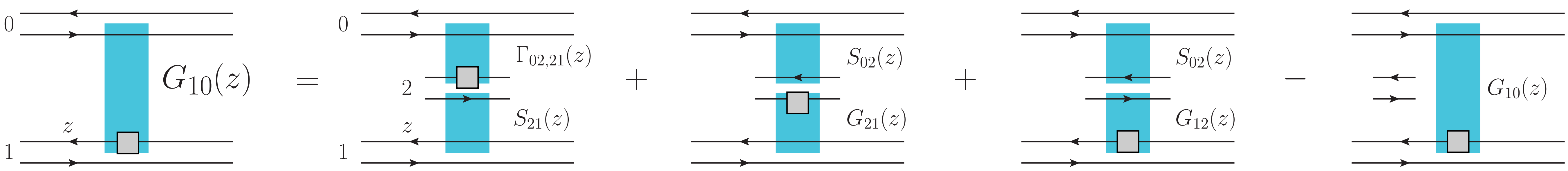


- The evolution yields another infinite operator hierarchy

➡ Closes in the large N_c limit, like BK evolution.

➡ But not physically relevant: neglects quark exchange

Trying to Solve It: The Large N_c Approximation



- The evolution yields another infinite operator hierarchy

➡ Closes in the large N_c limit, like BK evolution.

➡ But not physically relevant: neglects quark exchange

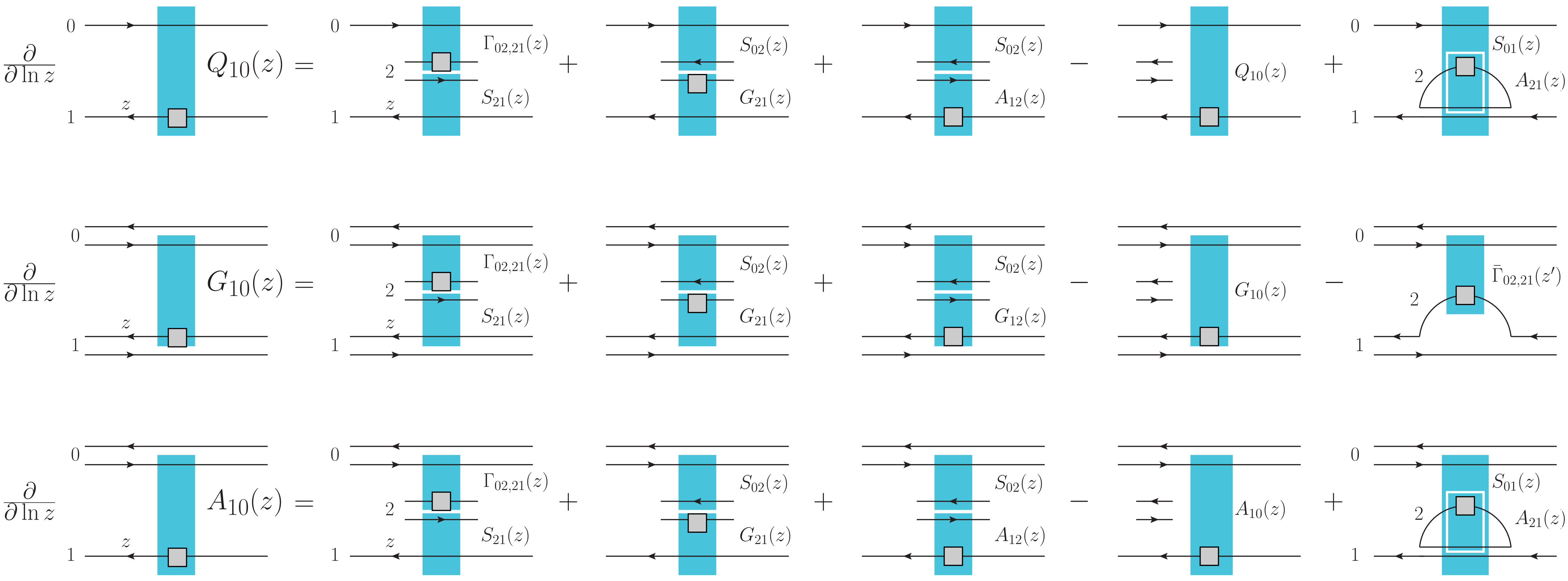
- The transverse ordering condition is not automatically satisfied.

$$Q^2 \ll \frac{k_{1T}^2}{z_1} \ll \frac{k_{2T}^2}{z_2} \ll \cdots$$

➡ Polarized dipoles can depend on their "neighbors"

➡ More complex than the large N_c BK equation.

A Better Approximation: Large N_c, N_f



- To keep quark contributions, must also take N_f large.

➡ Must distinguish between dipoles made of actual quarks vs. large N_c gluons.

➡ Evolution equation closes, but even more complicated....

Outlook: The Truth is Out There!

- Can we solve the helicity evolution in ANY systematic approximation?
  - Large N_c ,N_f ? Only large N_c?
  - Does the growth persist at small x?

Outlook: The Truth is Out There!

- Can we solve the helicity evolution in ANY systematic approximation?
  - Large N_c ,N_f ? Only large N_c?
  - Does the growth persist at small x?

- What is the role of saturation?
  - Does multiple unpolarized scattering reduce the intercept?
  - Does saturation keep the IR sector from becoming nonperturbative?

Outlook: The Truth is Out There!

- Can we solve the helicity evolution in ANY systematic approximation?
  - Large N_c ,N_f ? Only large N_c?
  - Does the growth persist at small x?

- What is the role of saturation?
  - Does multiple unpolarized scattering reduce the intercept?
  - Does saturation keep the IR sector from becoming nonperturbative?

- Do we need single log corrections?
  - Leading log evolution of the unpolarized gauge link.
  - Subleading evolution of the polarized matrix element.

Outlook: The Truth is Out There!

- Can we solve the helicity evolution in ANY systematic approximation?
  - Large N_c ,N_f? Only large N_c?
  - Does the growth persist at small x?

- What is the role of saturation?
  - Does multiple unpolarized scattering reduce the intercept?
  - Does saturation keep the IR sector from becoming nonperturbative?

- Do we need single log corrections?
  - Leading log evolution of the unpolarized gauge link.
  - Subleading evolution of the polarized matrix element.

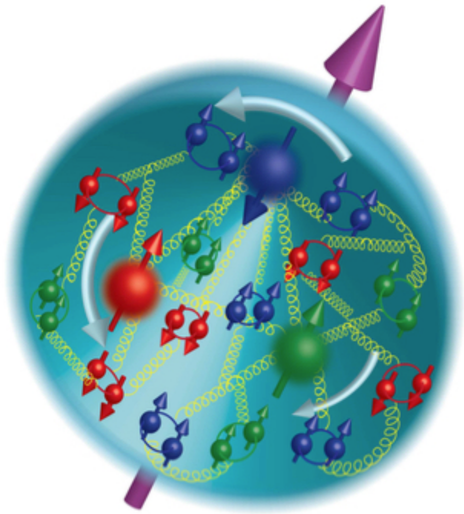
- What about other polarization observables like transversity?

Summary

- Up to 35% of the proton angular momentum is unaccounted for.
  - ➡ Is there significant polarization at small x?

$$0.001 < x < 1$$

$$\Delta\Sigma \approx 0.25 \;\; (25\%)$$
$$\Delta G \approx 0.2 \;\; (40\%)$$

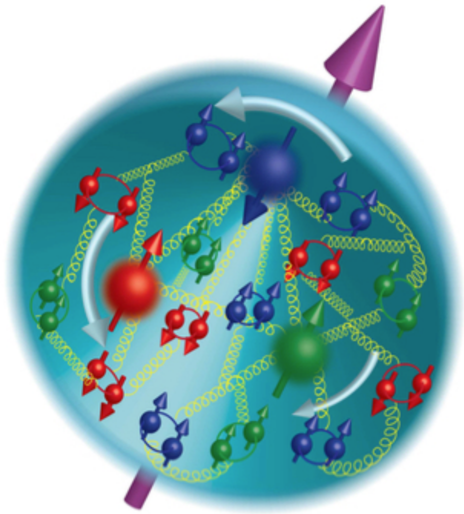


Summary

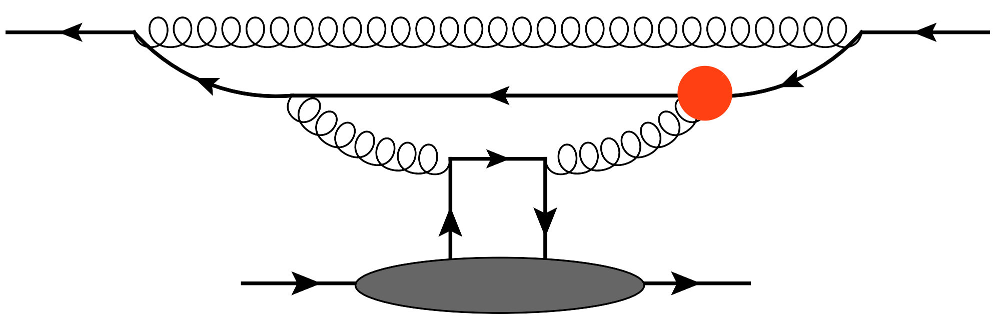
- Up to 35% of the proton angular momentum is unaccounted for.
  - ➡ Is there significant polarization at small x?

$$0.001 < x < 1$$

$$\Delta\Sigma \approx 0.25 \ \ (25\%)$$
$$\Delta G \approx 0.2 \ \ (40\%)$$



- Quark / gluon splitting leads to double-logarithmic evolution
  - ➡ Ladder graphs: rapid growth of polarization with small x!

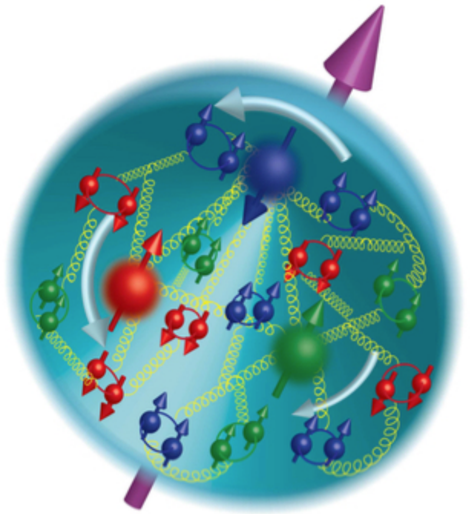


$$G_{xy}(s) \sim \left(\frac{s}{Q^2}\right)^{1.46}$$

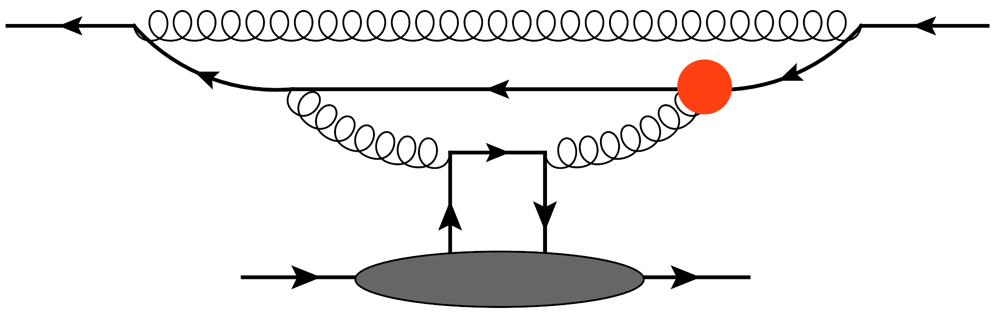
- Up to 35% of the proton angular momentum is unaccounted for.
  - ➡ Is there significant polarization at small x?

$$0.001 < x < 1$$

$$\Delta\Sigma \approx 0.25 \;\; (25\%)$$
$$\Delta G \approx 0.2 \;\; (40\%)$$



- Quark / gluon splitting leads to double-logarithmic evolution
  - ➡ Ladder graphs: rapid growth of polarization with small x!



$$G_{xy}(s) \sim \left(\frac{s}{Q^2}\right)^{1.46}$$

- Massive complications due to non-ladder gluons and IR phase space.
  - ➡ Much more to discover just around the corner!