Towards a unified description of the nuclear electroweak response

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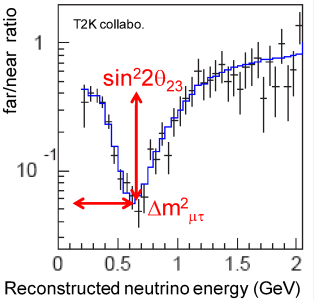
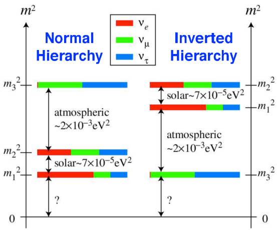
Outline

- ⋆ Motivations
- ⋆ Lessons from electron scattering
- ⋆ Neutrino-nucleus scattering
  - ▹ High energy regime: can the models developed to describe electron scattering data be extended to the case of neutrino scattering?
  - ▹ Low energy regime: can the relevant recation mechanisms be consistently described within *ab initio* many body approaches?
- ⋆ Summary & Outlook

Motivation I: detection of neutrino oscillations





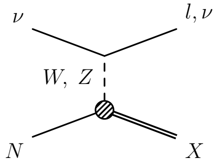
- Probability that a neutrino oscillate from flavor α to flavor β after travelling a distance L

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\theta \ \sin^2 \left(\frac{\Delta m^2 L}{4E_\nu} \right)$$

- Addressing a number of fundamental issues, such as the mass hyerarchy, leptonic CP violation and the existence of sterile neutrinos, will require *precise* measurements of neutrino and antineutrino oscillations.

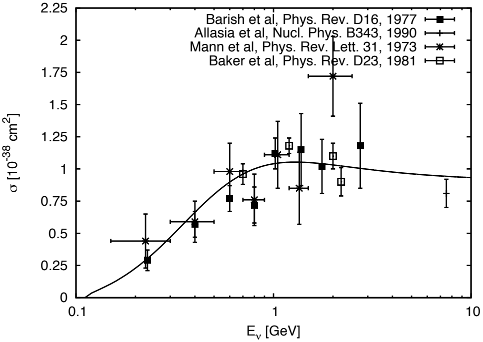
Neutrino interactions are very weak

- Neutrino-nucleon scattering



- Total cross section of the process

$$\nu_\mu + n \to \mu^- + p$$



$$\sigma_{\nu N} \sim 10^{-38}\ \mathrm{cm}^2 \quad , \quad \sigma_{\nu N}/\sigma_{eN} \sim 10^{-6}$$

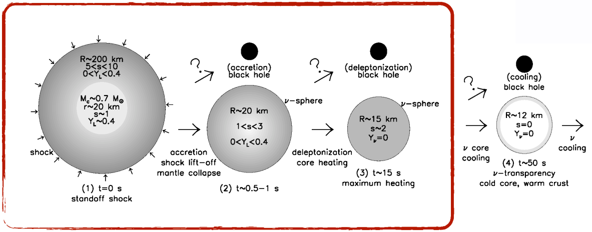
Detecting neutrinos requires big detectors

- The SUPER-K detector, in Japan, is filled with 12.5 million gallon of ultra-clean water
- The MiniBooNE detector,at FNAL, is filled with 800 tons of mineral oil
- The detected signal results from neutrino interactions with Oxygen and Carbon nuclei
- A quantitative understanding of their response to neutrino interactions is required for the interpretation of the measured cross sections

Relevant energy scale of accelerator-based experiments: $E_\nu \sim 1$ GeV

Motivation II: neutron star evolution

- ★ At much lower energies, ($E_\nu \lesssim 5$MeV), neutrino interactions with nuclear matter play a critical role in determining both the evolution of newly formed proto-neutron stars and cooling of aged stars.



- ★ The gravitational energy released in a supernova collapse is $\sim$ 200-300 times higher than that produced by the Sun over its entire lifetime.
- ★ $\sim$ 99% of it is radiated over a timescale of a few tens of seconds in the form of an immense flux of low-energy neutrinos.

Neutrino-nucleus x-section

- Differential cross section of the charged current process $\nu_\ell + A \to \ell^- + X$

$$\frac{d\sigma_A}{d\Omega_{\ell^-} dE_{\ell^-}} \propto L^{\mu\nu} W_{\mu\nu}$$

  - $L^{\mu\nu}$ is fully specified by the lepton kinematical variables. Same as in scattering, but in this case *the beam energy is not known*.
  - the calculation of the target response tensor

$$W_{\mu\nu} = \sum_X \langle 0|J_\mu^\dagger|X\rangle\langle X|J_\nu|0\rangle\delta^{(4)}(P_0 + k_{\nu_\ell} - P_X - k_{\ell^-})$$

    requires a *consistent* description of the target internal dynamics, determining the initial and final states, as well as of the nuclear weak current

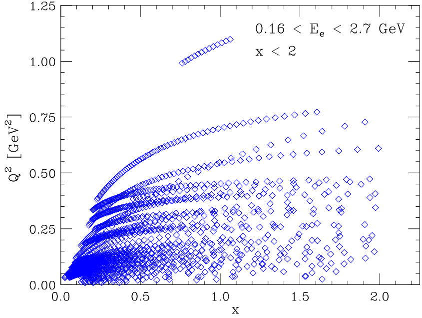
$$J_\mu = J_\mu^V - J_\mu^A = \sum_i j_\mu(i) + \sum_{j>i} j_\mu(ij) + \ldots$$

Information from electron scattering

- Vast supply of precise data available

$$Q^2 = 4E_e E_{e'} \sin^2 \frac{\theta_e}{2} \quad , \quad x = \frac{Q^2}{2M\omega}$$

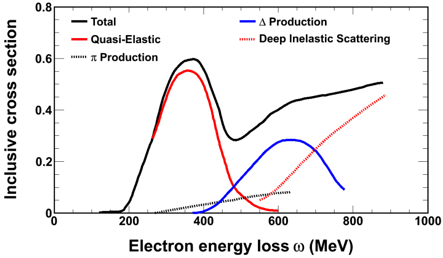
- Carbon target



- Different rection mechanisms contributimg to the mesured cross sections can be readily identified

$$e + A \to e' + X$$

$$E_e \sim 1 \text{ GeV}$$



ab initio calculations of the electromagnetic response

- The nucleus is seen as a collection of pointlike protons and neutrons interacting through the non relativistic hamiltonian

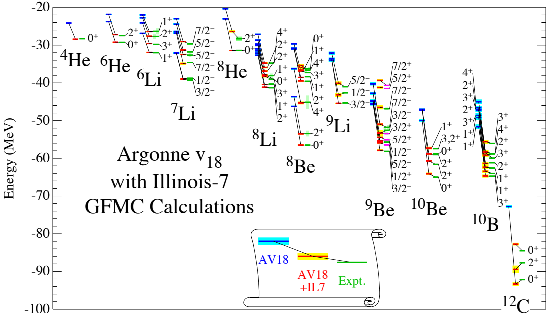
$$H = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} \mathrm{v}_{ij} + \sum_{k>j>i} U_{ijk}$$

- the potentials are determined by a fit to the properties of the *exactly solvable* two- and three-nucleon systems
- the nuclear electromagnetic current $J_\mu \equiv (\rho_{\rm ch}, \mathbf{J})$ is constructed in such a way as to fulfill the continuity equation

$$\boldsymbol{\nabla} \cdot \mathbf{J} + i[H, \rho_{\rm ch}] = 0$$

- at low to moderate momentum transfer, typically $|\mathbf{q}| \lesssim 400$ MeV, the non relativistic approach provides a set of electroweak charge and current operators consistent with the hamiltonian.

- Non relativistic nuclear hamiltonians can be used to carry out *exact* Quantum Monte Carlo (QMC) calculations of the energies of the ground and low-lying excited states of nuclei with $A \leq 12$ using the Green's Function Monte Carlo (GFMC) technique.



J. Carlson et al, arXiv:1412.3081v2 [nucl=th]

GFMC calculation of the nuclear response

- the GFMC technique is ideally suited for the calculations of the Euclidean responses

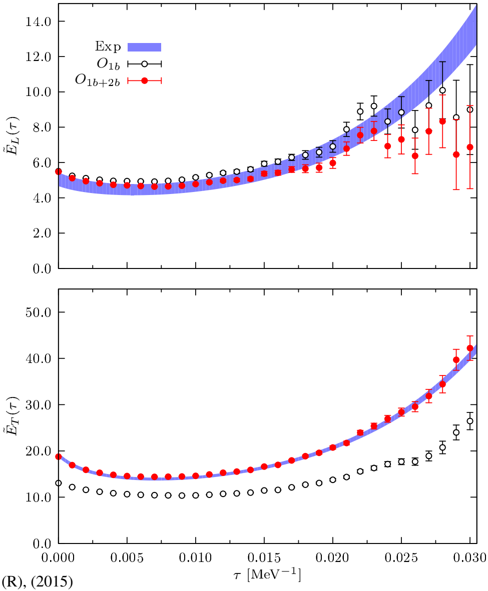
$$E_{\mu\nu}(|\mathbf{q}|,\tau) \propto \int_{\omega_{\rm thr}}^{\infty} d\omega e^{-\omega\tau}\, W_{\mu\nu}(|\mathbf{q}|,\omega)$$

- Contributions to the logitudinal (L) and transverse (T) channels

$$E_L(|\mathbf{q}|,\tau) = E_{00}(|\mathbf{q}|,\tau) \quad , \quad E_T(|\mathbf{q}|,\tau) = E_{11}(|\mathbf{q}|,\tau)$$

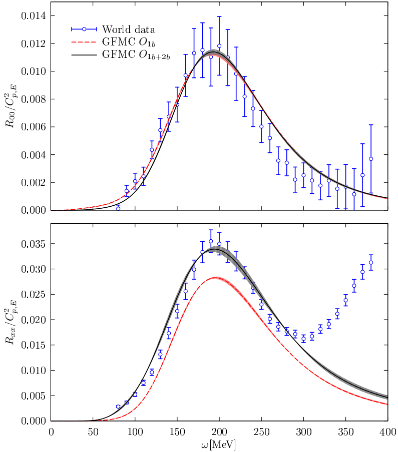
- The inversion of the euclidean responses of light nuclei has been recently obtained exploiting the maximum entropy method

Euclidean responses of carbon



A. Lovato et al, PRC 91 062501(R), (2015)

Inversion of the Euclidean responses of ^{4}He



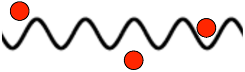
A. Lovato et al, PRC 91 062501(R), (2015)

- Significant two-nucleon current contribution in the transverse channel

High momentum transfer: the Impulse Approximation (IA)

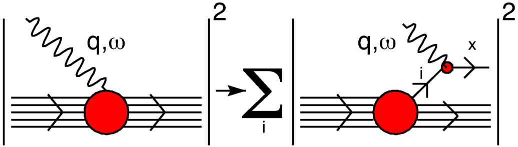
- In the kinematical regime corresponding to $|\mathbf{q}| \gtrsim 500$ MeV

$$\lambda \sim \frac{\pi}{|\mathbf{q}|} \ll d$$



where d is the average nucleon-nucleon distance

- Nuclear scattering reduces to the incoherent sum of elementary scattering processes involving individual nucleons



- The IA amounts obvioulsy implies the replacement

$$J_\mu \to \sum_i j_\mu(i)$$

- Assuming that Final State Interactions (FSI) between the struck nucleon and the spectators be negligible leads to the *factorized* final state

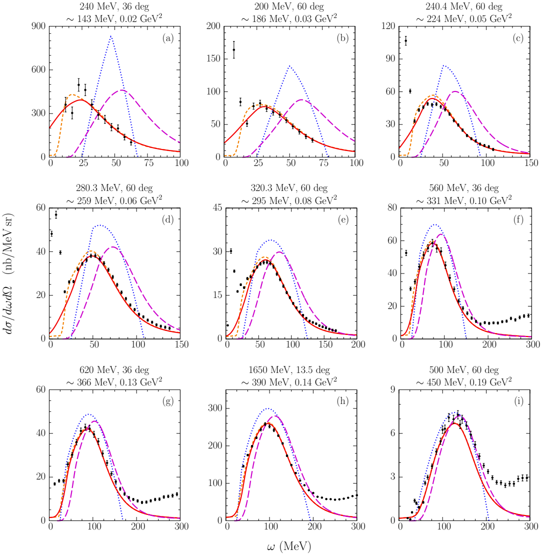
$$|X\rangle \to |x, \mathbf{p}_x\rangle \otimes |\mathcal{R}, \mathbf{p}_\mathcal{R}\rangle$$

- Nuclear dynamics and interaction vertex are decoupled

$$d\sigma_A = \int d^3k dE \, d\sigma_N \, P(\mathbf{k}, E)$$

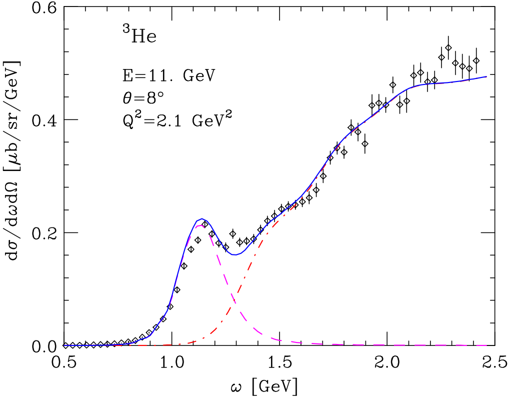
  - The electron-nucleon cross section $d\sigma_N$ can be written in terms of stucture functions extracted from electron-proton and electron-deuteron scattering data
  - The nuclear spectral function $P(\mathbf{k}, E)$, yielding the *momentum and energy distribution* of the knocked out nucleon, is an intrinsic property of the target nucleus, independent of momentum transfer, calculable within the *ab initio* many-body approach.

Carbon quasi elastic cross section within IA



- FSI corrections included [A. Ankowski et al, PRD 91 033005, (2015)]

Inclusion of inelastic channels



OB and V.R. Pandharipande, PRC 47 2218, (1993)

Generalized factorization *ansatz*

- GFMC calculations strongly suggest that the contributions arising from two-nucleon currents is important in the transverse channel
- Use of relativistic currents and a realistic description of the nuclear ground state requires the extension of the factorization *ansatz* undelrying the IA

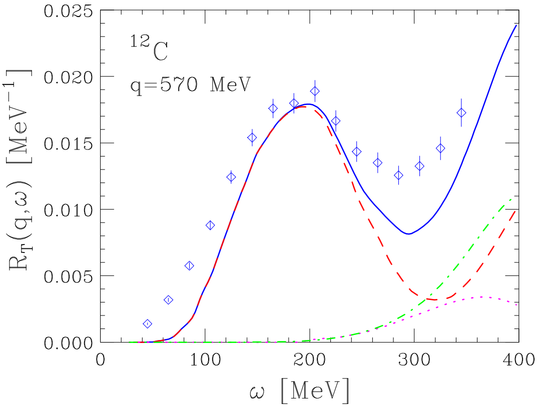
$$|X\rangle \to |\mathbf{p},\mathbf{p}'\rangle \otimes |m_{(A-2)}\rangle$$

$$\langle X|j_{ij}^{\mu}|0\rangle \to \int d^3k d^3k' M_m(\mathbf{k},\mathbf{k}') \, \langle \mathbf{p}\mathbf{p}'|j_{ij}^{\mu}|\mathbf{k}\mathbf{k}'\rangle$$

- ▹ The matrix elements of the two-nucleon current between states describing non interacting nucleons can be computed using the fully relativistic expression.
- ▹ The amplitude $M_m(\mathbf{k},\mathbf{k}') = \{\langle n_{(A-2)}| \otimes \langle \mathbf{k},\mathbf{k}'|\}|0\rangle$ is independent of $|\mathbf{q}|$ and can be obtained from non relativistic many-body theory

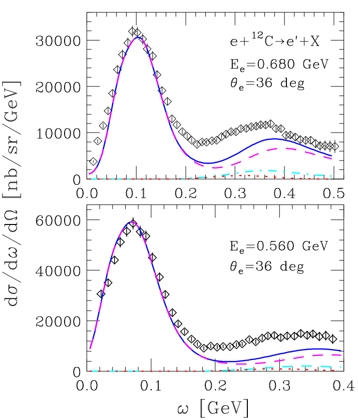
Results of the generalized factorization ansatz

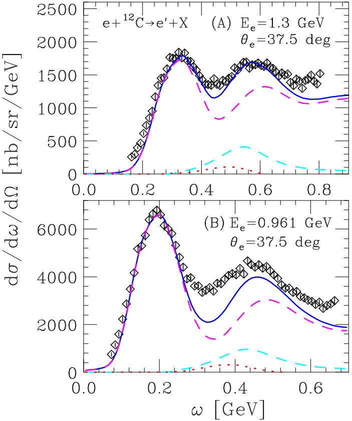
- Transverse response of carbon, not corrected for FSI



OB et al, PRC 92 024602, (2015)

Comparison to measured carbon cross sections





N. Rocco et al, preliminary

Electron *vs* neutrino nucleus scattering in the QE sector

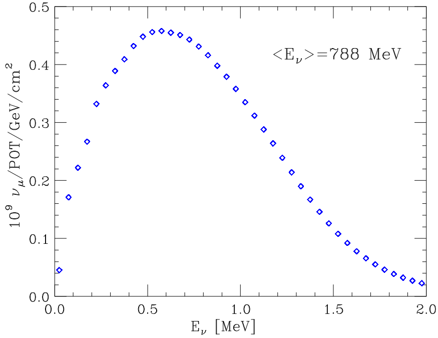
- In neutrino experiments, the measured double differential cross section is averaged over the energy of the incoming neutrino, broadly distributed according to the flux Φ

$$\frac{d\sigma_A}{dT_\mu d\cos\theta_\mu} = \frac{1}{N_\Phi}\int dE_\nu \Phi(E_\nu) \frac{d\sigma_A}{dE_\nu dT_\mu d\cos\theta_\mu}$$

- In addition to F_1 and F_2 , the QE electron-nucleon cross section is determined by the axial form factor F_A, assumed to be of dipole form and parametrized in terms of the axial mass M_A
- According to the *paradigm* succesfully employed to describe electron scattering data within the IA, M_A must be determined from measurement carried out using a deuterium target. The resulting value is $M_A \ = \ 1.03 \text{ GeV}$

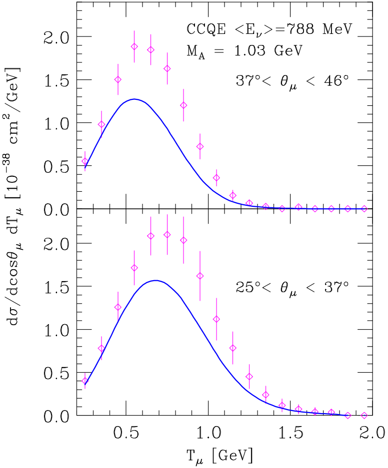
Analysis of CCQE data

- MiniBooNE flux



- MiniBooNe CCQE data

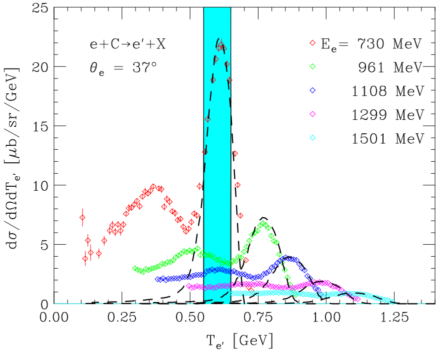
OB et al, PRL 105, 132301 (2010)



- Theoretical calculations carried out setting $M_A = 1.03$

“Flux averaged” electron-nucleus x-section

- Electron scattering x-sections off Carbon at $\theta_e = 37°$ and different beam energies



- Owing to flux average, a single bin of energy of the outgoing charged lepton picks up strength corresponding to different reaction mechanism
- All relevant mechanisms must be included in a consistent fashion

Opacity of neutron matter to low-energy neutrinos

- The opacity of nuclear matter to neutrinos of energy $E_\nu \lesssim 10$ MeV, which plays a critical role in determining the evolution of compact stars, is parametrized in terms of neutrino mean free path

$$\lambda_\nu = \frac{1}{\sigma\rho}$$

with

$$\sigma \propto \int \frac{d^3q}{(2\pi)^3} L_{\mu\nu} W^{\mu\nu}$$

- Consider weak neutral current interactions. In the extreme non relativistic approximation, the vector and axial vector weak currents reduce to

$$\overline{\psi}_n \gamma^\mu \psi_n \to \psi_n^\dagger \psi_n \delta_o^\mu = O_F : \qquad \text{Fermi (F) operator}$$

$$\overline{\psi}_n \gamma^\mu \gamma^5 \psi_n \to \psi_n^\dagger \sigma^i \psi_n \delta_i^\mu = O_{GT}^i : \qquad \text{Gamow–Teller (GT) operator}$$

Neutron matter response to low-energy neutrino interactions

- The nuclear cross section is computed from

$$L_{\mu\nu} W^{\mu\nu} \propto [(1 + \cos\theta) S(\mathbf{q}, \omega) + C_A^2 (3 - \cos\theta) \mathcal{S}(\mathbf{q}, \omega)]$$

with

$$S(\mathbf{q}, \omega) = \sum_n |\langle n | O_F | 0 \rangle|^2 \delta(\omega + E_0 - E_n) \quad , \quad \text{density response}$$

$$\mathcal{S}(\mathbf{q}, \omega) \sum_i \sum_n |\langle n | O^i_{GT} | 0 \rangle|^2 \delta(\omega + E_0 - E_n) \quad , \quad \text{spin response}$$

- The expression of the mean free path in terms of the response functions is

$$\frac{1}{\lambda_\nu} = \frac{G_F^2}{4} \rho \int \frac{d^3 q}{(2\pi)^3} \left[(1 + \cos\theta) S(\mathbf{q}, \omega) + \; C_A^2 (3 - \cos\theta) \mathcal{S}(\mathbf{q}, \omega) \right]$$

Interaction effects in the low-energy regime

- In the absence of interactions, nuclear matter reduces to a degenerate Fermi gas, and the final state is a one particle-one hole state

$$|n\rangle = |ph) \quad , \quad E_n - E_0 = e^0_p - e^0_h = \frac{|\mathbf{p}|^2}{2m} - \frac{|\mathbf{h}|^2}{2m}$$

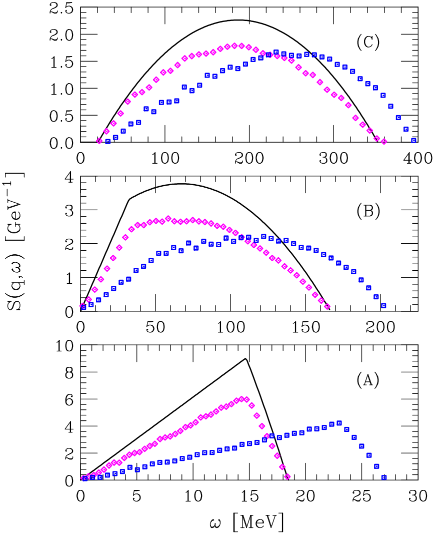
- Interactions can be included using an *effective interaction*, $V_{\rm eff}$, and consistently replacing the Fermi and Gamow-Teller operators with *effective operators*, $\widetilde{O}_F$ and $\widetilde{O}_{GT}$
- Interaction lead to a modification of the spectrum

$$e^0_k \to e_k = \frac{k^2}{2m} + \sum_{\mathbf{k}'} (\mathbf{k}\mathbf{k}'|V_{\rm eff}|\mathbf{k}\mathbf{k}' - \mathbf{k}'\mathbf{k})$$

as well as to a *quenching of the transition matrix elements* $(ph|\widetilde{O}_F|0)$ and $(ph|\widetilde{O}_{GT}|0)$, arising from correlations, coupling between $1p1h$ states and more complex, *npnh* final states

Interaction effects on the neutron matter response

- ⋆ (A), (B), (C) → $|\mathbf{q}| = 0.3, 1.8, 3.0 \text{ fm}^{-1}$



Excitation of collective (phonon-like) modes

- At low neutrino energy $\lambda \sim \pi/|\mathbf{q}| > d$, and the interaction process may involve many nucleons
- propagation of particle-hole states produced at the interactions vertex, leading to the occurrence of collective excitations, must be taken into account replacing

$$|ph\rangle \to |n\rangle = \sum_{i=1}^{N} C_i \, |p_i h_i)$$

★ The energy of the state $|n\rangle$ and the coefficients C_i are obtained diagonalizing the $N \times N$ hamiltonian matrix

$$H_{ij} = (E_0 + e_{p_i} - e_{h_i})\delta_{ij} + (h_i p_i | V_{\rm eff} | h_j p_j - p_j h_j)$$

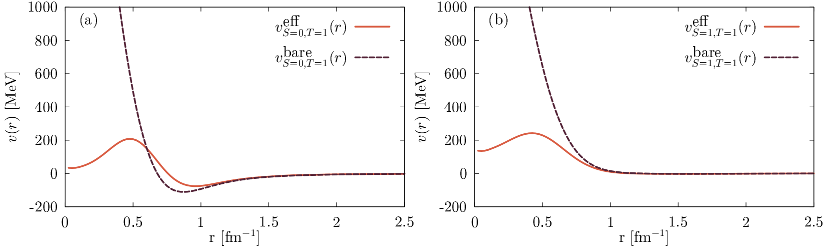
Effective interaction from realistic nuclear hamiltonian

- The formalism of Correlated Basis Functions (CBF) and the cluster expansion technique can be exploited to obtain $V_{\rm eff}$ from a realistic nuclear hamiltonian
- The effective interaction is defined through the relation

$$\langle 0|H|0\rangle = \frac{3}{5}\frac{k_F^2}{2m} + \sum_n (\Delta E)_n = \frac{3}{5}\frac{k_F^2}{2m} + \langle 0_{FG}|V_{\rm eff}|0_{FG}\rangle$$

  where $|0_{FG}\rangle$ is the ground state of the non interacting Fermi gas, and the ground state expectation value of the hamiltonian is expanded in a series whose terms correspond to contributions of clusters containing an increasing numbef of particles
- The effective interaction constructed retaining two- and three-body cluster contributions includes the effects of three-nucleon interactions, which are known to be needed to explain saturation of isospin-symmetric nuclear matter

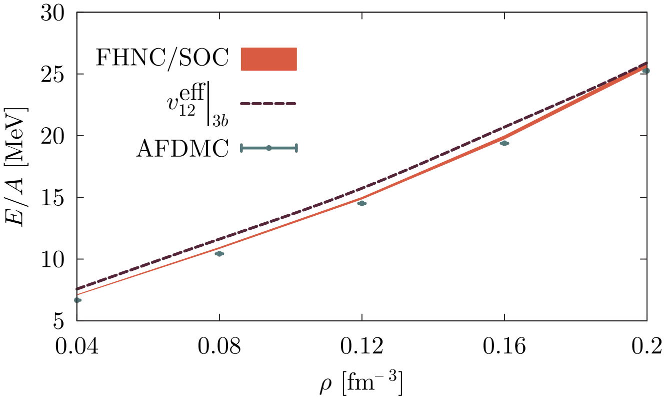
- CBF effective interaction obtained from the cluster expansion of the relevant matrix elements, keeping two- or two- and three-body terms



- Being well-behaved (unlike the bare nucleon-nucleon potential) the CBF effective interaction can be used to carry out perturbative calculations in the Femi gas basis

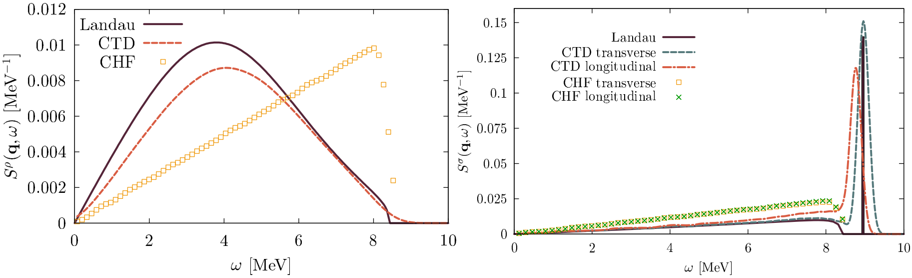
Equation of state of cold neutron matter

- Comparison between the EOS obtained using the CBF effective interaction and those resulting from advanced many-body approaches



Excitation of collective modes at low-momentum transfer

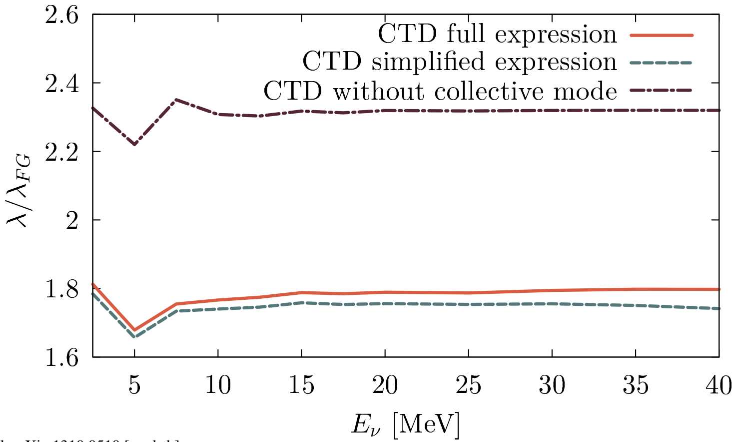
- Fermi (density, left) and Gamow-Teller (spin, right) contributions to the response of pure neutron matter at nuclear matter equilibrium density and momentum transfer $|\mathbf{q}| = 0.1 \text{ fm}^{-1}$



A. Lovato et al. PRC 89, 025804 (2014)

- the collective mode is only excited in the spin channel

Neutrino mean free path in neutron matter



A Lovato et al, arXiv 1310.0510 [nucl-th]

★ Both short and long range correlations important

Summary & Outlook

- ⋆ Ab initio calculations based on nuclear many-body theory and the available experimantal information on electron-nucleon interactions provide a remarkably accurate description of the nuclear cross sections in a broad kinematical range
- ⋆ The generalization to neutrino-nucleus scattering, needed to reduce the systematic uncertainty of LBL neutrino oscillation experiments, while being feasible, involves additional difficulties, arising from the flux average, and requires a consistent description of all relevant reaction mechanisms
- ⋆ The same formalism and dynamical input can be used to investigate neutrino interactions with nuclear matter in the low energy region, relevant to astrophysical applications.
- ⋆ The development of a unified treatment of the nuclear response to electroweak interactions at energies ranging from few MeV to few GeV is well under way