Superconductivity from repulsion

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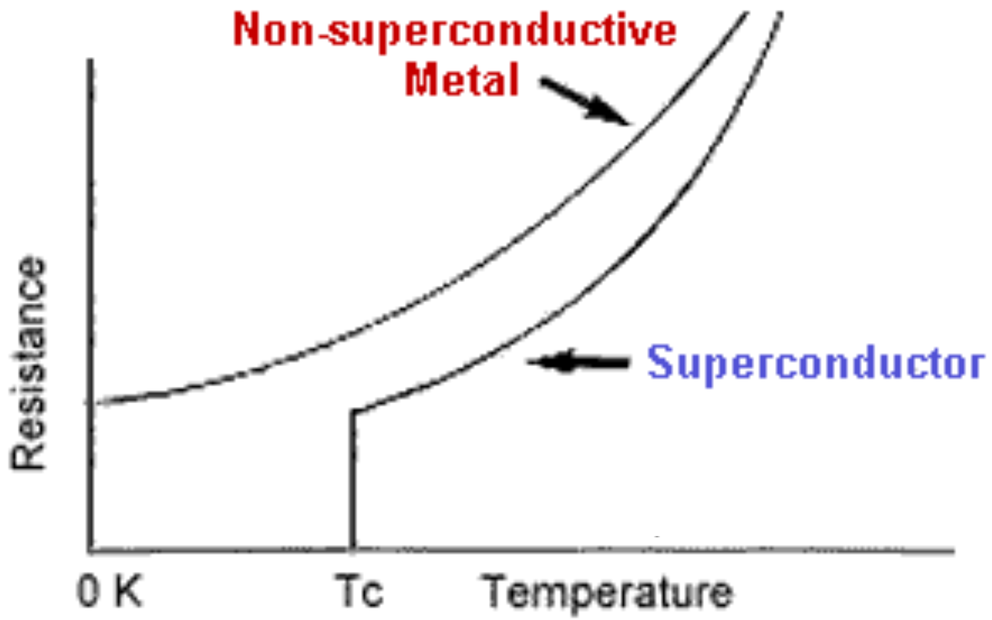
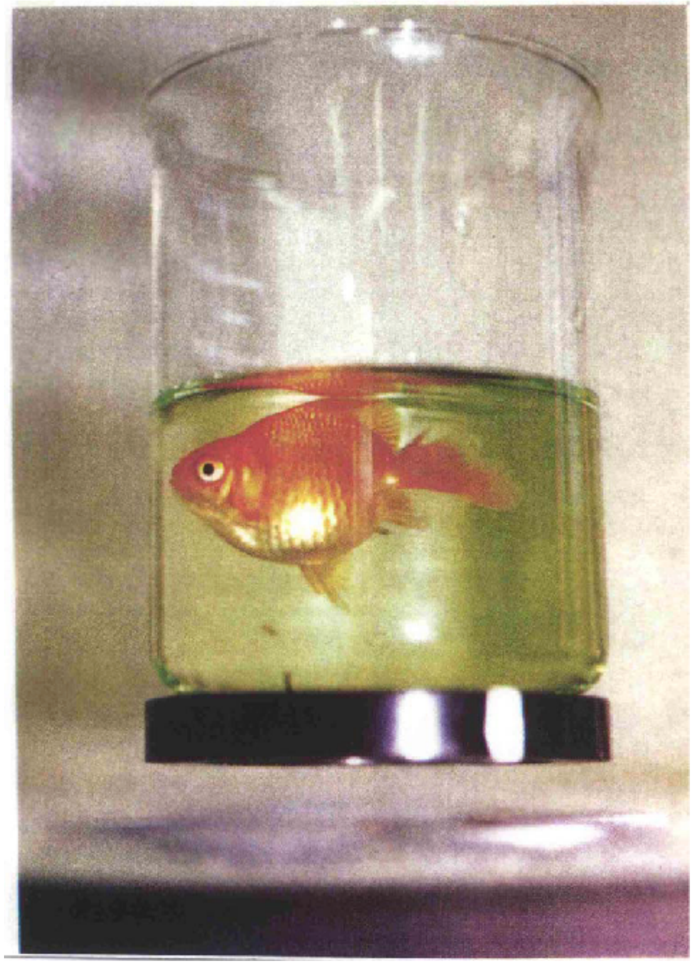
Superconductivity:

Zero-resistance state of interacting electrons



A superconductor expels a magnetic field



What we need for superconductivity?

Drude theory for metals predicts that resistivity should remain finite at T=0

Ohm's law

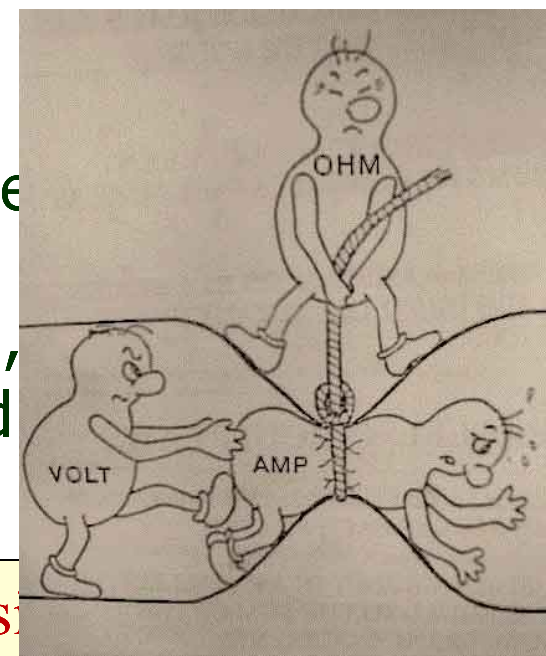
$$j = \frac{ne^2\tau}{m_e}E$$

$$j = \sigma E = \frac{E}{\rho}$$

$$\rho = \frac{m_e}{ne^2\tau}$$

This is a dissipative current: to sustain j we need to borrow energy ($\sim \sigma E^2$) from the source of the electric field

If the system had a macroscopic condensate there would be an additional current $\boxed{j \propto \nabla\varphi}$, accompanied by energy dissipation and would thermodynamic equilibrium at E=0



A nonzero current at E =0 means that resi

Once we have a condensate (with a fixed phase),
we have superconductivity

For bosons, the appearance of a condensate is natural,
because bosons tend to cluster at zero momentum
(Bose-Einstein condensation)

But electrons are fermions, and two fermions simply
cannot exist in one quantum state.

However, if two fermions form a bound state at zero momentum,
a bound pair becomes a boson, and bosons do condense.

We need to pair fermions into a bound state.

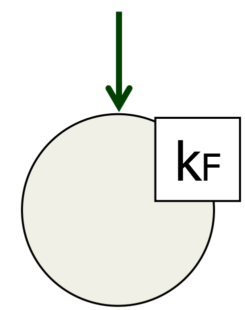
For pair formation, there must be attraction between fermions!

Nobel Prize 1972

J. Bardeen, L. Cooper, R. Schrieffer

An arbitrary small attraction between fermions is already capable to produce bound pairs with zero total momentum in any dimension because pairing susceptibility is logarithmically singular at small temperature (Cooper logarithm)

Zero energy



Reason: low-energy fermions live not near $k=0$, but near a Fermi surface at a finite $k=k_F$, $d^3k = 4\pi(k_F)^2\, d(k-k_F)$

J. Bardeen, L. Cooper, R. Schrieffer

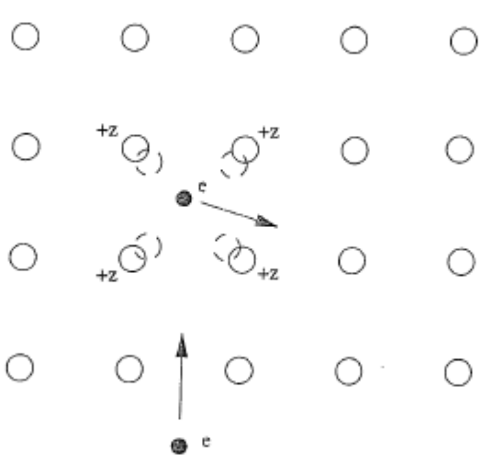
A. Abrikosov, V. Ginzburg, A. Leggett

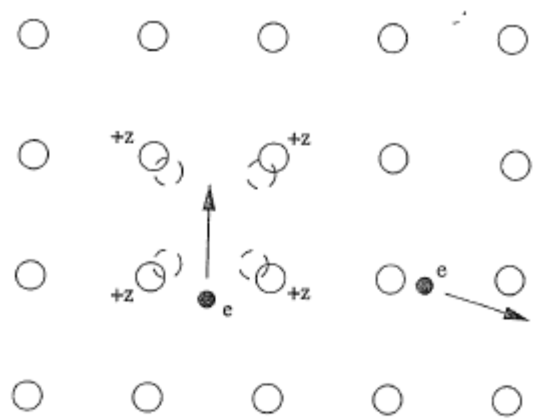
Nobel Prize 1972

L. Gorkov

Nobel Prize 2003

Two electrons attract each other by exchanging phonons – quanta of lattice vibrations





Phonon-mediated attraction competes with Coulomb repulsion between electrons and under certain conditions overshadows it

New era began in 1986: cuprates

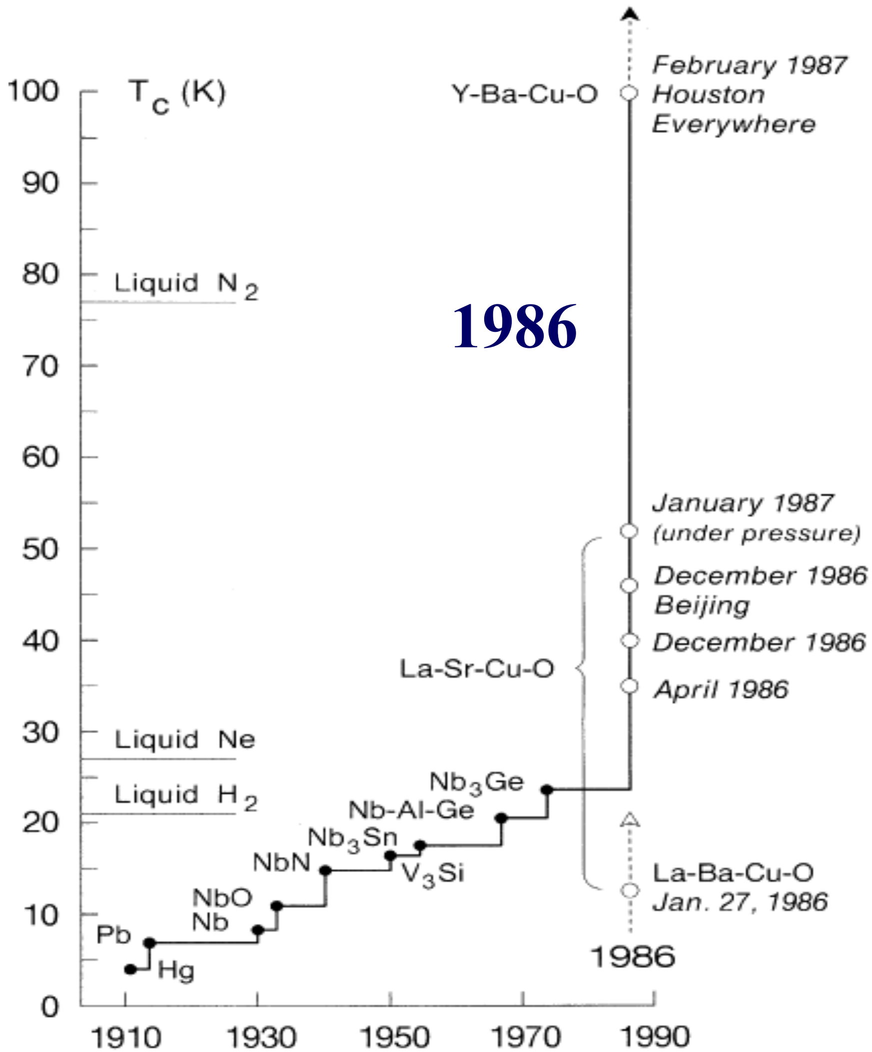
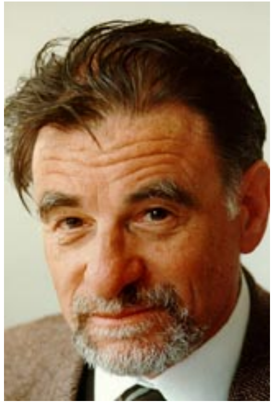


Fig. 1. Evolution of the superconductive transition temperature subsequent to the discovery of the phenomenon.



Alex Muller and Georg Bednortz

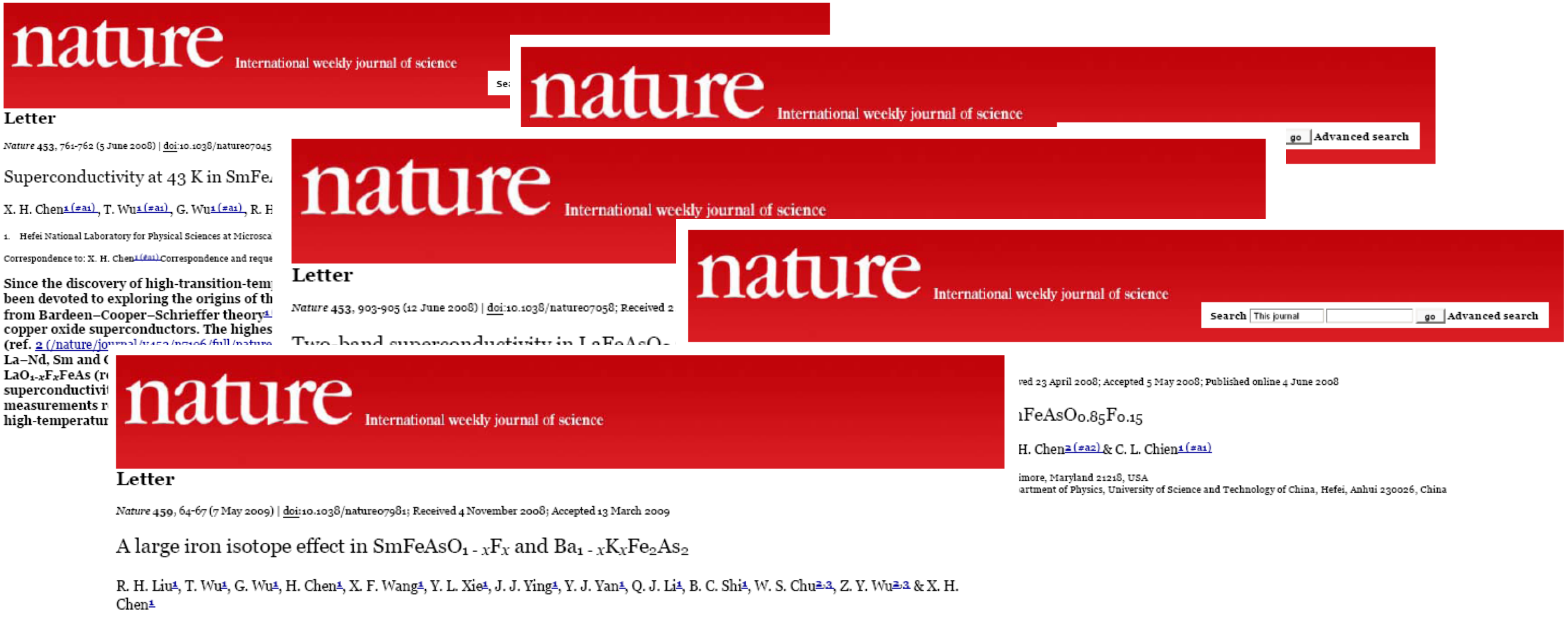
Nobel prize, 1987

New breakthrough in 2008: Fe-pnictides

$LaFeAsO_{1-x}F_x$, $Tc = 26K$

$SmFeAsO_{1-x}F_x$, $Tc = 43K$

Hideo Hosono


nature International weekly journal of science

Letter

Nature 453, 761-762 (5 June 2008) | doi:10.1038/nature07045

Superconductivity at 43 K in SmFe

X. H. Chen[1 (#a1)], T. Wu[1 (#a1)], G. Wu[1 (#a1)], R. H

1. Hefei National Laboratory for Physical Sciences at Microsca

Correspondence to: X. H. Chen[1 (#a1)] Correspondence and reque

Since the discovery of high-transition-tem been devoted to exploring the origins of th from Bardeen–Cooper–Schrieffer theory[1] copper oxide superconductors. The highes (ref. 2 (/nature/journal/v453/n7196/full/nature La–Nd, Sm and ($LaO_{1-x}F_xFeAs$ (r superconductivi measurements r high-temperatur

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nature International weekly journal of science

Letter

Nature 453, 903-905 (12 June 2008) | doi:10.1038/nature07058; Received 2

Two-band superconductivity in LaFeAsO

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ved 23 April 2008; Accepted 5 May 2008; Published online 4 June 2008

$FeAsO_{0.85}F_{0.15}$

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nature International weekly journal of science

Letter

Nature 459, 64-67 (7 May 2009) | doi:10.1038/nature07981; Received 4 November 2008; Accepted 13 March 2009

A large iron isotope effect in $SmFeAsO_{1-x}F_x$ and $Ba_{1-x}K_xFe_2As_2$

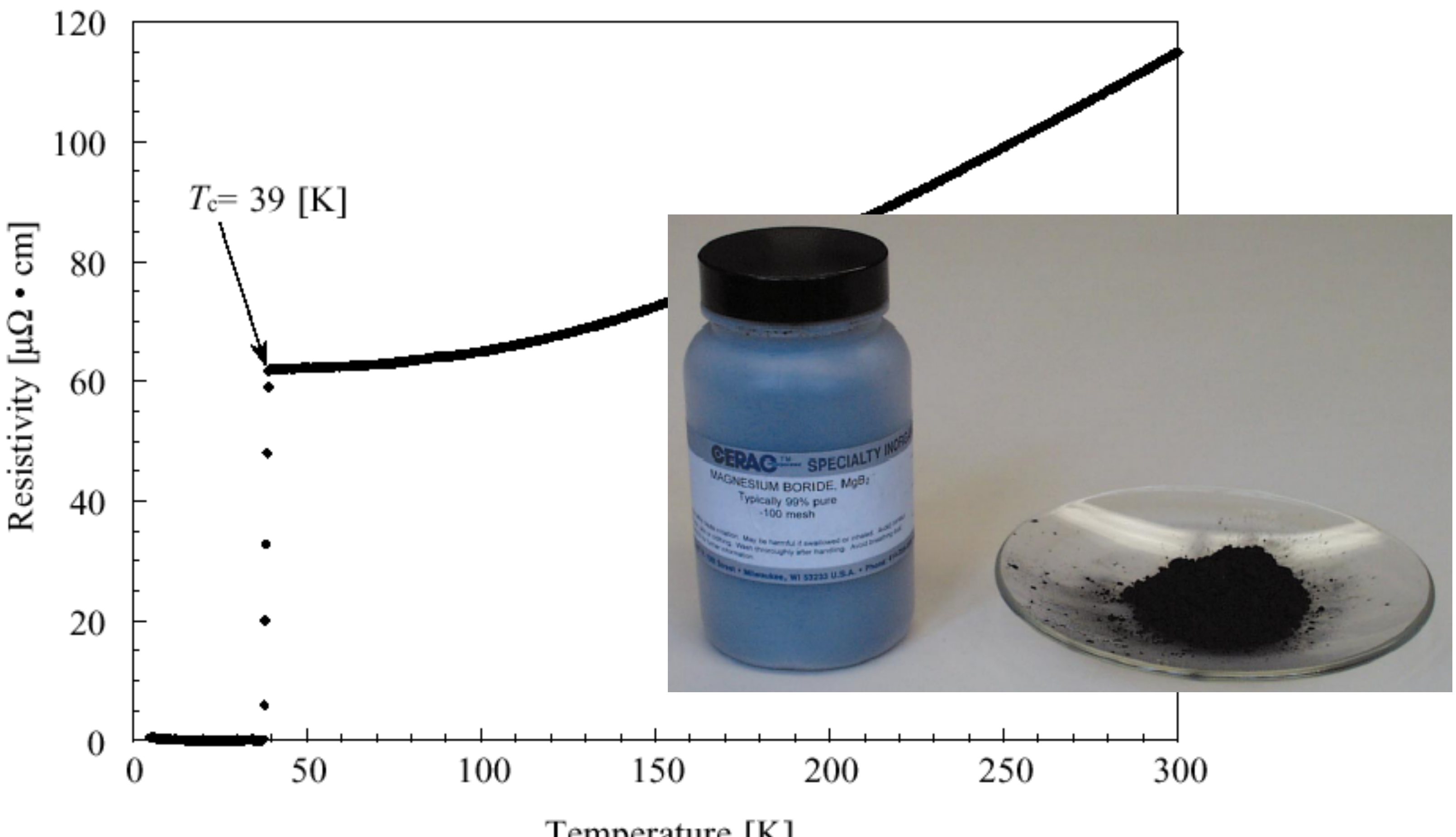
R. H. Liu[1], T. Wu[1], G. Wu[1], H. Chen[1], X. F. Wang[1], Y. L. Xie[1], J. J. Ying[1], Y. J. Yan[1], Q. J. Li[1], B. C. Shi[1], W. S. Chu[2,3], Z. Y. Wu[2,3] & X. H. Chen[1]

Is only high Tc relevant? No

MgB_2: A phonon Superconductor at 40 K

Fig.4 Nagamatsu et.al

T_c=39 K Akimitsu et al (2001)



Then what is relevant?

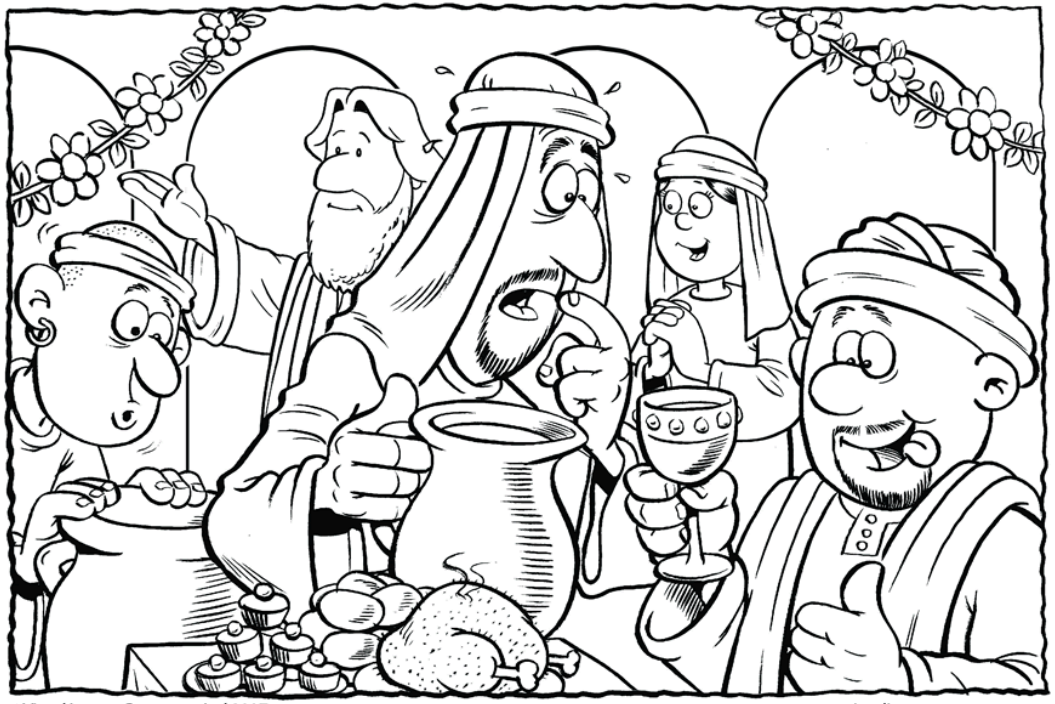
In Cuprates, Fe-pnictides, as well as in
Ruthenates (Sr_2RuO_4) ,
Heavy fermion materials ($CeIn_5$, UPl_3, $CePd_2Si_2$),
Organic superconductors ($(BEDT\text{-}TTF)_2\text{-}Cu[N(CN)_2]Br$)
…

electron-phonon interaction most likely is <u>NOT</u> responsible for the pairing, either by symmetry reasons, or because it is just too weak (Tc would be 1K in Fe-pnictides)

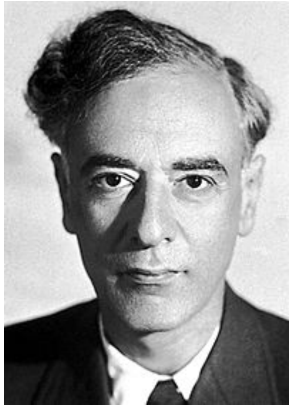
If so, then the pairing must somehow come from electron-electron Coulomb interaction, which is <u>repulsive</u>

Superconductivity from a repulsive interaction

How one possibly get a bound fermion pair out of repulsion?



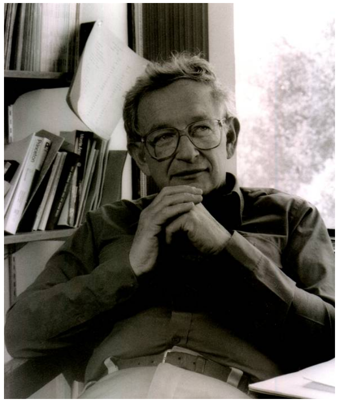
The story began in early 60th



Lev Landau

Pairing due to a generic interaction U(r)

Lev Pitaevskii



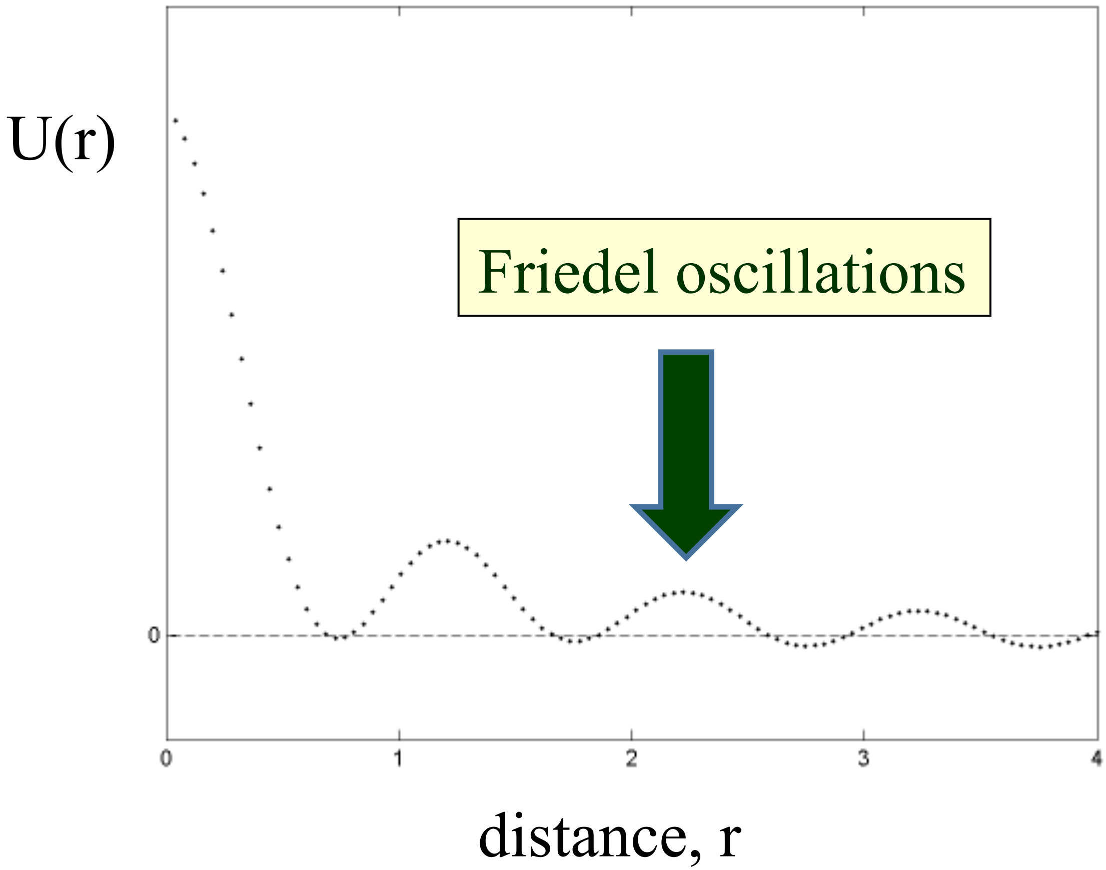
P.W. Anderson (with P. Morel)

- Fermions can form bound pairs with arbitrary angular momentum, m, not only with m=0, as was thought before them.

- The pairing problem decouples between different m
  It is sufficient to have attraction for just one value of m!

- Components of the interaction with large m come from large distances.

Screened Coulomb potential



At large distances, Coulomb interaction oscillates and occasionally gets over-screened $[U(r) = \cos(2k_F r)/r^3]$

(the oscillations are often called Friedel oscillations)

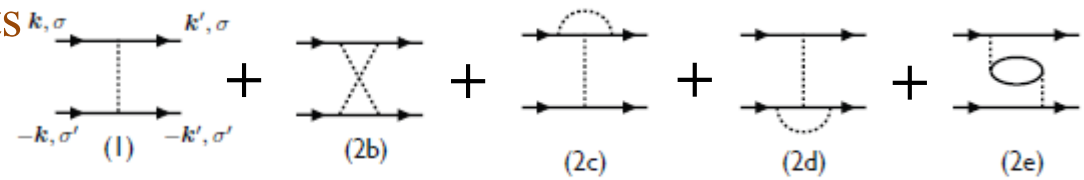
Walter Kohn

Joaquin Luttinger

Kohn-Luttinger story (1965)

Arbitrary regularily screened Coulomb interaction U(r)

Rigorously added the effects of Friedel oscillations to the pairing interaction



Components of the fully screened Coulomb interaction with large m are definitely attractive, at least for odd m

Then a bound state with some large angular momentum m necessary forms, and superconductivity develops below a certain Tc

This was the first example of "superconductivity from repulsion"

A (somewhat) simplified version of Kohn-Luttinger (KL) analysis applies to systems with small Hubbard interaction U (screening is so strong that repulsion acts only at r=0)

To first order in U, there is a repulsion in the s-wave (m =0) channel and nothing in channels with other angular momenta

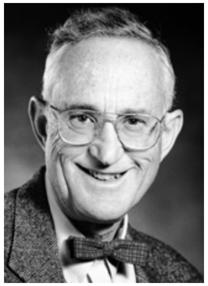
To second order in U, attraction emerges in all other channels, the largest one for m=1 (p-wave)

Fay and Layzer, 1968
M. Kagan and A.C., 1985

The Importance of Being Earnest

In 1965, most theorists believed that the pairing in ^3He should be with m=2 (d-wave).
KL obtained $T_c \sim E_F \exp[-2.5\, m^4]$,
substituted m=2, found $T_c \sim 10^{-17}$ K

A few years later experiments found that for ^3He, m=1.
If Kohn and Luttinger substituted m=1 into their formula,
they would obtain $T_c\,(m=1) \sim 10^{-3}\, E_F \sim 10^{-3}$ K
(close to $T_c \sim 3$ mK in ^3He)



Lee

Osherov

Richardson

?

Kohn

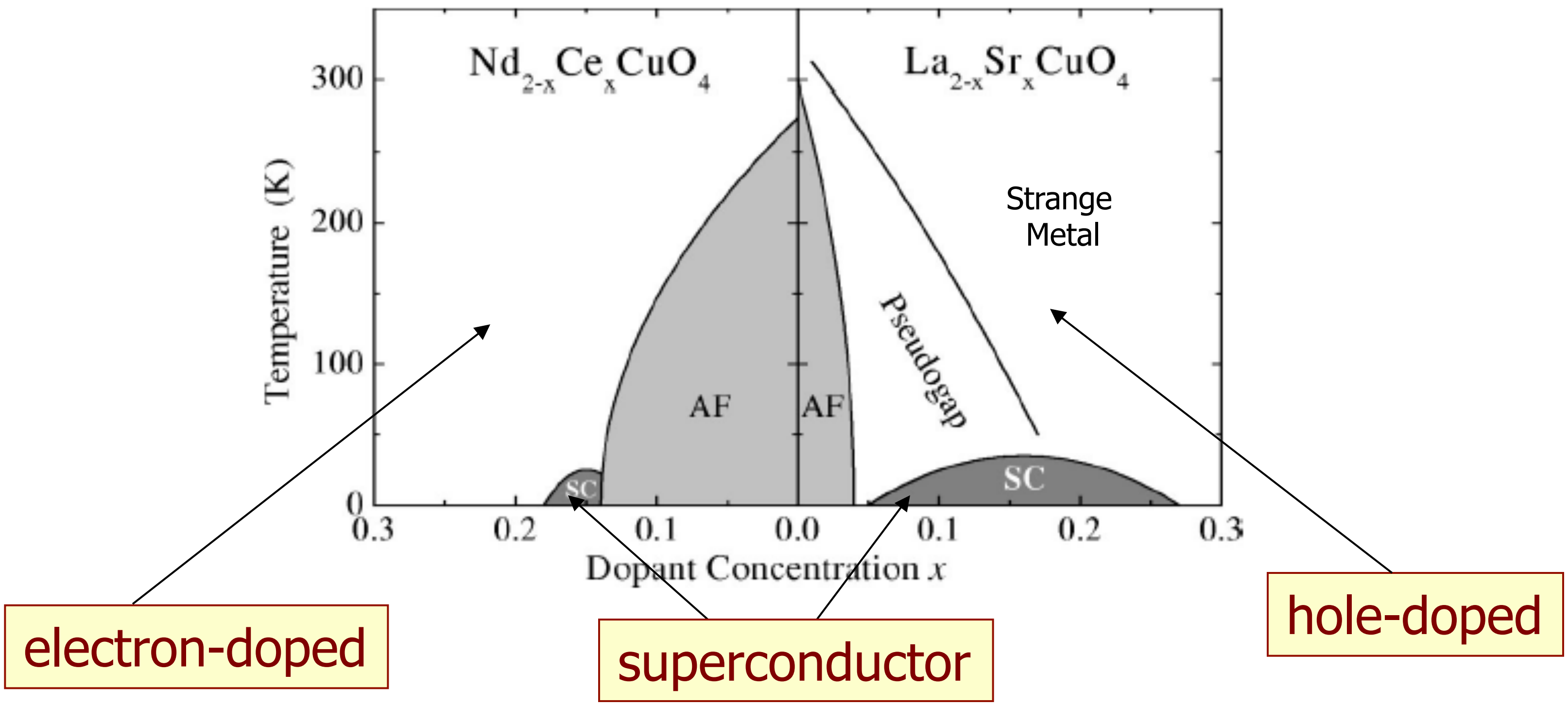
Luttinger

For the rest of this talk, I will explore KL idea that the effective pairing interaction is different from a bare repulsion U due to screening by other fermions, and may have attractive components in some channels

For lattice systems we cannot expand in angular harmonics (they are no longer orthogonal), and there is NO generic proof that "any system must become a superconductor at low enough T".

Nevertheless, KL-type reasoning gives us good understanding of non-phononic superconductivity

The cuprates (1986…)



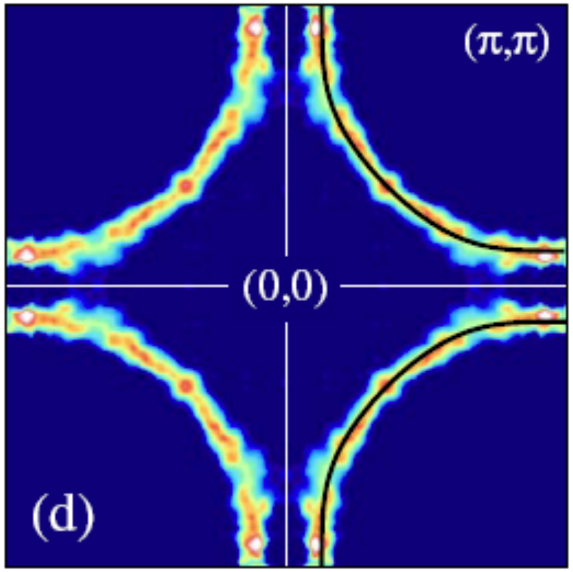
Parent compounds are antiferromagnetic insulators

Superconductivity emerges upon either hole or electron doping

Overdoped compounds are metals and Fermi liquids

$Tl_2Ba_2CuO_{6+\delta}$

Photoemission



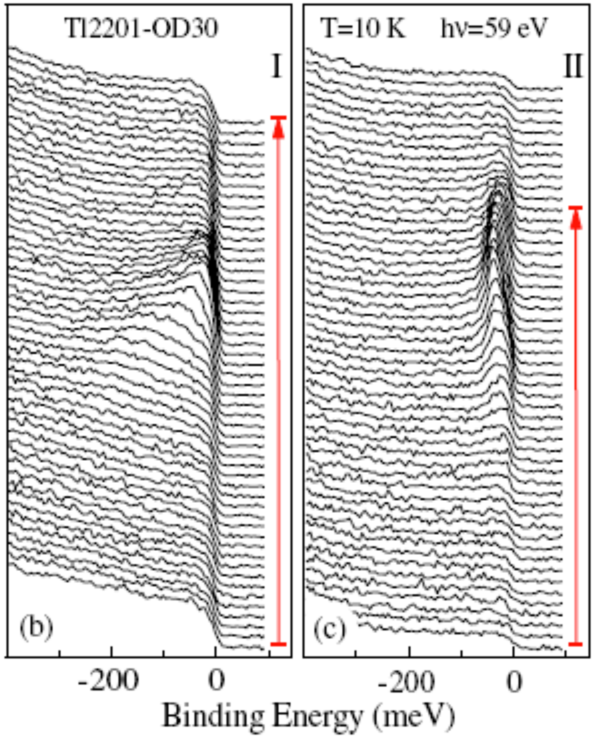
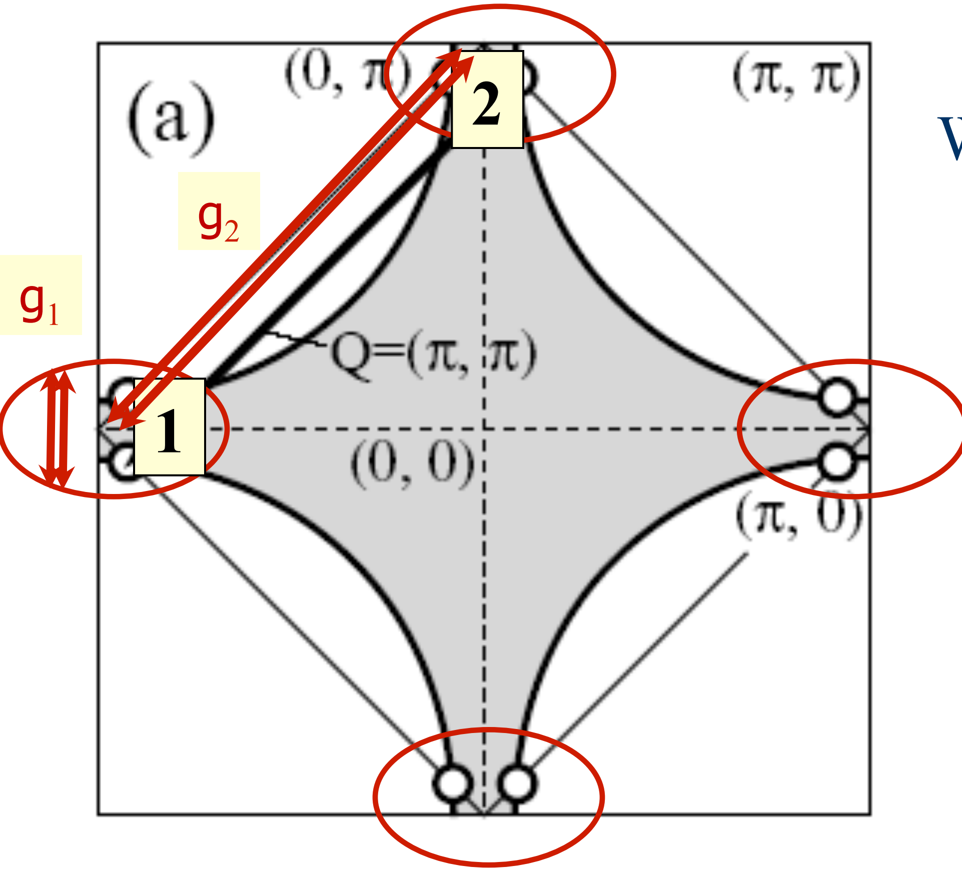


Plate et al

Kohn-Luttinger-type consideration (lattice version)



We have repulsive interactions within a patch

$$g(1,1) = g(2,2) = g_1$$

and between patches

$$g(1,2) = g_2$$

Δ = wave function of a pair

$$\Delta_1 = - g_1 \Delta_1 L - g_2 \Delta_2 L$$

$$\Delta_2 = - g_1 \Delta_2 L - g_2 \Delta_1 L$$

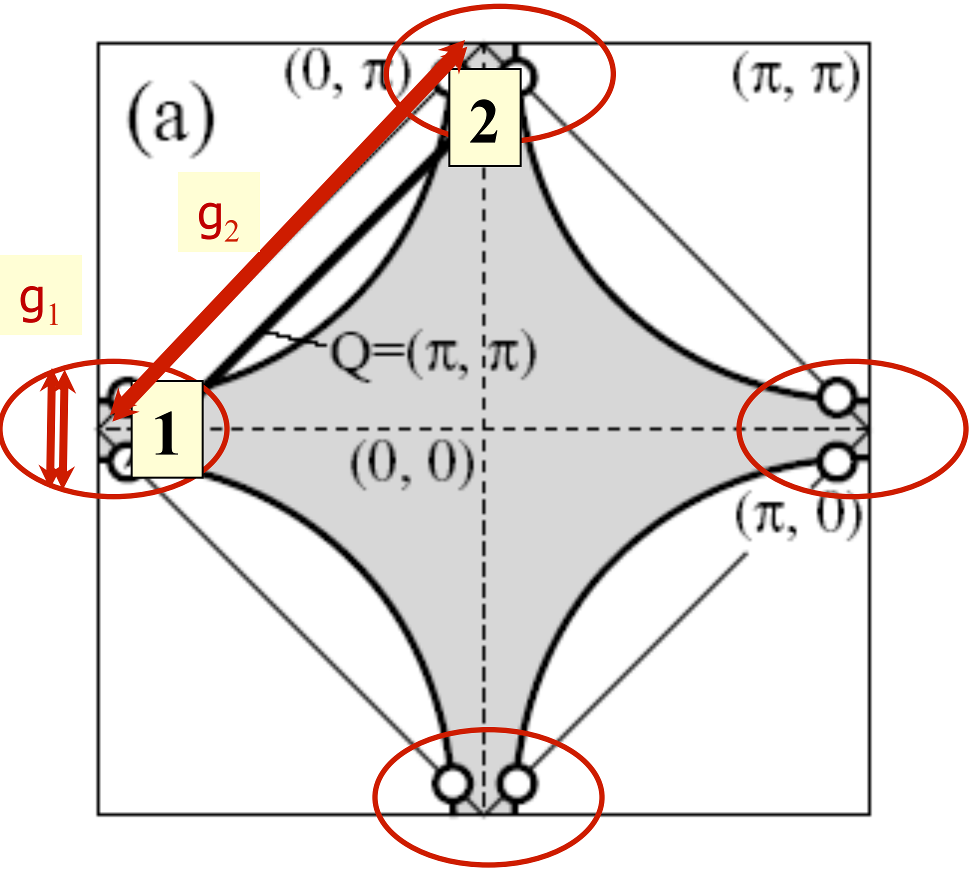
$$L = \log \frac{\Lambda}{T} \quad \text{Cooper logarithm}$$

Two pairing channels with couplings

$$\lambda_a = g_1 + g_2, \quad \lambda_b = g_1 - g_2,$$

need $\lambda < 0$ for the pairing

$g(1,1) = g(2,2) = g_1 \qquad g(1,2) = g_2$



Two pairing channels :

$\lambda_a = g_1 + g_2, \quad \lambda_b = g_1 - g_2,$

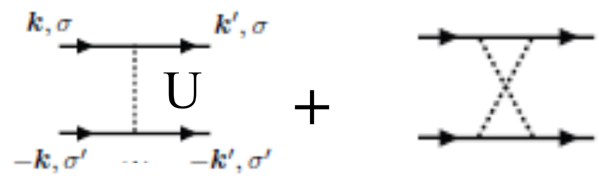
need $\lambda < 0$ for pairing

Do Kohn-Luttinger analysis for on-site repulsion U

To first order, we have a constant repulsive interaction – $g_1 = g_2 = U$, hence $\lambda_a > 0$, $\lambda_b = 0$

To order U^2

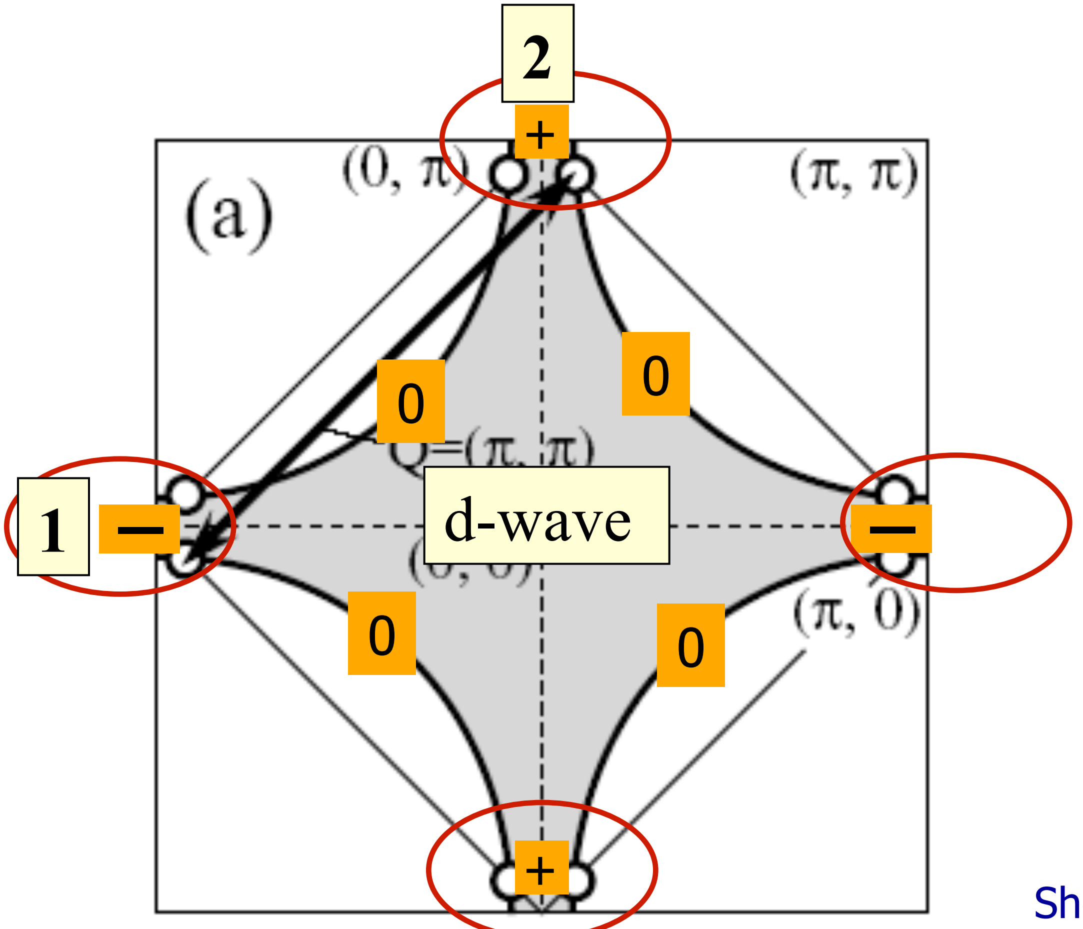
$\lambda_{a,b} =$

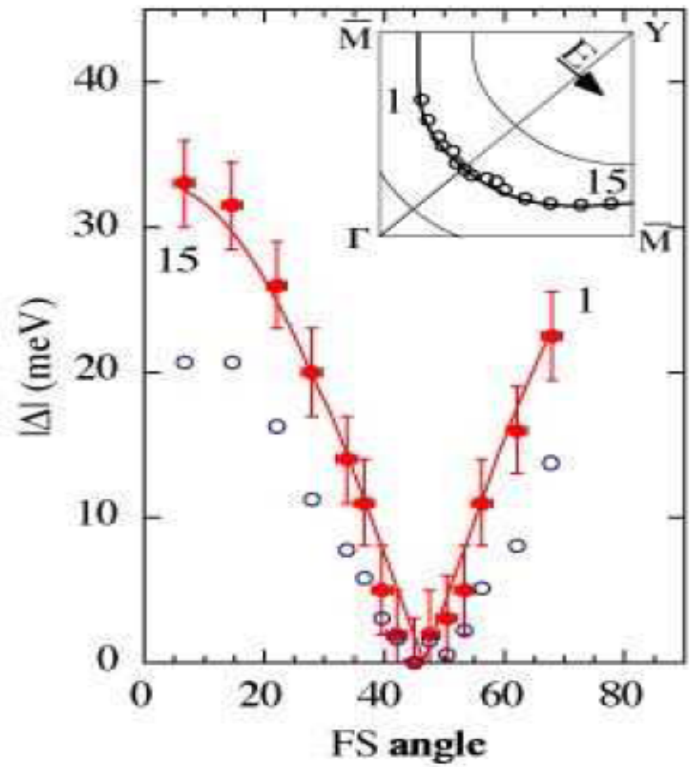


$g_2 > g_1$, hence $\lambda_b < 0$

Eigenvector for $\lambda_b = g_1 - g_2 < 0$: superconducting order parameter changes sign between patches

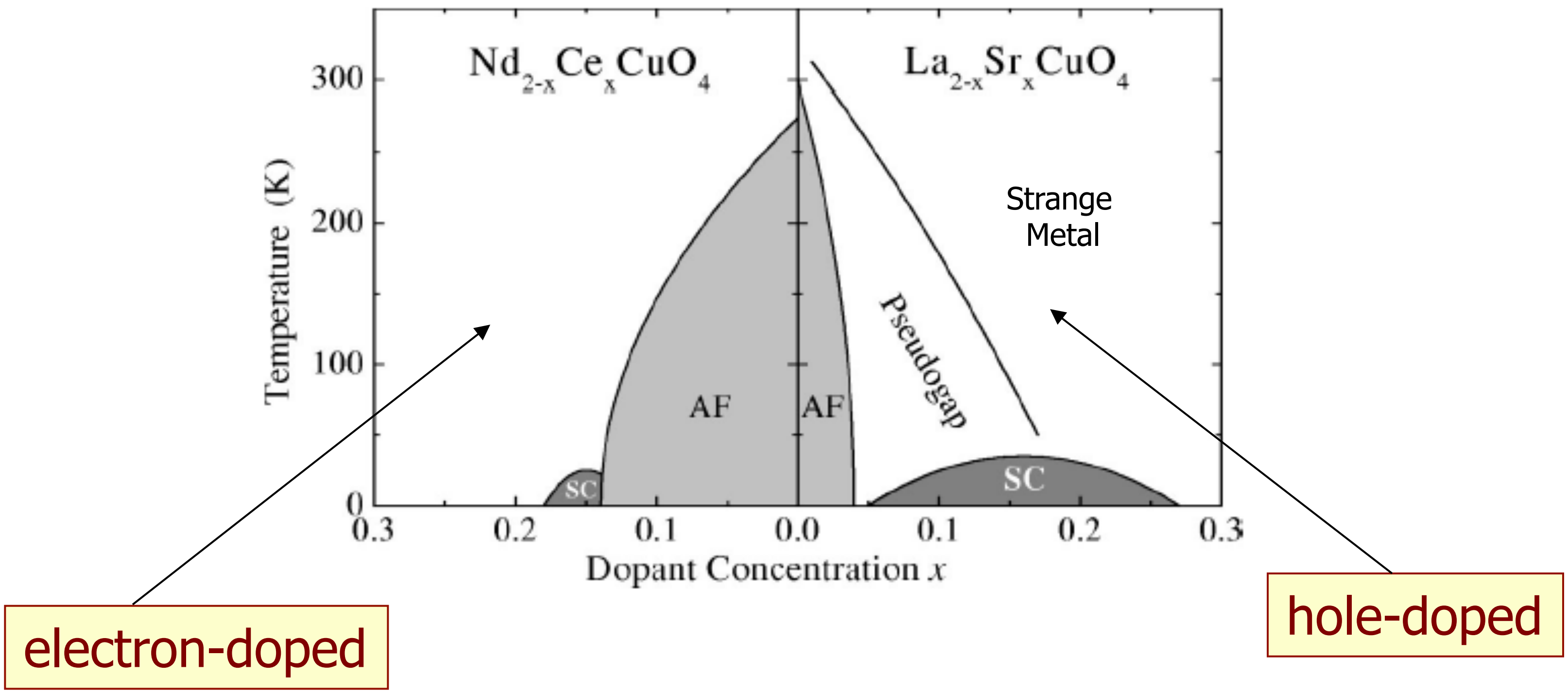
Spin fluctuation scenario: enhancement of KL effect by higher-order terms





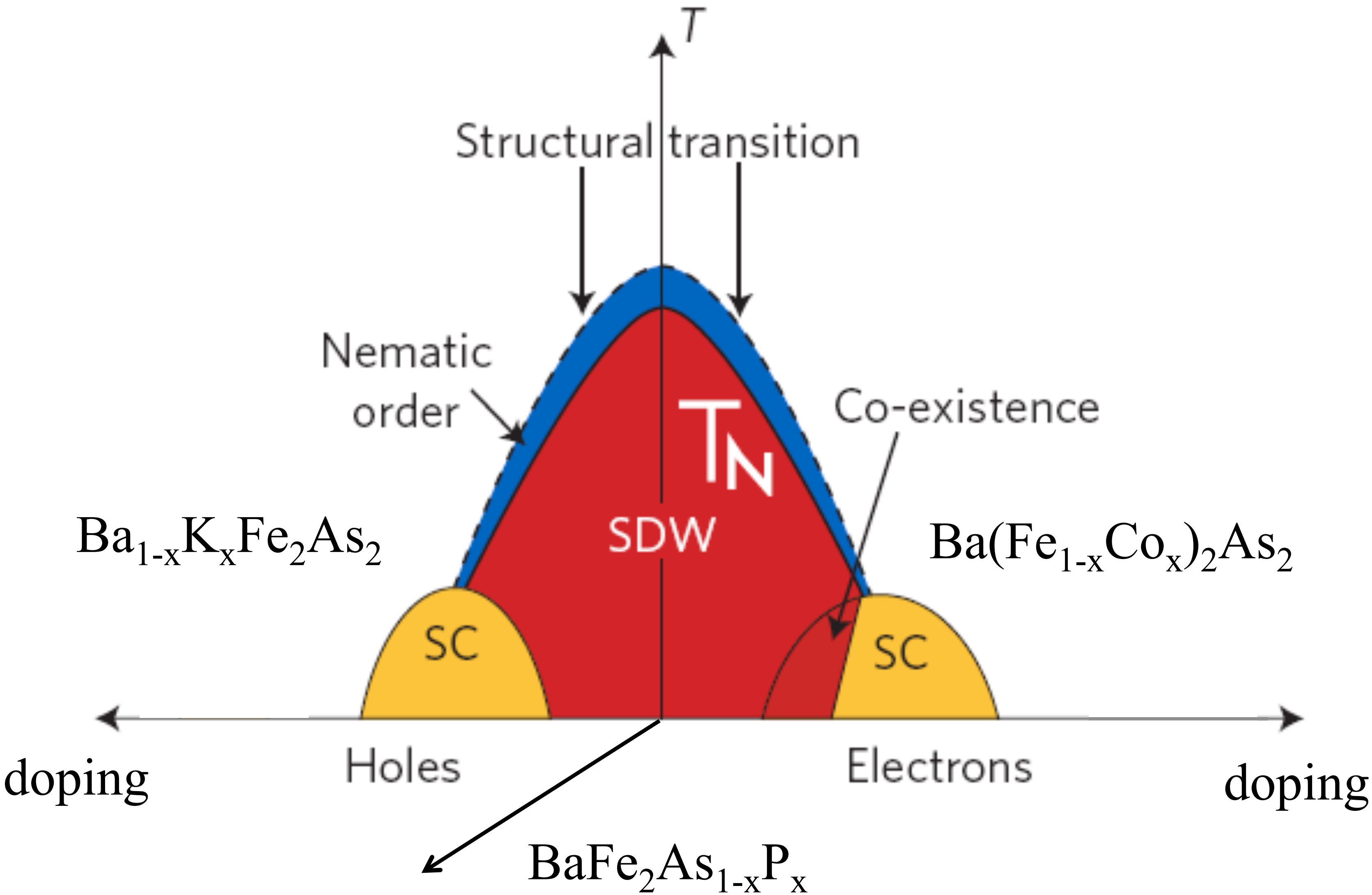
Shen, Dessau et al 93, Campuzano et al, 96

There is much more interesting physics in the cuprates than just d-wave pairing

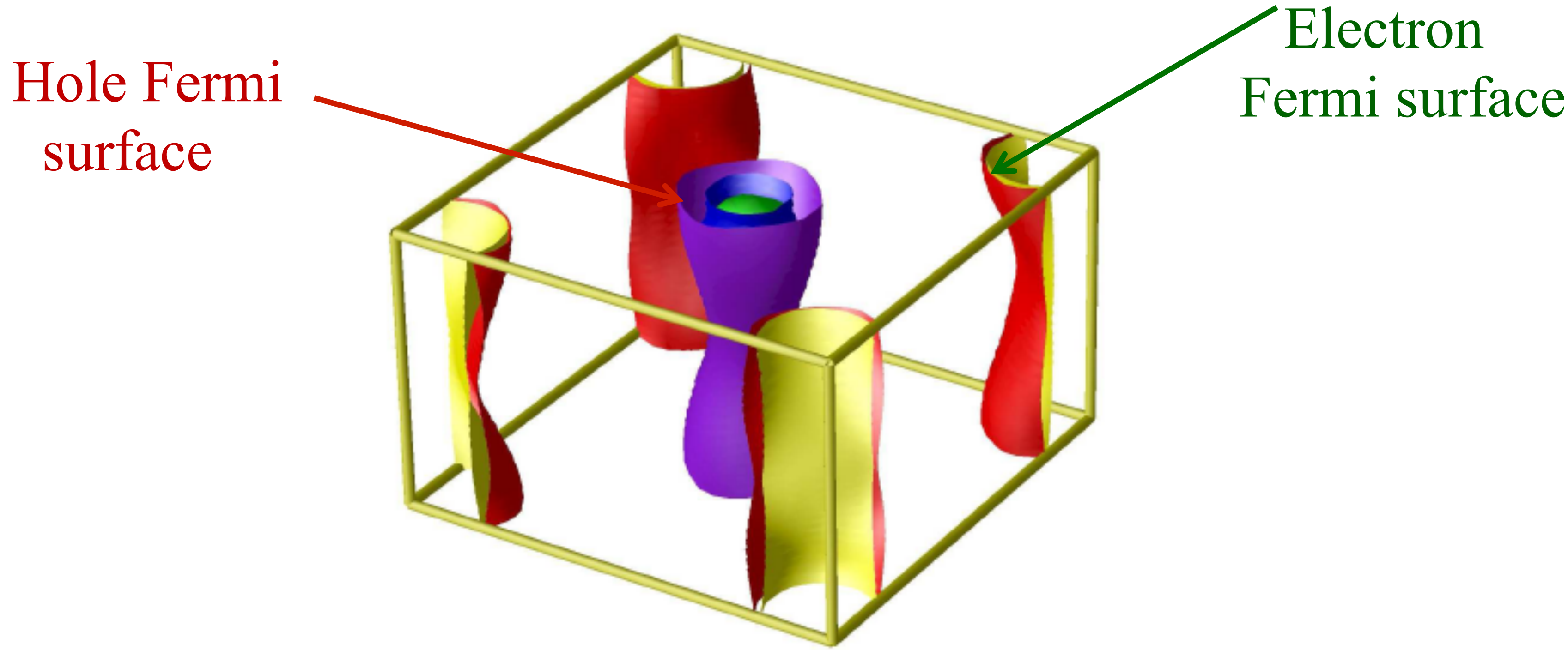


- Mott physics near zero doping
- Pseudogap phase, charge order…
- Fermionic decoherence (non-Fermi liquid physics)…
- Spin dynamics is crucial to determine Tc

The pnictides (2008…)



These are multi-band systems



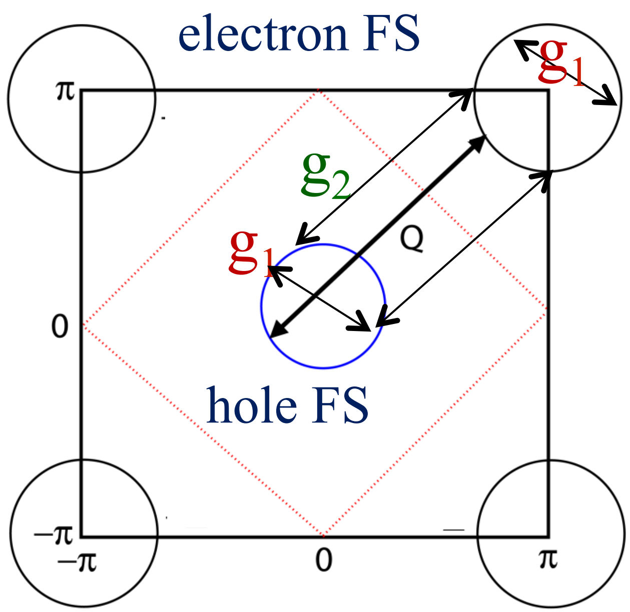
2-3 near-circular hole pockets around (0,0)

2 elliptical electron pockets around (π,π)

The minimal model: one hole and one electron pocket

Inter-pocket repulsion g_1

Intra-pocket repulsion g_2



Very similar to the cuprates, only "a patch" becomes "a pocket"

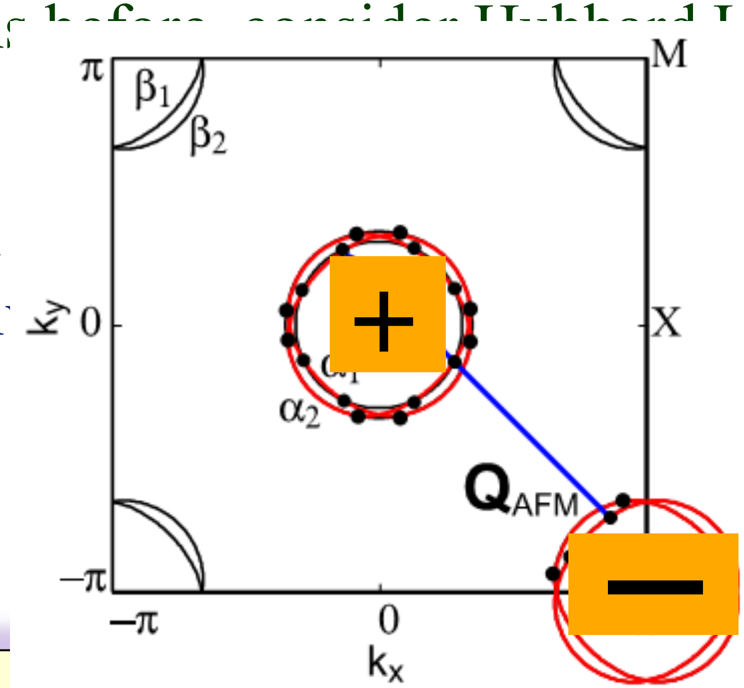
$\boxed{\lambda_a = g_1 + g_2,}$ $\boxed{\lambda_b = g_1 - g_2,}$

$\boxed{\lambda < 0 \text{ is needed for SC}}$

$$\lambda_a = g_1 + g_2,$$
$$\lambda_b = g_1 - g_2,$$
$\lambda < 0$ is needed for SC

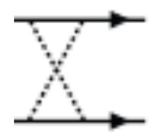
Do Kohn-Luttinger analysis:

As before, consider Hubbard U



To first ... e only have a
repulsi ... λ_b =0

To order U^2



Inter-pocket repulsion g_2 exceeds intra-pocket repulsion g_1, and λ_b becomes negative, i.e., superconductivity develops

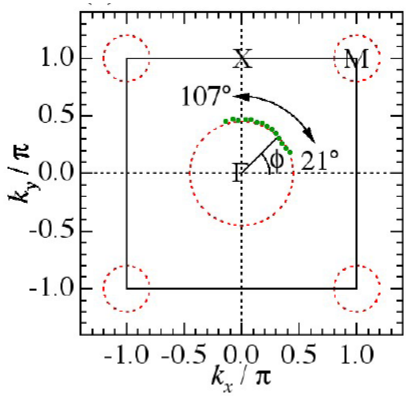
sign-changing s-wave gap s+-

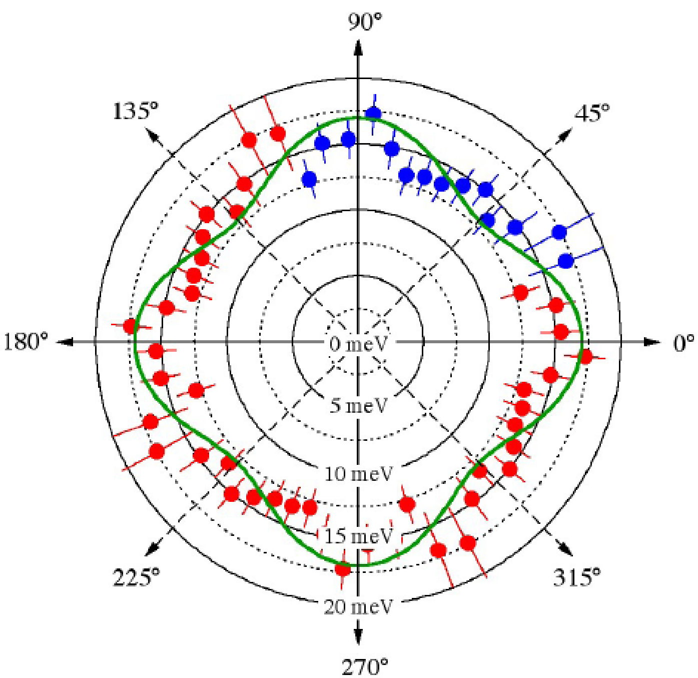
S-wave

Photoemission in 1111 and 122 FeAs

Data on the hole Fermi surfaces

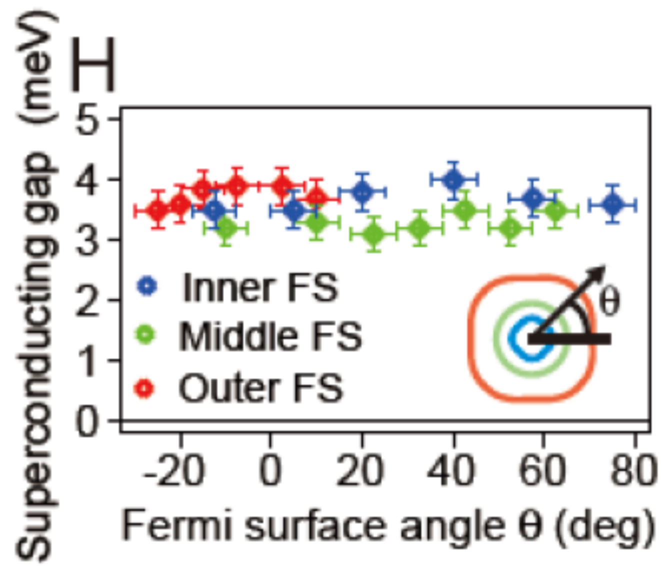
$NdFeAsO_{1-x}F_x$





T. Kondo et al.

$BaFe_2(As_{1-x}P_x)_2$



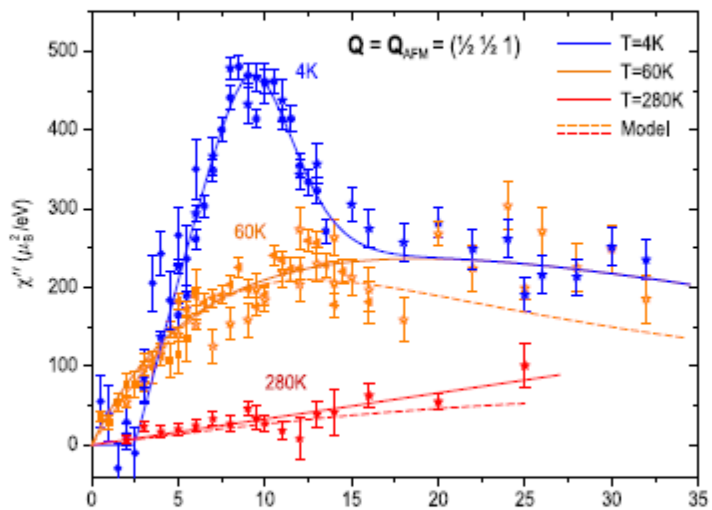
laser ARPES

T. Shimojima et al

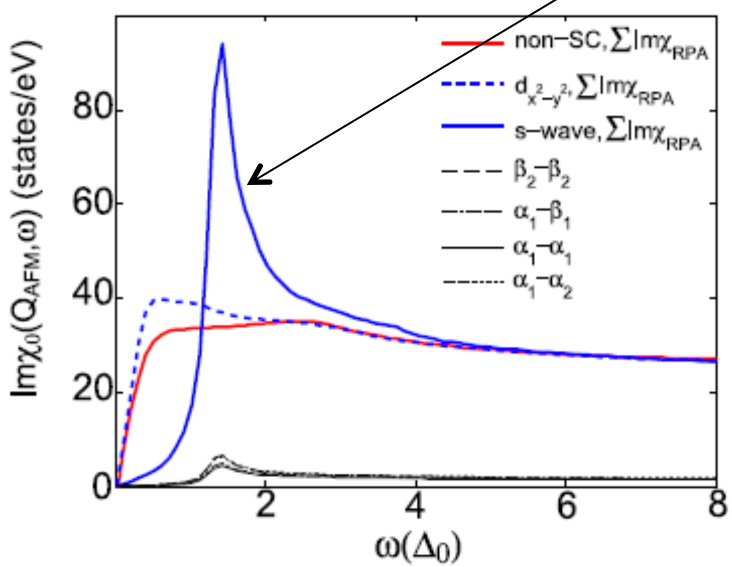
Almost angle-independent gap (consistent with s-wave) s+- gap

Neutron scattering – resonance peak below 2D

$BaFe_{1.85}Co_{0.15}As_2$ ($T_c = 25$ K)



D. Inosov et al



s_{+-} gap

Theorists say :

one needs $\Delta_{k+\pi} = -\Delta_k$

The "plus-minus" gap is the best candidate

Eremin & Korshunov Scalapino & Maier…

This story is a little bit too good to be true.

In both cases we assumed that bare interaction is a Hubbard U, in which case, in a relevant channel $\lambda = 0$ to order U and becomes negative (attractive) to order U^2

In reality, to first order U, $\lambda = g_1 - g_2 = U_{small} - U_{large}$

small (large) is a momentum transfer

For any realistic interaction, $U_{small} > U_{large}$

Then bare $\lambda > 0$, and the second order term has to overcome it

And this essentially what we try to understand, one way or the other!

Physicists, we have a problem

Two approaches:

One approach is to abandon perturbation theory and assume that inter-patch (inter-pocket) interaction is large because the system is close to a spin-density-wave instability

Effective fermion-boson model – superconductivity near a QCP

Another approach is to keep interactions weak, but see whether we can enhance Kohn-Luttinger effect in a controllable way, due to interplay with other channels.

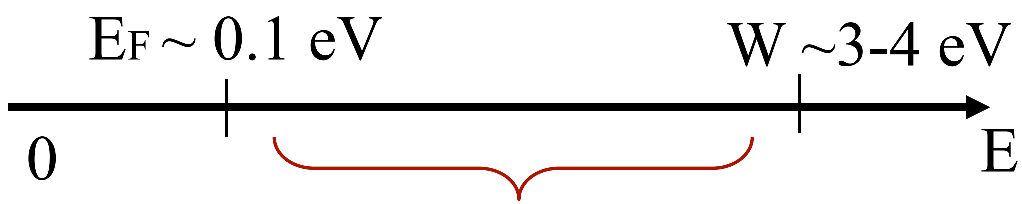
Renormalization group approach

$$\lambda_a = g_1 + g_2,$$
$$\lambda_b = -g_2 + g_1,$$
$\lambda < 0$ is needed for SC

Consider Fe-pnictides as an example

g_1 and g_2 are bare interactions, at energies of order bandwidth

For SC we need interactions at energies smaller than the Fermi energy

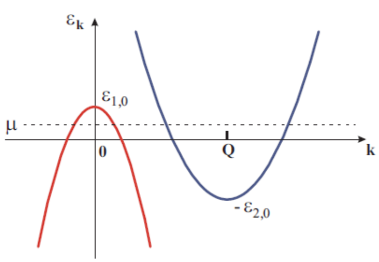


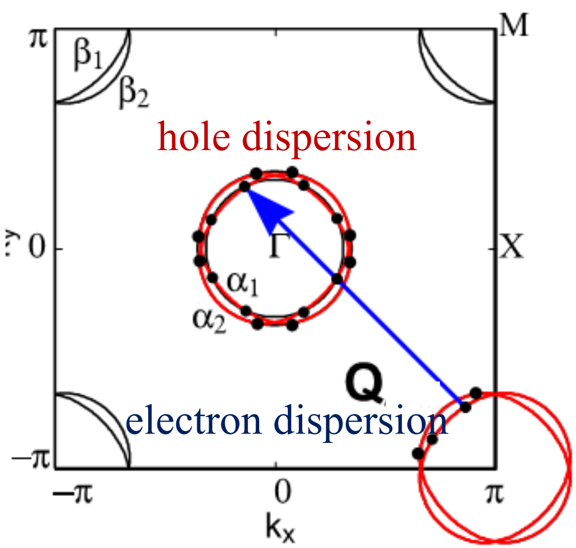
Couplings flow due to renormalizations in all channels (particle-particle AND particle-hole)

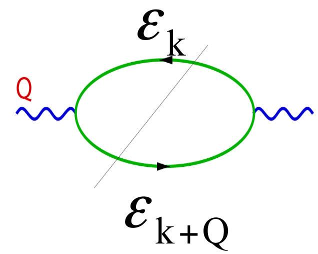
Because in Fe-pnictides one pocket is electron-type and another is hole-type, renormalizations in the particle-particle (Cooper) channel and in the particle-hole channel are both logarithmically singular

particle-particle channel – Cooper logarithm

particle-hole cannel – logarithm due to signs of dispersions







$$= \iint_T \frac{d\omega\, d\varepsilon_k}{\omega^2 + \varepsilon_k^2} = \log \frac{E_F}{T}$$

Then we have to treat particle-particle and particle-hole channels on equal footings

The presence of logarithms is actually a blessing

Conventional perturbation theory: expansion in g.
We can do controllable expansion when $g \ll 1$

When there are logarithms in perturbation theory, we can extend theoretical analysis in a controllable way by summing up infinite series in perturbation theory in $g * \log W/T$ and neglecting $g^2 * \log W/T$, etc…

The most known example – BCS theory of superconductivity (summing up Cooper logarithms in the particle-particle channel)

$$g + g^2 * \log W/T + g^3 *(\log W/T)^2 + \ldots = g/(1-g * \log W/T)$$

superconductivity at $T_c = W e^{-1/g}$

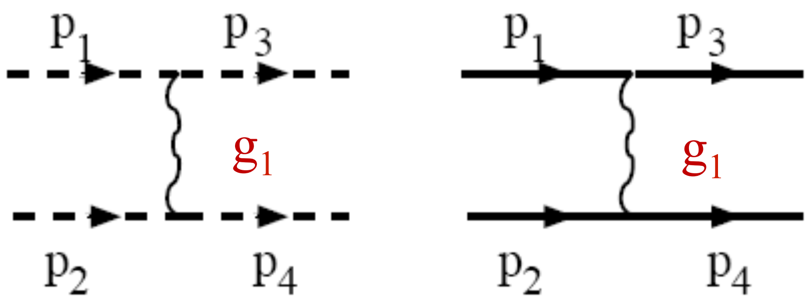
Summing up the logarithms == solving RG equation

Now we want to do the same when there are log W/E terms in both particle-particle and particle-hole channel

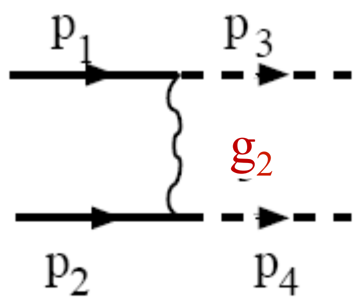
$$\frac{dg(E)}{d(\log W/E)} = g^2(E), \quad g(E) = \frac{g}{1-g \log W/E};$$

$$E_c = T_c = W e^{-1/g}$$

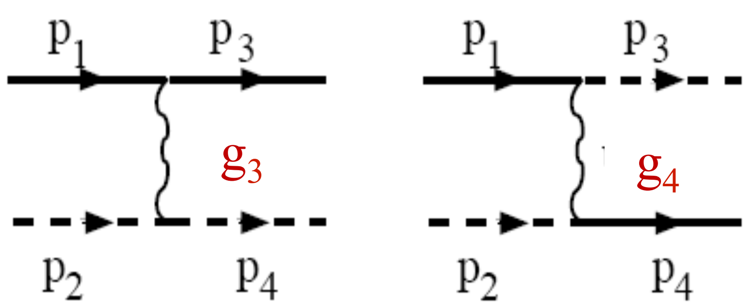
Strategy: introduce all relevant couplings between low-energy fermions



Intra-pocket repulsion



Inter-pocket repulsion



Inter-pocket forward and
backward scattering

Interactions leading to density-wave orders

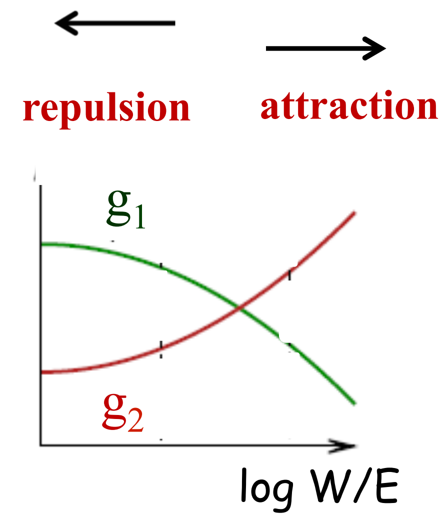
Renormalization group equations

$$\frac{dg_3}{d(\log \mathrm{W/E})} = g_3^2 + g_2^2$$

$$\frac{dg_4}{d(\log \mathrm{W/E})} = 2g_4\,(g_3 - g_4)$$

$$\frac{dg_2}{d(\log \mathrm{W/E})} = g_2\,(4g_3 - 2g_4 - 2g_1)$$

$$\frac{dg_1}{d(\log \mathrm{W/E})} = -g_1^2 - g_2^2$$

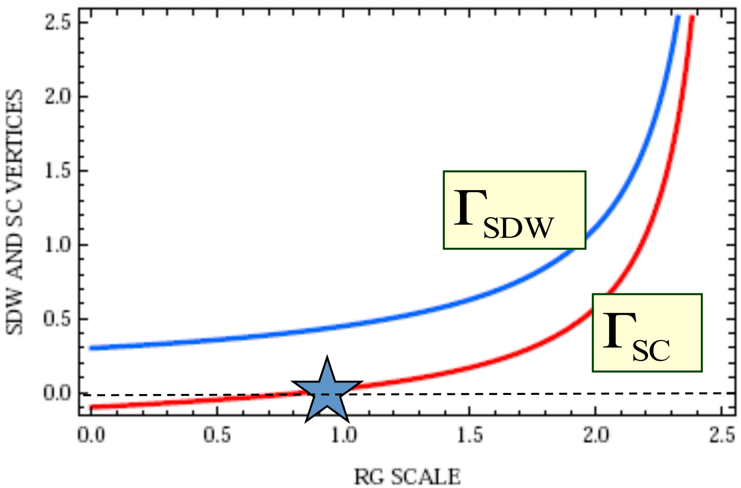


Emergence of the KL effect in a controllable calculation:
below some energy, inter-pocket repulsion exceeds intra-pocket one

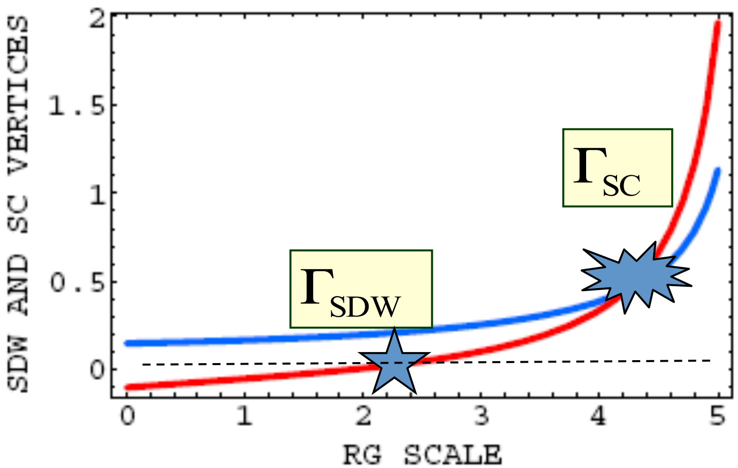
Physics: inter-pocket pairing interaction g_2 is pushed up by density-density interaction g_3, which favors SDW order

What happens after SC interaction in s+- channel becomes attractive depends on geometry of the Fermi surface

1 hole and 1 electron FSs



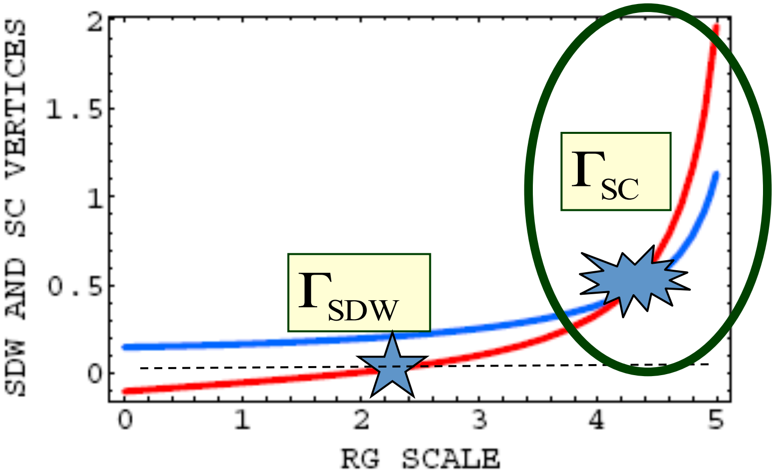
2 hole and 2 electron FSs

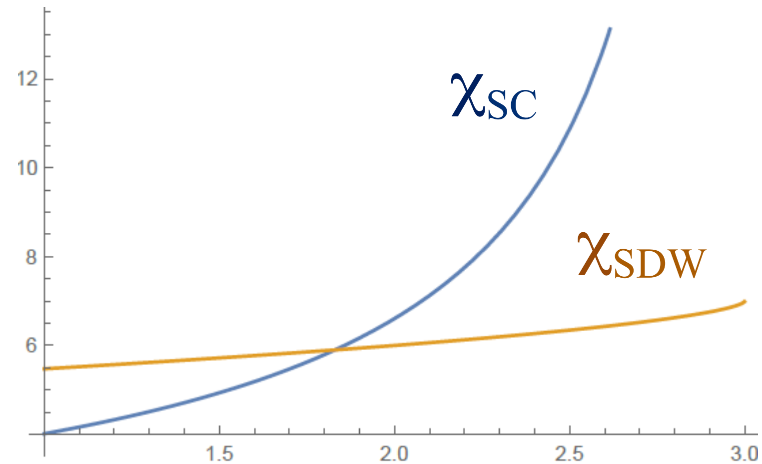


SC vertex can overshoot SDW vertex, in which case SC becomes the leading instability already at zero doping

More sophisticated analysis: calculation of the susceptibilities

2 hole and 2 electron FSs





Only SC susceptibility diverges at some critical RG scale == Tc

SDW susceptibility does not diverge (SDW order does not develop) due to negative feedback effect from increasing SC fluctuations.

The source creates the response, the growing response destroys the source

Similar phenomena in other fields

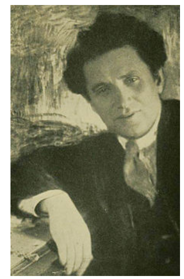
Russian politics (and not only Russian)

leading Bolsheviks after the revolution

Trotsky

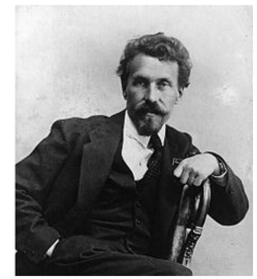
Bukharin

Kamenev



Zinoviev

Sokolnikov

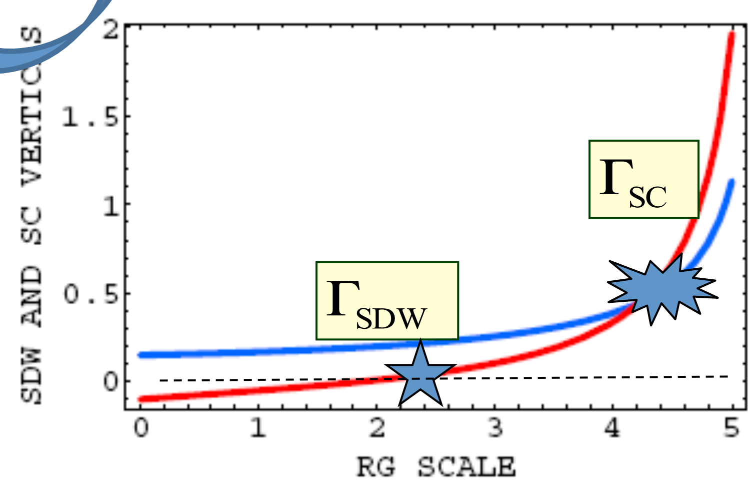


Rykhov

sources

Stalin

response



Conclusions

There are numerous examples when superconductivity (which requires pairing of fermions into bound states) comes from repulsive electron-electron interaction

The generic idea how to get superconductivity from repulsion goes back to Kohn and Luttinger

KL physics leads to:

d-wave superconductivity in cuprates
s-wave superconductivity in Fe-pnictides (s^{+-})

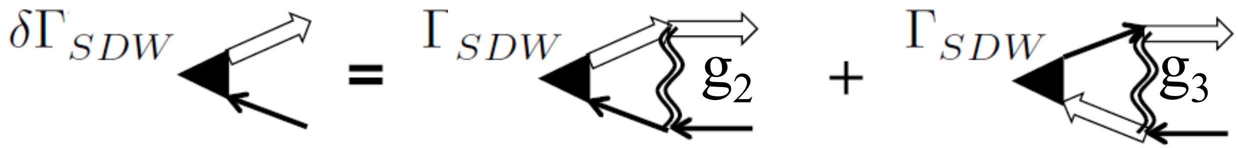
Also: d+id superconductivity in graphene near van-Hove point

In all cases, fluctuations in the spin-density-wave channel enhance tendency to superconductivity by reducing the repulsive part of the interaction and enhancing the attractive part => the system self-generates an attraction below some scale.

Growing SC fluctuations may block the development of spin-density-wave

THANK YOU

More sophisticated analysis: calculation of the susceptibilities

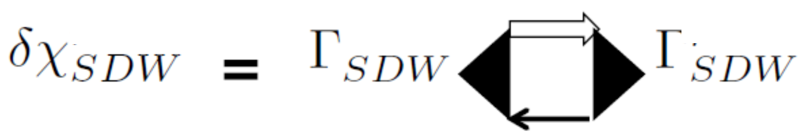


$$\frac{d\Gamma_{\rm SDW}}{d(\rm L)} = \Gamma_{\rm SDW}\,(g_2 + g_3)$$

$g_2 = \gamma\, g_3$ along fixed trajectory (from RG eqs), $\gamma = \sqrt{15}$

$$\frac{dg_3}{d(\rm L)} = g_3^2\,(1+\gamma^2), \quad g_3 = \frac{1}{1+\gamma^2}\,\frac{1}{\rm L_0 - L}, \; g_2 = \frac{\gamma}{1+\gamma^2}\,\frac{1}{\rm L_0 - L}$$

$$\Gamma_{\rm SDW} \sim \frac{1}{(\rm L_0 - L)^{\beta_{\rm sdw}}}, \quad \beta_{\rm sdw} = \frac{1+\gamma}{1+\gamma^2}$$



$$\chi_{\rm SDW} = \frac{1}{(\rm L_0 - L)^{\alpha_{\rm sdw}}}, \quad \alpha_{\rm sdw} = 2\beta_{\rm sdw}$$

$$\chi_{\rm SC} = \frac{1}{(\rm L_0 - L)^{\alpha_{\rm sc}}}, \quad \alpha_{\rm sc} = 2\beta_{\rm sc} - 1$$

Two hole and two electron Fermi surfaces: $\boxed{\alpha_{\rm sc} > 0, \quad \alpha_{\rm sdw} < 0}$

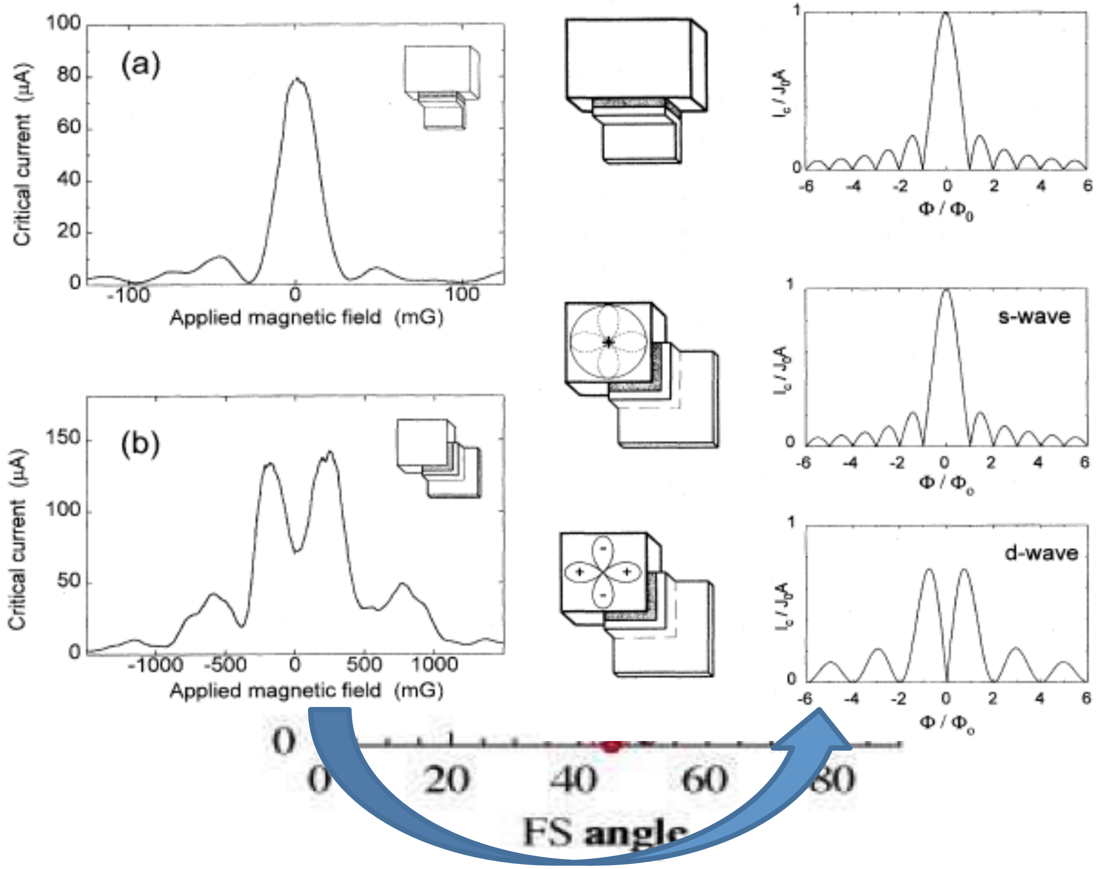
d-wave pairing is a well established phenomenon

Oliver E. Buckley Condensed Matter Physics Prize

Campuzano

Johnson

Z-X Shen



Tsuei

Van Harlingen

Ginsberg

Kirtley