Ground-state properties of artificial bosonic atoms, the Bose interaction blockade, and the atomic pipette

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Introduction

We analyze the ground-state properties of repulsive bosons in an attractive potential for the case when all the interactions are short-ranged [1]. We call the resulting bound state an **artificial bosonic atom**. Such a system can be created experimentally by optically trapping ultra-cold alkali atoms. In this scenario, the artificial bosonic atom is composed of atoms. The dependence of the ground-state energy on the number of particles has a minimum whose position is experimentally tuneable. This implies that the number of bosons bound to the central potential can be controlled by tuning external parameters. In particular, the number of bound bosons should have a staircase dependence on external parameters. Since it is the competition between Bose condensation and the repulsive boson-boson interactions, along with the discreteness of particles, that leads to this staircase dependence, we dub the phenomenon the **Bose interaction blockade**. By making use of this phenomenon it is possible to control the transport of atoms, one at a time, into and out of a reservoir. We call this application the **atomic pipette**.

I. How many repulsive bosons will a short-ranged potential accept into a bound state?

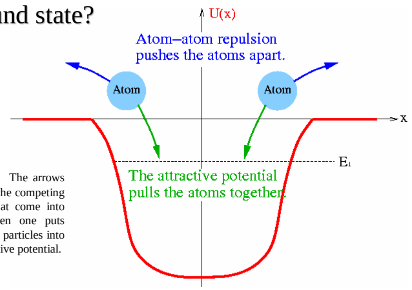

Fig. 1. The arrows indicate the competing forces that come into play when one puts repulsive particles into an attractive potential.

What we would like to know is the ground-state energy for *N* repulsive bosons in a short-ranged attractive potential. Since there are obviously some competing forces at play in such a system (see Fig. 1), we expect these features to appear in our answer. **Here we introduce a simple heuristic model that encapsulates the key physics.** Consider a short-ranged potential with a single particle energy level $-|E_1|$, as shown in Fig. 1. If for a moment we neglect the interactions between bosons, we can put *N* bosons into that same single-particle state for a **net energy gain of N times the single-particle energy**. This is basically just your "Bose condensation" energy. But, since the bosons we are dealing with are repulsive, there should be an energy cost associated with confining them together. This **energy cost can be argued to go as the number of pair interactions times a typical pair interaction energy**, which we can call ν_{pair}. Putting these two terms together gives the heuristic guess of

$$E(N) = -|E_1|N + \nu_{pair}\frac{N(N-1)}{2} \qquad (1)$$

for the ground state energy for *N* repulsive bosons in a short-ranged potential. The competition between the two terms leads to a minimum in (1). This minimum is a result of the competition between Bose condensation and repulsive interactions--the two competing forces highlighted in Fig. 1.

So how many repulsive bosons will a short-ranged potential choose for its ground state? In answering this question, assume that the short-ranged potential is coupled to a reservoir of particles such that it can accept or reject particles as it chooses (we will talk about reservoirs in more detail in section III). Then the short-ranged potential will accept the integer number of particles that allows it to minimize its energy. **Two qualitatively different scenarios can then be identified, and these two cases are shown above in Fig. 2 -- either a finite number or a single-particle ground state will be chosen, depending on the value of the parameters.**

Underlying Physics

- *Attractive* short-ranged potential
- Particles are:
  - *repulsive*
  - bosons ==> Bose Condensation
  - discrete ==> possible to see single-particle effects

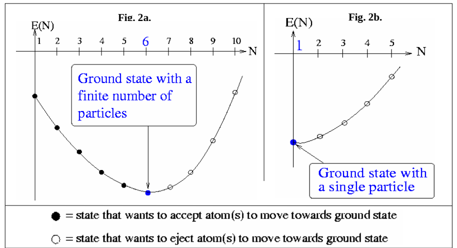

Fig. 2. The energy vs. particle number, *N*, for *N* bosons in a short-ranged potential. In Fig. 2a, the short-ranged potential accepts a finite number of particles from the reservoir in order to minimize its energy. In Fig. 2b, the potential accepts a single particle to minimize its energy.

II. Methods of calculation

The two regimes highlighted in Fig. 2 are qualitatively correct, but there are a couple of obvious shortcomings with the simple heuristic model (1) that one would like to be able to improve upon. For one thing, this simple model makes no mention of the dependence of the results on the **spatial dimension**. Since the binding properties of short-ranged potentials depend on dimension, we treat the cases of general dimension between $d=1$ and $d=3$. We also expect that the **localization length**, or the size of the bound state, will swell in size as more and more repulsive bosons are added to it. This changing of the localization length with the number of bound particles is also addressed in our calculation.

To address these issues our main method of attack is a self-consistent (or Hartree-Fock-Gross-Pitaevskii) treatment of the problem. Since the many-body problem is in general unsolvable, we instead deal with the particle density distribution, $\phi^2(r)$, of the bosons. In this formalism, we work with the energy functional

$$E[\phi] = \int d^d r\left[\frac{\hbar^2}{2m}(\nabla\phi)^2 + U_{ext}(r)\phi^2 + \frac{g(N-1)}{2N}\phi^4\right] \qquad (2)$$

where the three terms are seen to correspond, respectively, to the zero-point energy, the interaction with the short-ranged potential $U_{ext}(r)$, and the boson-boson interactions handled self-consistently. The coupling strength *g* is proportional to the *s*-wave scattering length for two-body collisions. The energy functional (2) should be minimized with respect to ϕ. This is accomplished using both a direct minimization solution and variational minimization in $d=1$ [2,3] and variational minimization in general *d* [1]. The end-result of the calculation is the ground-state energy, $E(N,Z)$, as a function of the number of particles and external parameters (here denoted by *Z*).

III. The Bose interaction blockade and the atomic pipette

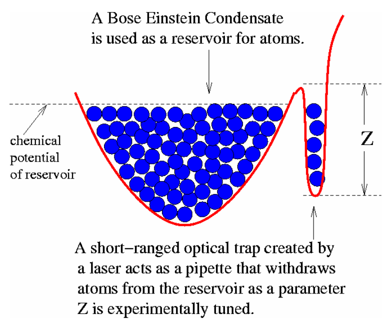

Fig. 3. A "pictogram" that illustrates the operation of the atomic pipette.

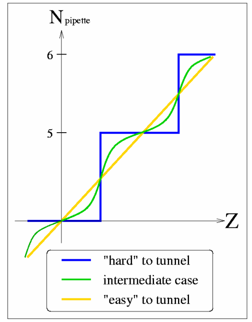

Fig. 4. A plot of the pipette occupation number vs. a tuning parameter *Z* (see Fig. 3 for an example of what *Z* can be) for three different regimes. The blue staircase is the hallmark of the strong form of the Bose interaction blockade.

Uploading an atomic pipette from a BEC reservoir

Now consider bringing a short-ranged potential, which is created in the focus of a detuned laser, up to the edge of a Bose Einstein condensate (BEC). The laser cannot be inserted *into* the BEC because this would destroy the superfluidity of the condensate and, ultimately, the condensate itself [1,4]. But the BEC is mechanically stable so the laser can be brought up near the edge of the condensate. If the laser is tuned so that the short-ranged potential develops an energy level below the chemical potential of the BEC (as shown in Fig. 3), then it becomes **possible for particles to tunnel from the BEC into the short-ranged potential of the laser**. This allows one to upload atoms from the BEC into the short-ranged potential. Thus, in a very real sense, the short-ranged potential acts as an **atomic pipette** that can suck up atoms from a reservoir. Once the atoms are loaded onto the pipette, they can be controlled by simply moving the laser around.

The way that this uploading of atoms occurs is dictated by energy concerns, which is why we needed to accurately calculate the energy of the short-ranged potential as described in Section II. It is easy to picture how the process works though by simply considering the "pictogram" of the pipette shown in Fig. 3. If one tunes the depth of the short-ranged potential (which can be done in practice by simply tuning the power of the laser), then one will tune how many atoms the pipette wants to accept in order to minimize its energy. The upshot of the tuning is that the minimum of the *E(N)* curve shown in Fig. 2 is being moved around.

As an example, let us say that the pipette has five atoms occupying it, as represented schematically in Fig. 3. Now imagine that we tune the parameter *Z* by making the pipette potential deeper. For a while, nothing will happen as the state with five atoms on the pipette ($N_{pipette}=5$) remains lower in energy than the state with six atoms ($N_{pipette}=6$). But at some point as we continue to make the pipette potential deeper, the *condensation energy gain* to add the sixth particle exceeds the *interaction energy cost*, and a sixth particle will be accepted. This process of tuning *Z* and seeing the occupation number of the pipette, $N_{pipette}$, suddenly jump up by one particle is exactly the process illustrated by the **blue staircase in Fig. 4**.

References

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[2] E.H. Lieb, J.P. Solovej, J. Yngvason, Phys. Rev. Lett. **69**, 749 (1992).
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[4] R.B. Deiner, B. Wu, M.G. Raizen, and Q. Niu, Phys Rev. Lett. **89**, 070401 (2002). This is a superficially similar proposal with both practical and conceptual flaws which we address in [1].

A staircase vs. a ramp: the role of tunneling

If the pipette could select exactly the number of particles that minimized its energy for a given set of parameters, then the blue staircase of Fig. 4 would be the end of the story. This staircase is the **strong form of the Bose interaction blockade** where the interaction cost of adding an extra particle restricts the pipette occupation number to be an integer. In practice, tunneling events, where particles tunnel frequently from the BEC to the pipette and vice versa, can make the occupation number of the pipette a continuous variable. In this case one no longer sees an integer number of particles on the pipette; one sees instead the time-averaged number of particles. This scenario where tunneling events occur frequently is the **washed-out form of the Bose interaction blockade** in Fig. 4. Whether or not you have the strong form or the washed-out form of the blockade depends on how easy it is to tunnel between the BEC and the pipette. Basically one can think of the strong form as occurring when the pipette is farther away from the BEC and it is difficult to tunnel, and the washed-out form when the pipette is closer to the BEC and it is easy to tunnel. Also indicated in Fig. 4 in **green** is a case somewhere between the two extreme versions.

How low can you go? & Conclusions

An obvious question relating to the real-world three-dimensional version of the pipette: Is it possible to tune the pipette so that both regimes, where you have either a finite number of particles in the pipette or a single particle in the pipette (the two regimes highlighted in Fig. 2), are possible? As it turns out, one of the two regimes will be much easier to achieve than the other. To have a finite number of particles in the pipette (say, of order of tens or hundreds) is easy compared to having a single particle. Experimentally it is possible to tune the atom-atom interaction strength (this corresponds to tuning either ν_{pair} or *g* above) using a magnetic field, as well as being able to tune the depth of the pipette potential. This may make it possible to approach the case where one can build an atomic pipette that holds a single atom in place, but it will not be an easy thing to do. This is interesting because the control of a single atom is something that people would like to be able to accomplish for applications like **quantum computing**. In any case, the atomic pipette containing a finite number of particles and the physics of the Bose interaction blockade should be experimentally realizable and we hope that experiments will be able to achieve these effects in the not-so-distant future.