Spin Crisis and Angular Momentum in Hadrons

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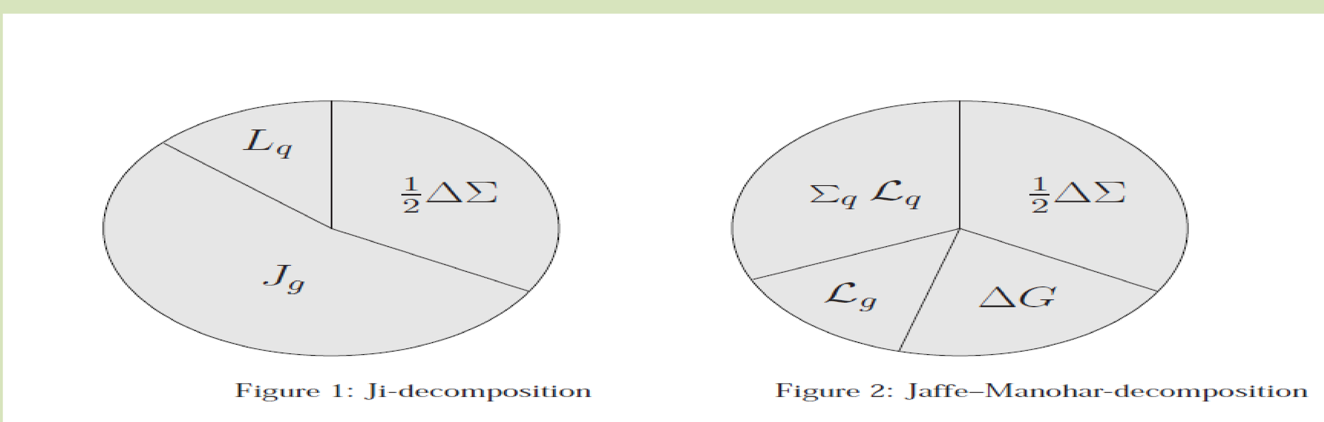
Introduction

The "spin" of the proton is composed of the total angular momentum of its constituents, i.e. quarks and gluons. The spin of the quarks accounts for only about 30% of the total spin of the proton. The rest, presumably, is somehow expressible in terms of the OAM of quarks and the OAM and spin of the gluons. If one could decompose the spin of the proton into these constituents' individual contributions, one could gain further insight into the proton structure. There are two decompositions in the literature, one due to Jaffe and the other due to Ji. They are both given below (Burkardt, 1999).



Of course, for a physically measurable quantity to be meaningful, it should be gauge-invariant. In either decomposition above, the quark spin contribution is the same and gauge-invariant but the OAM of the quark differs. Further, in Ji's decomposition, the gluon contribution to the proton spin cannot be meaningfully broken into spin and orbital components. Finally, the orbital contributions in Jaffe's decomposition are *not* gauge-invariant but the gluon spin is physically measurable in a meaningful way..

Brief GPD Aside: Caricature Sketch

Deeply Virtual Compton Scattering

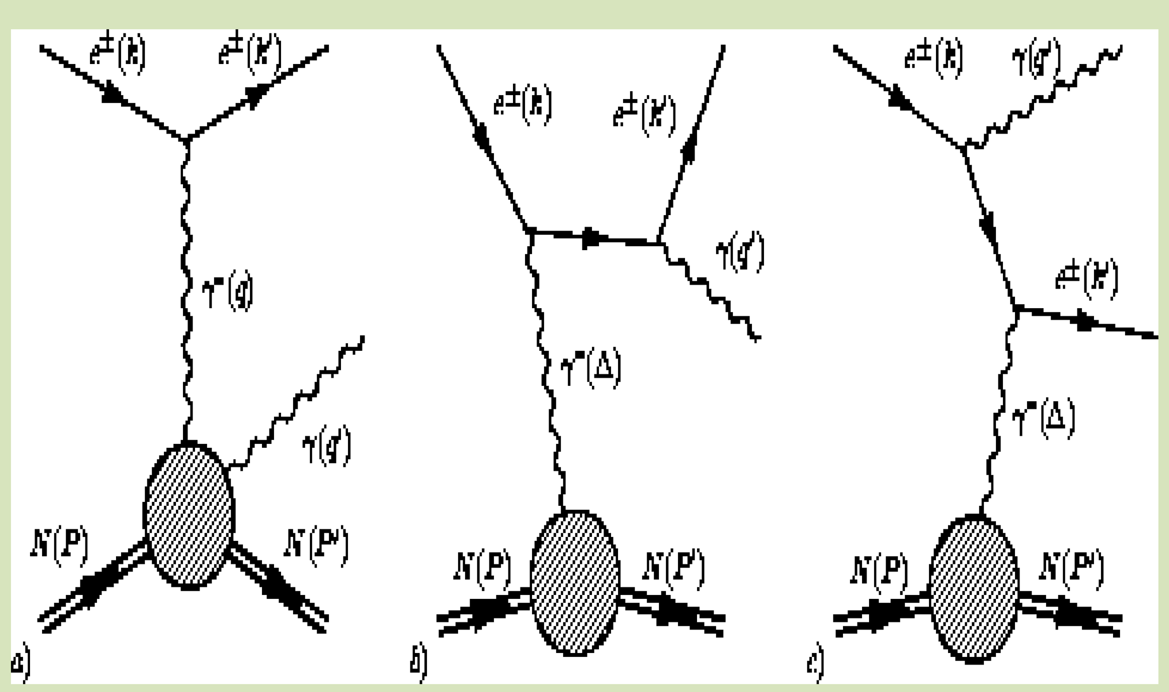


Fig. 1: Leading contributions to Deeply Virtual Compton Scattering. Making use of the factorization theorem of QCD one can express the soft proton-quark amplitude (imagine bottom part of blobs) in terms of GPDs, to be connected to the hard electromagnetic quark scattering off photons (top part of blobs) (www.j-lab.org images, 2000).

Ji's Decomposition of Angular Momentum

The energy-momentum tensor of QCD, whose zeroth components are obtained by invariance under translations, and whose cross products with x are the total angular momentum operator (quantities manifestly conserved under rotations) is:

$$T^{uv} = T_g^{\ uv} + T_q^{\ uv},$$

$$T_q^{\mu\nu} = \frac{1}{2}\left[\overline{\Psi}\gamma^{(\mu} i\vec{D}^{\nu)}\Psi + \overline{\Psi}\gamma^{(\mu} i\overleftarrow{D}^{\nu)}\Psi\right]$$

for quarks; and,

$$T_g^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^2 - F^{u\alpha} F^{\nu}{}_{\alpha}$$

for gluons as per Ji's decomposition (Physics Review Letters, 1997).

The angular momentum operator is analogously given by

$$J = J_q + J_g,$$

where the quark and gluon part are separately given by

$$J^i_{q,g} = \frac{1}{2}\varepsilon^{ijk}\int d^3x\,(T^{0k}_{q,g}x^j - T^{0j}_{q,g}x^k).$$

Specifically, $< P \mid J \mid P >= \frac{1}{2}.$

Form Factor Decomposition

This energy-momentum tensor is decomposed into form factors for the proton that involve all possible Lorentz structures allowed by various restrictions including Lorentz Invariance, parity Conservation, tensor symmetry , time-reversal symmetry etc.

$$\begin{aligned}\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \overline{U}(P')[&A_{q,g}(\Delta^2)\gamma^{(\mu}\overline{P}^{\nu)} \\ &+ B_{q,g}(\Delta^2)\overline{P}^{(\mu} i\sigma^{\nu)\alpha}\Delta_\alpha/2M \\ &+ C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M \\ &+ \overline{C}_{q,g}(\Delta^2)g^{\mu\nu}M U(P),\end{aligned}$$

(Ji, Phys Rev 78, 4, 1997)

When *P=P'* (or $\Delta^2 = 0$), we obtain expectation values of the energy-momentum tensor.

Proton Sum Rule

These form factors are used to express expectation values of the **angular momentum tensor.** We obtain "spin sum rules" when these expectation values are expressed in terms of the very same GPDs as in DVCS. This can be done since the energy-momentum tensor can also be written in terms of GPDs, which involve matrix elements of similar operators.

For the proton, setting $\Delta^2 = 0$ we obtain the sum rule

$$\int_{-1}^{1} dxx\{H(x,\xi,0\ ) + E(x,\xi,0)] = A(0) + B(0),$$

where $A(0) = A_q(0) + A_g(0)$

and $B(0) = B_q(0) + B_g(0).$

Generalization to Other Bound States

This formalism can be generalized to other bound states in QCD as well. The authors of the currently reviewed paper (Taneja et al, 2011) considered the deuteron, a spin-1 hadronic bound state. The formalism will be extended to spin-0 (pion) as well. The framework is essentially the same– the difference lies in relating the form factors of the energy-momentum tensor to the GPDs specific to the target and its spin degrees of freedom.

As an example, here is the result of a similar calculation, but for the spin-1 bound state of the deuteron. There are many more GPDs here since there are more independent spin degrees of freedom and thus more independent scattering cross-sections involved than for the proton. However, only one of the GPDs contributes, as shown below.

$$< P_{deut} \mid J \mid P_{deut} >= \int_{-1}^{1} dx\{H_2(x,\xi,0\ )x\}.$$

Measurement Suggestions: Transverse Asymmetry

The GPD above can be accessed experimentally via the transverse target asymmetry A_{UT}.

$$A_{UT} \sim \mathrm{Im}\{H_1^* H_5 + (H^*_1 + \frac{1}{6}H^*_5)(H_2 - H_4)\}.$$

Final Note: The quadrupole moment of the deuteron is also being considered, since it has one unlike the spin-1/2 proton. It is treated and constructed analogously, in operator terms, to the dipole moment or the angular momentum above, considering intrinsic and orbital parts .