Introduction

- Generation of massively entangled states is of great interest for quantum information. For quantum communication, multiparty quantum teleportation and quantum secret sharing are good examples.
- Here in order to generate multipartite entangled state we consider simplest possible case of an optical parametric oscillator (OPO) with a single, nondegenerate nonlinear interaction.

Hamiltonian of such a simple OPO when it is operating well above oscillation threshold needs to be written in three-wave mixing interaction as:

\[ H = i\hbar(2\chi) \sum_n p^n a_n a_{-n} - p_n a^\dagger_n a_n \]

Based on input-output theory heisenberg equations for modes inside the cavity can be written as:

- For input:
  \[ \dot{a}_i = 2\chi p_a a_i + 2\chi p_n a_{-i} - 2\chi p_{a^\dagger} a_{i^\dagger} - 2\chi p_{a^\dagger} a_{i^\dagger} \]

- For output:
  \[ \dot{a}_o = 2\chi p_{a^\dagger} a_{o^\dagger} + 2\chi p_{a^\dagger} a_{o^\dagger} \]

\[ A_o = \frac{p_{a^\dagger} a_{o^\dagger}}{\gamma + 2\chi p_{a^\dagger} a_{o^\dagger}} \]

\[ P_{a^\dagger} = \frac{p_{a^\dagger} a_{o^\dagger}}{\gamma + 2\chi p_{a^\dagger} a_{o^\dagger}} \]

Squeezing terms

- Near oscillation threshold squeezing terms for the output modes of the OPO are equivalent to squeezing terms for EPR state:
  \[ Q_n = 0, P_n \to 0 \]
  \[ Q_n + P_n \to \infty \]
  \[ \alpha_n (P_n + P_{-n}) - \alpha (P_n + P_{-n}) \to 0 \]

- However increasing the relative input pump power increases the variance of phase sum operator from zero. It means in the OPO operating well above threshold phase sum operator is not perfectly squeezed.

\[ \Delta^2 \sum_n (P_n + P_{-n}) \]

\[ \Delta^2 \sum_n (P_n + P_{-n}) - \Delta^2 \sum_n (P_n + P_{-n}) \to 0 \]

- Here we compared the variance of phase sum operator to the phase fluctuations of signal and idler modes are correlated to the phase fluctuations of pump mode.

van Loock Furusawa inequality

The van Loock-Furusawa (VLF) separability criteria provides necessary condition for separability of density operator.

- The density operator for the partially separable state can be written as:
  \[ \hat{\rho} = \sum_k \rho_k \otimes \rho_{n+1,k} \]

- Defining:
  \[ x = (\Delta \mu)^2 + \frac{1}{2} (\Delta \beta)^2 \]
  \[ y = \Delta \kappa \]
  \[ v = \Delta \delta \]

- necessary condition for this kind of separability is:
  \[ (\Delta \mu)^2 + (\Delta \beta)^2 \geq 4 |\Delta \kappa| + \Delta \delta \]

Evaluating the VLF criteria

- Evaluating van Loock-Furusawa separability criteria on all possible partitions on modes and violating them can verify the multiparticle nature of the entanglement.

\[ S_1 = (\Delta^4 \sum_n (Q_n + Q_{-n}) + \Delta^4 \sum_m (P_m + P_{-m}) - \Delta^4 \sum_n (P_n + P_{-n}) ) \geq 8n \]

Violation of \( S_1 \)

\[ S_1 - 8n \]

\[ S_2 - 8(n - 1) \]

Inseparability of \( (a_i - a_j) \) pair from rest

The necessary separability condition for separability of \( (a_i - a_j) \) mode pair from rest is:

\[ S_2 = |\Delta^4 \sum_n (Q_n + Q_{-n}) + \Delta^4 \sum_m (P_m + P_{-m}) - \Delta^4 \sum_n (P_n + P_{-n}) ) | \geq 8(n - 1) \]

Violation of \( S_2 \)

\[ S_2 - 8(n - 1) \]

Conclusion

We theoretically analyzed the generation of multiparticle continuous-variable entanglement in a single optical parametric oscillator (OPO) operating well above threshold. We verified the multiparticle nature of the entanglement by evaluating the van Loock-Furusawa separability criteria on all possible partitions on modes and violating them.