# Real-Time Finite-Temperature Holography and its Applications

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## Introduction : Failure of Traditional Method

It is widely believed that our Universe was born in a Big Bang around 13.7 billion years ago, and microseconds after that, before the formation of hadrons, there existed a state of free flowing liquid of de-confined quarks and gluons, usually referred to as Quark-Gluon Plasma (QGP). Recent experiments at the Relativistic Heavy Ion Collider (RHIC) were able to reproduce such a state of matter at an extremely high temperature of 4 trillion degrees Kelvin by colliding high energy gold ions. More measurements revealed that the QGP produced at RHIC is not a gas, as initially anticipated, but rather a nearly perfect liquid because the constituent quarks and gluons interact strongly with each others.

The study of this QGP offers us a good opportunity to understand the fundamental structures of matter as well as the early Universe. However, it poses a huge challenge to theorists. In Quantum Chromo-dynamics (QCD), the traditional perturbative method of computing Feynman diagrams works well only in weak interaction regime, but breaks down in strong interaction regime, where the QGP produced at RHIC lives. Thus a completely new approach must be developed to carry out analytical calculation for the gauge theory at its strong interaction regime.

## New Approach : Holography (String/Gauge Duality)

String theory describes the dynamics of strings living in a background of higher-dimensional curved spacetime. It was traditionally viewed as a quantum theory for gravity. In 1997, it was discovered that string theory can also serve as an efficient computational tool, alternative to the Feynman diagram approach, to carry out analytical calculations for a gauge theory like QCD, especially in its strong interaction regime! This is the so called *holography*, sometimes also referred to as *string/gauge duality* or *AdS/CFT correspondence*.

More specifically, holography states that, *a **strongly coupled** gauge theory living in a **flat** spacetime **mathematically** has a holographic dual description in the guise of a **weakly coupled** string theory [or its low energy limit supergravity (SUGRA)] in a **higher dimensional curved** spacetime background, with the gauge theory living **on its boundary**.*

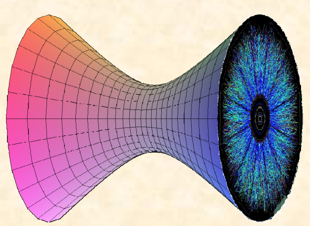

*Fig.1 An illustration to holography: hyperboloid denotes the curved background of string theory; its flat boundary, seen here as a collision event at the RHIC, is where the gauge theory lives*

There is a one-to-one mapping of everything in gauge theory to its counterpart in the dual string theory. When calculating a quantity in a gauge theory, instead of doing it directly in the gauge theory, an alternative calculation can be done in its dual string theory and be converted back to the gauge theory using this mapping. The most promising feature of this duality is that, when the gauge theory is in its strong coupling regime, where the traditional perturbative approach fails, the dual string theory is in its weak coupling regime, where the perturbative approach works out perfectly. So the worst computational regime in a gauge theory is the easiest computational regime in its dual string theory!

## Review : What has NOT been done ?

Over the last decade since its discovery, many computations based on holography had been done to study various aspects of strongly-interacting gauge theories. However, there are still many gaps in the literature:

- Finite-temperature holography had been much less studied than zero temperature holography because of the complication of the dual black hole background.
- Most holographic computations were carried out in Euclidean background, dual to an imaginary time gauge theory. For real-time holography, only the simplest quantities – 2-point correlators had been previously studied, which allowed only a limited access to leading order transport coefficients in the hydrodynamics regime of QGP. No higher n-point correlators ($n \geqslant 3$) had yet been computed, so many interesting quantities were still not calculated via this holographic approach.
- An important application of real-time holography is the study of the evolution of energetic jet in the hot strongly-interacting QGP. Previous studies done using classical string configurations lacked a clear gauge theory setup. This makes it hard to link the theoretical results to experimental data.
- Quite recently holography had been extended to condensed matter to analytically study phenomena such as superconductivity, superfluidity and cold atoms, all involving strong coupling. Since this study is at its beginning, there are still many unanswered questions, some being very fundamental.

Our research is aimed to address all the four problems above, to refine the technique of holographic computation and to extend its applications to various interesting subjects in modern physics.

## Real-Time Finite-Temperature 3-Point Correlators

In real time at finite temperature, the dual SUGRA background is a five dimensional Anti-de Sitter (AdS)-Schwarzschild black hole:

$$ds^2 = \frac{(\pi TR)^2}{u}\left[-f(u)dt^2 + dx^2 + dy^2 + dz^2\right] + \frac{R^2}{4uf(u)}du^2 \qquad (1)$$

where $f(u) = 1 - u^2$ and $u = 0, 1, \infty$ are the boundary, horizon and singularity, respectively. The global structure of this black hole is shown in Fig.2. It has four quadrant: past (P) and future (F) are inside the horizon; left (L) and right (R) are outside the horizon. The SUGRA fields live in the whole space (bulk). Their solutions to the classical EOMs are boundary-to-bulk propagators, and their boundary values and the classical on-shell SUGRA action are mapped to sources and generating functional of the gauge theory in the following way:

$$\begin{cases} \bar{\phi}(x) = J(x) \\ iS_{cl}[\bar{\phi}] = \ln Z[J] \end{cases} \qquad (2)$$

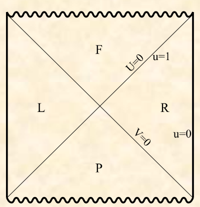


*Fig.2 Penrose diagram of AdS-Schwarzschild black hole*

Starting here, we built up a complete prescription for calculating real-time n-point correlators [1]. We proposed the SUGRA action is only integrated over R- and L-quadrants with a relative minus sign, and the boundary fields living on R- and L-boundaries are identified with type 1 and 2 sources of Schwinger-Keldysh formalism. This amounts to integrating only the R-quadrant, then adapting the finite temperature analog of Veltman's circling rules to gravity tree-level diagrams to calculate the real-time correlators, with the R- and L-boundary fields identified with circled and un-circled operators. Thus we constructed a complete mapping between all types of real-time field theory correlators (Feynman, causal, Wightman etc) and their SUGRA dual. We subjected our prescription to several checks: Kubo-Martin-Schwinger (KMS) relations, largest time identity, analyticity and causality considerations, and zero-temperature limit.

For the first time, we gave concrete formulas for all real-time 3-pt functions [1]. Of particular interest, the causal 3-pt function in momentum space has the following general form:

$$G_R(p_1,p_2,p_3) \sim \delta^{(4)}(p_1+p_2+p_3)\int_0^1 du\sqrt{-g}\,F(p_1,u)F(p_2,u)F^*(p_3,u) \qquad (3)$$

where $F(p,u)$'s are the incoming wave solutions to the (linearized) SUGRA EOM, and they are indentified with the retarded boundary-to-bulk propagators. The structure is simple: three causal boundary-to-bulk propagators meeting at a vertex point in the bulk, which is integrated only up to the horizon in R-quadrant (Fig. 3). It agrees with causality considerations and analytic continuation of the imaginary-time calculation. This is the starting point of our follow-up research.

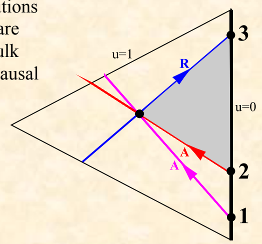


*Fig.3 Structure of causal 3-pt function in position space, integration only over the shadowed area in R-quadrant.*

This part of our research addresses the first two problems in review section.

## Application (1) : Jet Quenching in Hot QGP

This research is to study the third problem in review section. A refined approach based on holography was developed [3,4], specifically using our causal 3-point function result, to study how a high energy jet (R-current) deposits charges in a strongly-coupled hot $\mathcal{N}=4$ SYM QGP. Charge deposit function was computed (Fig. 4) and a new scale that characterizes how far the jet can travel was discovered, proportional to $E^{1/4}/T$, where E is the energy of the jet and T is the temperature of QGP. This scale did not appear in the previous studies, which lacked a clear gauge theory setup.

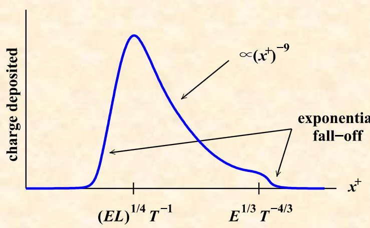


*Fig.4 The deposition of charges as a function of light-cone coordinate $x^+ = t + z$, with the momentum along $z$-direction*

We next generalized this calculation to the case where the QGP has a finite chemical potential. On the SUGRA side this is a more challenging enterprise because now: (a) the background is an AdS-Reissner-Nordström black hole, significantly more complicated than AdS-Schwarzschild black hole; (b) the metric fluctuations highly couple with the vector field fluctuations, which source the R-current in the dual gauge theory. These two facts lead to a set of very complicated EOMs and thus more complex correlators. We have managed to solve them and the research is near its conclusion [5]. We found the stopping distance has a similar $E^{1/4}/T$ dependence, dressed by a factor involving $\zeta$, which is related to dimensionless chemical potential $\mu/T$:

$$l_{stop} \sim \frac{2-\zeta}{(1+\zeta)^{1/4}}\frac{E^{1/4}}{T} \qquad (4)$$

## Application (2) : 2nd Order Hydrodynamic Coefficients

Hydrodynamic coefficients of QGP can be computed from causal n-point correlators of energy-stress tensor using Kubo formulae. We recently refined and extended the work of [7] by solving equations of energy-stress tensor conservation (i.e. Ward identity) rigorously in the hydrodynamic regime to 2nd order in fluctuations (sources), and obtained a set of Kubo formulae for 2nd order hydrodynamic coefficients [6], notably:

$$\lambda_3 = -4\lim_{p_1,p_2\to 0}\frac{\partial^2}{\partial p_1\partial p_2}G_{raa}^{xy,tx,ty}(p_1,p_2)\Big|_{E_1,E_2=0} \qquad (5)$$

In dual SUGRA energy-stress tensor is sourced by metric fluctuations at the boundary. So we compute the generating functional by calculating on-shell Einstein-Hilbert action, plus Gibbons-Hawking term and counter terms, using our real-time holographic prescription, and then obtain the causal 3-point correlators of energy-stress tensor. This has partially been done [6] and the rest is being worked out intensively. For example, we have found:

$$G_{raa}^{xy,tx,ty}(p_1,p_2)\Big|_{E_1,E_2=0} = -\frac{\pi^2N^2T^4}{8} + \frac{N^2T^2}{16}\left(p_1^2+p_2^2\right) + O\left(p^3\right) \qquad (6)$$

Thus from Eq.(5) we obtain the vorticity coefficient $\lambda_3$:

$$\lambda_3 = 0 \qquad (7)$$

This agrees with previous result in the literature obtained from other method. We are currently computing other 3-pt functions to obtain $\lambda_1$ and $\lambda_2$.

## Application (3) : Extension to Condensed Matter

This subject is to study the last problem mentioned in the review section. We concentrated on a non-relativistic scale-invariant field theory (usually referred to as Schrödinger field theory). We generalized our prescription for real-time relativistic holography to this non-relativistic case. For zero-temperature, we computed the Feynman, retarded and Wightman scalar correlators with arbitrary conformal dimension in two independent ways: our real-time holography prescription and light-like Fourier transform of relativistic correlators (similar to Discrete Light-Cone Quantization), and we found that they agree and satisfy Källén-Lehmann relations. We then computed finite-temperature Feynman and retarded 2- and 3-point scalar correlators with arbitrary conformal dimension via holography [2].

## References & List of Publications

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[3] ***Jet quenching in hot strongly coupled gauge theories revisited: 3-pt correlators with gauge-gravity duality***
P. Arnold & D. Vaman. JHEP 1010:099, 2010; arXiv:1008.4023 [hep-th]

[4] ***Jet quenching in hot strongly coupled gauge theories simplified***
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[6] ***2nd order hydrodynamic coefficients of hot $\mathcal{N}=4$ super Yang-Mills plasma from supergravity correlators***
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[7] ***Kubo formulae for 2nd order hydrodynamic coefficients***
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