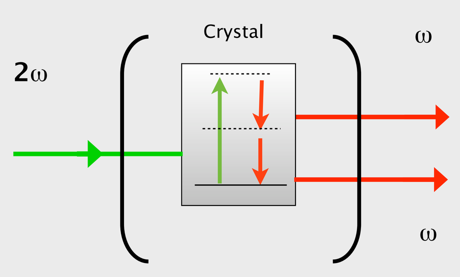
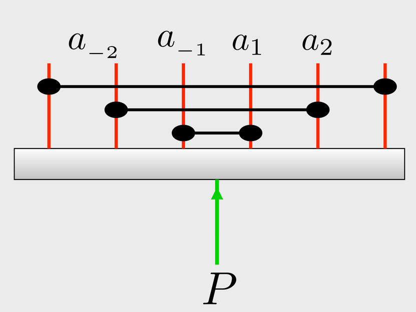
Multipartite entanglement in the optical frequency comb of a depleted-pump optical parametric oscillator

Reihaneh Shahrokhshahi , Olivier Pfister University of Virginia

Introduction

- Generation of massively entangled states is of great interest for quantum information. For quantum communication, multiparty quantum teleportation and quantum secret sharing are good examples.
- Here in order to generate multipartite entangled state we consider simplest possible case of an optical parametric oscillator (OPO) with a single, nondegenerate nonlinear interaction.



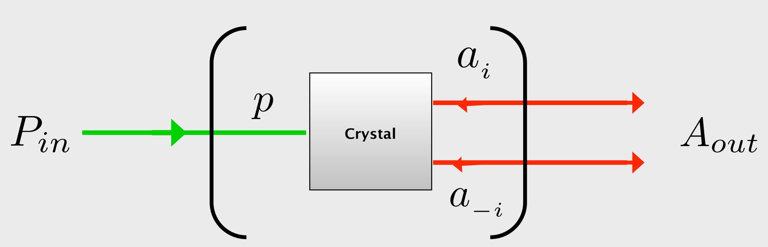


- Hamiltonian of such a simple OPO when it is operating well above oscillation threshold needs to be written in three-wave mixing interaction as.:

$$H = i\hbar(2\chi)\sum_i^n p^\dagger a_i a_{-i} - p a_i^\dagger a_{-i}^\dagger$$

Based on input-output theory heisenberg equations for modes inside the cavity can be written as:

$$\dot{a_i} = 2\chi^{(2)} p a_{-i}^\dagger - k_a a_i + \sqrt{2k_a} A_{i_{in}}$$
$$\dot{a_{-i}} = 2\chi^{(2)} p a_i^\dagger - k_b a_{-i} + \sqrt{2k_a} A_{-i_{in}}$$
$$\dot{p} = -2\chi^{(2)}\sum_i^n a_i a_{-i} - k_p p + \sqrt{2k_p} p_{in}$$



We solved heisenberg equations by linearizing them in the vicinity of classical mean values.

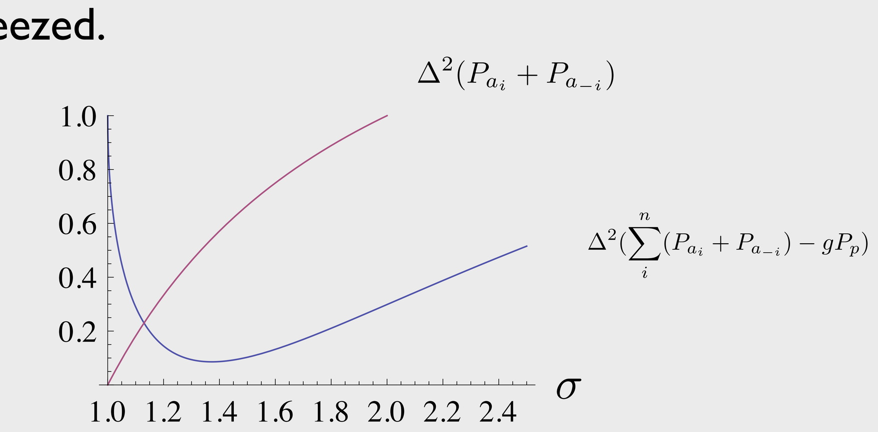
$$a_i = \alpha_i + \delta a_i \qquad a_{-i} = \beta_i + \delta a_{-i}$$
$$p = \gamma + \delta p \qquad p_{in} = \gamma_{in} + \delta p_{in}$$
$$A_{i_{in}} = 0 + \delta A_{i_{in}} \qquad A_{-i_{in}} = 0 + \delta A_{-i_{in}}$$

Squeezing terms

- Near oscillation threshold squeezing terms for the output modes of the OPO are equivalent to squeezing terms for EPR state :

$$Q_{a_i} - Q_{a_{-i}} \longrightarrow 0 \qquad P_{a_i} + P_{a_{-i}} \longrightarrow 0$$
$$P_{a_i} - P_{a_{-i}} \longrightarrow \infty \qquad Q_{a_i} + Q_{a_{-i}} \longrightarrow \infty$$
$$\alpha_i(P_{a_j} + P_{a_{-j}}) - \alpha_j(P_{a_i} + P_{a_{-j}}) \longrightarrow 0$$

- However increasing the relative input pump power increases the variance of phase sum operator from zero. It means in the OPO operating well above threshold phase sum operator is not perfectly squeezed.



- Here we compared the variance of phase sum operator to $\Delta^2(\sum_i^n(P_{a_i} + P_{a_{-i}}) - gP_p)$.

This comparison demonstrates the fact that for the OPO operating well above threshold phase fluctuations of signal and Idler modes are correlated to the phase fluctuations of pump mode.

van Loock Furusawa inequality

The van Loock-Furusawa (VLF) separability criteria provides necessary condition for separability of density operator.

- The density operator for the partially separable state can be written as

$$\hat{\rho} = \sum_i^n \eta_i \hat{\rho}_{i,1m} \otimes \hat{\rho}_{i,m+1,n}$$

- Defining

$$u = h_1q_1 + h_2q_2 + \ldots\ldots + h_nq_n$$
$$v = g_1p_1 + g_2p_2 + \ldots\ldots + g_np_n$$

- necessary condition for this kind of Separability is :

$$\langle(\Delta u)^2\rangle_\rho + \langle(\Delta v)^2\rangle_\rho \geq 4(|h_1g_1 + \ldots + h_mg_m| + |h_{m+1}g_{m+1} + \ldots + h_ng_n|)$$

Evaluating the VLF criteria

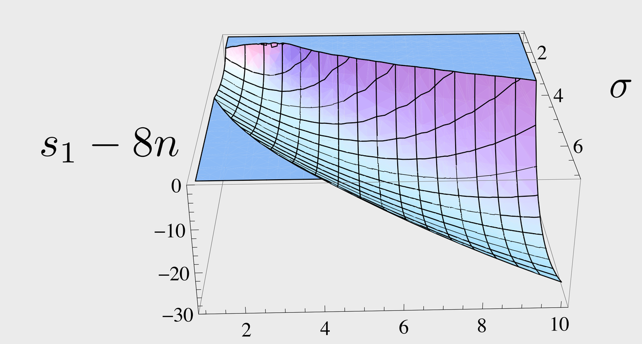
Evaluating the van Loock-Furusawa separability criteria on all possible partitions on modes and violating them can verify the multipartite nature of the entanglement.

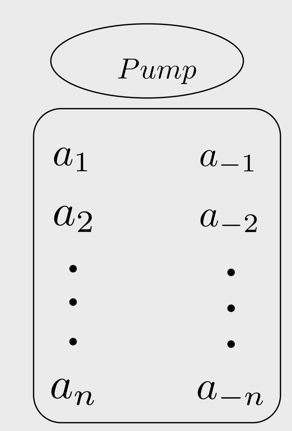
Violation of the van Loock-Furusawa inequality to show pump is entangled with pairs.

The necessary separability condition for separability of pump mode from rest is :

$$S_1 = \langle\Delta^2(\sum_i^n \alpha_i(Q_{a_i} + Q_{a_{-i}}) + \frac{2}{g}\sum_i^n \alpha_i Q_p)\rangle + \langle\Delta^2(\sum_i^n(P_{a_i} + P_{a_{-i}}) - gP_p)\rangle \geq 8n$$

Violation of S_1





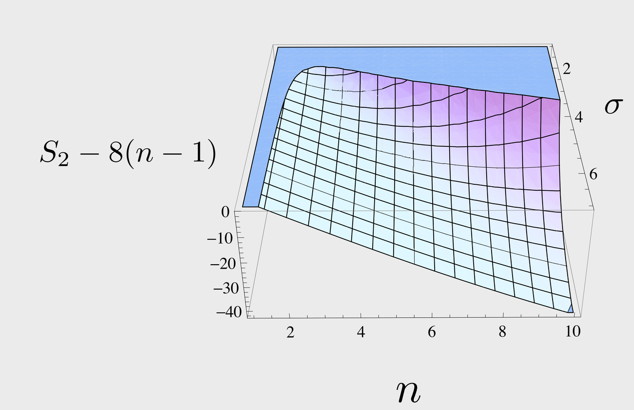
σ = Pump power relative to oscillation threshold

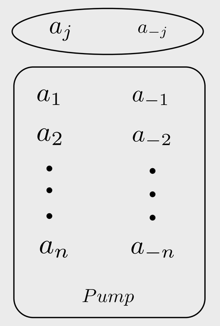
Inseparability of $(a_j - a_{-j})$ pair from rest

The necessary separability condition for separability of $(a_j - a_{-j})$ mode pair from rest is :

$$S_2 = \langle\Delta^2(\alpha_j Q_{+_j} + \sum_{i\neq j}^n \alpha_i Q_{+_i} + \frac{2}{g}\sum_i^n \alpha_i Q_p)\rangle + \langle\Delta^2(\sum_{i\neq j}^n(\alpha_i P_{+_j} - \alpha_j P_{+_i}))\rangle \geq 8(n-1)$$

Violation of S_2





Inseparability of k pairs from rest of n-k pairs and pump

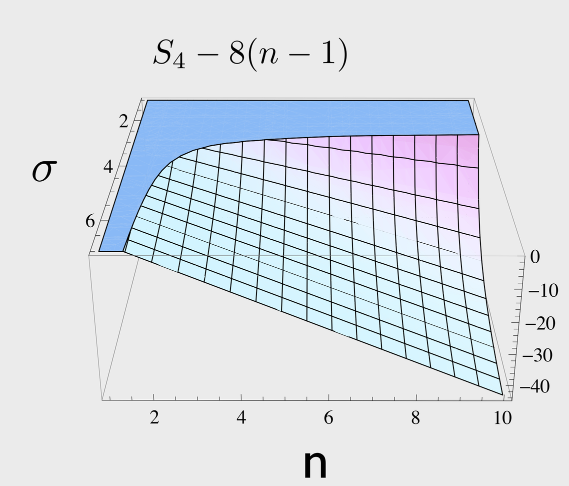
The necessary separability condition for such a partition on modes is :

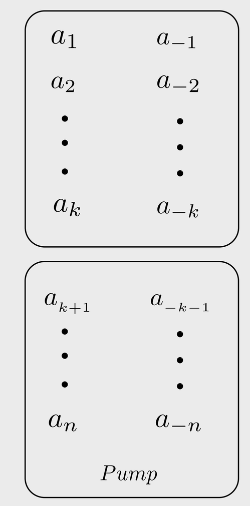
$$S_4 = \langle\Delta^2(\sum_i^k \alpha_i Q_{+_i} + \sum_{j=k+1}^n \alpha_j Q_{+_j})\rangle + \langle\Delta^2(\sum_{i=1}^k\sum_{j\neq i}^n(\alpha_j P_{+_i} - \alpha_i P_{+_j}))\rangle \geq 8k(n-k)$$

- Since

$$8(n-1) \leq 8k(n-k) \leq 2n^2$$

- $8(n-1)$ is a lower limit which needs to be violated





For partition on modes when mode a_j is in one part and mode a_{-j} is in other part the necessary separation criteria is :

$$S_3 = \langle\Delta^2(\alpha_j Q_{a_j} - \alpha_j Q_{a_{-j}} + \sum_{i\neq j}^n \alpha_i Q_{a_i} - \alpha_i Q_{a_{-i}})\rangle + \langle\Delta^2(\sum_{i\neq j}^n(\alpha_j P_{a_i} + \alpha_j P_{a_{-i}} - \alpha_i P_{a_j} - \alpha_i P_{a_{-j}}))\rangle \geq 4$$

Left hand side of this inequality is always equal to zero so it is violated for all relative input pump powers.

Conclusion

We theoretically analyzed the generation of multipartite continuous-variable entanglement in a single optical parametric oscillator (OPO) operating well above threshold. we verified the multipartite nature of the entanglement by evaluating the van Loock-Furusawa separability criteria on all possible partitions on modes and violating them.

Ref 1. S.Reynaud, C.Fabre, E.Giacobino J.Opt. Soc.Am.B/Vol 4, No. 10 /October1987
Ref 2. A.Villar, M.Martinelli, C.Fabre, P. Nussenzveig PRL 97, 140504 (2006)