Solutions

Print name______________________________
ID number______________________________
Section in which you are registered (sec 1 = 9 AM lec, sec 2 = 10 AM lec)__________

Physics 142E, Test No. 2, test session 1
March 24, 2004, 5:30-7:00 PM

On the bubble sheet, fill in your student id number, and in addition write your name and your section in the appropriate spot. After you have found an answer, fill in the appropriate bubbles on your bubble sheet—note any special instructions on this point in the problem itself. The last problem is a multipart problem whose solution should be written out neatly. You can get partial credit for this problem, so make sure that your answers are written out and contain no ambiguities.

No notes or books are allowed during the exam, nor is any consultation with anyone but me. You should be taking the 5:30 exam if you are in section 1 (9 AM lecture) and the 7:30 exam if you are in section 2 (10 AM lecture).

Write out an authorized form of the pledge here, and sign it.

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Signed________________________

1. (10) An object of mass M moves through outer space with speed V. It separates into two equal parts such that both pieces move in the same direction as the original object. The speed of one of the pieces is $V/3$. What is the speed of the second?
(a) $V/3$
(b) $V/2$
(c) $2V$
*(d) $5V/3$
(e) V
Soln (d) If v is the unknown speed, conservation of momentum reads (all velocities are positive)

$$MV = (M/2)(V/3) + (M/2)v$$

with soln $v = 2(V - V/6) = (5/3)V$.

2. (10) A man of mass M is standing on the left end of a uniform plank of mass $M/3$ and length L. The plank is on frictionless ice. The man walks on the plank to get to its right end. How far has he moved *relative to the ice*? [Hint: Think about what happens to the center of mass.]
(a) $L/2$
(b) $L/3$
*(c) $L/4$
(d) L
(e) cannot be determined.
Soln (c) The center of mass of the entire system doesn't move – all the forces involved in the walk from one end to the other are internal. Thus proceed by calculating the cm before and after. Set up a coord system fixed to the ice with the plank along x and the initial position of the man at the origin. The center of mass of the plank alone is at its center, a displacement $+L/2$ from the man at the beginning and $-L/2$ from the man at the end. Let the final x-position of the man be at x_f. Then the cm position of the system is at

$$\text{before} \quad X_i = \frac{0\times M + (L/2)(M/3)}{M + (M/3)} = \frac{\frac{L}{6}}{4/3} = \frac{3}{4}\frac{L}{6} = \frac{L}{8}$$

$$\text{after} \quad X_f = \frac{x_f \times M + (x_f - L/2)(M/3)}{M + (M/3)} = \frac{x_f\left(\frac{4}{3}\right) - \frac{L}{6}}{4/3} = x_f - \frac{L}{8}$$

Set these equal: $L/8 = x_f - L/8$, or $x_f = L/4$.

3. (10) We observed in the study of completely inelastic collisions (two point objects coalesce into one) in one dimension that the velocity of the final coalesced object is determined if the initial masses and velocities are known. Is this statement also true if the motion is not limited to one dimension but instead can occur in the full three-dimensional space?
(a) No.
*(b) Yes.
Soln. Yes. The cons of momentum equation, the only conservation law that applies here, consists of three equations, and that is enough to solve for the three components of the velocity of the coalesced object.

4. (10) A hollow cylinder and a solid cylinder, each of the same total mass and the same radius, roll down a ramp starting from rest and from the same initial position. Which one arrives at the bottom first?
(a) The hollow cylinder arrives first.
(b) You can't tell from the information given.
*(c) The solid cylinder arrives first.
(d) They arrive at the same time.
Soln. (c); with the larger rotational inertia the hollow cylinder lags. The ans is indep of mass and radius

5. (10) A particle moves freely, along a straight line, and at point A receives an impulse such that after the impulse the particle again moves freely, this time along a second straight line perpendicular to the first, as in the figure. The angular momentum of the particle with respect to point O remains constant throughout. the impulse is directed

(a) In the plane of the motion, along the second direction of motion.
(b) In the plane of the motion, directly away from point O.
*(c) In the plane of the motion, directly toward point O.
(d) out of the page.
(e) in a direction that cannot be determined from the information given.
Soln: (c) The impulse must be aligned along the line from the point where the trajectory turns, because that is the only way there is no angular impulse about O. To deflect the particle in the direction shown the impulse is towards O rather than away from it.

6. (10) A point object of mass $m = 2$ kg moves in one-dimension under the influence of a force for which the potential energy function has the form

$$U = \infty \text{ for } |x| > a.$$
$$U = 0 \text{ for } b < |x| < a.$$
$$U = 5 \text{ J for } |x| < b.$$

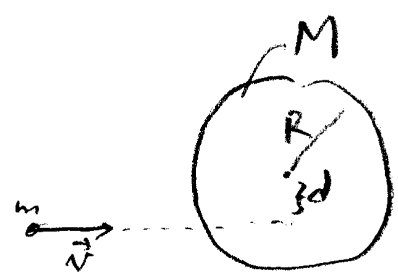
It is observed to be moving with speed $v = 3$ m/s in the region $-a < x < -b$. With what speed will it be moving at $x = 0$? [*Hint*: It helps to make a sketch of the potential energy function here.]
(a) Cannot be determined from the information given.
(b) 5 m/s
(c) 3 m/s
*(d) 2 m/s
(e) It is at rest in that region.
Soln. Cons of energy determines the result. In the region where it is observed to have $v = 3$ m/s, U is zero. We want the speed v'in the region where $U = 5$ J. Then cons of energy reads

$$\tfrac{1}{2} mv^2 = \tfrac{1}{2} mv'^2 + 5 \text{ J, or}$$
$$v'^2 = v^2 - (2/\text{m})(5 \text{ J}) = 9 \text{ m}^2/\text{s}^2 - 5 \text{ m}^2/\text{s}^2 = 4 \text{ m}^2/\text{s}^2,$$

or

$$v' = 2 \text{ m/s}.$$

7. (10) In the device shown for measuring the speed of a bullet, the bullet, of mass m, is fired so that it enters the edge of a stationary uniform solid disk as shown and does not pass through. The disk, which has mass $M >> m$ and radius R, is free to rotate about a fixed central axis. The disk is observed to rotate with angular velocity ω after the bullet hits it. The speed of the bullet is approximately



(a) $v = \dfrac{M}{m}\dfrac{R}{d}\omega$

(b) $\dfrac{1}{2}\dfrac{M}{m}R^2\omega$

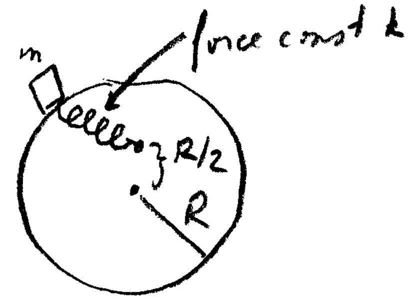
(c) ωR

(d) ωd

*(e) $\dfrac{1}{2d}\dfrac{M}{m}R^2\omega$

Solution. (e) If we consider the bullet and disk as a single system the forces are all internal and angular momentum is conserved. The initial angular momentum about the axis is due to the bullet alone and has magnitude $L_{initial} = mvd$. The final angular momentum about the axis is due to the rotating disk, and has magnitude given approximately ($M + m \cong M$) by $L_{final} = I\omega = ½\, MR^2\omega$, where we have used the result for the rotational inertia of a solid uniform disk about its axis. We set these equal and solve for v:

$$v = \frac{1}{2d}\frac{M}{m}R^2\omega$$



8. (30) A little toy car of mass m moves on the outside of a vertically oriented circular track of radius R. The forces acting on the car are the normal force of the track (a force that does no work on the car), gravity, and a spring that has one end attached to a point a distance $R/2$ directly above the center of the track. The spring has force constant k and has its relaxed length when the car is at the very top of the circle.

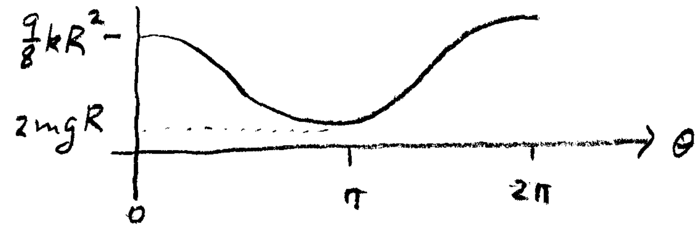
(a) (6) Which forces in this problem have potential energies associated with them, if any?
Answer: gravity and the spring force. Since the contact force from the track does no work, it is neither conservative nor nonconservative.

(b) (8) What is the expression for the total potential energy of the car? Use any variables that you think are relevant. Make sure you make it clear at what point(s) the potential energy terms have their zeros.
Answer: The PE assoc with grav is mgh, where h is the height above the ground level, so that this piece of the total PE is zero at ground level. There is also a PE assoc with the spring given by $½\, kx^2$, where x is the amount of stretch beyond $R/2$, the relaxed length. Thus we could write

$$U = mgh + ½\, kx^2.$$

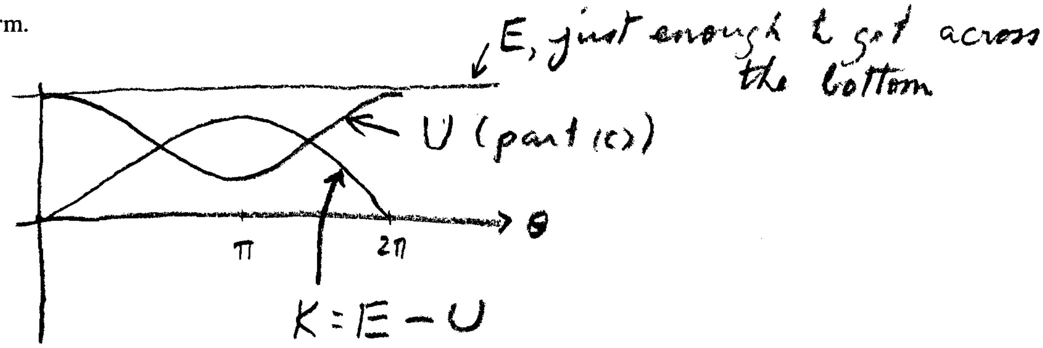
(c) (8) A convenient single variable that describes the position of the car around the track is the variable θ in the fig, which runs from 0 when the car is at the bottom of the track and increases as the car goes around counterclockwise. Assume that the spring is so strong that $kR >> mg$. *Sketch* the total potential energy as a function of θ. Use your knowledge of the PE at certain points around the circle to do so. I don't expect a mathematically exact curve here.

Answer. At the top, where $\theta = \pi$, there is no contribution from the spring, and $U = mgh = 2mgR$. At the bottom, where $\theta = 0$, there is no contribution from gravity, and $U = ½\, k(3R/2)^2 = 9/8\, kR^2$. This is much larger than the gravity term, and so the PE is a maximum at this point. We could suppose that the minimum potential energy occurs at the top.



(d) (8) Use your sketch of the potential energy as a function of θ or an algebraic expression to decide how much is the minimum kinetic energy the car would need at the top of the track to make it all the way around.

Answer in sketch form.



Algebraically, min K is ε greater than 0 at $\theta = 0$.
↑ tiny

Means $E_{min} = U_{bottom} + K_{bottom} = \frac{9}{8} kR^2$. ($K_{bottom} \to 0$)

Thus $K_{min,\,top} = E_{min} - U_{top} = \frac{9}{8} kR^2 - 2mgR$