Physics 152 Homework #7

Due Wednesday March 17, 2004

1. The differential equation for a damped oscillator is

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

(a) Construct a “leapfrog” Excel spreadsheet to solve this equation. (It turns out to be sufficiently accurate to have the damping term contribution at a time 0.5*delta_t behind the other terms.)

Put in your spreadsheet separate columns for the potential energy, the kinetic energy and the total energy.

(b) Put $m = k = 1$. Plot both position against time and total energy against time for low damping, critical damping and overdamping. Comment on the values of b you choose to illustrate these cases.

(c) For strong overdamping, magnify the initial movement (that is, take small delta_t) when the oscillator is released from a position away from the center. Measure the initial acceleration, and give a physical interpretation.

2. The differential equation for a driven undamped oscillator is

$$m\frac{d^2x}{dt^2} = -m\omega_0^2 x + F_0 \cos\omega t$$

where $\omega_0^2 = k/m$.

(a) Make a copy of your leapfrog spreadsheet from problem 1, and adapt it to solve this problem. Take $m = k = 1$ as before, and $x(t = 0) = 0$. Find a value of F_0 that gives a clear graph (showing several oscillations), and plot position against time and total energy against time for $\omega = \omega_0$ and for $\omega = 1.1\omega_0$. Interpret your energy graphs.

(b) Plot graphs for $\omega = 0.2\omega_0$ and for $\omega = 5\omega_0$. These graphs should show many oscillations of the fast frequency. Try to explain the graphs you come up with!

3. Copy your spreadsheet again, but now adapt it to solve the damped driven oscillator:

$$m\frac{d^2x}{dt^2} = -m\omega_0^2 x - b\frac{dx}{dt} + F_0 \cos\omega t$$

(a) For small damping, plot position against time and total energy against time for $\omega = \omega_0$ and for $\omega = 1.1\omega_0$. Interpret your energy graphs.

(b) For small damping, plot force, position and velocity against time on the same graph for $\omega = 0.2\omega_0$, $\omega = \omega_0$ and $\omega = 5\omega_0$. From your graphs, is the position in phase with the force in any of these situations? Is the velocity in phase with the source?

(c) The rate of working of the outside force is force × velocity. Construct a graph of the rate of working of the force as a function of time in these three situations.