Homework #8 Solutions

31.2 $\mathcal{E} = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{\Delta(\mathbf{B}\cdot\mathbf{A})}{\Delta t} = 1.60 \text{ mV}$ and $I_{\text{loop}} = \frac{\mathcal{E}}{R} = \frac{1.60 \text{ mV}}{2.00\ \Omega} = \boxed{0.800 \text{ mA}}$

31.3 $\mathcal{E} = -N \frac{\Delta BA \cos\theta}{\Delta t} = -NB \pi r^2 \left(\frac{\cos\theta_f - \cos\theta_i}{\Delta t}\right)$

$= -25.0\left(50.0\times10^{-6}\text{ T}\right)\pi\left(0.500\text{ m}\right)^2\left(\frac{\cos 180° - \cos 0}{0.200 \text{ s}}\right)$

$\text{E} = \boxed{+9.82 \text{ mV}}$

31.4 (a) $\mathcal{E} = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = \boxed{\frac{AB_{\max}}{\tau}e^{-t/\tau}}$

(b) $\mathcal{E} = \frac{(0.160 \text{ m}^2)(0.350 \text{ T})}{2.00 \text{ s}} e^{-4.00/2.00} = \boxed{3.79 \text{ mV}}$

(c) At $t = 0$, $\mathcal{E} = \boxed{28.0 \text{ mV}}$

31.5 $|\mathcal{E}| = N\frac{d\Phi_B}{dt} = \frac{\Delta(NBA)}{\Delta t} = 3.20 \text{ kV}$ so $I = \frac{\mathcal{E}}{R} = \boxed{160 \text{ A}}$

(a) $I_{\text{ring}} = \dfrac{\mathcal{E}}{R} = \boxed{\dfrac{\mu_0 n \pi r_2^2}{2R}\dfrac{\Delta I}{\Delta t}}$

(b) $B = \dfrac{\mu_0 I}{2r_1} = \boxed{\dfrac{\mu_0^2 n \pi r_2^2}{4 r_1 R}\dfrac{\Delta I}{\Delta t}}$

(c) The coil's field points downward, and is increasing, so $\boxed{B_{\text{ring}} \text{ points upward}}$.

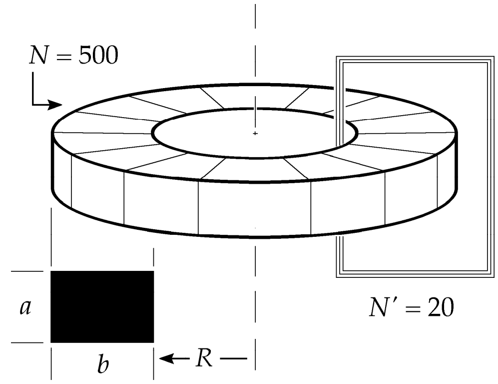
31.9 (a) $d\Phi_B = \mathbf{B}\cdot d\mathbf{A} = \dfrac{\mu_0 I}{2\pi x} L dx$: $\quad \Phi_B = \displaystyle\int_{x=h}^{h+w} \dfrac{\mu_0 IL}{2\pi}\dfrac{dx}{x} = \boxed{\dfrac{\mu_0 IL}{2\pi}\ln\left(\dfrac{h+w}{h}\right)}$

(b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -\dfrac{d}{dt}\left[\dfrac{\mu_0 IL}{2\pi}\ln\left(\dfrac{h+w}{h}\right)\right] = -\left[\dfrac{\mu_0 L}{2\pi}\ln\left(\dfrac{h+w}{h}\right)\right]\dfrac{dI}{dt}$

$$\mathcal{E} = -\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(1.00\ \text{m})}{2\pi}\ln\left(\frac{1.00+10.0}{1.00}\right)\left(10.0\ \frac{\text{A}}{\text{s}}\right) = \boxed{-4.80\ \mu\text{V}}$$

The long wire produces magnetic flux into the page through the rectangle (first figure, above). As it increases, the rectangle wants to produce its own magnetic field out of the page, which it does by carrying $\boxed{\text{counterclockwise}}$ current (second figure, above).

31.17 In a toroid, all the flux is confined to the inside of the toroid.



$$B = \frac{\mu_0 NI}{2\pi r} = \frac{500\,\mu_0 I}{2\pi r}$$

$$\Phi_B = \int B\,dA = \frac{500\,\mu_0 I_{\max}}{2\pi}\sin\omega t \int \frac{dz\,dr}{r}$$

$$\Phi_B = \frac{500\,\mu_0 I_{\max}}{2\pi} a \sin\omega t \ln\left(\frac{b+R}{R}\right)$$

$$\mathcal{E} = N'\frac{d\Phi_B}{dt} = 20\left(\frac{500\,\mu_0 I_{\max}}{2\pi}\right)\omega a \ln\left(\frac{b+R}{R}\right)\cos\omega t$$

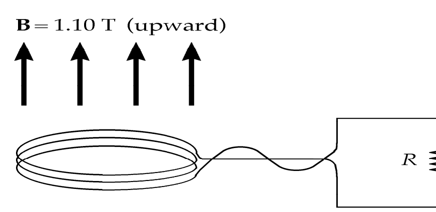
$$\mathcal{E} = \frac{10^4}{2\pi}\left(4\pi\times10^{-7}\ \frac{\text{N}}{\text{A}^2}\right)(50.0\ \text{A})\left(377\,\frac{\text{rad}}{\text{s}}\right)(0.0200\ \text{m})\ln\left(\frac{(3.00+4.00)\ \text{cm}}{4.00\ \text{cm}}\right)\cos\omega t = \boxed{(0.422\ \text{V})\cos\omega t}$$

31.18 The field inside the solenoid is: $B = \mu_0 nI = \mu_0\left(\frac{N}{\ell}\right)I$

Thus, through the single-turn loop $\Phi_B = BA_{\text{solenoid}} = \mu_0\left(\frac{N}{\ell}\right)(\pi r^2)I$

and the induced emf in the loop is $\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\mu_0\left(\frac{N}{\ell}\right)(\pi r^2)\left(\frac{\Delta I}{\Delta t}\right) = \boxed{-\frac{\mu_0 N\pi r^2}{\ell}\left(\frac{I_2 - I_1}{\Delta t}\right)}$

31.19 $\mathcal{E} = -N\frac{d\Phi_B}{dt}$ $\qquad IR = -N\frac{d\Phi_B}{dt}$

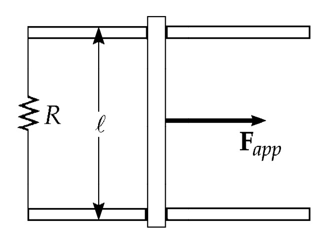


$Idt = -\frac{N}{R}d\Phi_B$ $\qquad \int Idt = -\frac{N}{R}\int d\Phi_B$

$$Q = -\frac{N}{R}\Delta\Phi_B = -\frac{N}{R}A\left(B_f - B_i\right)$$

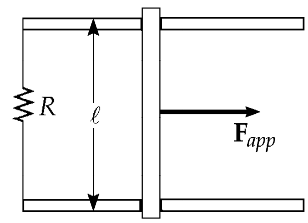
$$Q = -\left(\frac{200}{5.00\ \Omega}\right)(100\times10^{-4}\ \text{m}^2)(-1.10-1.10)\ \text{T} = \boxed{0.880\ \text{C}}$$

31.20 $I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$



$\boxed{v = 1.00\ \text{m/s}}$

31.21 (a) $|\mathbf{F}_B| = I|\boldsymbol{\ell}\times\mathbf{B}| = I\ell B$. When $I = \mathcal{E}/R$ and $\mathcal{E} = B\ell v$, we get



$$F_B = \frac{B\ell v}{R}(\ell B) = \frac{B^2\ell^2 v}{R} = \frac{(2.50)^2(1.20)^2(2.00)}{6.00} = 3.00\text{ N}$$

The applied force is $\boxed{\text{3.00 N to the right}}$

(b) $\mathcal{P} = I^2R = \frac{B^2\ell^2v^2}{R} = 6.00\text{ W}$ or $\mathcal{P} = Fv = \boxed{6.00\text{ W}}$

***31.22** $F_B = I\ell B$ and $\mathcal{E} = B\ell v$

$I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$ so $B = \frac{IR}{\ell v}$

(a) $F_B = \frac{I^2\ell R}{\ell v}$ and $I = \sqrt{\frac{F_B v}{R}} = \boxed{0.500\text{ A}}$

(b) $I^2R = \boxed{2.00\text{ W}}$

(c) For constant force, $\mathcal{P} = \mathbf{F}\cdot\mathbf{v} = (1.00\text{ N})(2.00\text{ m/s}) = \boxed{2.00\text{ W}}$

31.23 The downward component of **B**, perpendicular to **v**, is $(50.0\times10^{-6}\text{ T})\sin 58.0° = 4.24\times10^{-5}\text{ T}$

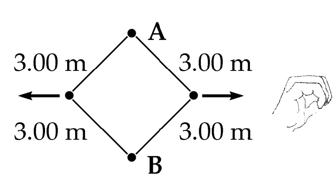
$$\mathcal{E} = B\ell v = (4.24\times10^{-5}\text{ T})(60.0\text{ m})(300\text{ m/s}) = \boxed{0.763\text{ V}}$$

The $\boxed{\text{left wing tip is positive}}$ relative to the right.

31.24 $\mathcal{E} = -N\frac{d}{dt}BA\cos\theta = -NB\cos\theta\left(\frac{\Delta A}{\Delta t}\right)$

$$\mathcal{E} = -1(0.100\text{ T})\cos 0°\,\frac{(3.00\text{ m}\times3.00\text{ m}\sin 60.0°) - (3.00\text{ m})^2}{0.100\text{ s}} = 1.21\text{ V}$$

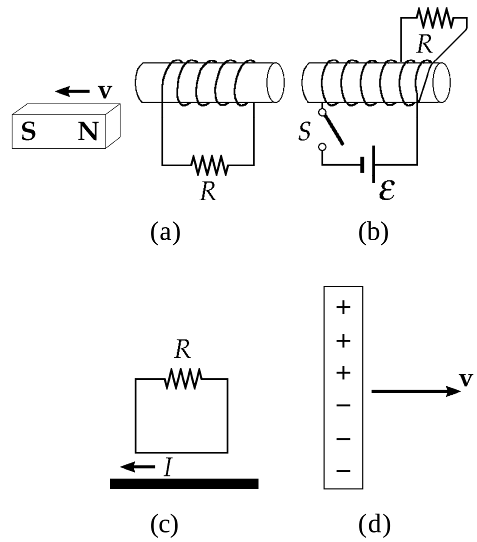
$$I = \frac{1.21\text{ V}}{10.0\ \Omega} = \boxed{0.121\text{ A}}$$



The flux is into the page and decreasing. The loop makes its own magnetic field into the page by carrying $\boxed{\text{clockwise}}$ current.

31.25 $\omega = (2.00\text{ rev/s})(2\pi\text{ rad/rev}) = (4.00)\pi\text{ rad/s}$

$$\mathcal{E} = \tfrac{1}{2}B\omega\ell^2 = \boxed{2.83\text{ mV}}$$

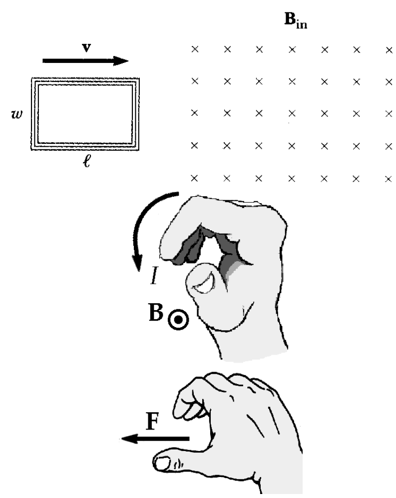


31.26 (a) $\mathbf{B}_{ext} = B_{ext}\,\mathbf{i}$ and B_{ext} decreases; therefore, the induced field is $\mathbf{B}_0 = B_0\,\mathbf{i}$ (to the right). Therefore, the current is **to the right** in the resistor.

(b) $\mathbf{B}_{ext} = B_{ext}\,(-\mathbf{i})$ increases; therefore, the induced field $\mathbf{B}_0 = B_0\,(+\mathbf{i})$ is to the right, and the current is **to the right** in the resistor.

(c) $\mathbf{B}_{ext} = B_{ext}\,(-\mathbf{k})$ into the paper and B_{ext} decreases; therefore, the induced field is $\mathbf{B}_0 = B_0\,(-\mathbf{k})$ into the paper. Therefore, the current is **to the right** in the resistor.

(d) By the Lorentz force law, $F_B = q(\mathbf{v}\times\mathbf{B})$. Therefore, a positive charge will move to the top of the bar if $\mathbf{B}$ is **into the paper** .



31.27 (a) The force on the side of the coil entering the field (consisting of N wires) is

$$F = N(ILB) = N(IwB)$$

The induced emf in the coil is

$$|\mathcal{E}| = N\frac{d\Phi_B}{dt} = N\frac{d(Bwx)}{dt} = NBwv,$$

so the current is $I = \dfrac{|\mathcal{E}|}{R} = \dfrac{NBwv}{R}$ counterclockwise.

The force on the leading side of the coil is then:

$$F = N\left(\frac{NBwv}{R}\right)wB = \boxed{\frac{N^2B^2w^2v}{R}\ \text{to the left}}$$

(b) Once the coil is entirely inside the field, $\Phi_B = NBA =$ constant, so $\mathcal{E} = 0$, $I = 0$, and $F = \boxed{0}$.

(c) As the coil starts to leave the field, the flux *decreases* at the rate Bwv, so the magnitude of the current is the same as in part (a), but now the current flows clockwise. Thus, the force exerted on the trailing side of the coil is:

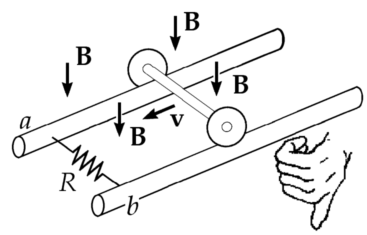
***31.58** (a) $I = \frac{dq}{dt} = \frac{\mathcal{E}}{R}$ where $\mathcal{E} = -N\frac{d\Phi_B}{dt}$ so $\int dq = \frac{N}{R}\int_{\Phi_1}^{\Phi_2} d\Phi_B$

and the charge through the circuit will be $|Q| = \frac{N}{R}(\Phi_2 - \Phi_1)$

(b) $Q = \frac{N}{R}\left[BA\cos 0 - BA\cos\left(\frac{\pi}{2}\right)\right] = \frac{BAN}{R}$

so $B = \frac{RQ}{NA} = \frac{(200\ \Omega)(5.00\times10^{-4}\ \text{C})}{(100)(40.0\times10^{-4}\ \text{m}^2)} = \boxed{0.250\ \text{T}}$

31.59 (a) $\mathcal{E} = B\ell v = 0.360$ V $\quad I = \frac{\mathcal{E}}{R} = \boxed{0.900\ \text{A}}$

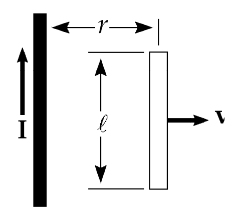


(b) $F_B = I\ell B = \boxed{0.108\ \text{N}}$

(c) Since the magnetic flux $\mathbf{B}\cdot\mathbf{A}$ is in effect decreasing, the induced current flow through R is from b to a. $\boxed{\text{Point } b}$ is at higher potential.

(d) $\boxed{\text{No}}$. Magnetic flux will increase through a loop to the left of ab. Here counterclockwise current will flow to produce upward magnetic field. The in R is still from b to a.

31.60 $\mathcal{E} = B\ell v$ at a distance r from wire



$$|\mathcal{E}| = \left(\frac{\mu_0 I}{2\pi r}\right)\ell v$$

31.61 (a) At time t, the flux through the loop is $\Phi_B = BA\cos\theta = (a+bt)(\pi r^2)\cos 0° = \pi(a+bt)r^2$

At $t = 0$, $\Phi_B = \boxed{\pi a r^2}$

(b) $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\pi r^2\frac{d(a+bt)}{dt} = \boxed{-\pi b r^2}$

(c) $I = \frac{\mathcal{E}}{R} = \boxed{-\frac{\pi b r^2}{R}}$

(d) $\mathcal{P} = \mathcal{E}I = \left(-\frac{\pi b r^2}{R}\right)\left(-\pi b r^2\right) = \boxed{\frac{\pi^2 b^2 r^4}{R}}$