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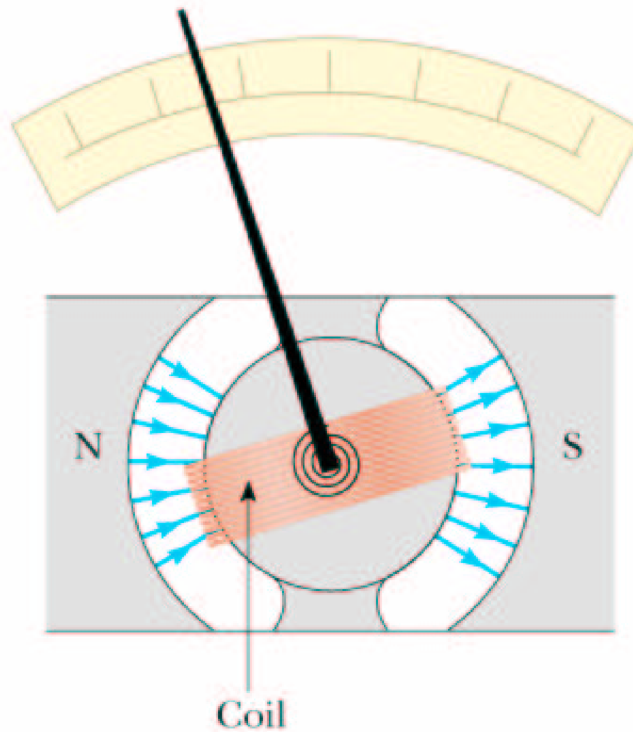
Figure 29.16



Motion of a charged particle in a uniform Magnetic Field

- Consider a positively charged particle moving in a magnetic field perpendicular to the velocity of the particle.

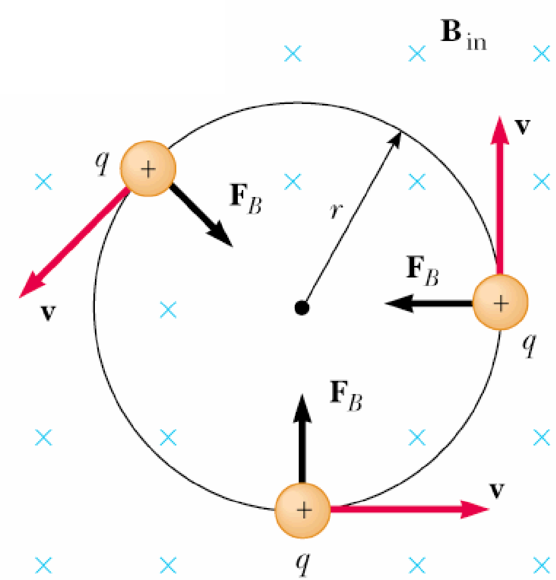
$$\sum F_r = m a_r$$

$$F_B = q v B = \frac{m v^2}{r}$$

$$r = \frac{m v}{q B}$$

- The angular speed of the particle:

$$\omega = \frac{v}{r} = \frac{q B}{m}$$



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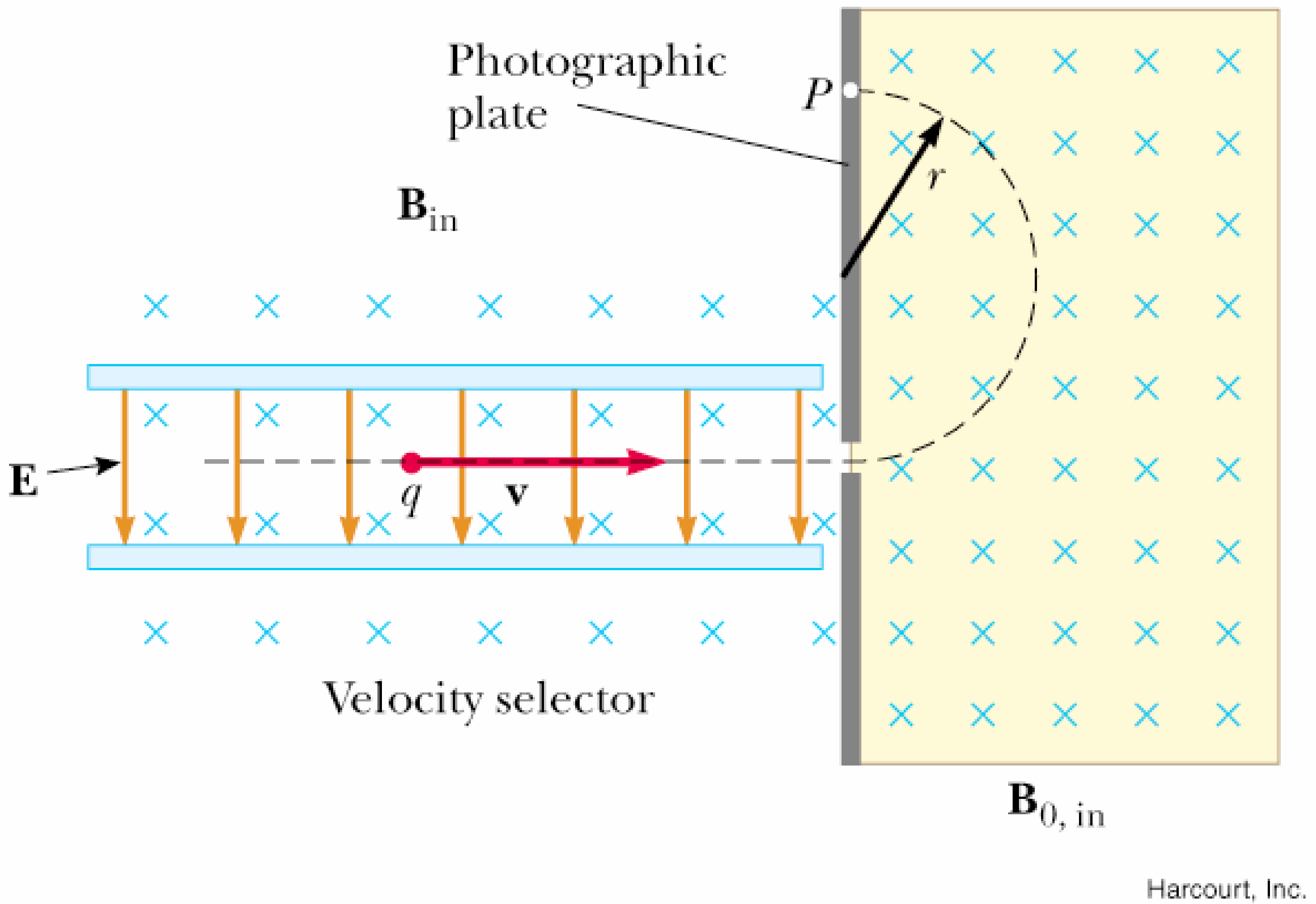
Mass Spectrometer



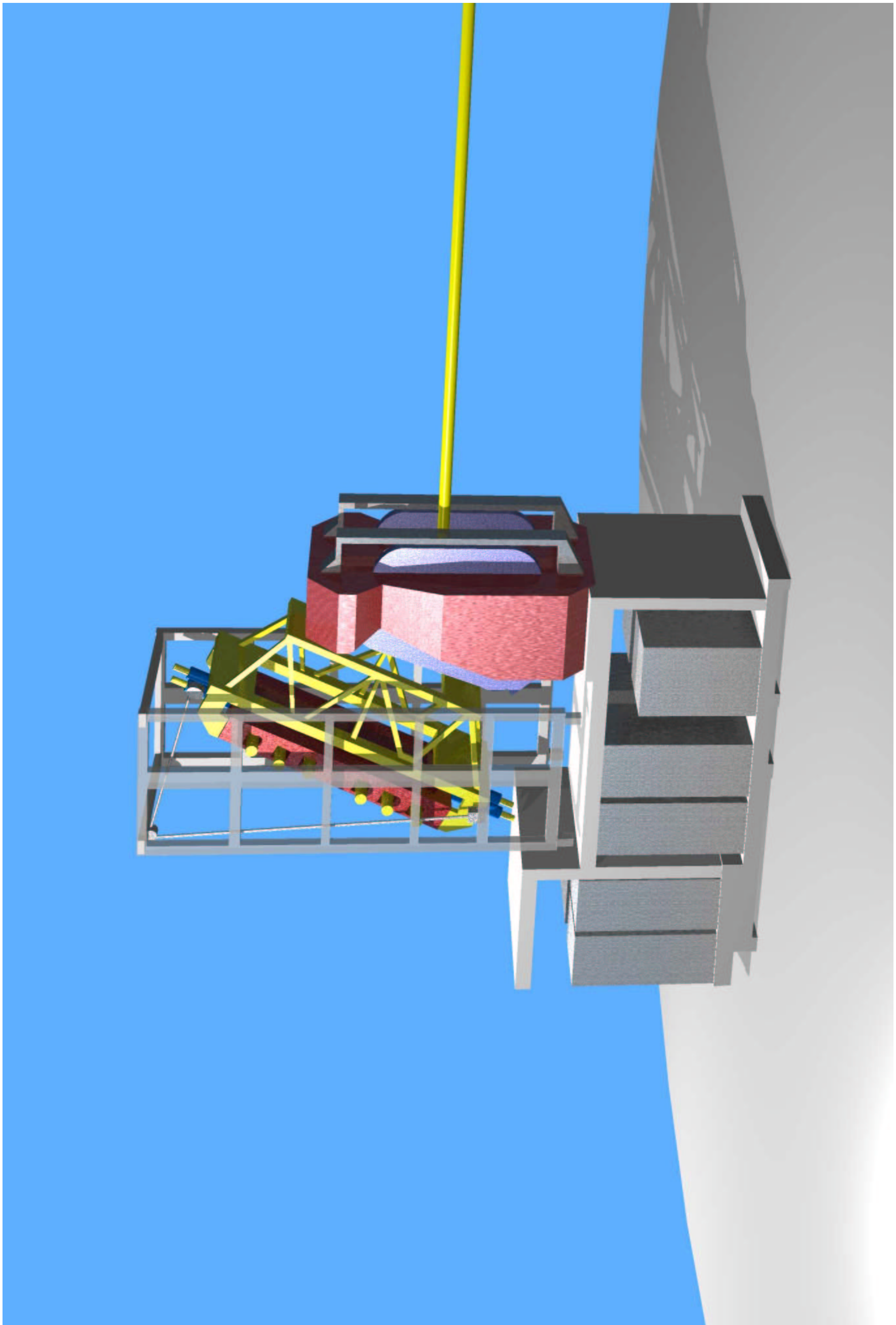


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Figure 30.8

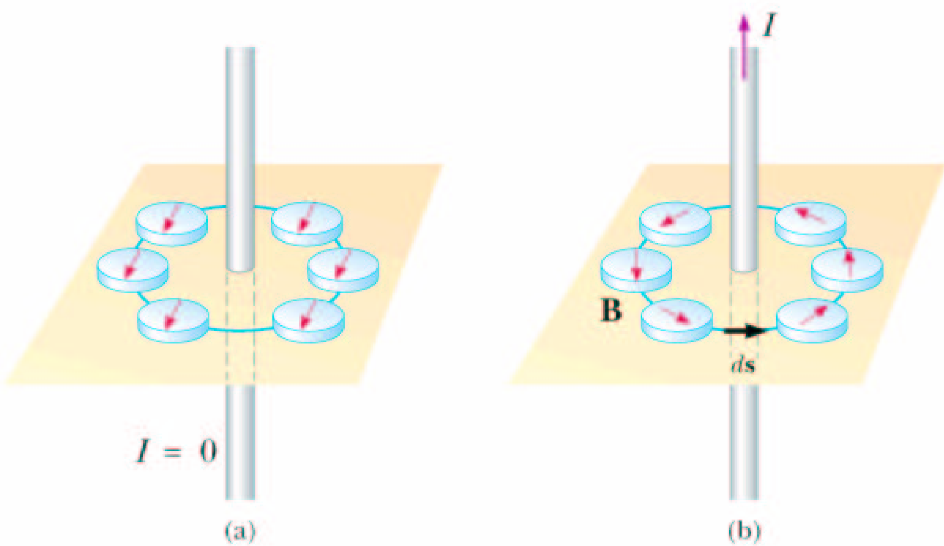

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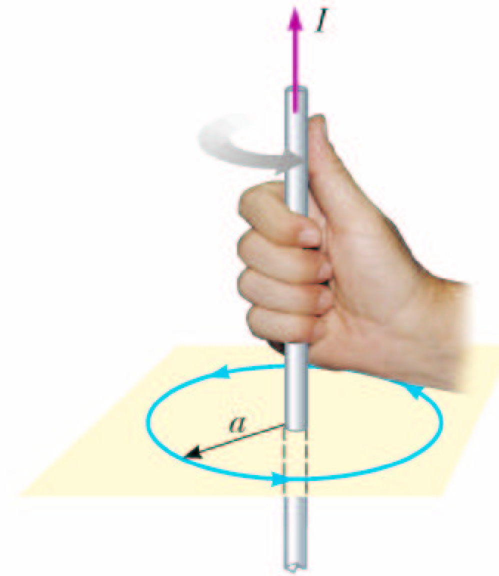
Figure 30.3



Amphere's law

- The line integral of $\mathbf{B} \cdot d\mathbf{s}$ around any closed path equals $\mu_0 I$, where I is the total continuous current passing through any surface bounded by the closed path and μ_0 is a constant called the **permeability of free space**

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$$

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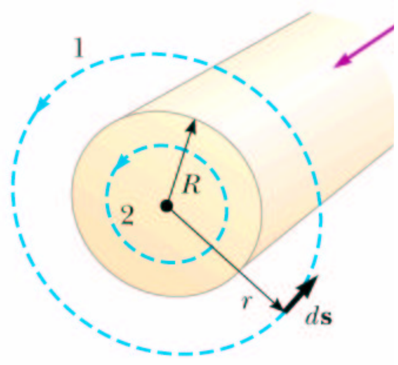
Figure 30.11



Example 30.4

A long straight wire of radius R carries a current I_0 which is uniformly distributed through the cross section of the wire. Calculate the magnetic field at a distance r inside and outside the wire:

- Assume a circular loop around the wire and use cylindrical symmetry around the wire: $\mathbf{B}$ is along the loop and constant at every point of the loop.:

  For $r > R$

$$\mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 I_0 \tag{5}$$

$$B = \frac{\mu_0 I_0}{2\pi r}$$

For $r < R$, since the current is distributed uniformly, the amount of current enclosed by a circular loop of radius r:

$$I = \frac{\pi r^2}{\pi R^2} I_0 = \frac{r^2}{R^2} I_0 \tag{6}$$

$$\Rightarrow \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 \left(\frac{r^2}{R^2} I_0 \right)$$

$$B = \left(\frac{\mu_0 I_0}{2\pi R^2} \right) r$$

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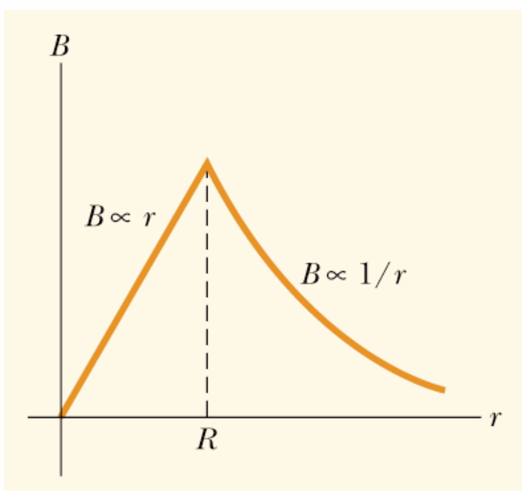
Figure 30.12



Biot-Savart law

Magnetic field $d\mathrm{B}$ at a point P due to a current I passing through a length element ds:

- The vector $d\mathbf{B}$ is perpendicular to both $d\mathbf{s}$ and the unit vector $\hat{\mathbf{r}}$ directed from $d\mathbf{s}$ to P
- dB is proportional to I and ds
- dB is inversely proportional to r^2
- dB is proportional to $\sin\theta$

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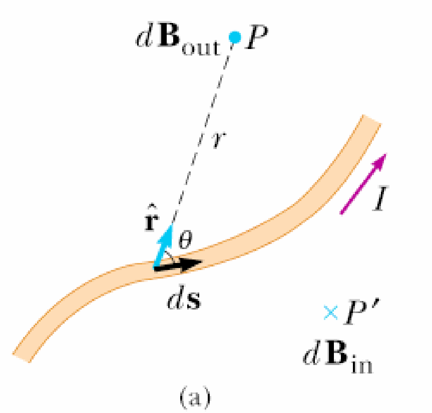
Figure 30.1



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$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$

So the current due to a finite wire segment that goes from a to b

$$\mathbf{B} = \frac{\mu_0 I}{4\pi}\int_a^b \frac{d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$

Note that $\hat{\mathbf{r}}$ is a unit vector with magnitude 1

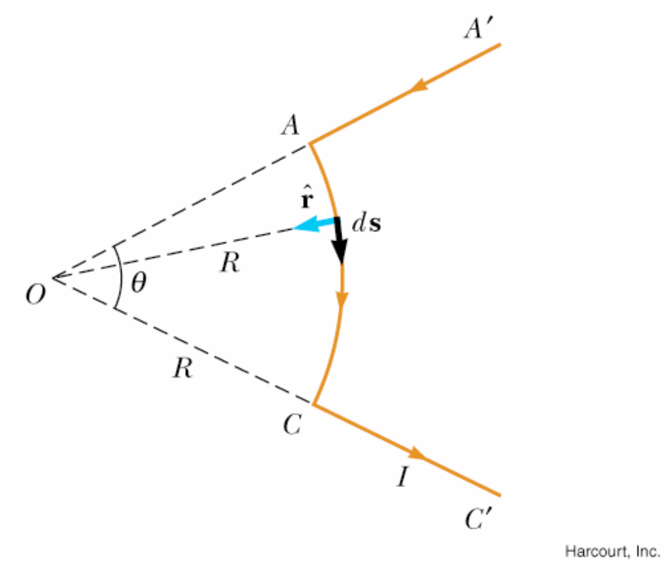
Example 30.2

Calculate the magnetic field at point O.

- Note that segments AA' and CC' are radial and AC is an arc centered at O.
- For the segments AA' and CC', $d\mathbf{s}$ is parallel to $\hat{\mathbf{r}}$ and $d\mathbf{s} \times \hat{\mathbf{r}} = 0$. Hence these two segments do not contribute to the field at O.
- For AC, $d\mathbf{s}$ is perpendicular to $\hat{\mathbf{r}}$, and $d\mathbf{s} \times \hat{\mathbf{r}} = ds$:

$$\begin{aligned} B &= \frac{\mu_0 I}{4\pi} \int_A^C \frac{ds}{R^2} \\ &= \frac{\mu_0 I}{4\pi R^2} \int_A^C ds = \frac{\mu_0}{4\pi R^2} s \\ &= \frac{\mu_0 I}{4\pi R} \theta \end{aligned} \tag{7}$$

- Where $s = R\theta$, with θ measured in radians.



Example 30.3

Calculate the magnetic field at an axial point P a distance x from the center of the loop.

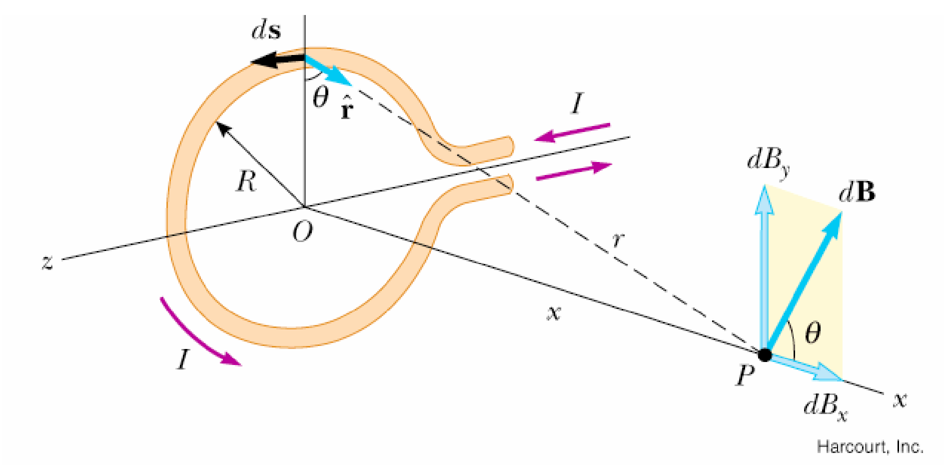
- For every length segment of the loop, $d\mathbf{s}$ is perpendicular to $\hat{\mathbf{r}}$, and $d\mathbf{s} \times \hat{\mathbf{r}} = ds$
- For every segment r is a constant such that $r^2 = x^2 + R^2$; for one such segment,

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \tag{8}$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{ds}{x^2 + R^2}$$

$$dB_y = dB \,.\, \sin\theta$$

$$dB_x = dB \,.\, \cos\theta$$



- dB_y components due to different segments around the loop cancel each other, and dB_x segments add.

- $\cos\theta = R/(x^2 + R^2)^{1/2}$:

$$B = \oint dB_x = \frac{\mu_0 I}{4\pi} \oint \frac{ds\,R}{(x^2 + R^2)^{3/2}} \qquad (9)$$
$$= \frac{\mu_0 I R}{4\pi(x^2 + R^2)^{3/2}} \oint ds$$
$$= \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

- Magnetic field at the center of the loop.

$$B = \frac{\mu_0 I}{2R}$$

- Field along the axis for a current carrying coil of N turns, assuming that the thickness of the coil is small compared to the radius:

$$B = \frac{N \mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

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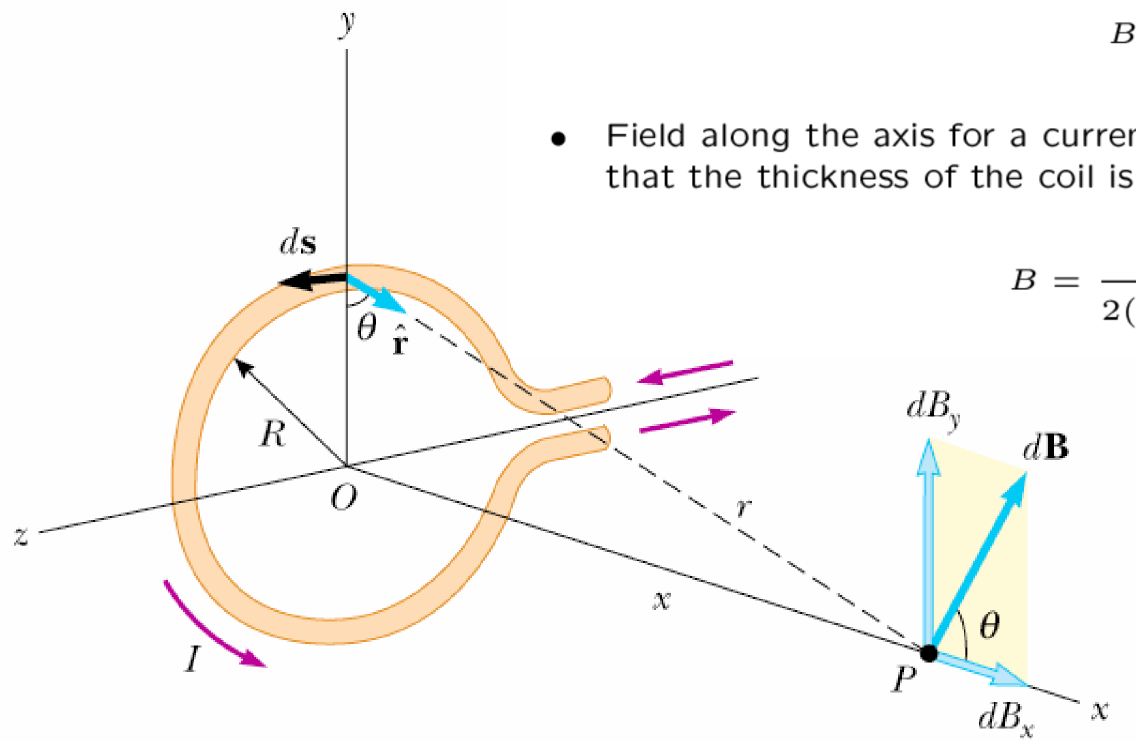
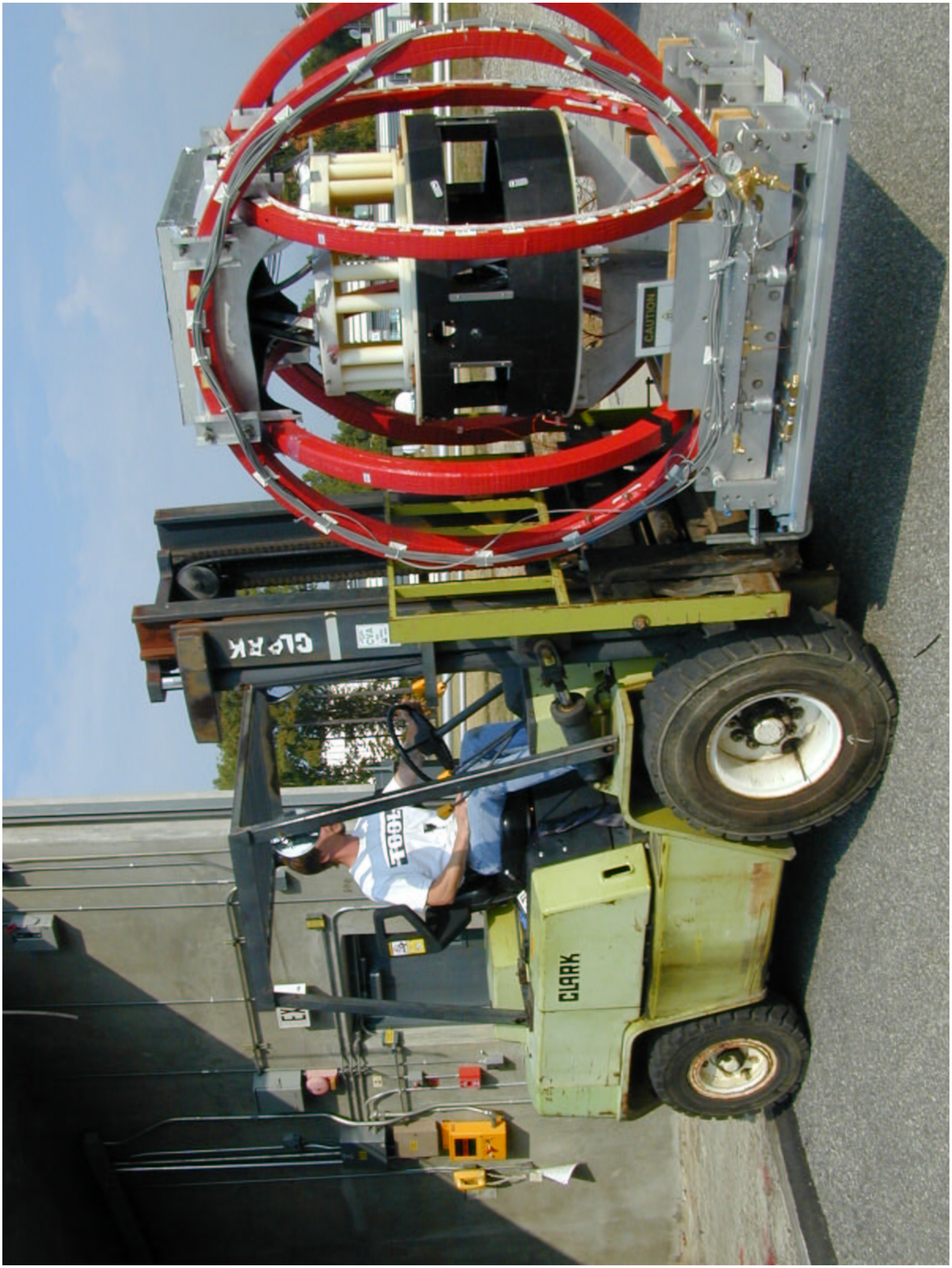
Figure 30.5




CAUTION
CLARK
CLARK

Magnetic force between two parallel conductors

- The magnetic field due to one current carrying conductor exerts a force on the other current carrying conductor: The force F_1 on conductor 1;

$$\mathbf{F}_1 = I_1 \mathbf{l} \times \mathbf{B}_2 \tag{10}$$

$$F_1 = I_1 l B_2 = I_1 l \left(\frac{\mu_0 I_2}{2\pi a} \right) = \frac{\mu_0 I_1 I_2}{2\pi a} l$$

- $F_2 = F_1$

- If the currents are in the same direction the wires attract each other, and if the currents are in opposite directions they repel.