PHYS 232
Optional Problem Set Solutions

24.28 (a) $E = \frac{2k_e \lambda}{r}$

$$3.60 \text{ x } 10^4 = \frac{2(8.99 \times 10^9)(Q/2.40)}{(0.190)}$$

$Q = +9.13 \infty 10^{-7}$ C = $\boxed{+913 \text{ nC}}$

(b) $\boxed{E = 0}$

25.58 From Example 25.5, the potential created by the ring at the electron's starting point is

$$V_i = \frac{k_e Q}{\sqrt{x_i^2 + a^2}} = \frac{k_e (2\pi \lambda a)}{\sqrt{x_i^2 + a^2}}$$

while at the center, it is $V_f = 2\pi k_e \lambda$. From conservation of energy,

$$0 + (-eV_i) = \frac{1}{2} m_e v_f^2 + (-eV_f)$$

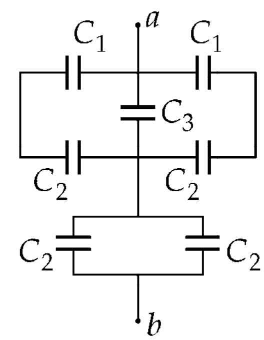
$$v_f^2 = \frac{2e}{m_e}(V_f - V_i) = \frac{4\pi e k_e \lambda}{m_e}\left(1 - \frac{a}{\sqrt{x_i^2 + a^2}}\right)$$

$$v_f^2 = \frac{4\pi(1.60 \times 10^{-19})(8.99 \times 10^9)(1.00 \times 10^{-7})}{9.11 \times 10^{-31}}\left(1 - \frac{0.200}{\sqrt{(0.100)^2 + (0.200)^2}}\right)$$

$$v_f = \boxed{1.45 \times 10^7 \, m/s}$$

26.28

$$C_s = \left(\frac{1}{5.00} + \frac{1}{10.0}\right)^{-1} = 3.33\ \mu\text{F}$$



$C_{p1} = 2(3.33) + 2.00 = 8.66\ \mu\text{F}$

$C_{p2} = 2(10.0) = 20.0\ \mu\text{F}$

$$C_{\text{eq}} = \left(\frac{1}{8.66} + \frac{1}{20.0}\right)^{-1} = \boxed{6.04\ \mu\text{F}}$$

26.29

$$Q_{\text{eq}} = C_{\text{eq}}(\Delta V) = \left(6.04 \times 10^{-6}\ \text{F}\right)\left(60.0\ \text{V}\right) = 3.62 \times 10^{-4}\ \text{C}$$

$$Q_{p1} = Q_{\text{eq}}, \quad \text{so} \quad \Delta V_{p1} = \frac{Q_{\text{eq}}}{C_{p1}} = \frac{3.62 \times 10^{-4}\ \text{C}}{8.66 \times 10^{-6}\ \text{F}} = 41.8\ \text{V}$$

$$Q_3 = C_3\left(\Delta V_{p1}\right) = \left(2.00 \times 10^{-6}\ \text{F}\right)\left(41.8\ \text{V}\right) = \boxed{83.6\ \mu\text{C}}$$

27.58

2 wires: $\ell = 100$ m

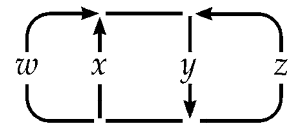
$$R = \frac{0.108\ \Omega}{300\ \text{m}}(100\ \text{m}) = 0.0360\ \Omega$$

(a) $(\Delta V)_{\text{home}} = (\Delta V)_{\text{line}} - IR = 120 - (110)(0.0360) = \boxed{116\ \text{V}}$

(b) $P = I(\Delta V) = (110\ \text{A})(116\ \text{V}) = \boxed{12.8\ \text{kW}}$

(c) $P_{\text{wires}} = I^2R = (110\ \text{A})^2(0.0360\ \Omega) = \boxed{436\ \text{W}}$

28.24 Name the currents as shown in the figure to the right. Then $w + x + z = y$. Loop equations are



$$-200w - 40.0 + 80.0x = 0$$

$$-80.0x + 40.0 + 360 - 20.0y = 0$$

$$+360 - 20.0y - 70.0z + 80.0 = 0$$

Eliminate y by substitution.
$$\begin{cases} x = 2.50w + 0.500 \\ 400 - 100x - 20.0w - 20.0z = 0 \\ 440 - 20.0w - 20.0x - 90.0z = 0 \end{cases}$$

Eliminate x:
$$\begin{cases} 350 - 270w - 20.0z = 0 \\ 430 - 70.0w - 90.0z = 0 \end{cases}$$

Eliminate $z = 17.5 - 13.5w$ to obtain $430 - 70.0w - 1575 + 1215w = 0$

$w = 70.0/70.0 =$ $\boxed{\text{1.00 A upward in 200 }\Omega}$

Now $z =$ $\boxed{\text{4.00 A upward in 70.0 }\Omega}$

$x =$ $\boxed{\text{3.00 A upward in 80.0 }\Omega}$

$y =$ $\boxed{\text{8.00 A downward in 20.0 }\Omega}$

and for the 200 Ω, $\Delta V = IR = (1.00\text{ A})(200\ \Omega) =$ $\boxed{200\text{ V}}$

30.22 (a) In $B = \frac{\mu_0 I}{2\pi r}$, the field will be one-tenth as large at a ten-times larger distance: $\boxed{400 \text{ cm}}$

(b) $\mathbf{B} = \frac{\mu_0 I}{2\pi r_1}\,\mathbf{k} + \frac{\mu_0 I}{2\pi r_2}(-\mathbf{k})$ so $B = \frac{4\pi \times 10^7\, T\cdot m(2.00A)}{2pA}\left(\frac{1}{0.3985m}\frac{1}{0.4015m}\right) = \boxed{7.50 \text{ nT}}$

(c) Call r the distance from cord center to field point and $2d$ = 3.00 mm the distance between conductors.

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r-d} - \frac{1}{r+d}\right) = \frac{\mu_0 I}{2\pi}\frac{2d}{r^2 - d^2}$$

$$7.50 \times 10^{-10}\text{ T} = \left(2.00 \times 10^7\,\frac{T\cdot m}{A}\right)(2.00A)\frac{(3.00 \times 10^3 m)}{r^2\, 2.25 \times 10^6 m^2} \quad \text{so} \quad r = \boxed{1.26 \text{ m}}$$

The field of the two-conductor cord is weak to start with and falls off rapidly with distance.

(d) The cable creates $\boxed{\text{zero}}$ field at exterior points, since a loop in Ampère's law encloses zero total current. Shall we sell coaxial-cable power cords to people who worry about biological damage from weak magnetic fields?

31.34

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \pi r^2\left(\frac{dB}{dt}\right) = \oint \mathbf{E}\cdot d1$$

$$E(2\pi R) = \pi r^2\frac{dB}{dt}, \quad \text{or} \quad E = \left(\frac{\pi r^2}{2\pi R}\right)\frac{dB}{dt}$$

$$B = \mu_0 n I \qquad \frac{dB}{dt} = \mu_0 n\frac{dI}{dt}$$

$$I = 3.00e^{0.200t} \qquad \frac{dI}{dt} = 0.600\, e^{0.200t}$$

At $t = 10.0$ s, $E = \frac{\pi r^2}{2\pi R}(\mu_0 n)(0.600\, e^{0.200t})$

becomes $E = \frac{(0.0200\text{ m})^2}{2(0.0500\text{ m})}(4\pi \times 10^{-7}\text{ N/ A}^2)(1000\text{ turns / m})(0.600)e^{2.00} = \boxed{2.23 \times 10^{-5}\text{ N/ C}}$

37.62 $\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8} \text{ m/s}}{1.50 \times 10^{6} \text{ Hz}} = 200 \text{ m}$

For destructive interference, the path difference is one-half wavelength.

Thus, $\frac{\lambda}{2} = 100 \text{ m} = x + \sqrt{x^2 + \left(2.00 \times 10^4 \text{ m}\right)^2} - 2.00 \times 10^4 \text{ m}$,

or $2.01 \times 10^4 \text{ m} - x = \sqrt{x^2 + \left(2.00 \times 10^4 \text{ m}\right)^2}$

Squaring and solving, $x = \boxed{99.8 \text{ m}}$

38.10 (a) Double-slit interference maxima are at angles given by $d \sin\theta = m\lambda$.

For $m = 0$, $\theta_0 = \boxed{0°}$

For $m = 1$, $(2.80\ \mu\text{m})\sin\theta = 1(0.5015\ \mu\text{m})$: $\theta_1 = \sin^{-1}(0.179) = \boxed{10.3°}$

Similarly, for $m = 2, 3, 4, 5$ and 6, $\theta_2 = \boxed{21.0°}$, $\theta_3 = \boxed{32.5°}$, $\theta_4 = \boxed{45.8°}$,
$\theta_5 = \boxed{63.6°}$, and $\theta_6 = \sin^{-1}(1.07) = \text{nonexistent}$.

Thus, there are $5 + 5 + 1 = \boxed{\text{11 directions for interference maxima}}$.

(b) We check for missing orders by looking for single-slit diffraction minima, at $a \sin\theta = m\lambda$.

For $m = 1$, $(0.700\ \mu\text{m})\sin\theta = 1(0.5015\ \mu\text{m})$ and $\theta_1 = 45.8°$.

Thus, there is no bright fringe at this angle. There are only $\boxed{\text{nine bright fringes}}$, at
$\boxed{\theta = 0°, \pm 10.3°, \pm 21.0°, \pm 32.5°, \text{ and } \pm 63.6°}$.

(c) $I = I_{\max}\left[\frac{\sin\left(\pi a \sin\theta/\lambda\right)}{\pi a \sin\theta/\lambda}\right]^2$

At $\theta = 0°$, $\frac{\sin\theta}{\theta} \to 1$ and $\frac{I}{I_{\max}} \to \boxed{1.00}$

At $\theta = 10.3°$, $\frac{\pi a \sin\theta}{\lambda} = \frac{\pi(0.700\ \mu\text{m})\sin 10.3°}{0.5015\ \mu\text{m}} = 0.785\ \text{rad} = 45.0°$

$$\frac{I}{I_{\max}} = \left[\frac{\sin 45.0°}{0.785}\right]^2 = \boxed{0.811}$$

Similarly, at $\theta = 21.0°$, $\frac{\pi a \sin\theta}{\lambda} = 1.57\ \text{rad} = 90.0°$ and $\frac{I}{I_{\max}} = \boxed{0.405}$

At $\theta = 32.5°$, $\frac{\pi a \sin\theta}{\lambda} = 2.36\ \text{rad} = 135°$ and $\frac{I}{I_{\max}} = \boxed{0.0901}$

At $\theta = 63.6°$, $\frac{\pi a \sin\theta}{\lambda} = 3.93\ \text{rad} = 225°$ and $\frac{I}{I_{\max}} = \boxed{0.0324}$

39.8 The observed length of an object moving at speed v is $L = L_p\sqrt{1 - v^2/c^2}$ with L_p as the proper length. For the two ships, we know $L_2 = L_1$, $L_{2p} = 3L_{1p}$, and $v_1 = 0.350c$

Thus, $L_2^2 = L_1^2$ and $9L_{1p}^{\ 2}\left(1 - \frac{v_2^{\ 2}}{c^2}\right) = L_{1p}^{\ 2}\left[1 - (0.350)^2\right]$

giving $9 - 9\frac{v_2^2}{c^2} = 0.878$, or $v_2 = \boxed{0.950c}$