## Homework \#1 Solutions

Due: Friday September 4, 1998
(a) When the rod makes contact, the spheres both become charged with the same sign as the rod. They therefore repel.
(b) When the rod is brought near to the right sphere, the double-sphere unit becomes polarized. The charges opposite in sign to those on the rod migrate to the right (i. e. the right sphere) while those of same sign migrate to the left (i. e. the left sphere). When the spheres are separated, they have opposite charge and attract each other. Note that in reality, electrons are the likely charge carriers, and migrate right for a positively charged rod or left for a negatively charged rod. The charge on the other sphere is then due to depletion of electrons.
(c)


We need to draw a force diagram and take into account all of the forces in action: Coulomb, gravity, and tension of the string. Since we know that the force is repulsive, the sphere on the right experiences the forces as shown (with angles exaggerated). At equilibrium, the forces for both the $x$ - and $y$-components cancel:

$$
\begin{align*}
T \cos \theta & =m g  \tag{1}\\
T \sin \theta & =F_{C}=\frac{k q^{2}}{x^{2}} \tag{2}
\end{align*}
$$

where $x \equiv x_{0}+2 R \sin \theta$ is the distance between the two spheres, and $x_{0}=2 \mathrm{~cm}, R \simeq 15 \mathrm{~cm}$. You can divide eq. (2) by (1) and solve for $q$ :

$$
\begin{equation*}
q= \pm \sqrt{\frac{m g}{k} \tan \theta}\left(x_{0}+2 R \sin \theta\right) \tag{3}
\end{equation*}
$$

Inserting the values given (all in SI),

$$
\begin{equation*}
q= \pm \sqrt{\frac{(9.8)\left(10^{-3}\right)}{9 \times 10^{9}} \tan \left(4^{\circ}\right)}\left((0.02)+2(0.15) \sin \left(4^{\circ}\right)\right) \tag{4}
\end{equation*}
$$

or $q \simeq \pm 1.1 \times 10^{-8} \mathrm{C}$. There are two possibilities for the value of the charge that are equal in magnitude. (d) We found in part (b) that the charges in this case must be equal and opposite on each sphere. If the magnitude of the charge on each sphere must be the result from part (c), then the spheres must be swung toward each other by more than $4^{\circ}$. This is because the Coulomb force is proportional to $1 / x^{2}$, so the closer they are together the stronger the force. In any case, $4^{\circ}$ is more than enough to cause the spheres to make contact $(2 R \sin \theta>2 \mathrm{~cm})$. When they make contact, they neutralize each other's charge, and then hang straight down, 2 cm apart.

